Accounting for activity-induced RV variations through Gaussian-process regression

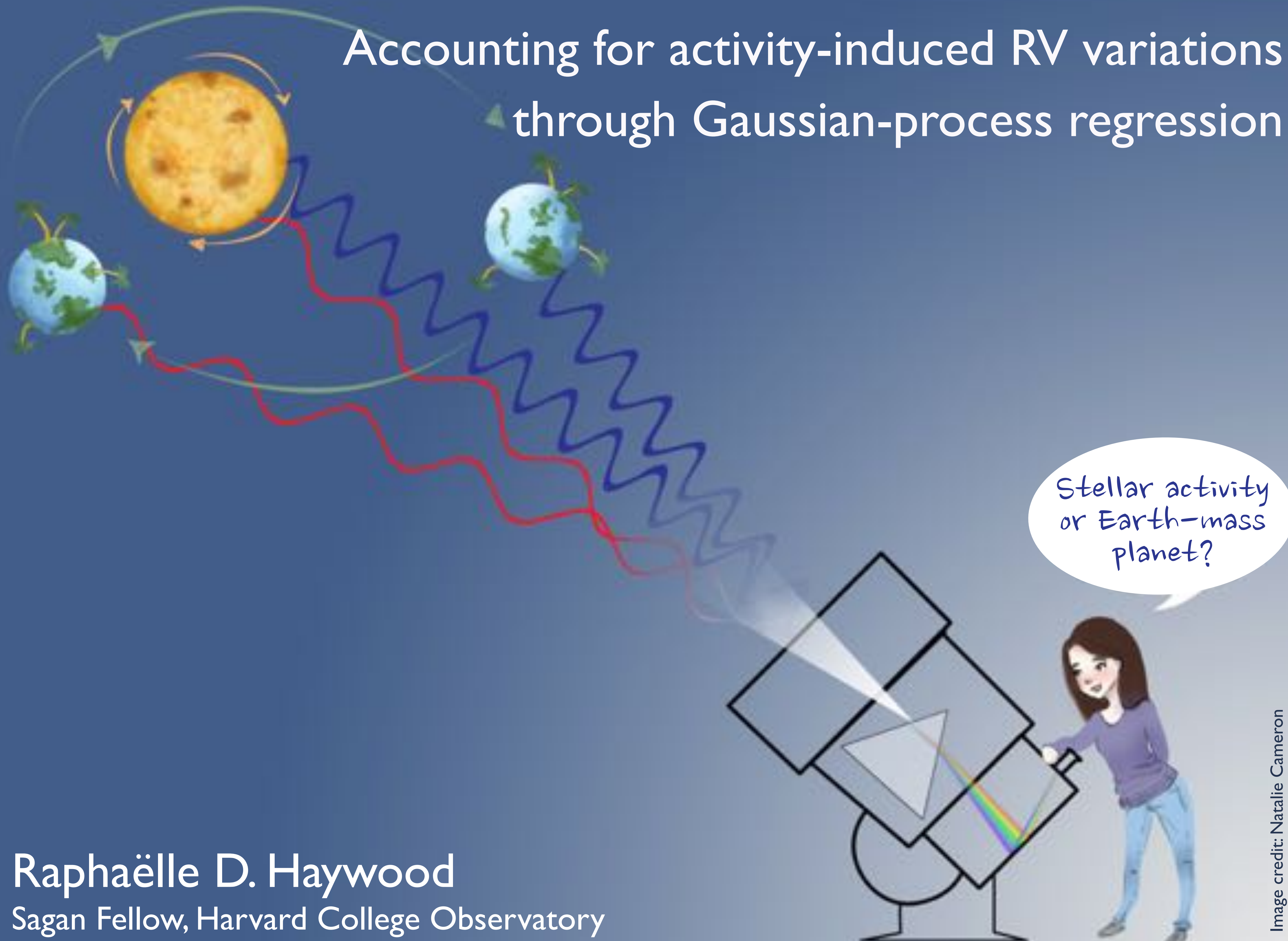


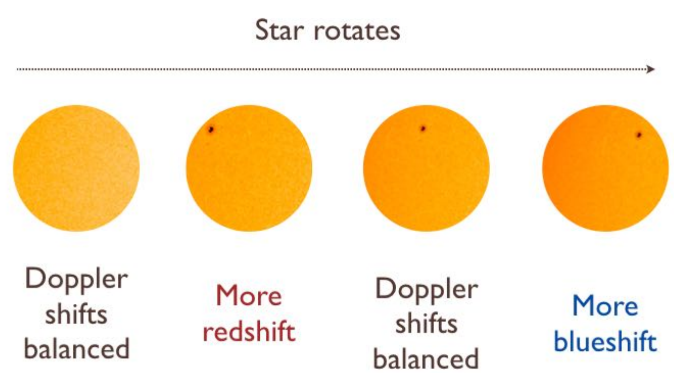
Image credit: Natalie Cameron

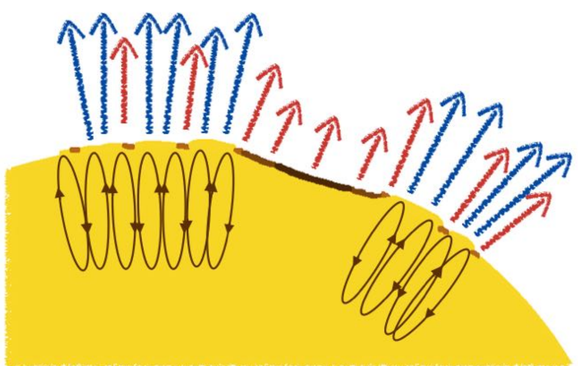
Raphaëlle D. Haywood
Sagan Fellow, Harvard College Observatory

How will we overcome the stellar activity barrier?

How will we overcome the stellar activity barrier?

By learning about the physical processes at play on the surfaces of stars

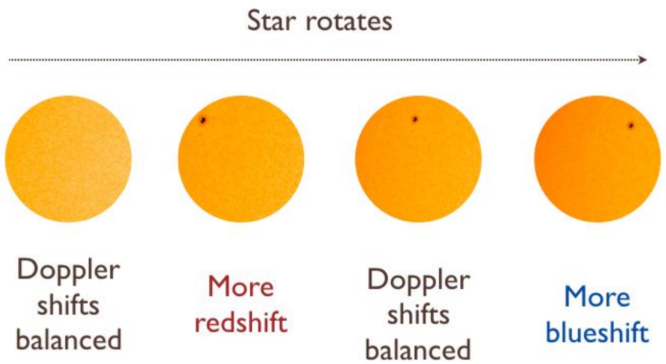


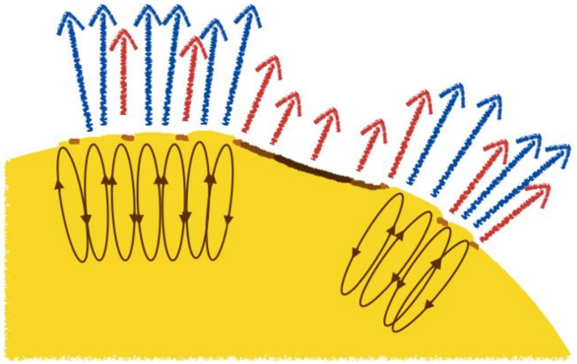


How will we overcome the stellar activity barrier?

By learning about the physical processes at play on the surfaces of stars

See tomorrow

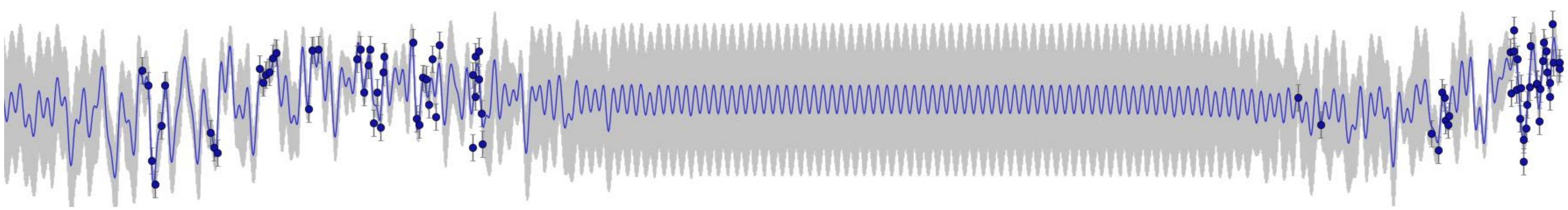




In the meantime, here is the best thing we've found:

Treat activity-induced RV variations as (correlated) "noise"

Baluev (2012), Tuomi et al. (2012), Haywood et al. (2014), Rajpaul et al. (2015), Faria et al. (2016), Anglada-Escudé et al. (2016), Mortier et al. (2016), López-Morales et al. (2016), Grunblatt et al. (2017) and many others.



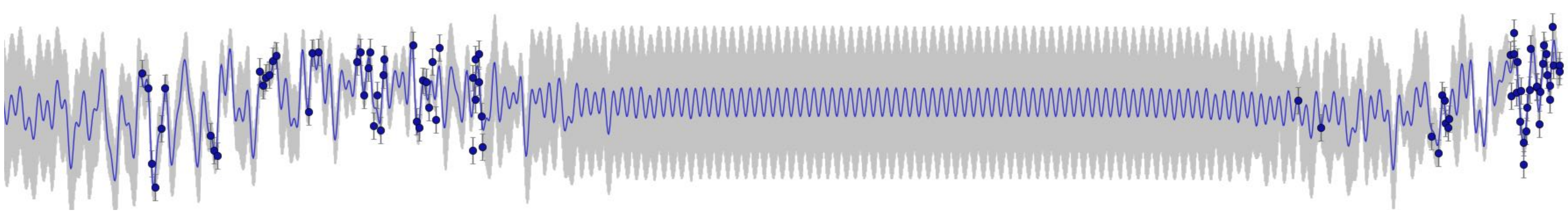
Gaussian process fit to RV data

In the meantime, here is the best thing we've found:

Treat activity-induced RV variations as (correlated) "noise"

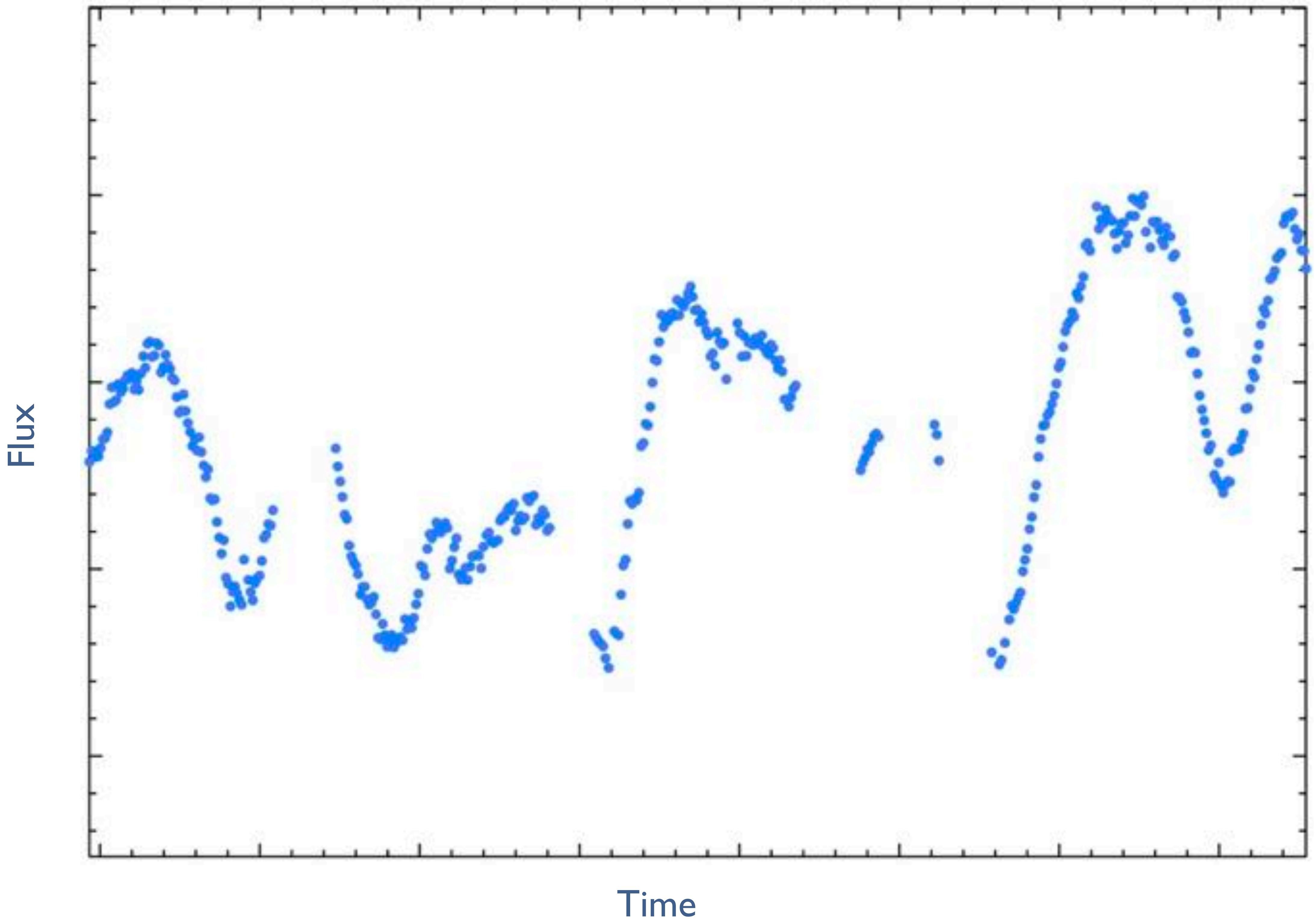
Baluev (2012), Tuomi et al. (2012), Haywood et al. (2014), Rajpaul et al. (2015), Faria et al. (2016), Anglada-Escudé et al. (2016), Mortier et al. (2016), López-Morales et al. (2016), Grunblatt et al. (2017) and many others.

Note: I am talking about rotation-modulated signals arising from surface features.

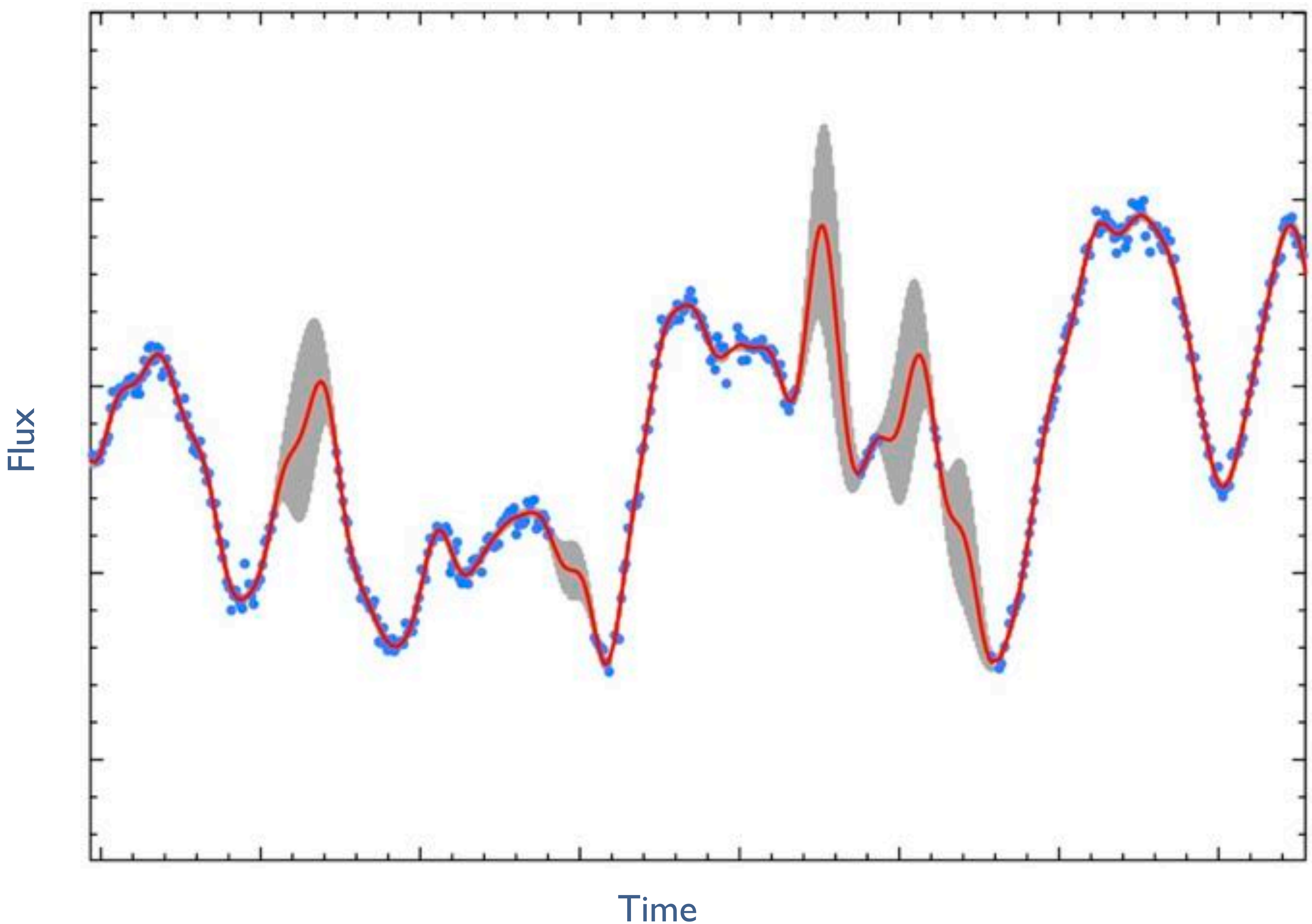


Gaussian process fit to RV data

What does a Gaussian process look like?



What does a Gaussian process look like?



Stellar activity as correlated noise

- In radial-velocity (RV) time-series:

Planet signals:
periodic and coherent

Activity signals:
quasi-periodic and non-coherent

- ~~white noise~~ correlated noise → ~~chi2~~ log $\mathcal{L}$

Stellar activity as correlated noise

• In radial-velocity (RV) time-series:

Planet signals:
periodic and coherent

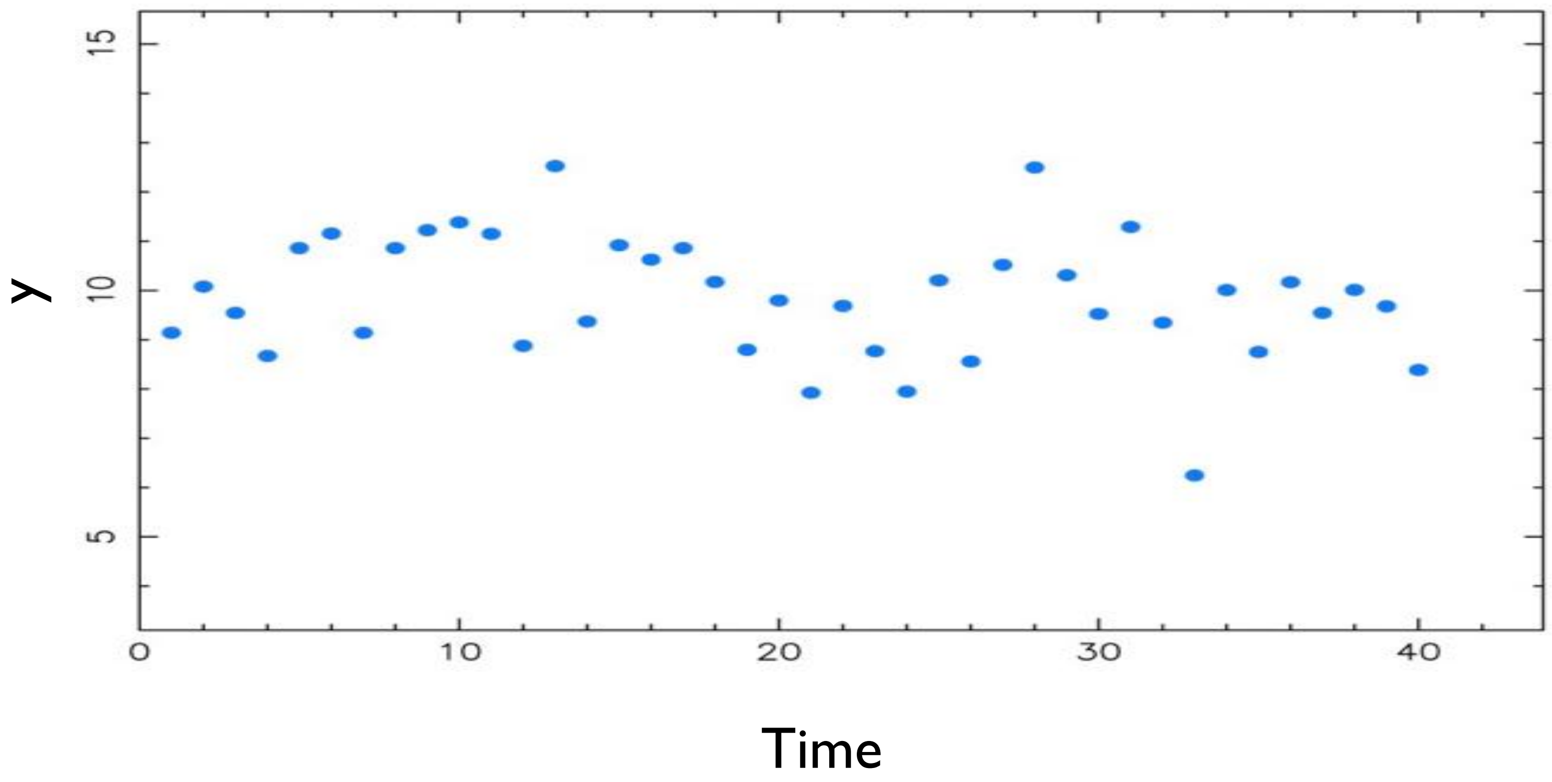
Activity signals:
quasi-periodic and non-coherent

• ~~white noise~~ correlated noise → ~~chi2~~ log $\mathcal{L}$

• Activity-induced RV variations are a signature of the intrinsic magnetic behaviour of a star. All observables, *eg.* the lightcurve, R'_{HK} index, FWHM,... share a common frequency structure and similar covariance properties.

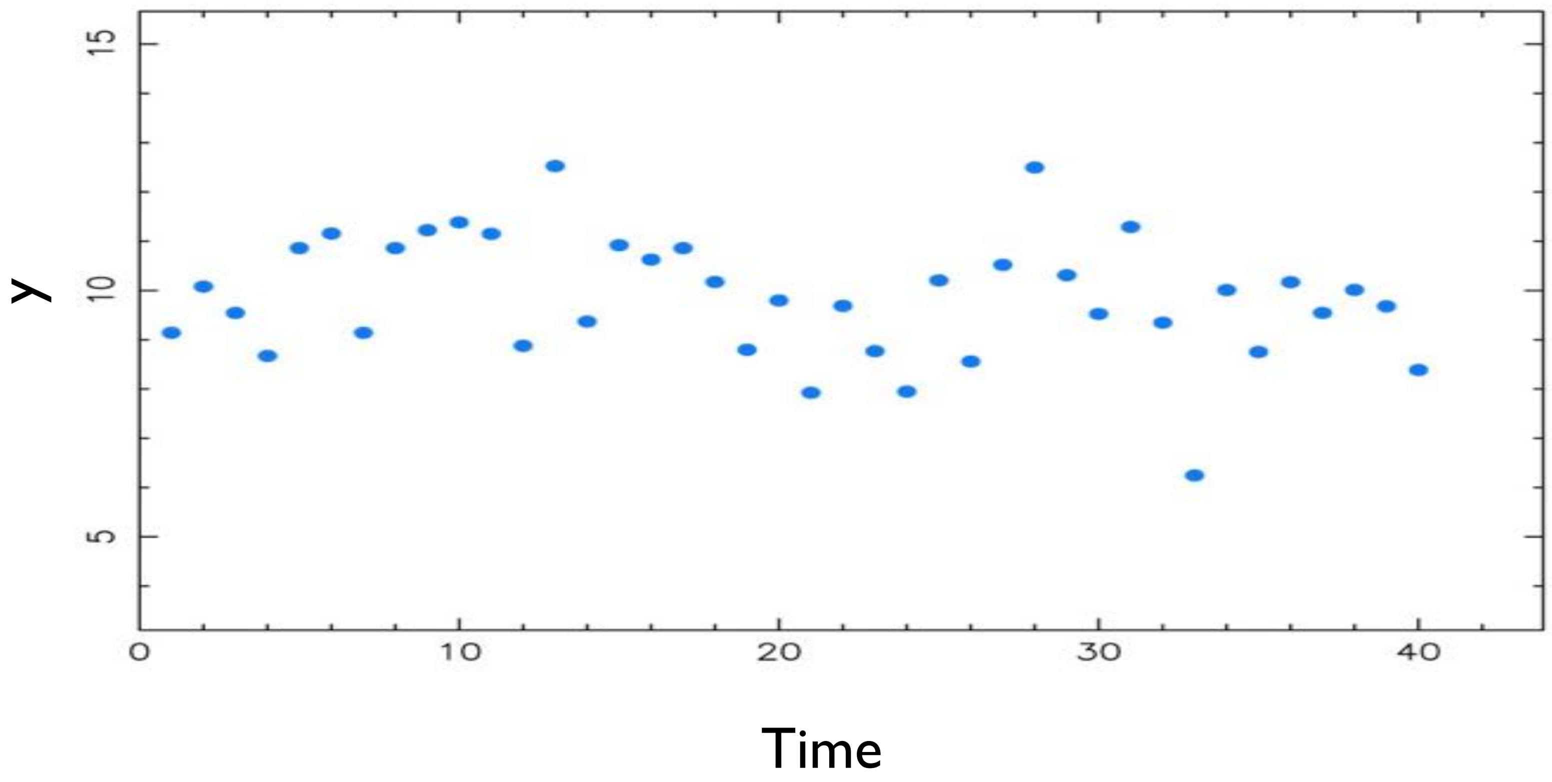
Uncorrelated noise ("white" noise, "jitter")

All data points are completely independent of each other

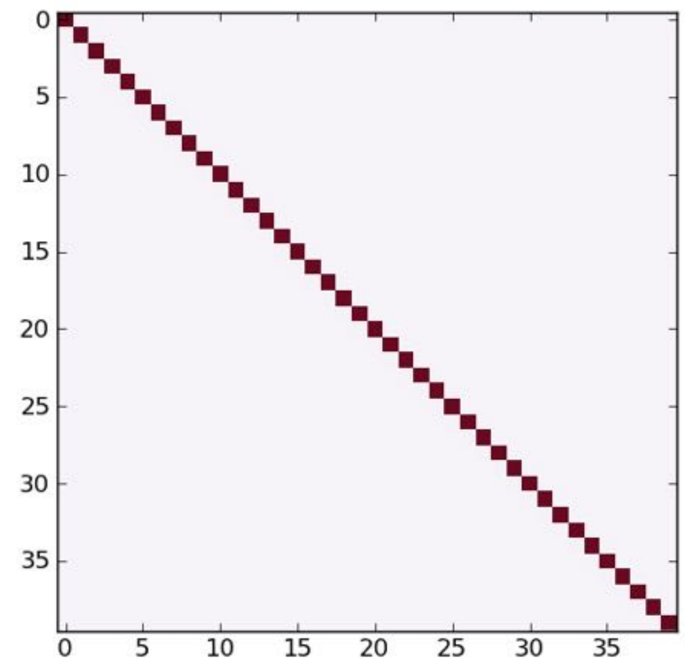


Uncorrelated noise (“white” noise, “jitter”)

All data points are completely independent of each other

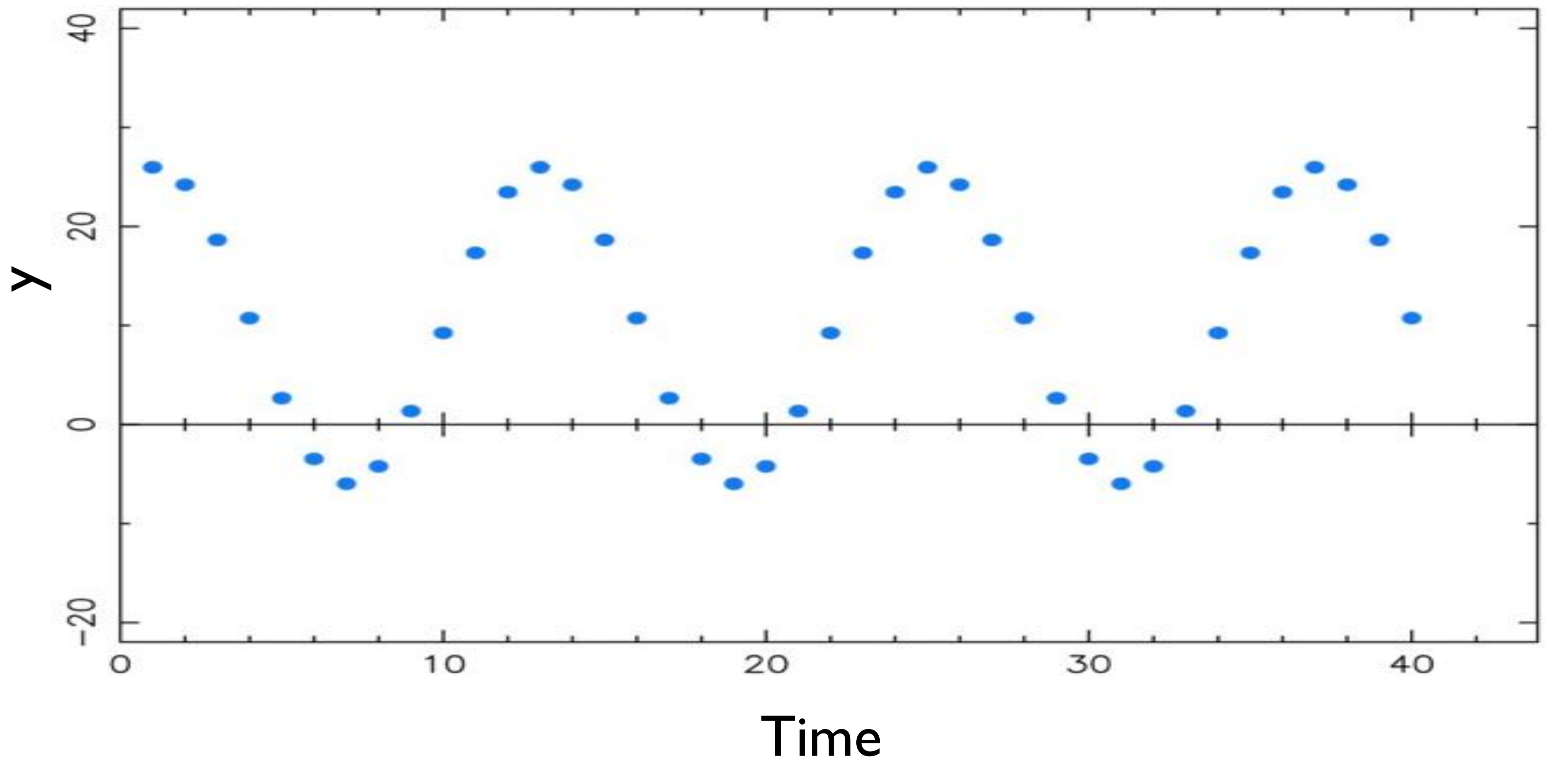


Covariance matrix



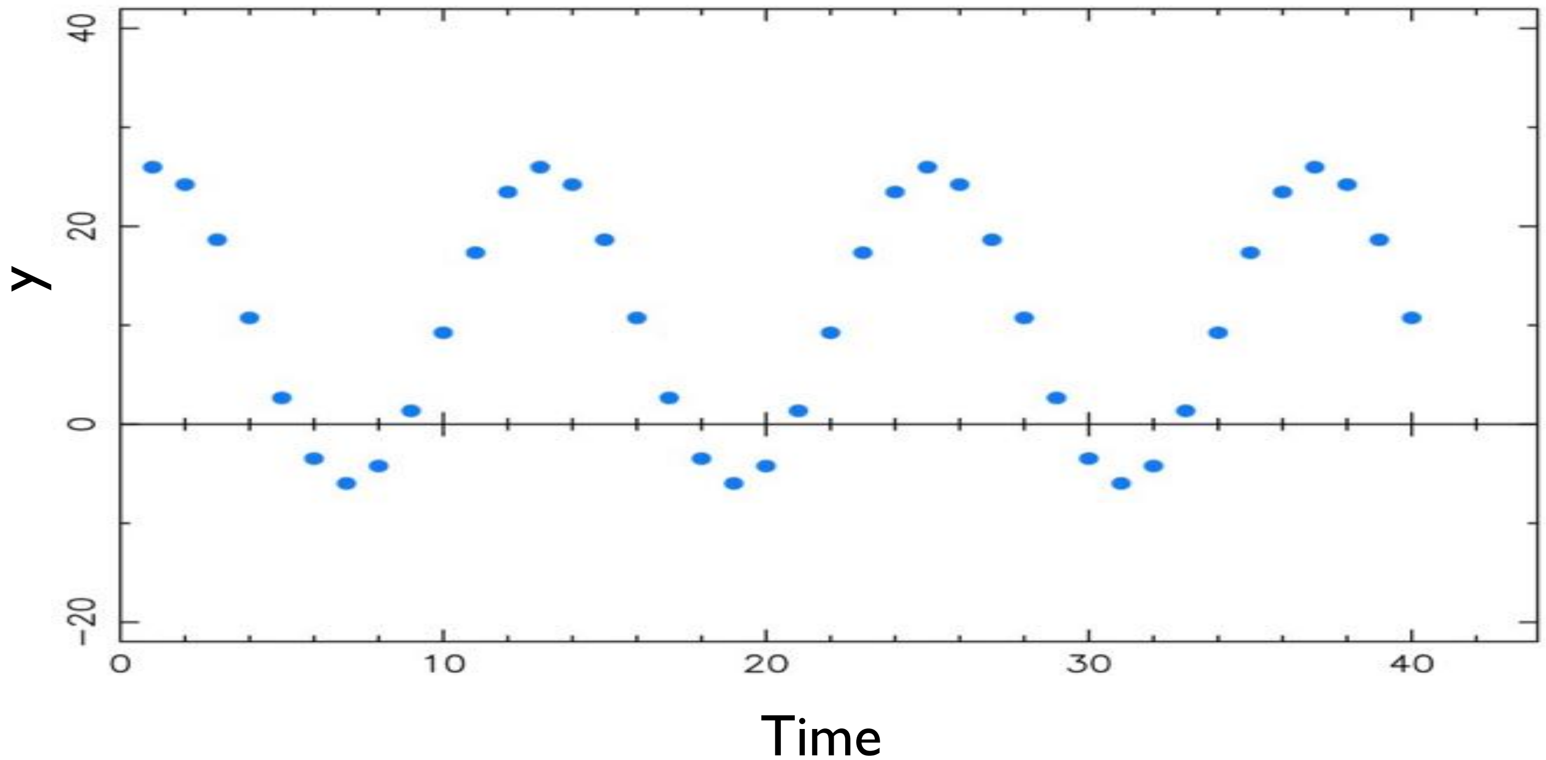
Correlated noise

Data points are correlated with each other

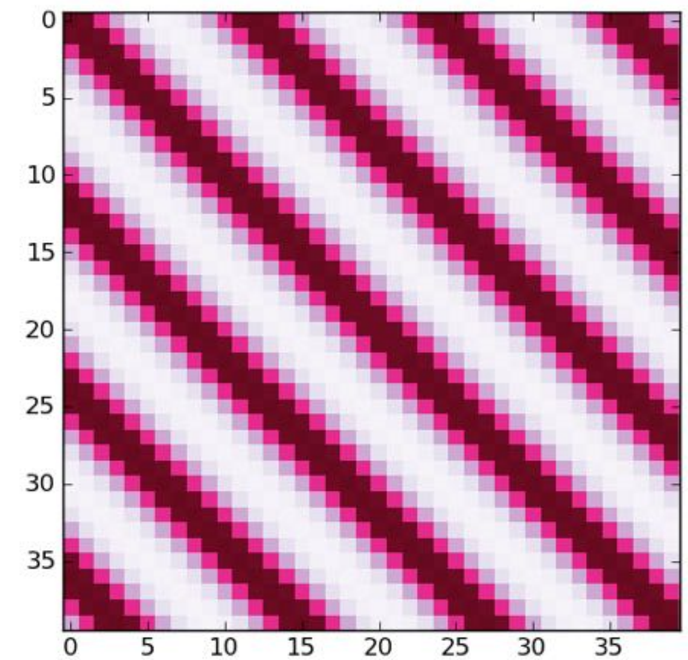


Correlated noise

Data points are correlated with each other

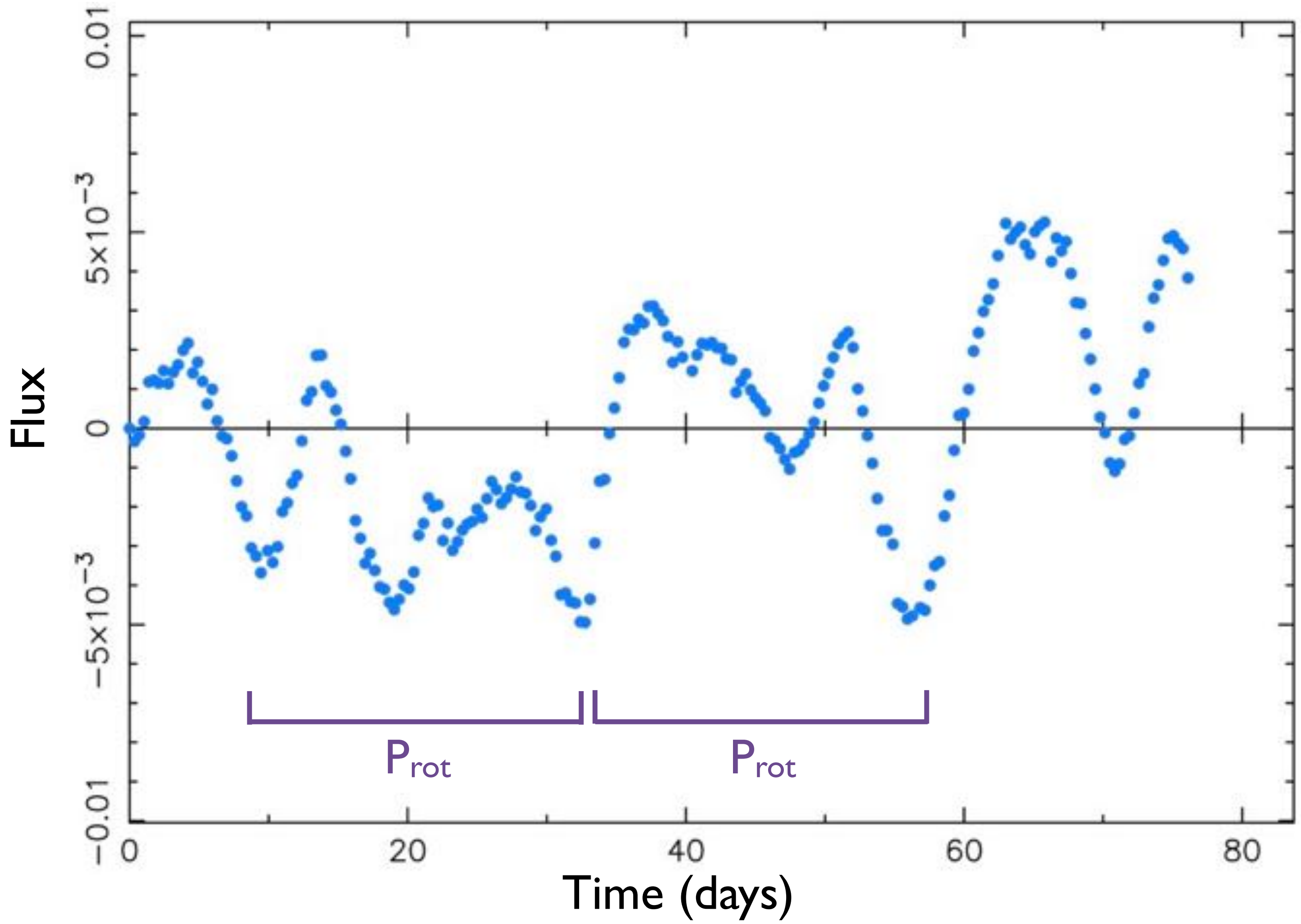


Covariance matrix

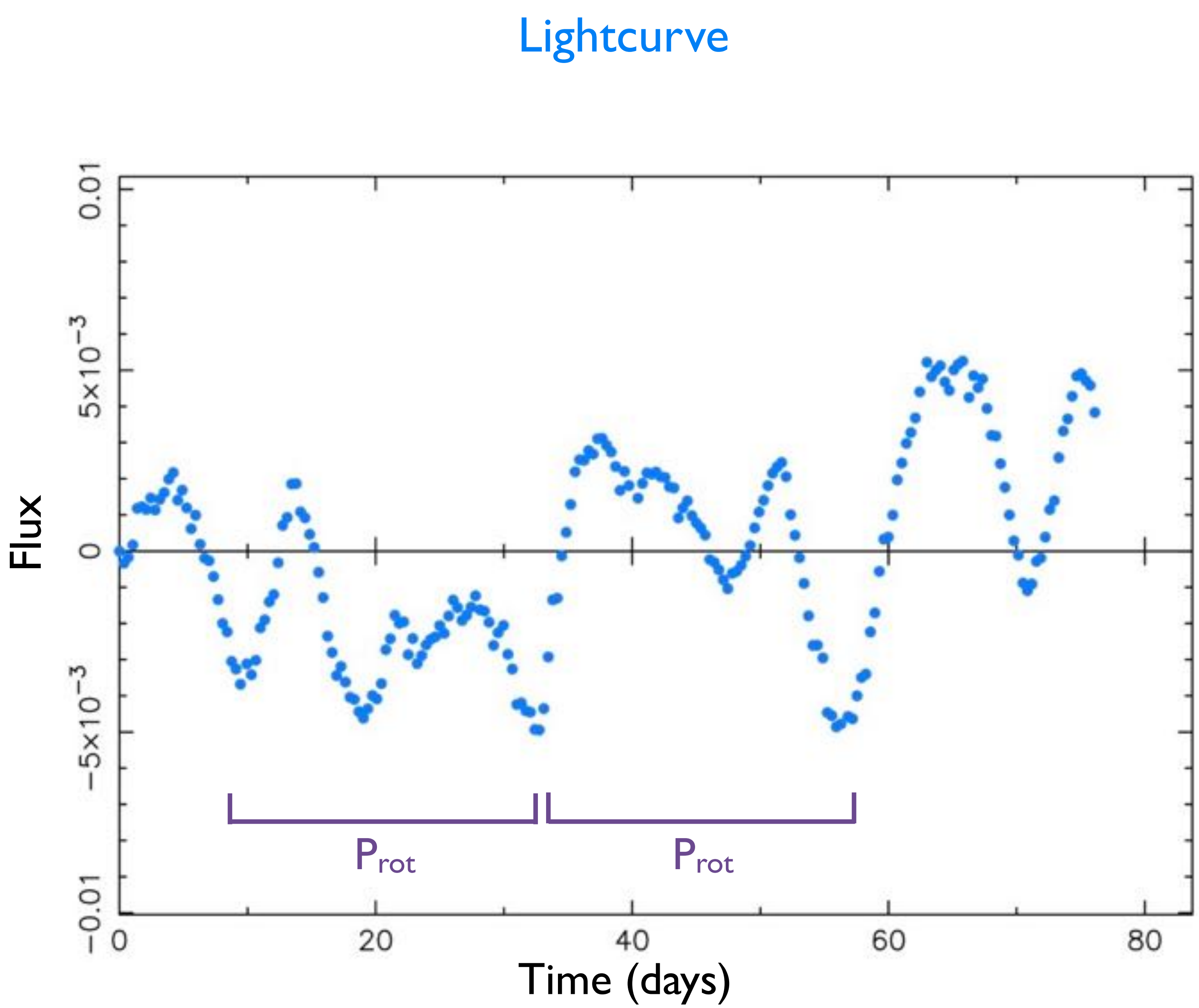


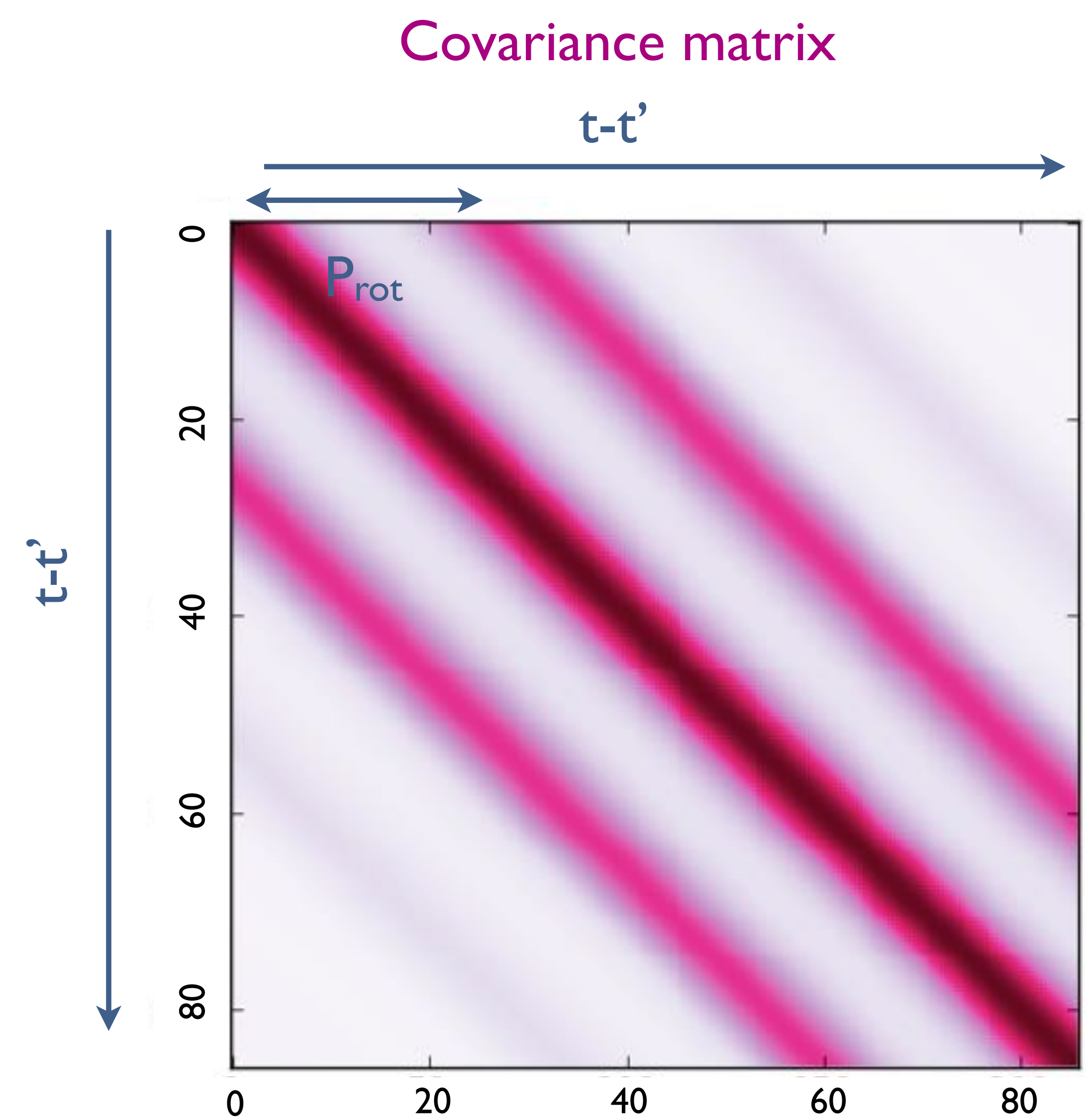
Rotation-modulated activity: quasi-periodic variations

Lightcurve



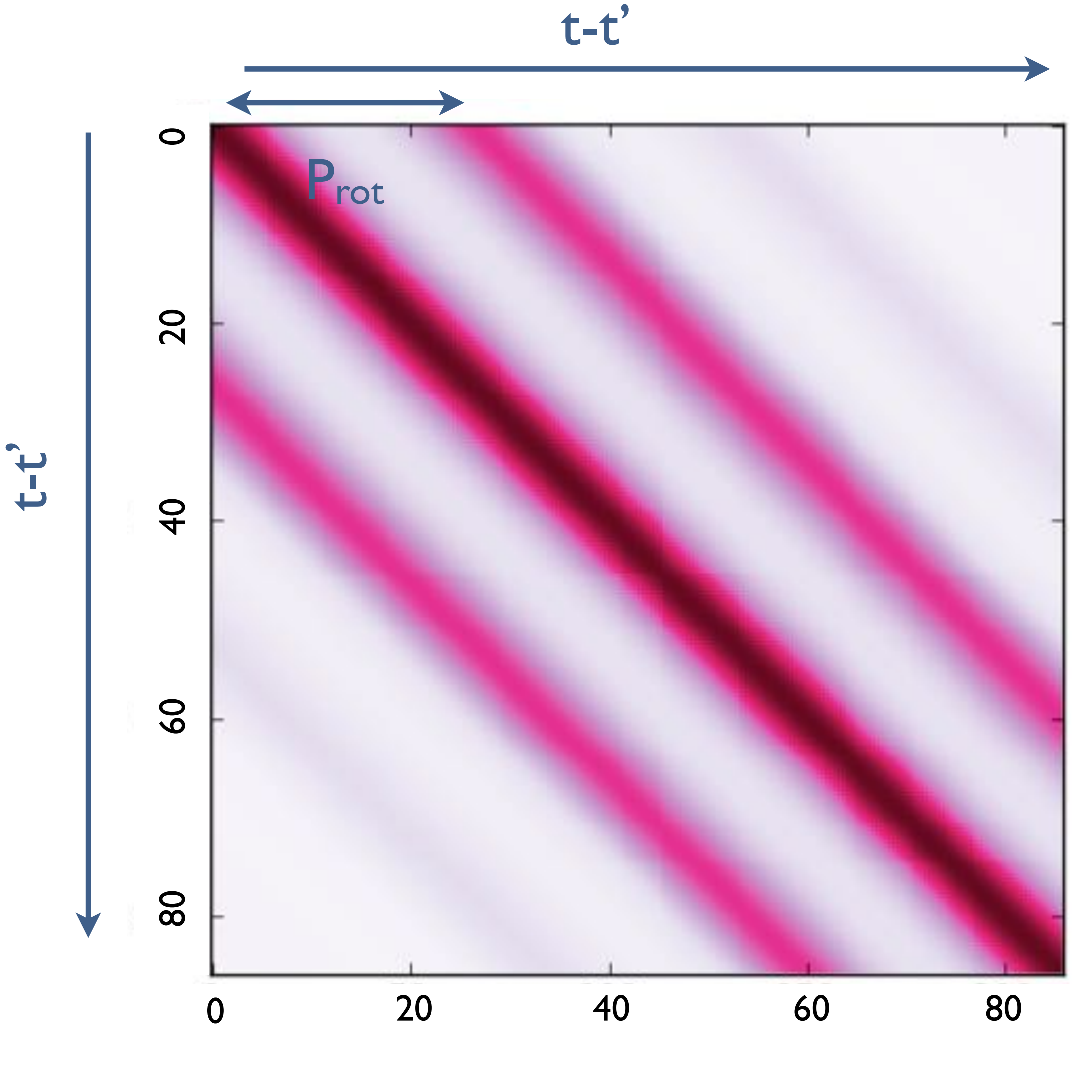
Rotation-modulated activity: quasi-periodic variations





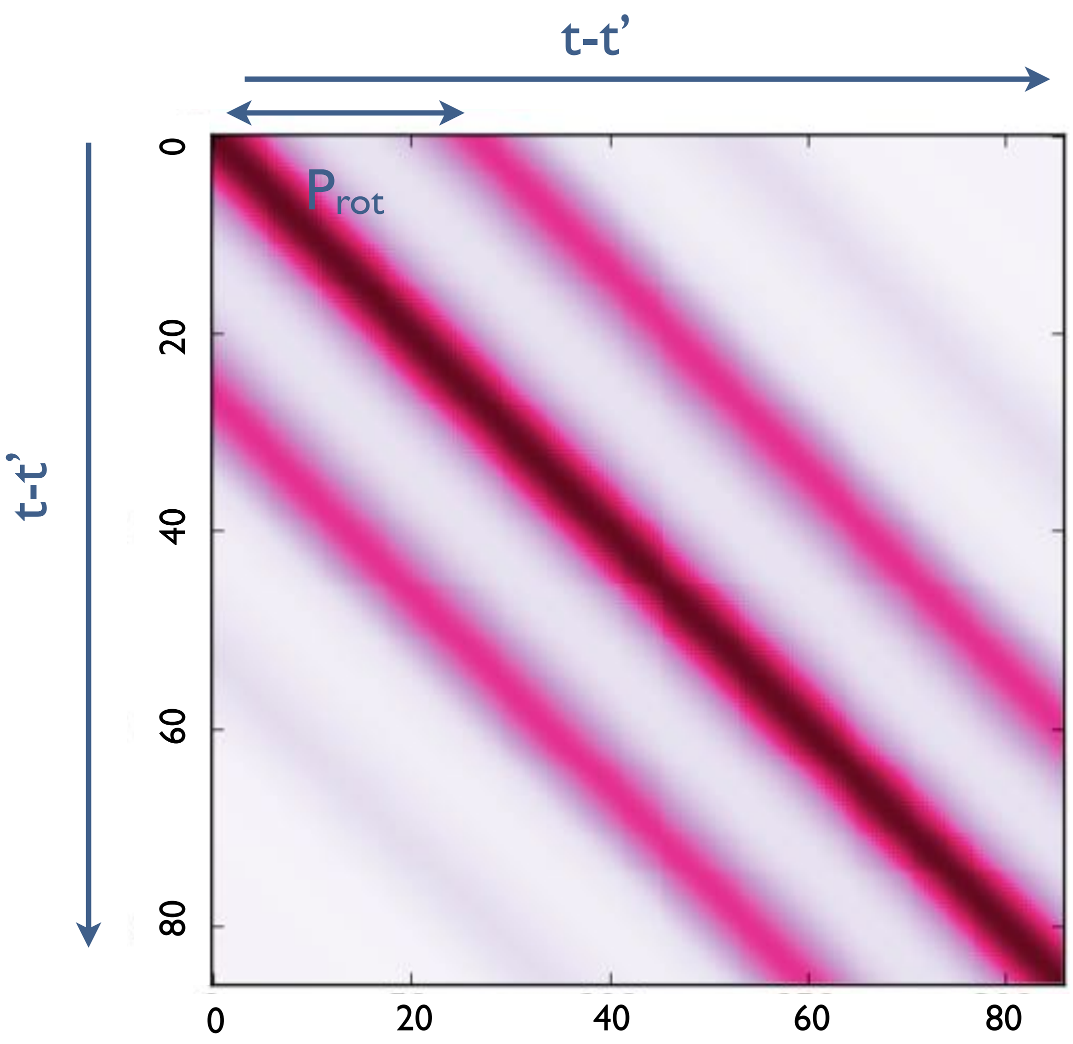
$\mathbf{K}_{i,j} = k\,(t_i, t_j)$

Covariance matrix

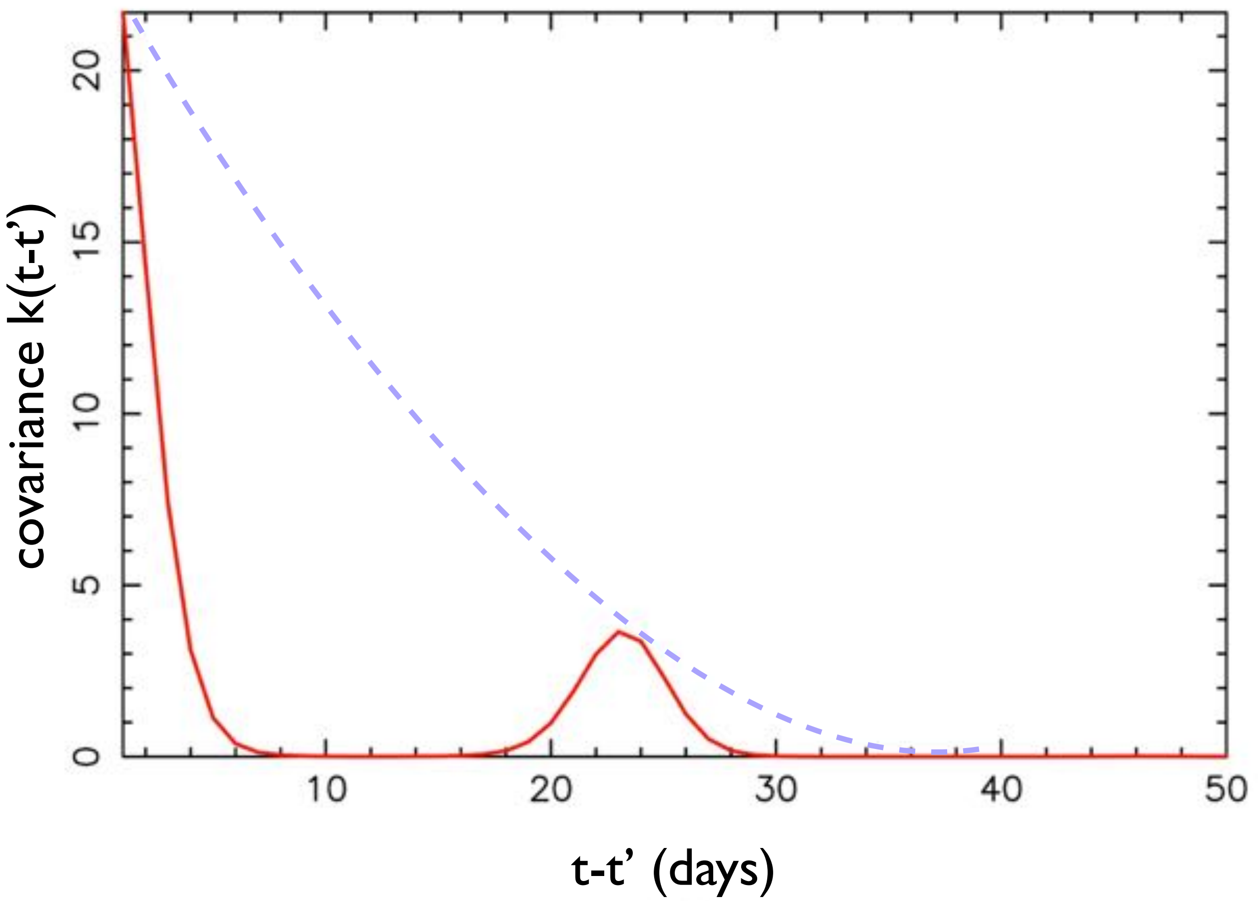


$\mathbf{K}_{i,j} = k\,(t_i, t_j)$

Covariance matrix



Covariance function k (t, t')

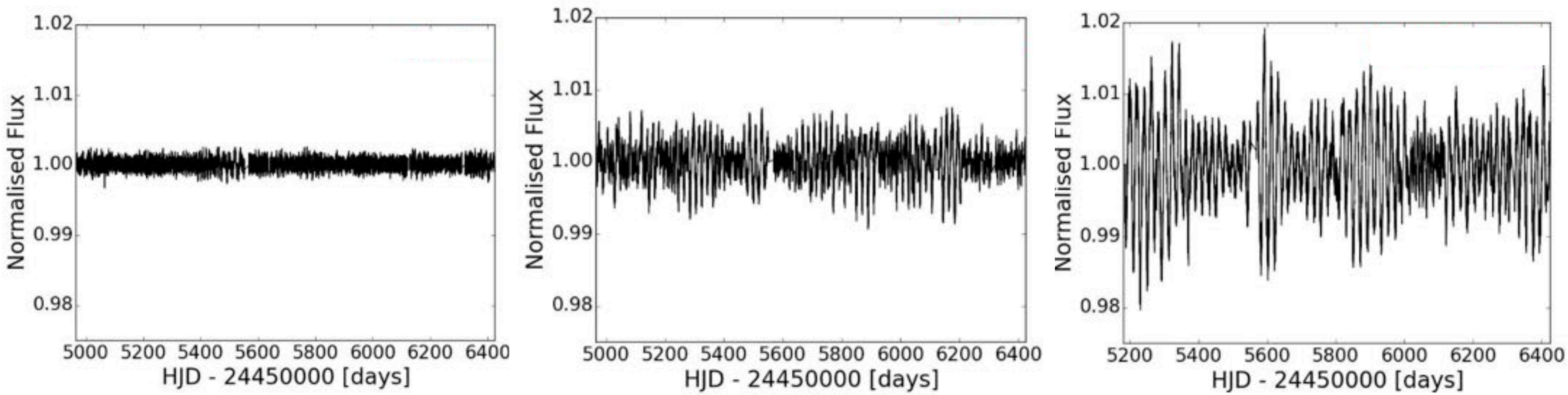


Quasi-periodic form:

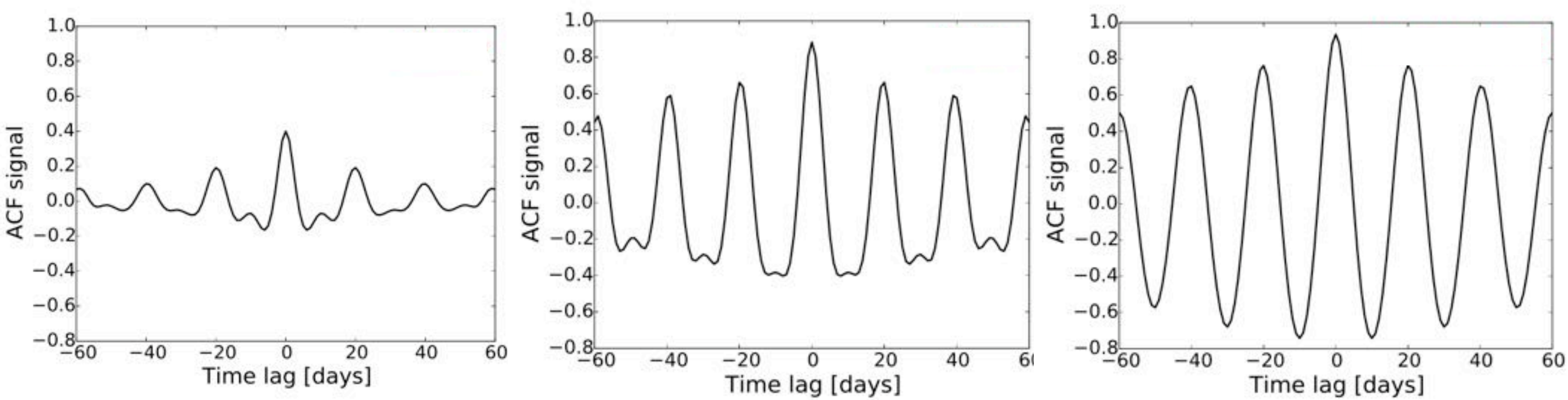
$$k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\,\eta_2^2} - \frac{2\,\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2}\right]$$

Why is this form of covariance function adequate?

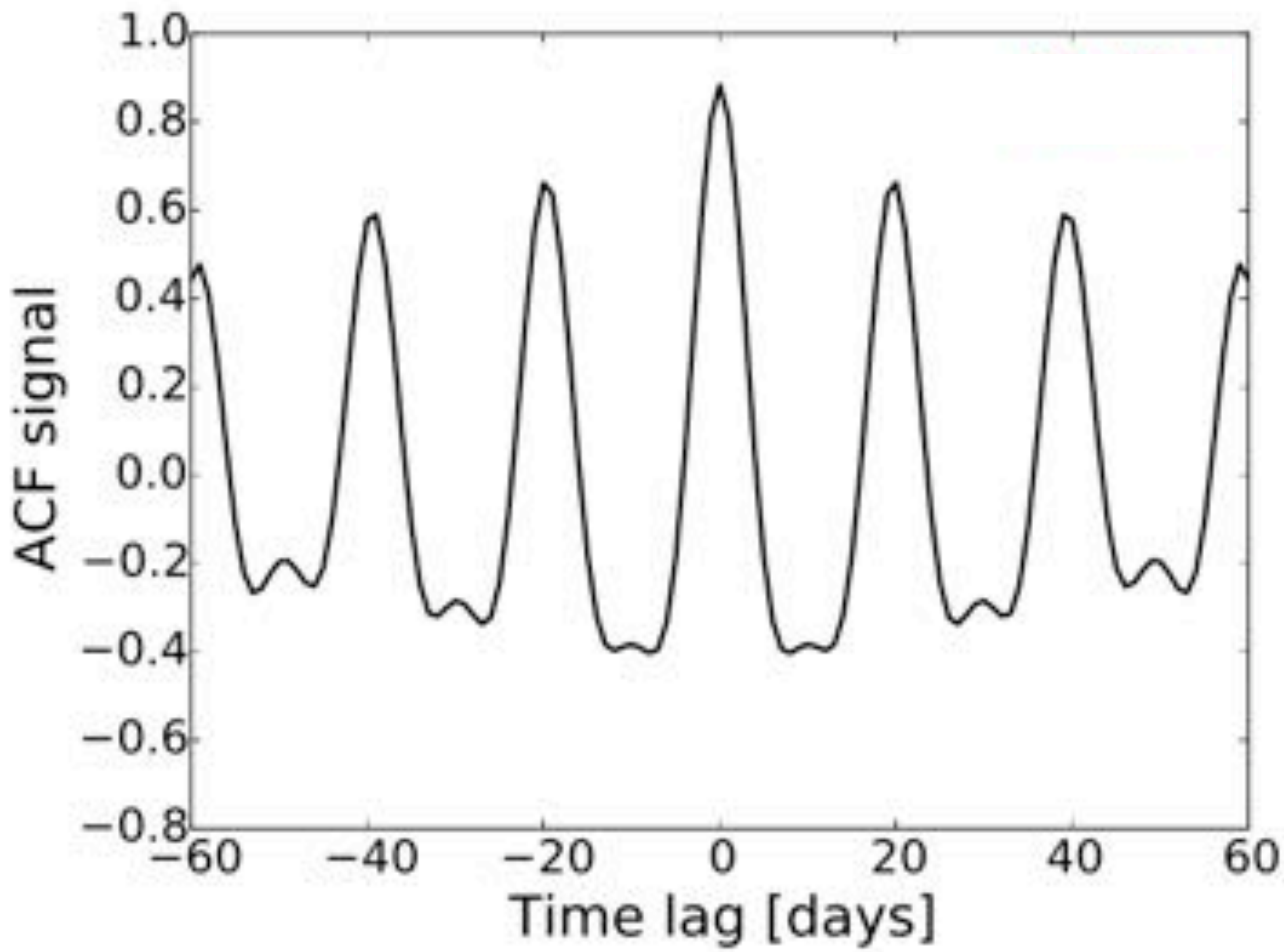
Typical *Kepler* lightcurves of FGKs stars



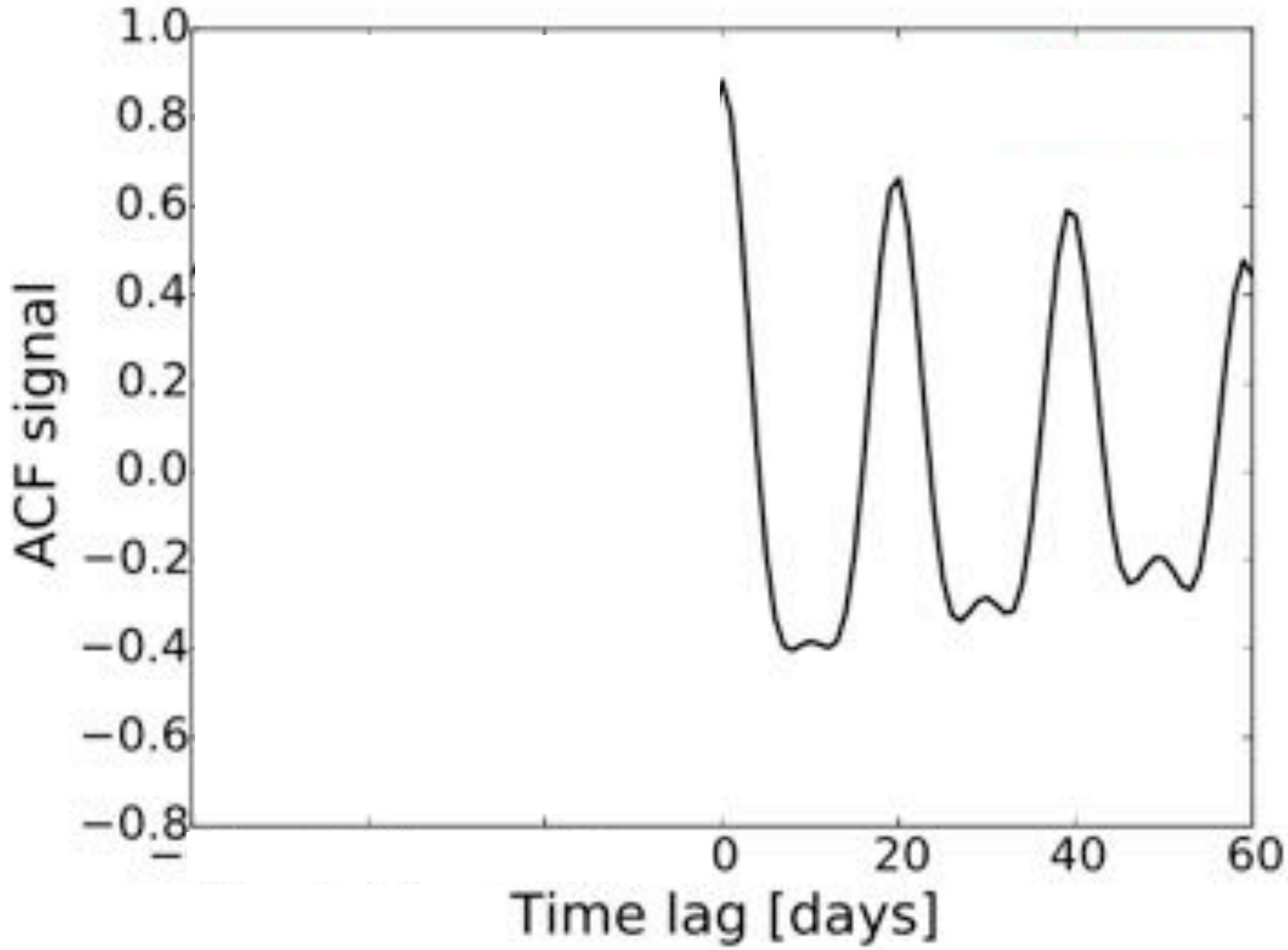
Corresponding autocorrelation functions (ACFs)



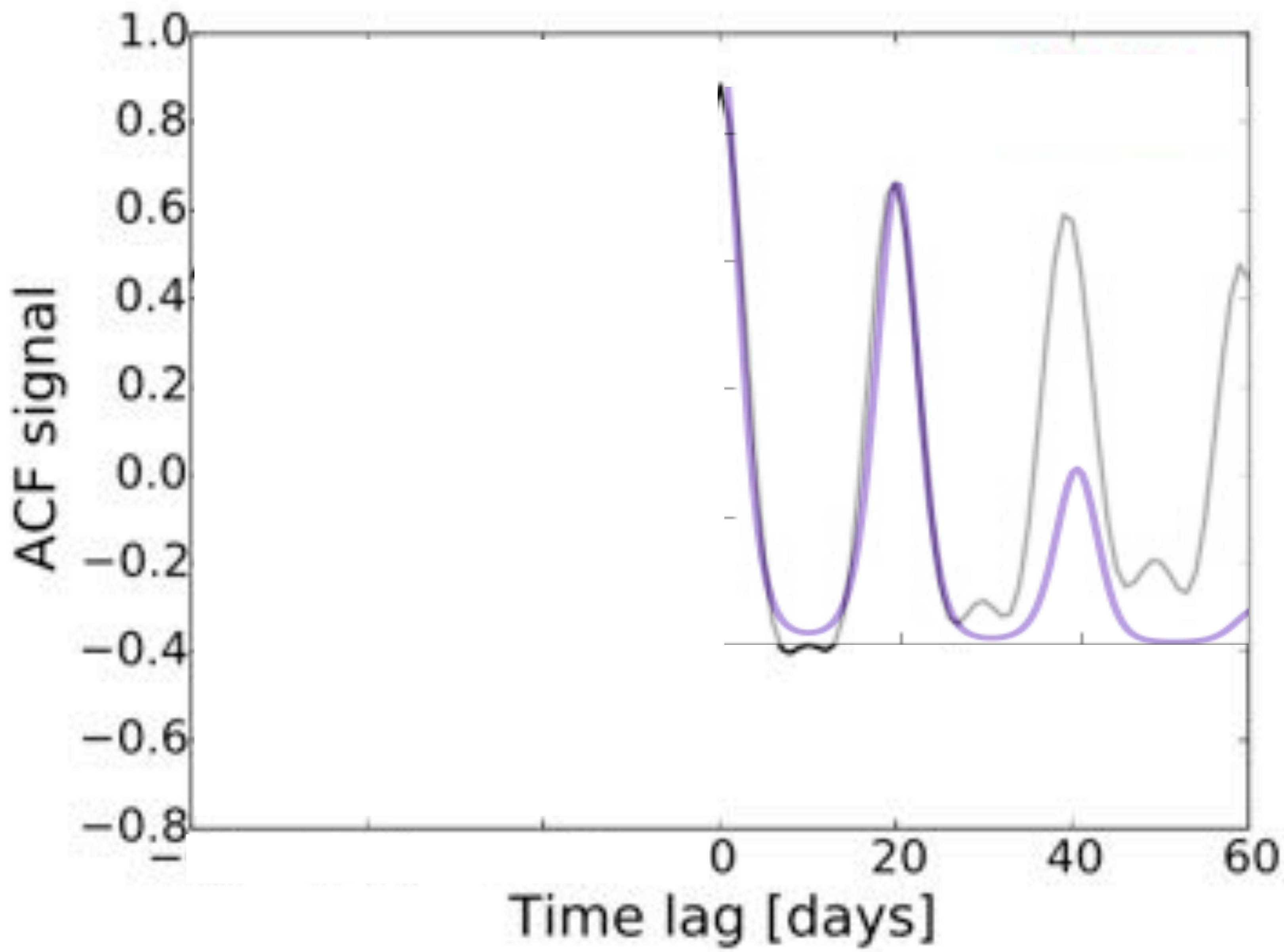
The covariance function is an “analytical model” for the autocorrelation function



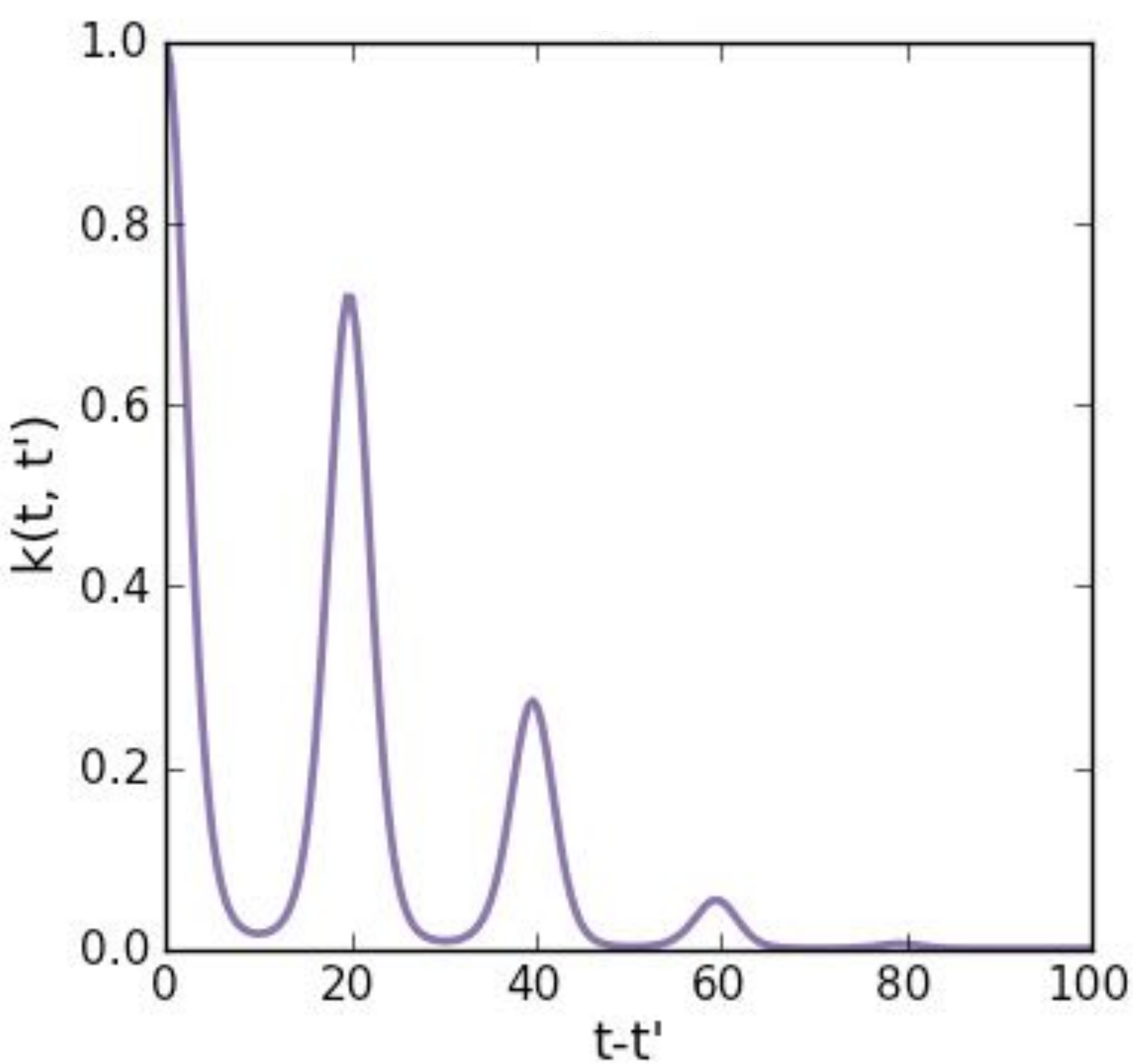
The covariance function is an "analytical model" for the autocorrelation function



The covariance function is an "analytical model" for the autocorrelation function

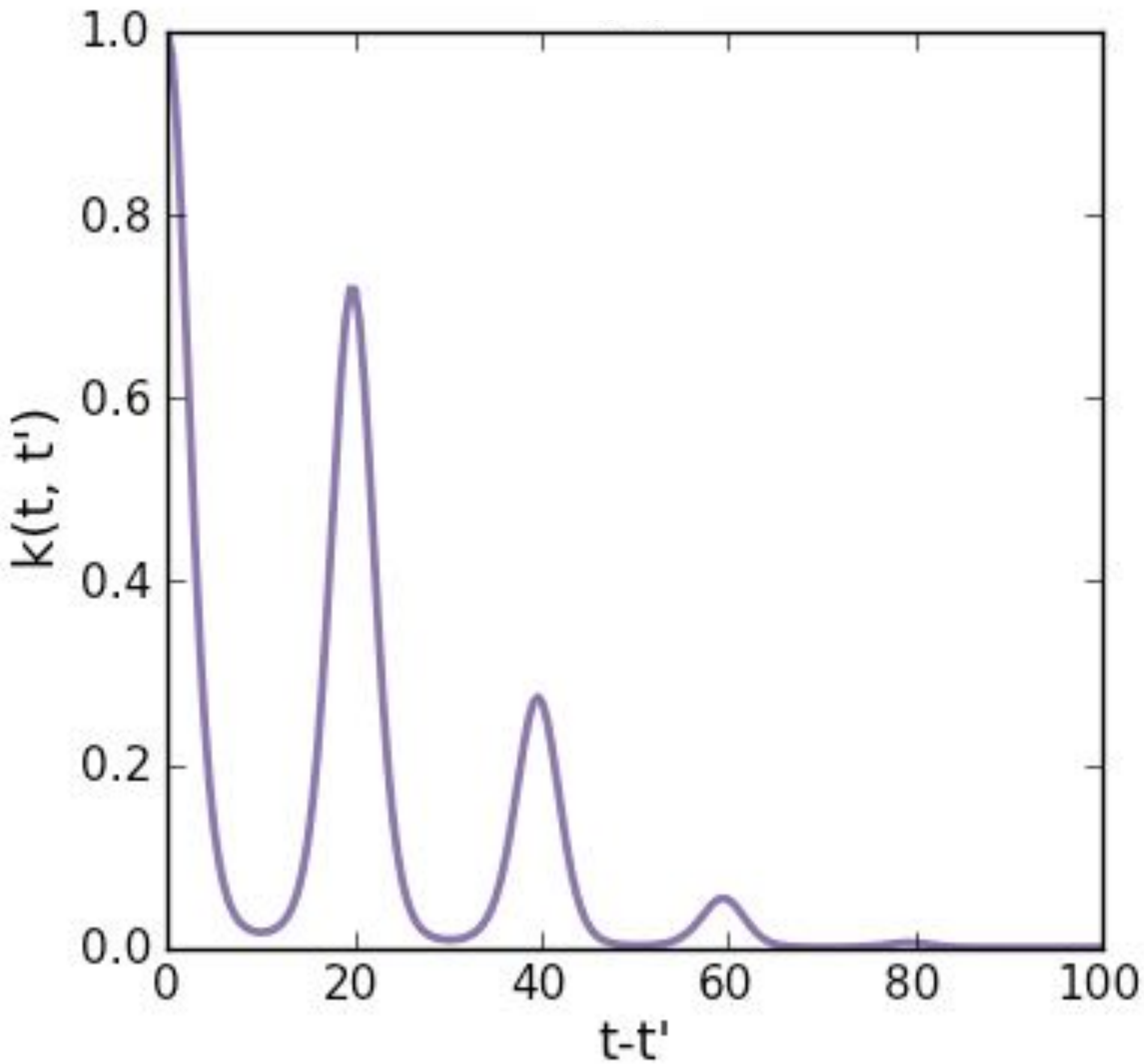


Why is this form of covariance function adequate?



Why is this form of covariance function adequate?

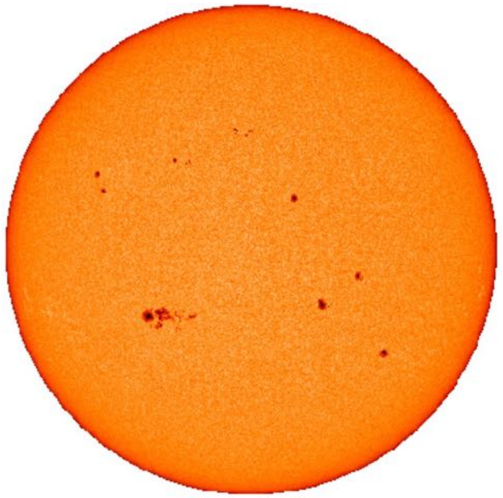
$$k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\,\eta_2^2} - \frac{2\,\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2}\right]$$

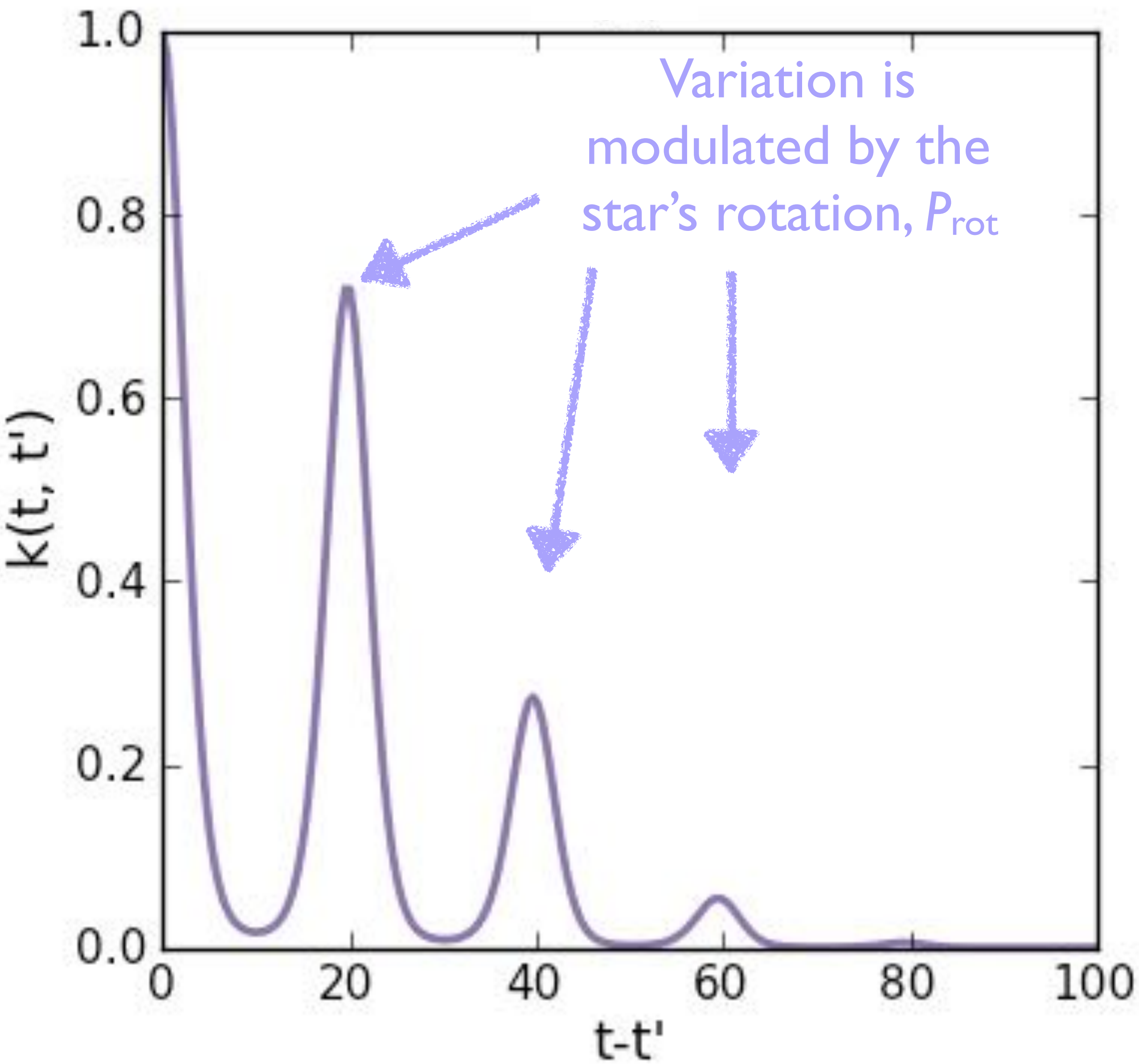


Why is this form of covariance function adequate?

$$k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\,\eta_2^2} - \frac{2\,\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2}\right]$$

recurrence timescale P_{rot}



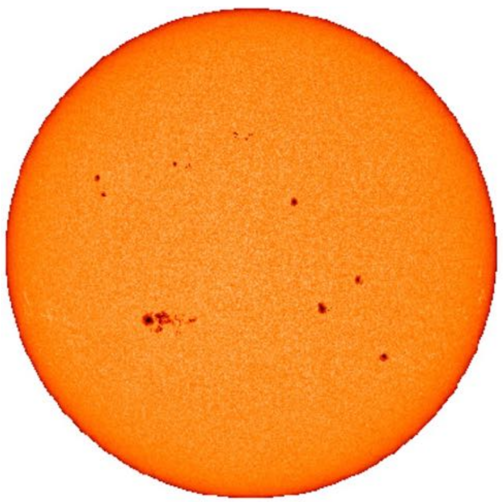


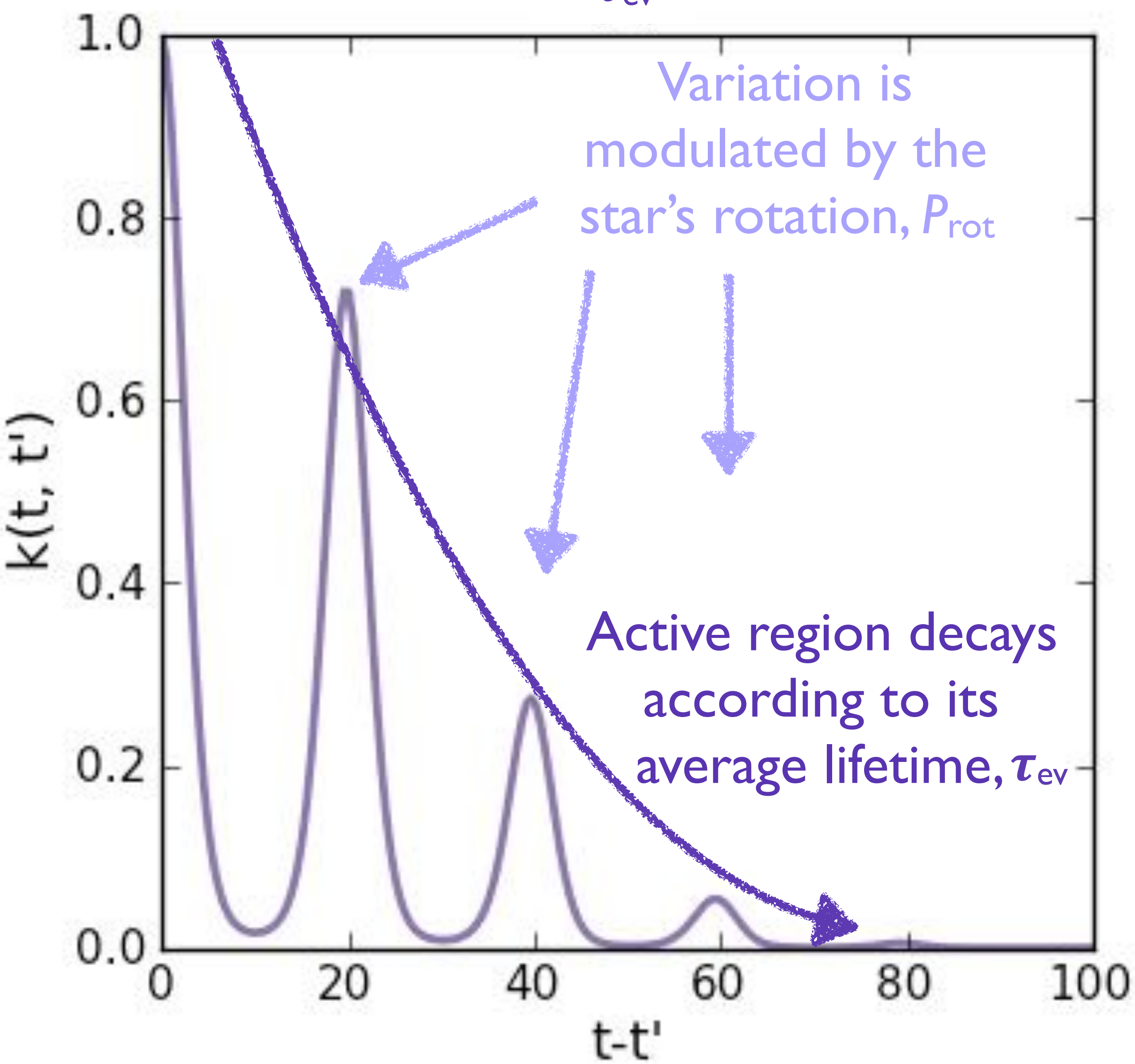
Why is this form of covariance function adequate?

$$k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\,\eta_2^2} - \frac{2\,\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2}\right]$$

decay timescale τ_{ev}

recurrence timescale P_{rot}



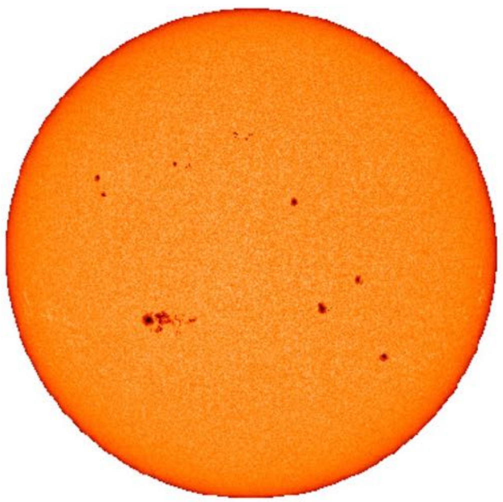


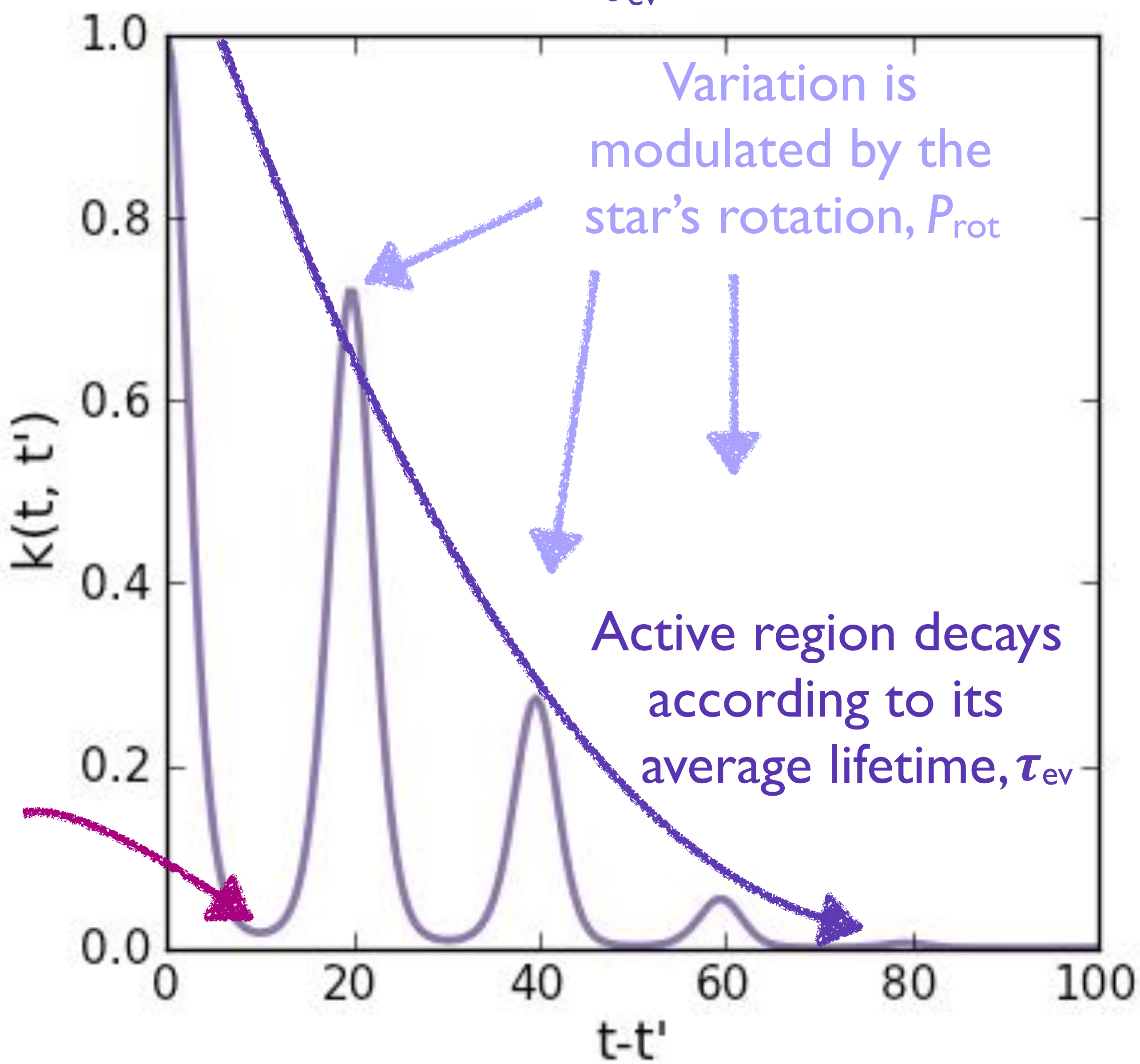
Why is this form of covariance function adequate?

$$k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\,\eta_2^2} - \frac{2\,\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2}\right]$$

decay timescale τ_{ev}

recurrence timescale P_{rot}





Active region is on the invisible side of the star

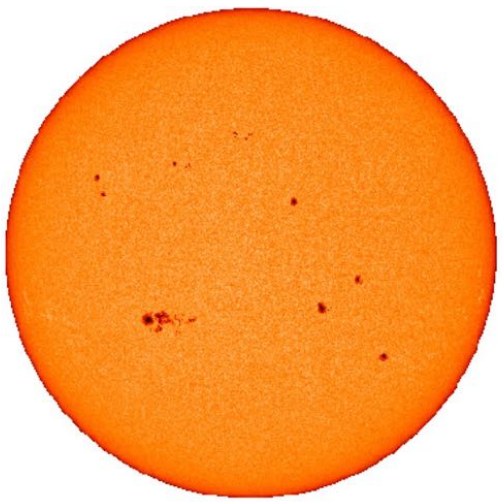
Why is this form of covariance function adequate?

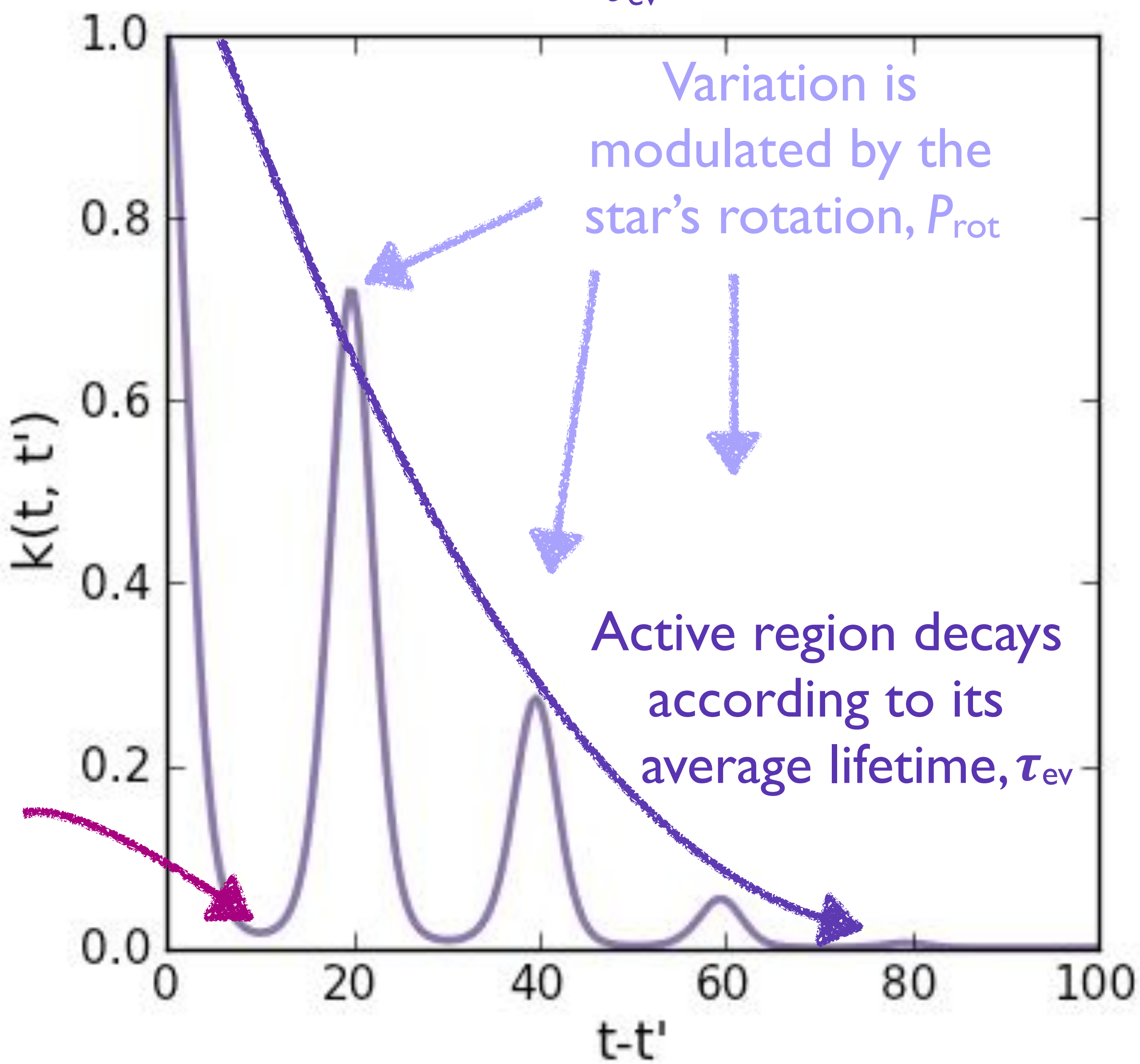
$$k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\,\eta_2^2} - \frac{2\,\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2}\right]$$

decay timescale τ_{ev}

"smoothing" factor

recurrence timescale P_{rot}





Active region is on the invisible side of the star

Why is this form of covariance function adequate?

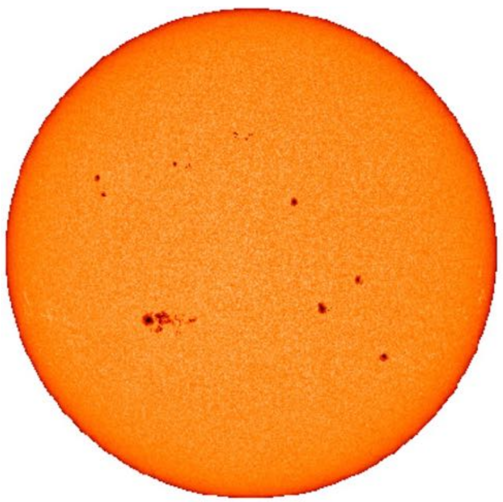
$$k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\,\eta_2^2} - \frac{2\,\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2}\right]$$

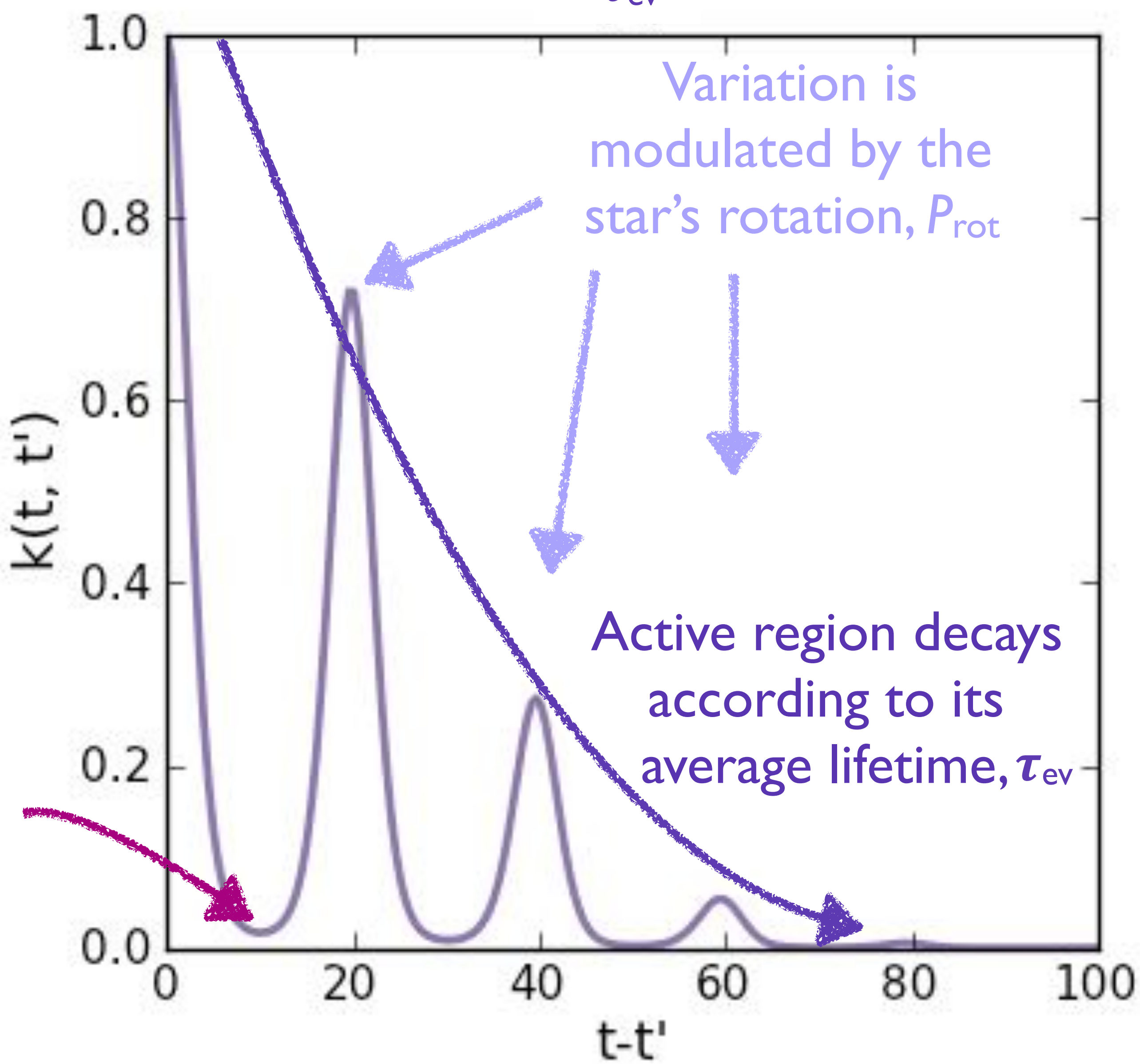
amplitude

decay timescale τ_{ev}

"smoothing" factor

recurrence timescale P_{rot}





Active region is on the invisible side of the star

Determining the hyperparameters

$$k(t,t') = \underbrace{\eta_1^2}_{\text{amplitude}} \cdot \exp\left[-\frac{(t-t')^2}{2\,\underbrace{\eta_2^2}_{\text{decay timescale } \tau_{ev}}} - \frac{2\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\underbrace{\eta_4^2}_{\text{"smoothing" factor}}}\right]$$

recurrence timescale P_{rot}

- Can "train" the GP on an auxiliary dataset, eg. the lightcurve (Haywood et al. 2014, Grunblatt et al. 2015), the spectroscopic indicators like FWHM, BIS (Rajpaul et al. 2015), in some cases even the RVs themselves (Faria et al. 2016)

- Can "fix" the hyperparameter values using Gaussian priors, based on prior knowledge/analysis (López-Morales et al. 2016)

Determining the hyperparameters

$$k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\,\eta_2^2} - \frac{2\,\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2}\right]$$

amplitude

decay timescale τ_{ev}

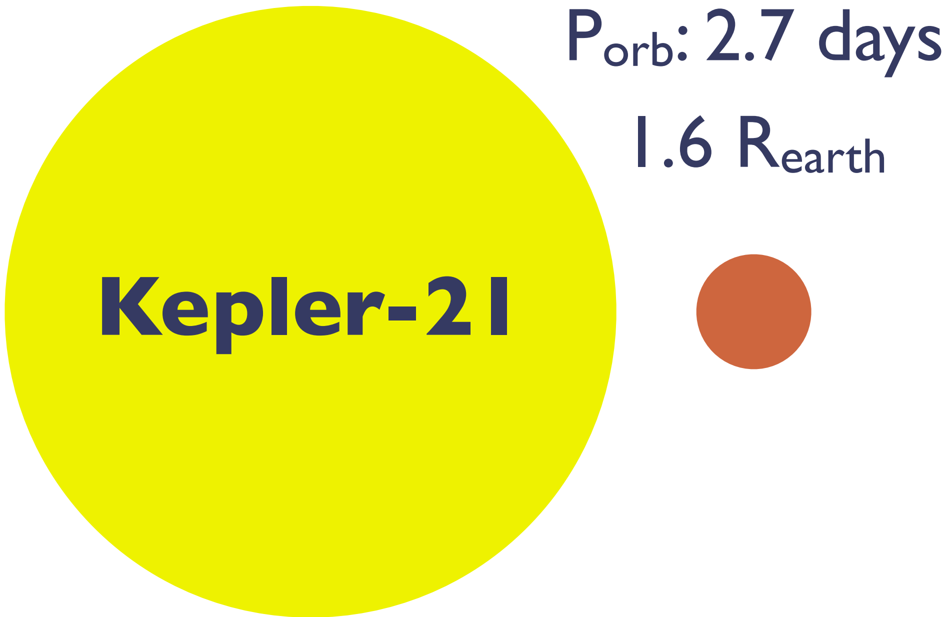
"smoothing" factor

recurrence timescale P_{rot}

- Can "train" the GP on an auxiliary dataset, eg. the lightcurve (Haywood et al. 2014, Grunblatt et al. 2015), the spectroscopic indicators like FWHM, BIS (Rajpaul et al. 2015), in some cases even the RVs themselves (Faria et al. 2016)

- Can "fix" the hyperparameter values using Gaussian priors, based on prior knowledge/analysis (López-Morales et al. 2016)

López-Morales, Haywood, Giles et al. (2016)

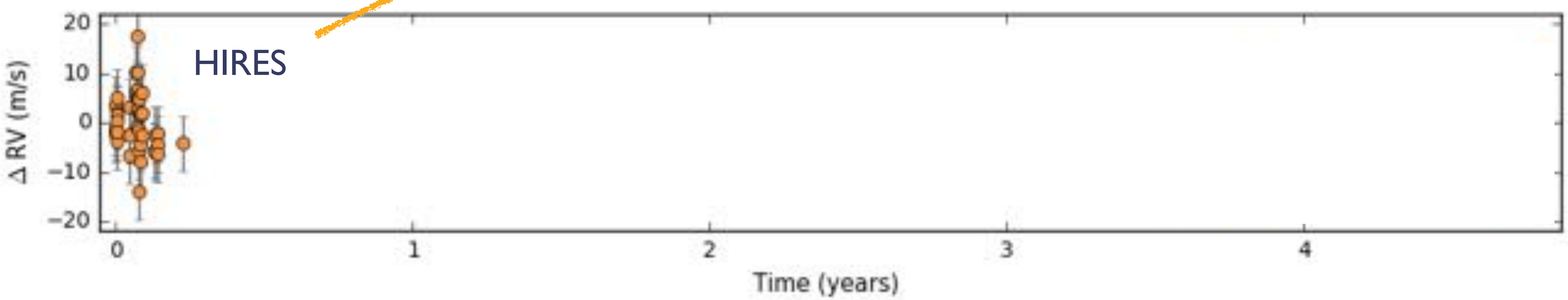


López-Morales, Haywood, Giles et al. (2016)

P_{orb}: 2.7 days

1.6 R_{earth}

Previous mass estimate by Howell et al. (2012): m_p < 10M⊕



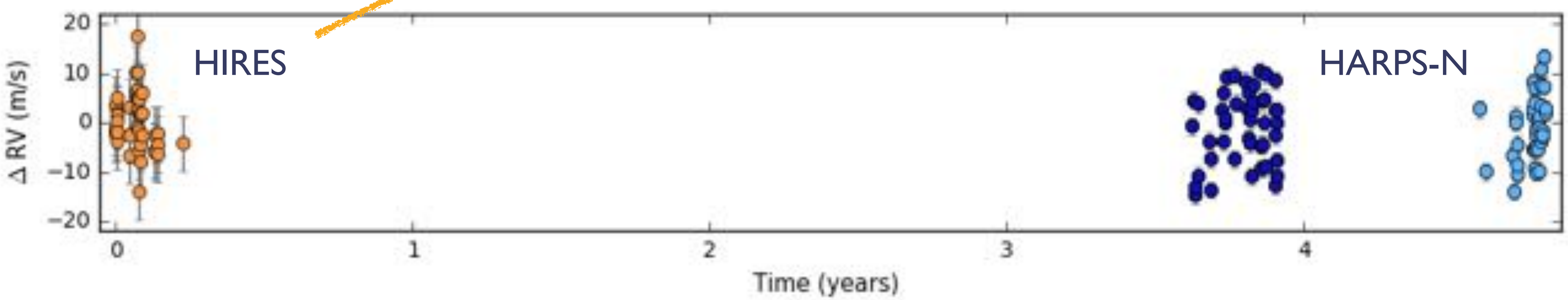
López-Morales, Haywood, Giles et al. (2016)

P_{orb}: 2.7 days

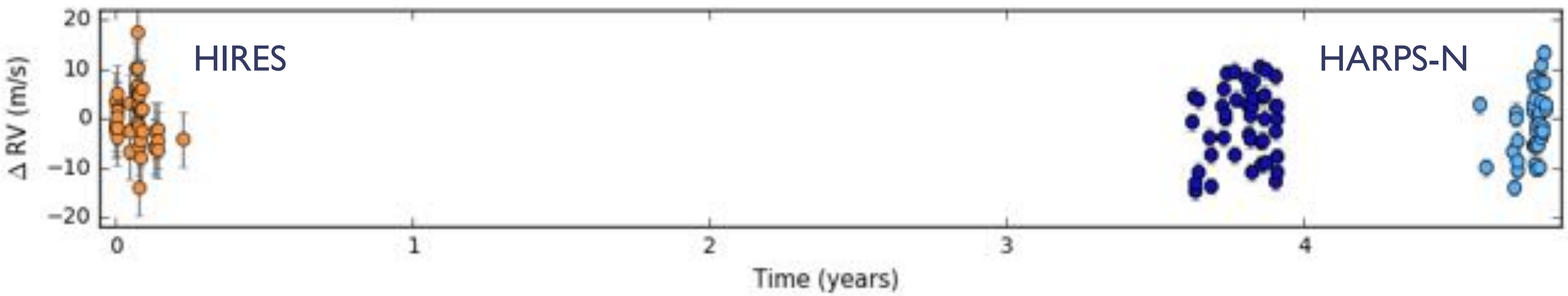
1.6 R_{earth}

Kepler-21

Previous mass estimate by
Howell et al. (2012): m_p < 10M⊕



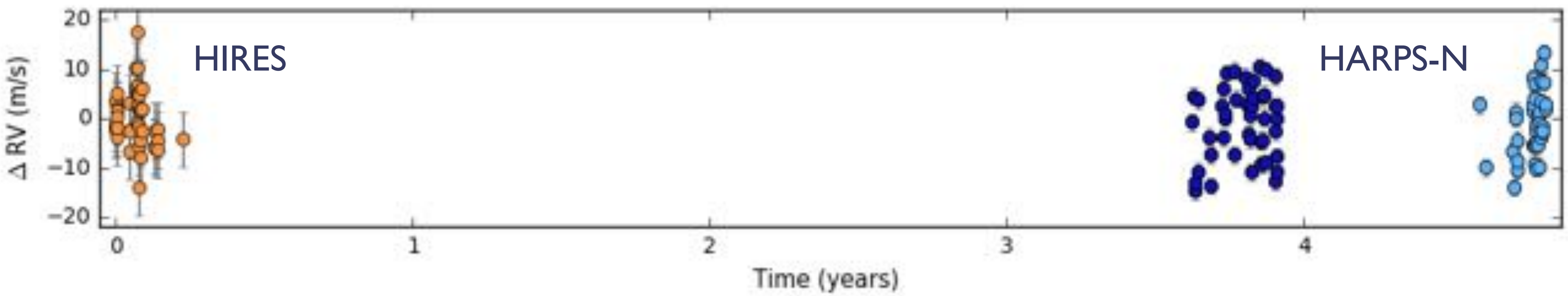
López-Morales, Haywood, Giles et al. (2016)



RV model = Keplerian orbit + RV0

Noise = Gaussian process + uncorrelated term

López-Morales, Haywood, Giles et al. (2016)



RV model = Keplerian orbit + RV0

Noise = Gaussian process + uncorrelated term

Maximize likelihood function to determine best-fit parameters and uncertainties, *eg.* via MCMC procedure:

$$\ln \mathcal{L} = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\ln(\det \mathbf{K}) - \frac{1}{2}(\underline{y} - \underline{\mu})^T \mathbf{K}^{-\mathbf{1}}(\underline{y} - \underline{\mu})$$

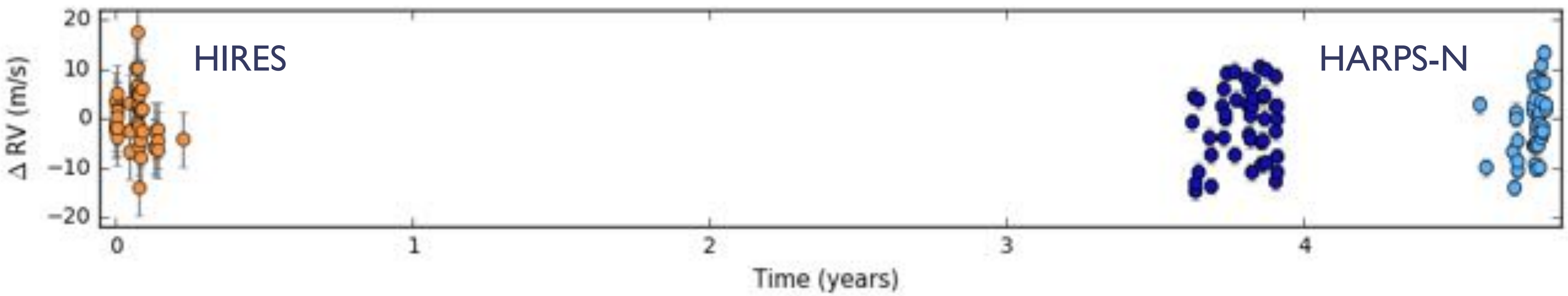
n: number of RV observations

y = RV

μ= model

$\mathbf{K}_{i,j}$ = $k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\,\eta_2^2} - \frac{2\,\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2}\right]$

López-Morales, Haywood, Giles et al. (2016)



RV model = Keplerian orbit + RV0

Noise = Gaussian process + uncorrelated term

Maximize likelihood function to determine best-fit parameters and uncertainties, *eg.* via MCMC procedure:

$$\ln \mathcal{L} = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\ln(\det \mathbf{K}) - \frac{1}{2}(\underline{y} - \underline{\mu})^T \mathbf{K}^{-1}(\underline{y} - \underline{\mu})$$

n: number of RV observations

y = RV

μ= model

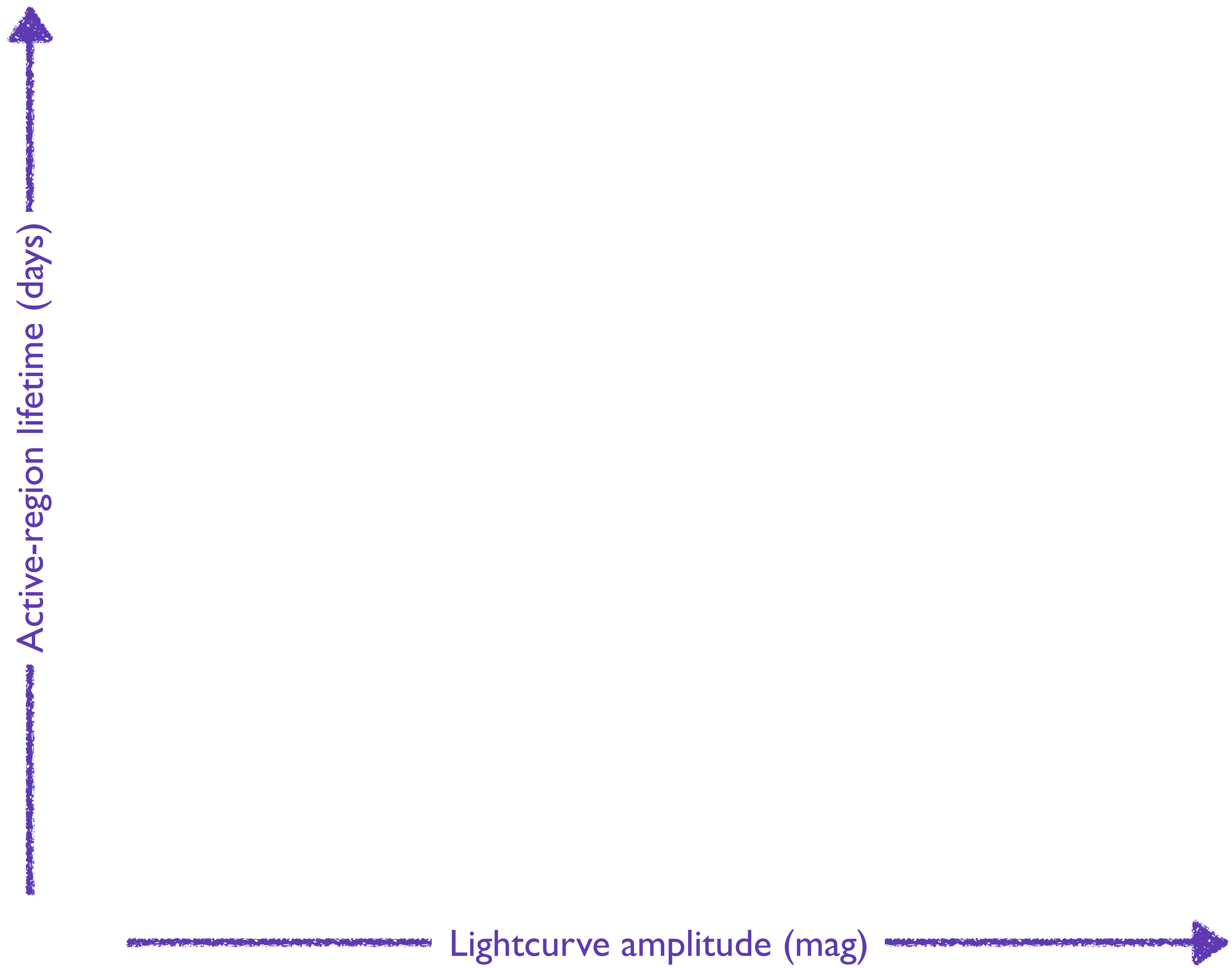
$\mathbf{K}_{i,j}$ = $k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\,\eta_2^2} - \frac{2\,\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2}\right]$

Use priors for stellar rotation and active region lifetime derived from *Kepler* lightcurve

Place strong prior on η_4

Active-region lifetimes τ_{ev}

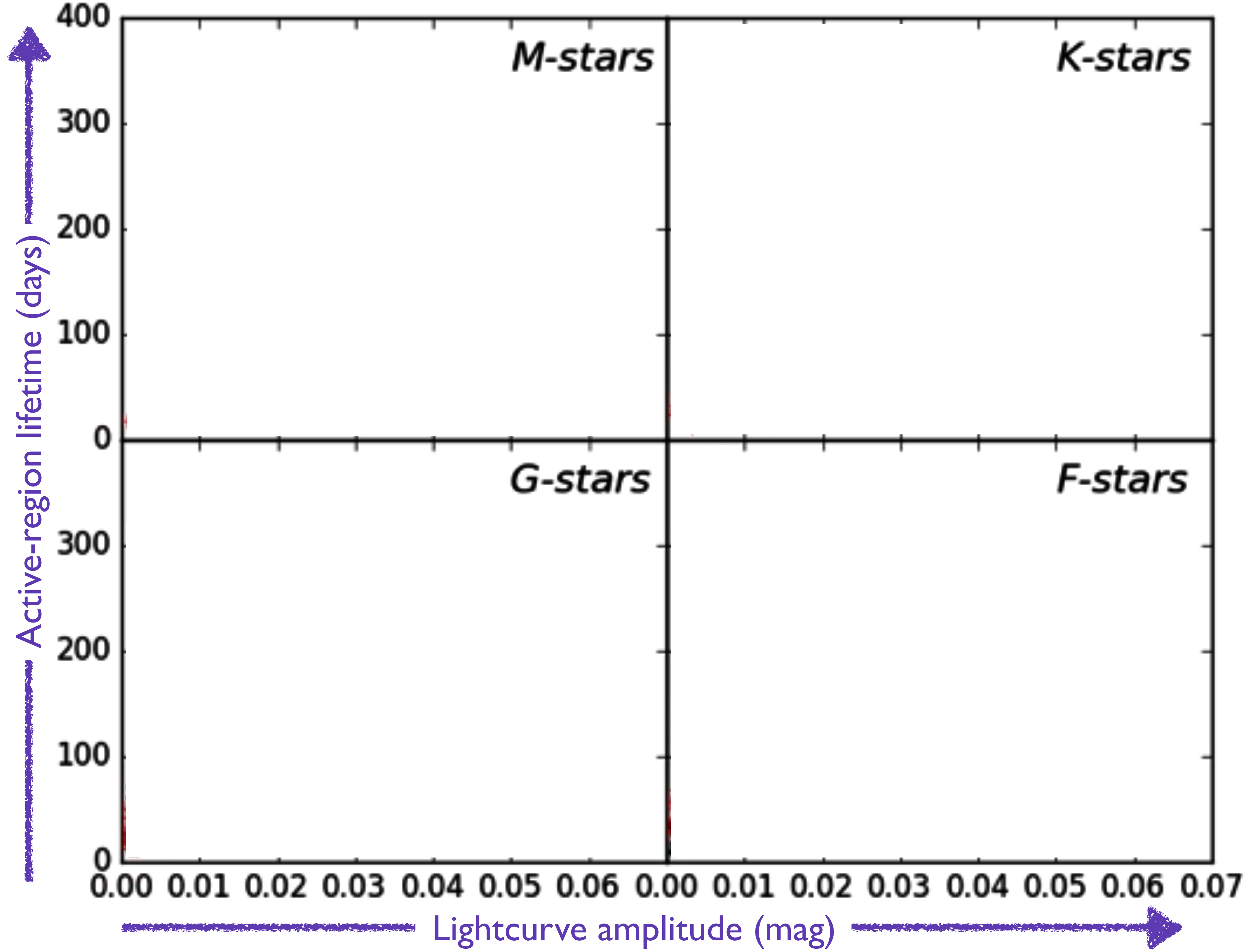
Giles, Collier Cameron & Haywood (accepted)



Kepler FGK stars sample from McQuillan et al., 2014

Active-region lifetimes τ_{ev}

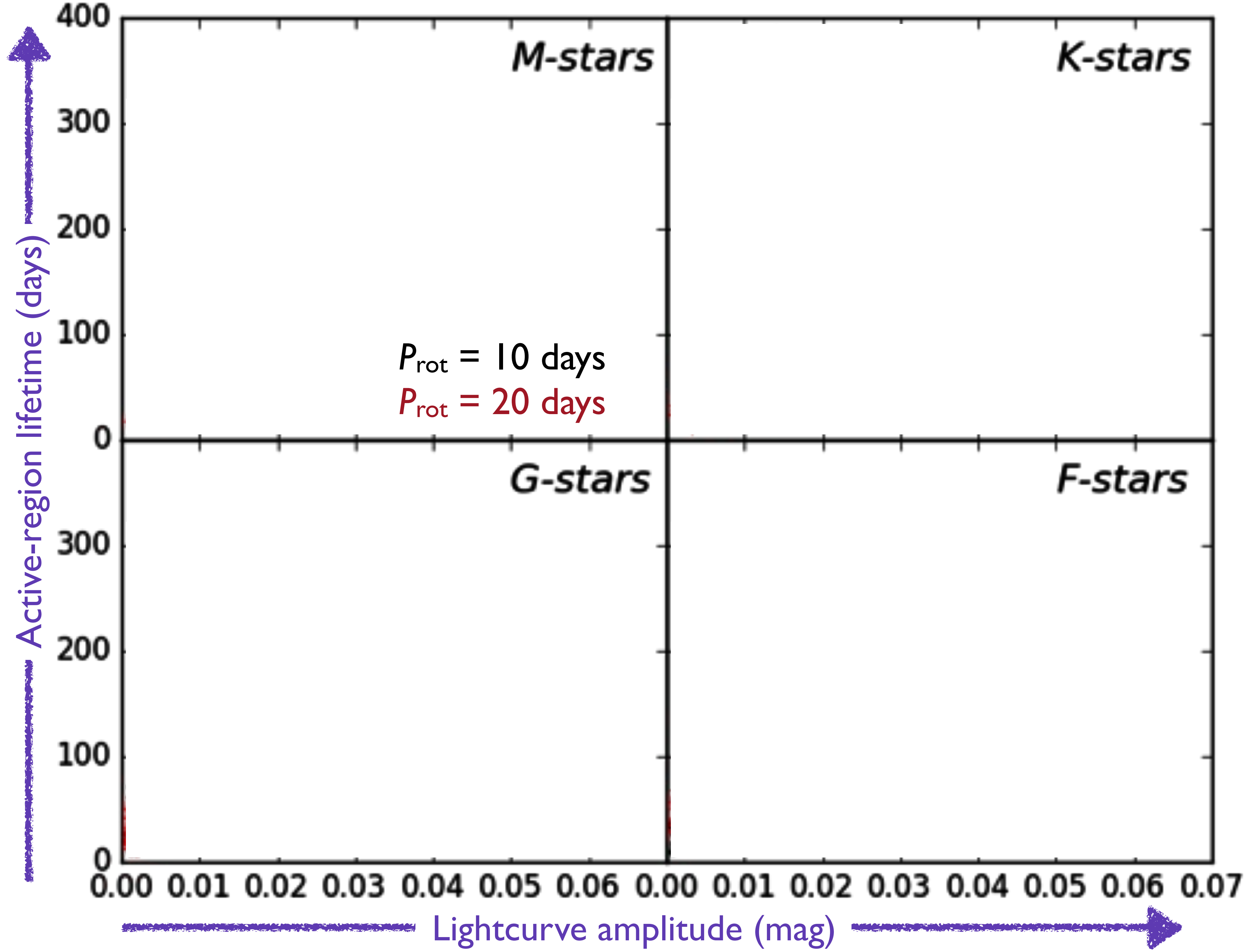
Giles, Collier Cameron & Haywood (accepted)



Kepler FGK stars sample from McQuillan et al., 2014

Active-region lifetimes τ_{ev}

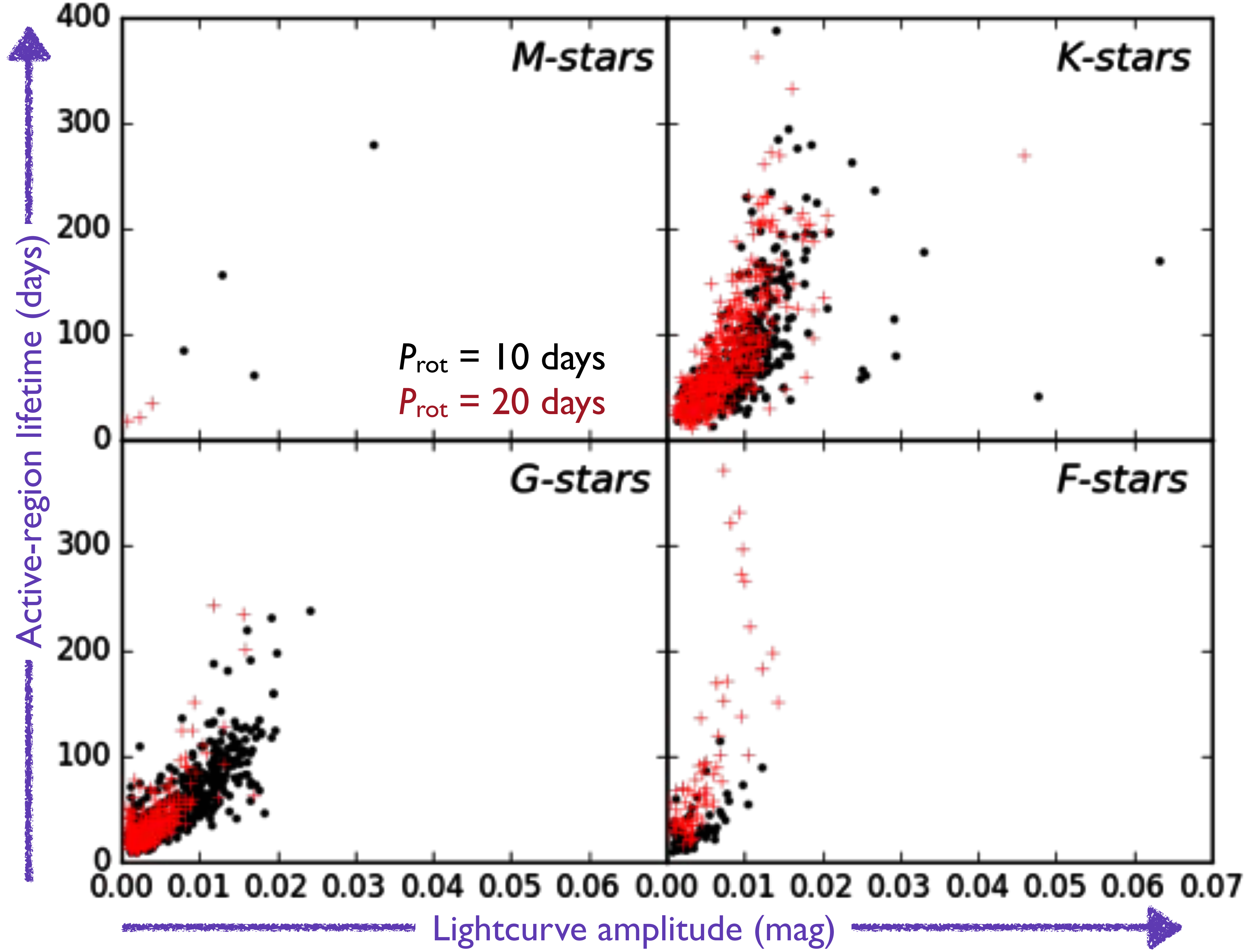
Giles, Collier Cameron & Haywood (accepted)



Kepler FGK stars sample from McQuillan et al., 2014

Active-region lifetimes τ_{ev}

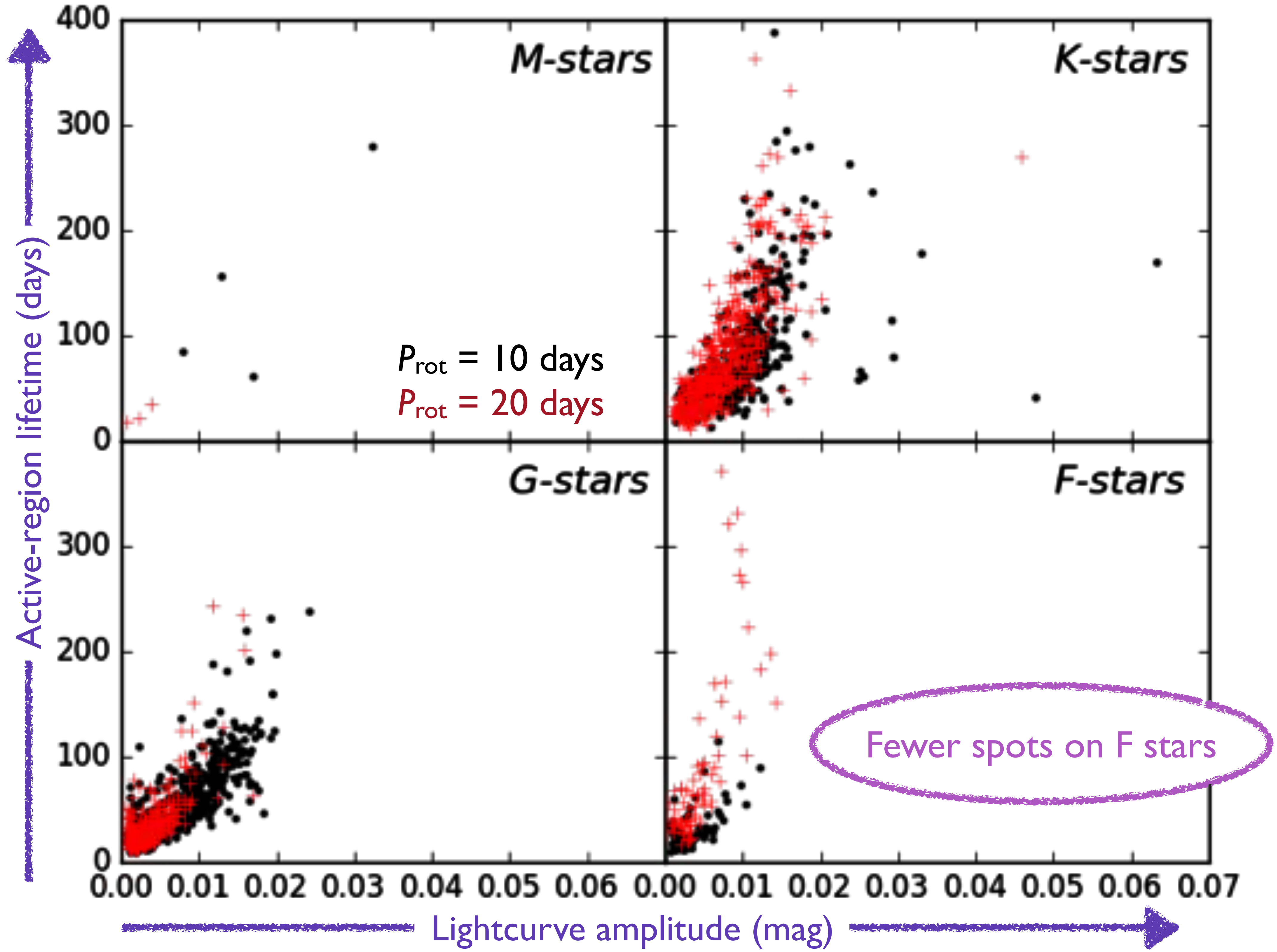
Giles, Collier Cameron & Haywood (accepted)



Kepler FGK stars sample from McQuillan et al., 2014

Active-region lifetimes τ_{ev}

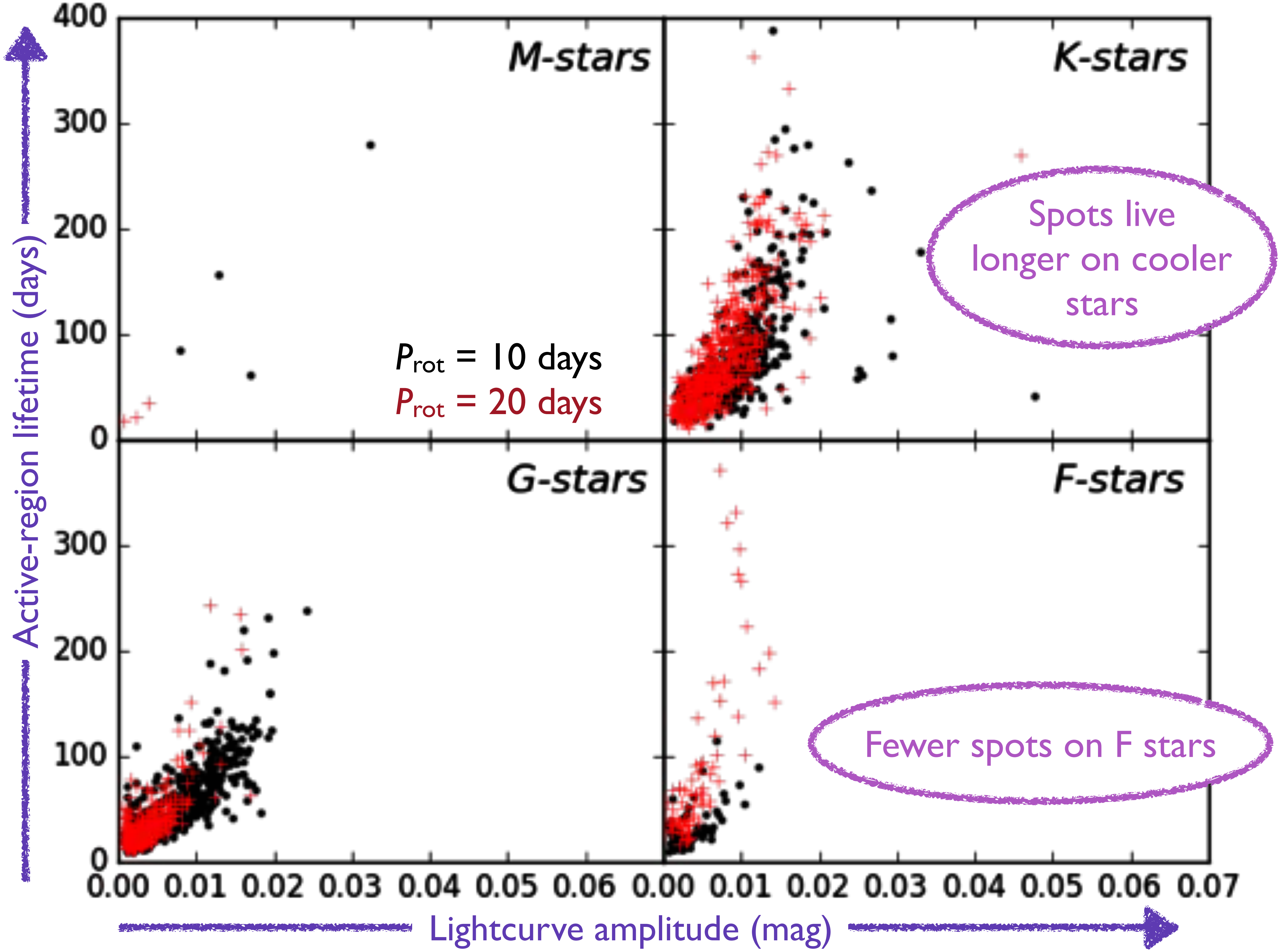
Giles, Collier Cameron & Haywood (accepted)



Kepler FGK stars sample from McQuillan et al., 2014

Active-region lifetimes τ_{ev}

Giles, Collier Cameron & Haywood (accepted)

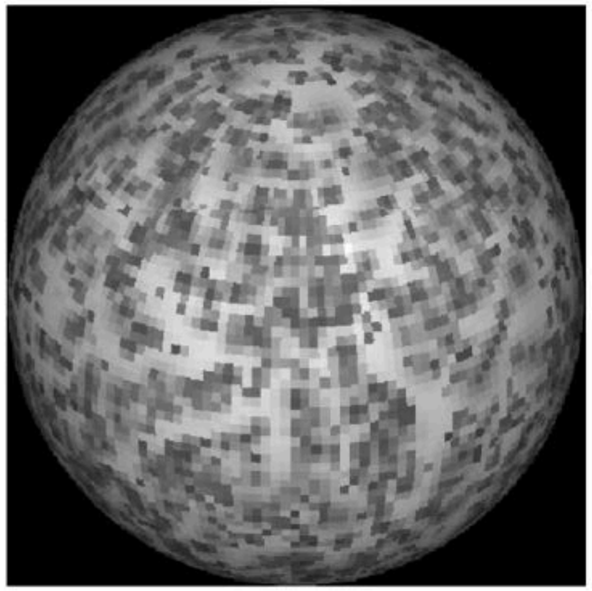


Kepler FGK stars sample from McQuillan et al., 2014

How can we constrain η_4 ?

Jeffers & Keller (2009)

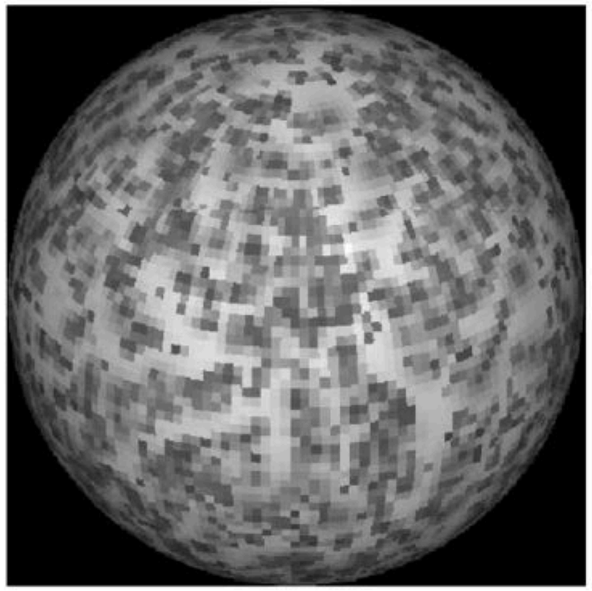
Synthetic stellar surface



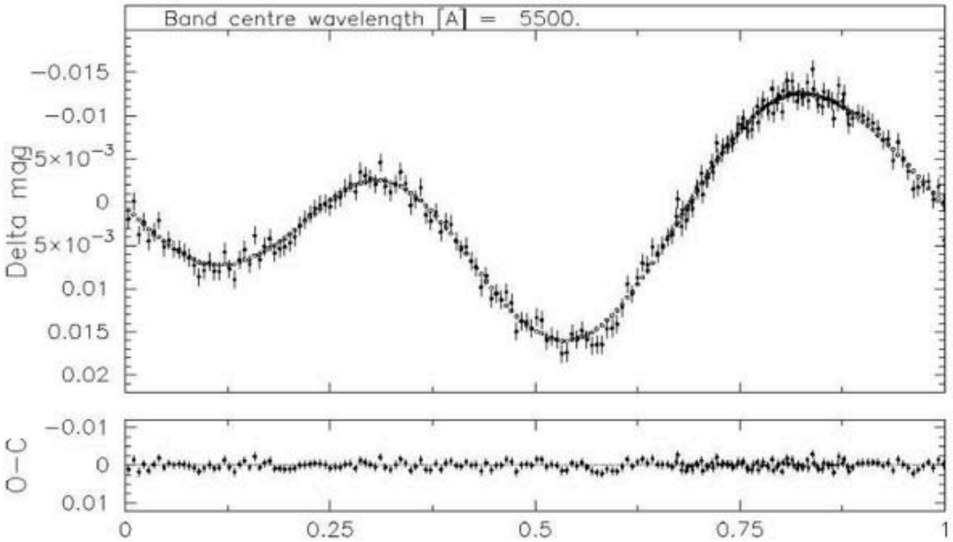
How can we constrain η_4 ?

Jeffers & Keller (2009)

Synthetic stellar surface



Reconstructed lightcurve

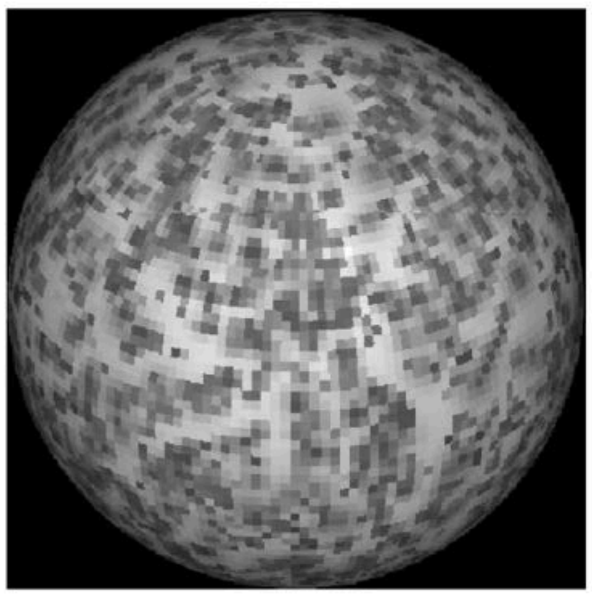


One rotation period

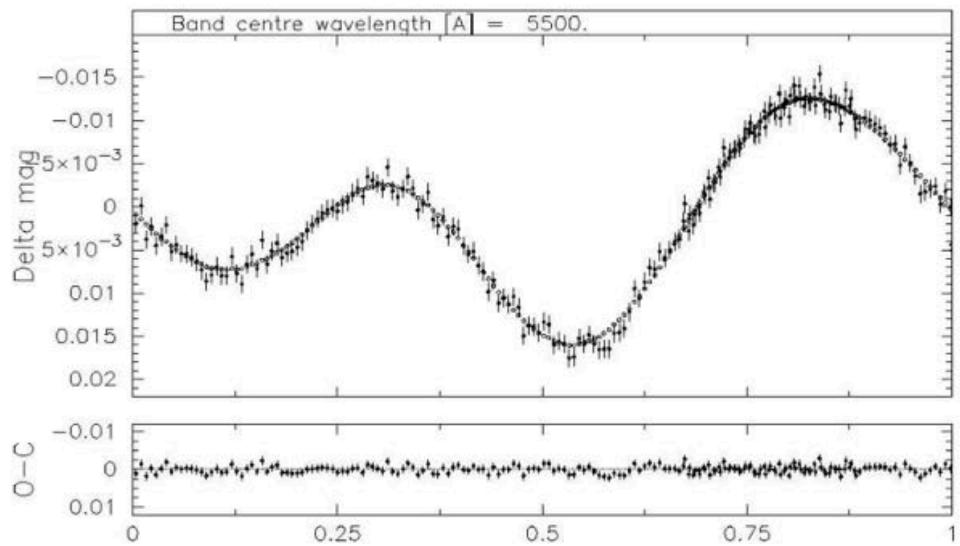
How can we constrain η_4 ?

Jeffers & Keller (2009)

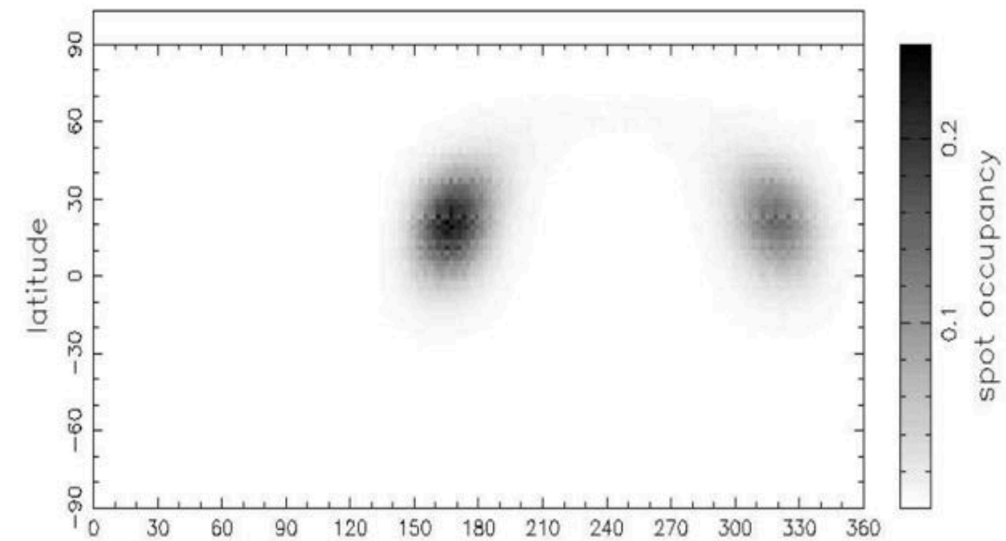
Synthetic stellar surface



Reconstructed lightcurve



Reconstructed surface map



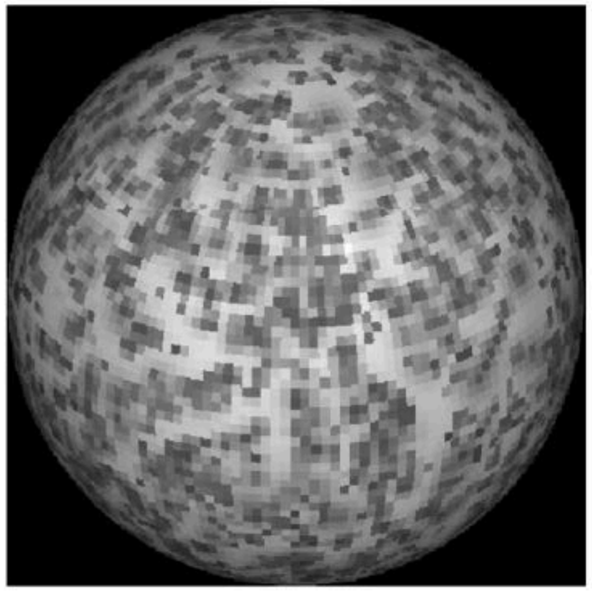
One rotation period

Longitude

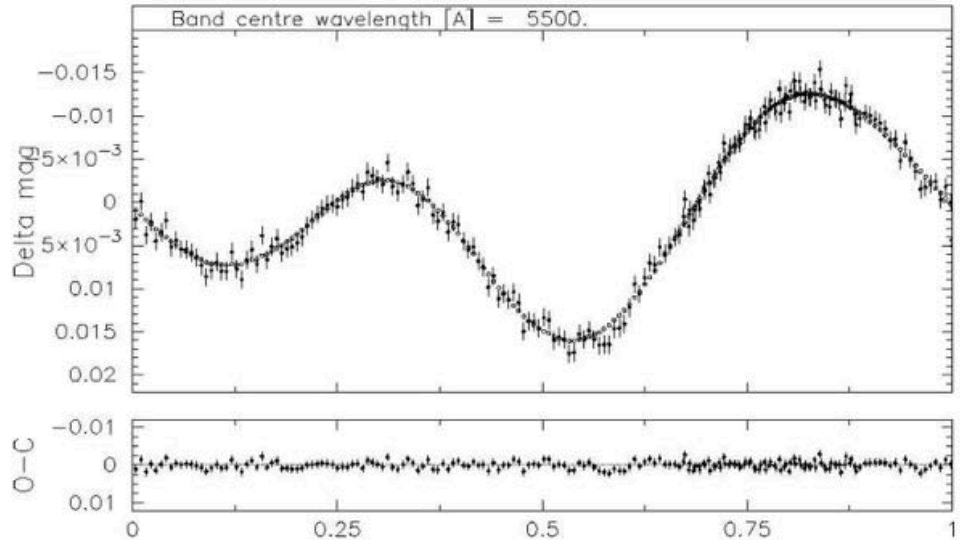
How can we constrain η_4 ?

Jeffers & Keller (2009)

Synthetic stellar surface

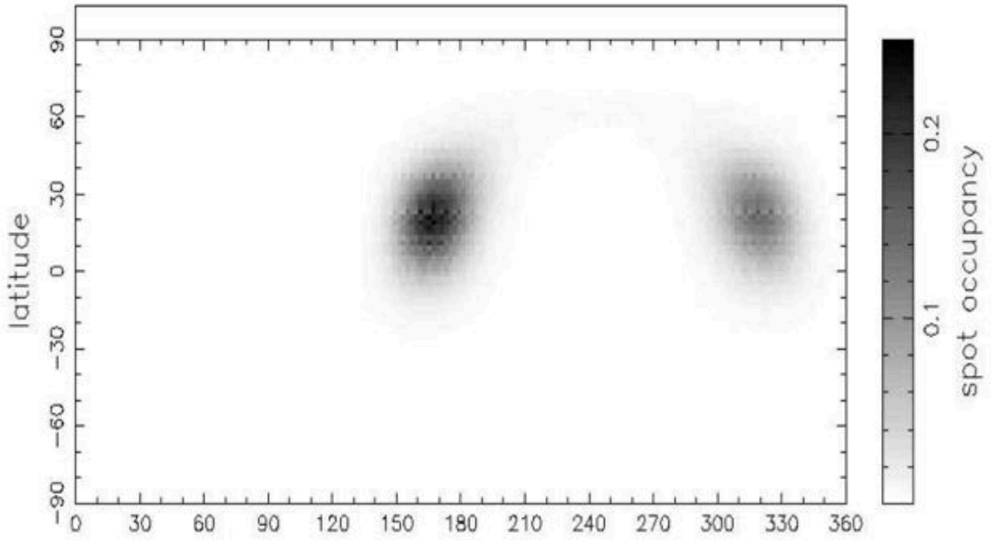


Reconstructed lightcurve



One rotation period

Reconstructed surface map



Longitude

A lightcurve, or an RV curve, will only ever show 2-3 peaks per stellar rotation.
This is equivalent to $\eta_4 \approx 0.5$.

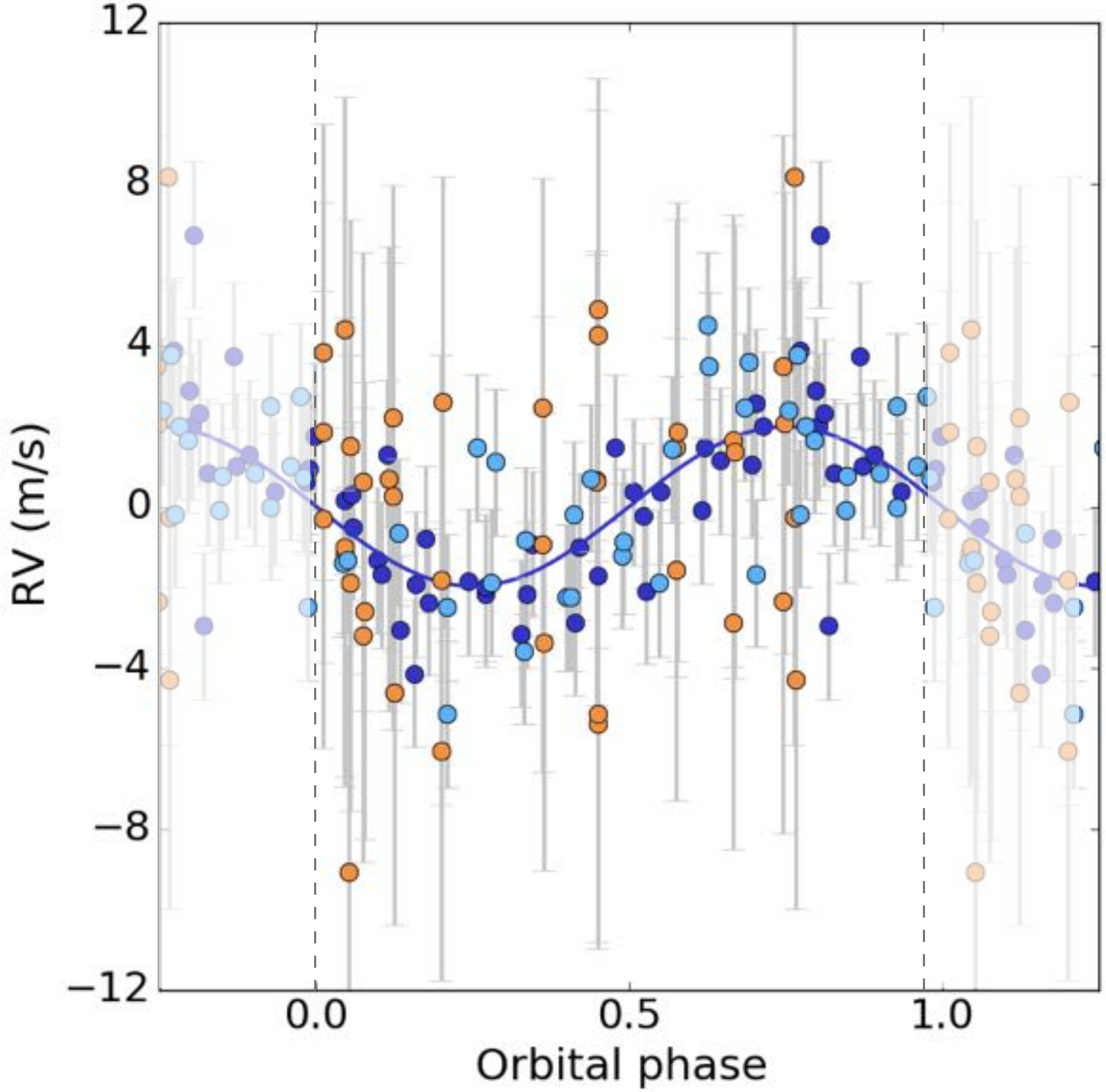
$$k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\,\eta_2^2} - \frac{2\,\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2} \right]$$

P_{orb}: 2.7 days

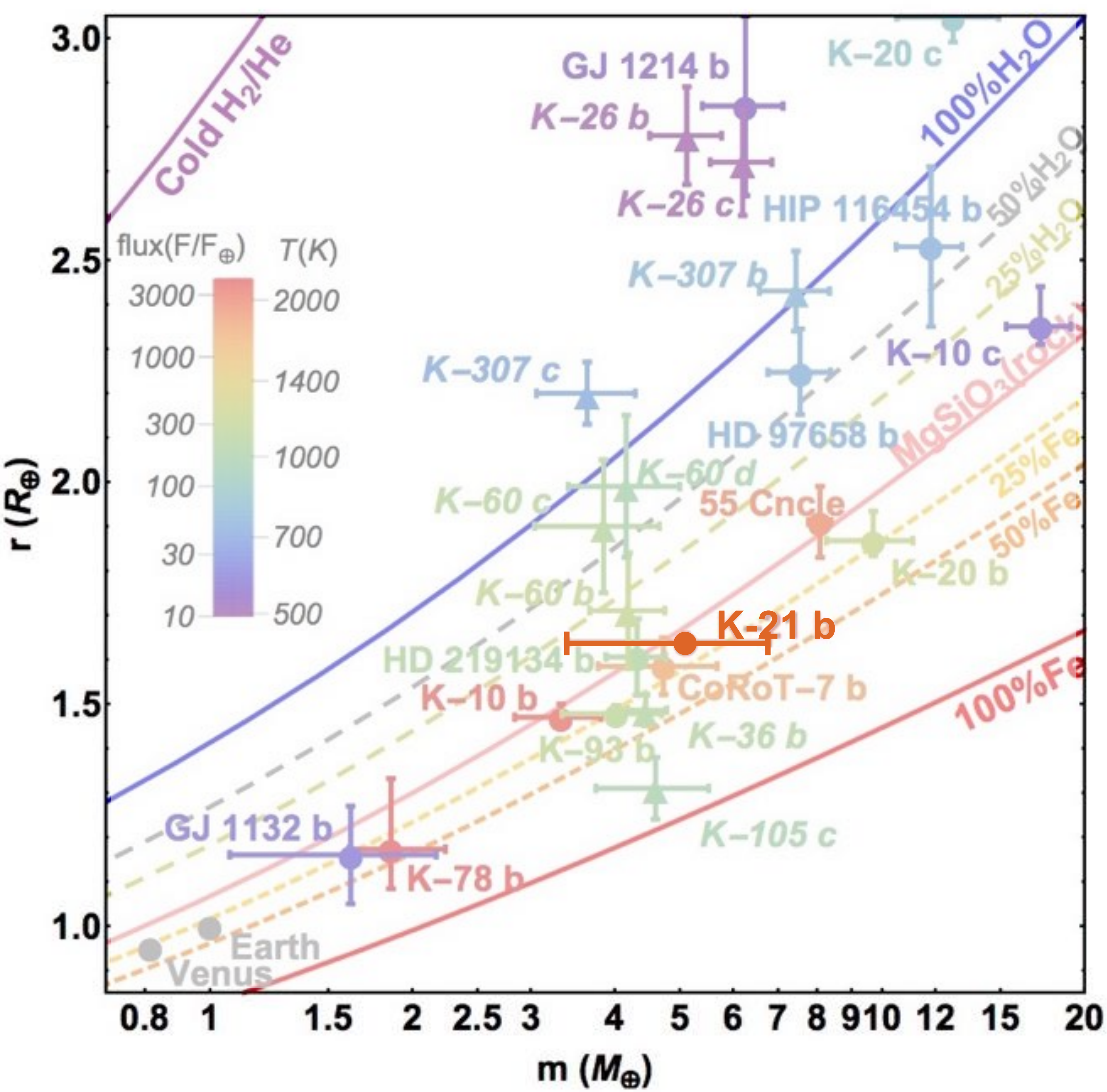
1.6 R_{earth}

López-Morales, Haywood, Giles et al. (2016)

Mass of Kepler-21b: 5.1 ± 1.7 M_{earth}



Kepler-21b in the mass-radius diagram



Adapted from López-Morales et al. (2016)

Words of caution regarding the use of Gaussian processes

- GPs are just like any other statistics tool: garbage in, garbage out!

Choice of covariance function/hyperparameters is crucial. Must think carefully of physical phenomena/instrumental sources to be accounted for.

- Precision ≠ accuracy

- There is only so much data can tell you

Degeneracies produced by disc-averaged measurements.
Must think about observing strategy. *See Friday*

- Statistics do not tell you about the nature of a signal.

For more on GPs: see online lectures/code by Dan Foreman-Mackey, among many others — just ask me!

rhaywood@cfa.harvard.edu