NYQUIST FREQUENCY, CHAOS DETECTION AND ORBIT DETERMINATION

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Detection of chaotic phenomena in non-linear dynamics, as well as the determination of chaotic orbits is mostly done by two methods: variational — based on the temporal evolution of the separation between two orbits with initial conditions infinitesimally close to each other, intimally related to the Lyapunov Exponent — and spectral methods — based on the analysis of the time evolution of individual orbits, which hinge on on the fact that for chaotic orbits in conservative systems, the fundamental frequencies are time-dependent.

The scope of this work consist on the study of the Spectral Number method, a representative of the Spectral Method. In this tool for chaos detection, the study is realized by the analysis of individal orbits by means of a Fourier Transform of it's time series. For regular orbits, the power spectra resulting from the transform has a definite number of frequency peaks, referring to the system's fundamental frequencies. On the other hand, for chaotic orbits, it's power spectra has a huge number of peaks, indicating the temporal dependence of it's fundamental frequencies. From the power spectra, a base value is defined — usually, at 10% of the most intense frequency detected, and, from this value, the frequencies above this value are counted. The more the quantity of peaks, the more chaotic the orbit is.

It's also presented a powerful tool for analysis, tied to the Spectral Number Method, the Dynamic Power Spectra, which consists in the mapping of the fundamental frequencies detected in the system as a function of the initial conditions of it's orbits, where upon it's possible to verify the evolution of the fundamental frequencies, the resonances within the system and the transitions to global chaos. This tool, however, has a limitation of pratical order: the Nyquist Frequency, defined as $f_{Nyquist} = \frac{1}{\Delta t}$, where Δt is the integration step or the sampling rate of the data from some observed system.

In continuous systems and in the integration of the equations of motion, Nyquist Frequency can be limited to higher frequencies by conveniently setting small integration steps. However, in discrete dynamical systems — such as the Taylor-Chirikov Standard Map — the integration step can't be modified to values aside of the unitary step $\Delta t = \Delta n = 1$, causing a phenomena known in the temporal series field as aliasing, which consists in an omission of the higher frequencies of the system. This limitations doesn't only affect computational astronomy, but also observational, as in the detection of exoplanets, where the large distances, observation difficulties and uncertainties inherent of the methods make the continuous observation to the planets' orbits unfeasible, making it hard to determine them with precision.

Milani, A. and Gronchi, G.; "Theory of Orbit Determination", Cambridge University Press (2010)

Ferraz-Mello, S.; Michtchenko, T. A.; Beaugè, C.; Callegari, N., "Chaos and Stability in Planetary Systems", em Lecture Notes in Physics, 683, 219-271 (2005).