Perguntas Frequentes:

Vc já defendeu?

Vc têm trabalhado com outras coisas?

Tem alguma coisa sobre seu trabalho de PCA que vc ainda não falou?

Algumas...

Análise por Componentes Principais: Detalhes e Curiosidades nas aplicações cosmológicas

Émille Ishida

Trabalho em colaboração com Ribamar Reis, Bruno Lago e Ioav Waga

IX Workshop Nova Física no Espaço

28 de fevereiro a 05 de março de 2010

Análise por Componentes Principais (PCA)

Dada uma amostra de: *n* objetos;
p observáveis - x_i (i=1,2,3,....,*p*)

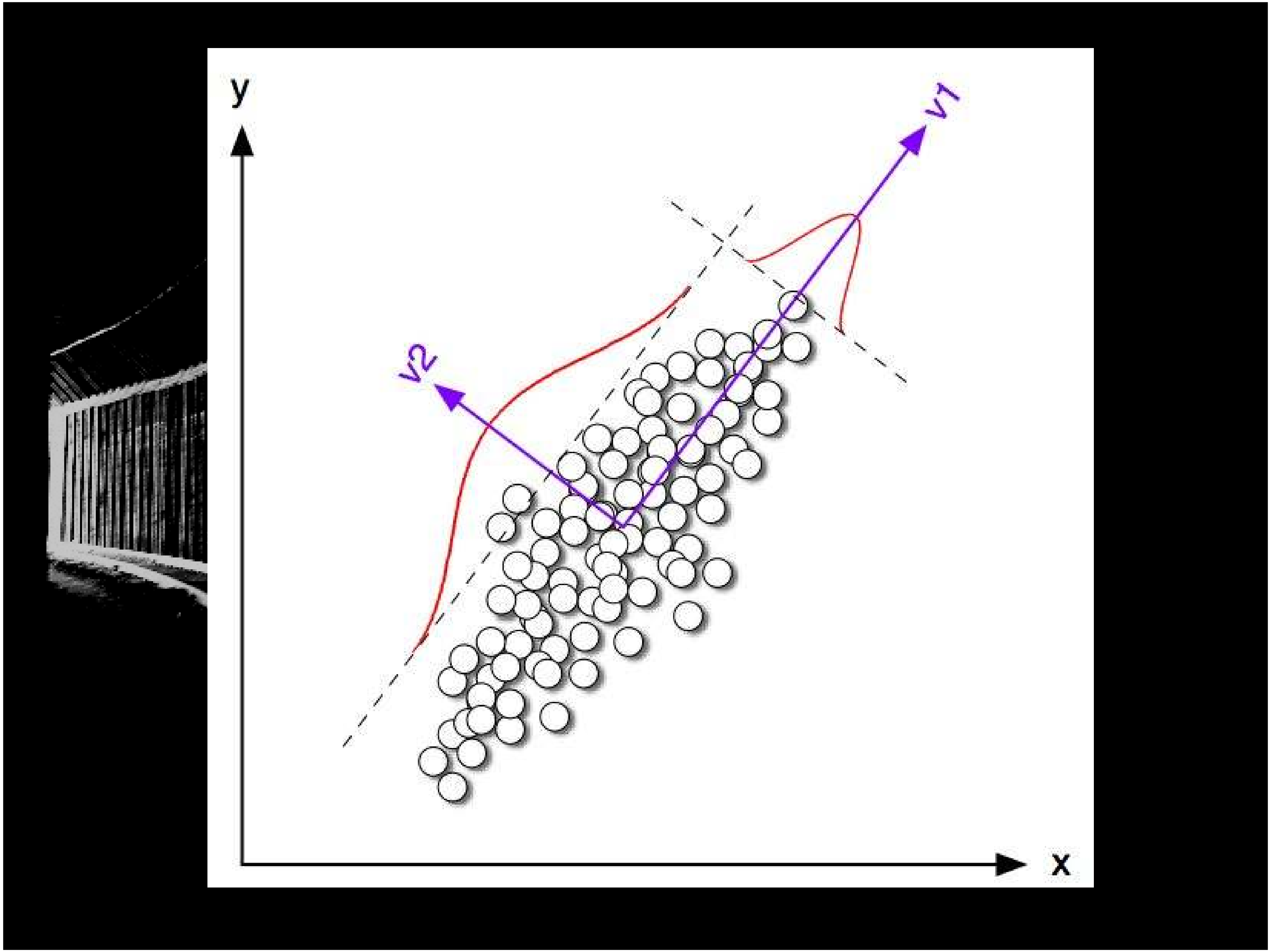
Procuramos um novo conjunto de *p* novas variáveis (ξ_i , ... ξ_p), cada uma formada por uma combinação linear das originais

Componentes Principais

$$\xi_i = \alpha_{i1} x_1 + \ldots + \alpha_{ij} x_j + \ldots. + \alpha_{ip} x_p$$

Determinamos então α_{ij} de modo que o menor número de variáveis contenha a maior parte da variância

–astro-ph/9905079

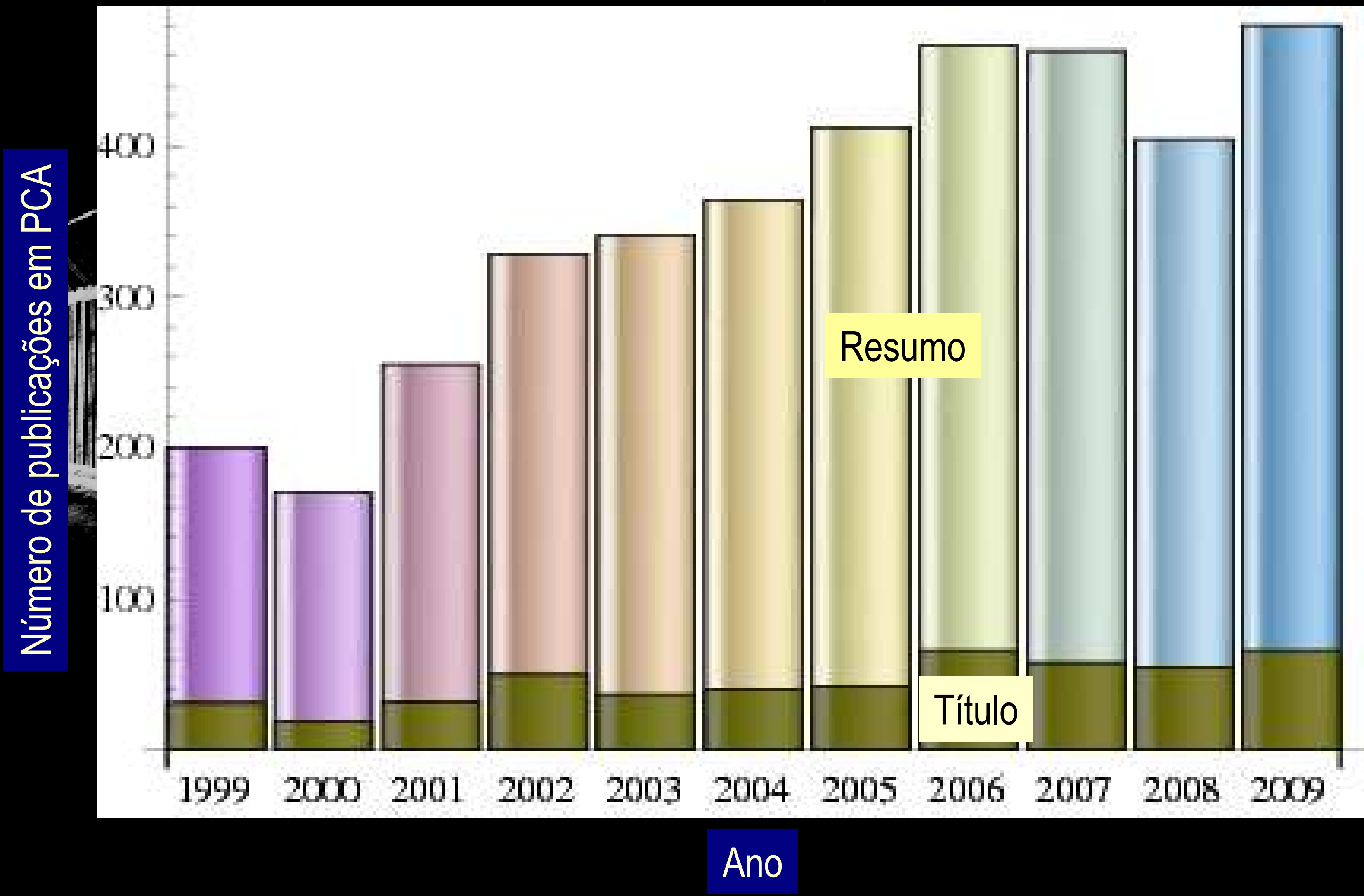


y
v1
v2
x

Solidão pré-tese

No processo de redação... as referências!

PCA no ADS nos últimos 10 anos...



Trabalhos importantes:

THE ASTROPHYSICAL JOURNAL, 480:22–35, 1997 May 1

KARHUNEN-LOÈVE EIGENVALUE PROBLEMS IN COSMOLOGY: HOW SHOULD WE TACKLE LARGE DATA SETS?

MAX TEGMARK[1] ANDY N. TAYLOR ALAN F. HEAVENS

VOLUME 90, NUMBER 3 PHYSICAL REVIEW LETTERS week ending 24 JANUARY 2003

Parametrization of Dark-Energy Properties: A Principal-Component Approach

Dragan Huterer and Glenn Starkman

THE ASTROPHYSICAL JOURNAL, 649:563–569, 2006 October 1

WHAT DO WE REALLY KNOW ABOUT COSMIC ACCELERATION?

CHARLES SHAPIRO[1,2] AND MICHAEL S. TURNER[1,2,3]

Findings of the
Joint Dark Energy Mission
Figure of Merit Science Working Group

Andreas Albrecht, Luca Amendola, Gary Bernstein, Douglas Clowe, Daniel Eisenstein,
Luigi Guzzo, Christopher Hirata, Dragan Huterer, Robert Kirshner, Edward Kolb, Robert Nichol

arXiv:0901.0721v1

Being PC: Principal Components and Dark Energy

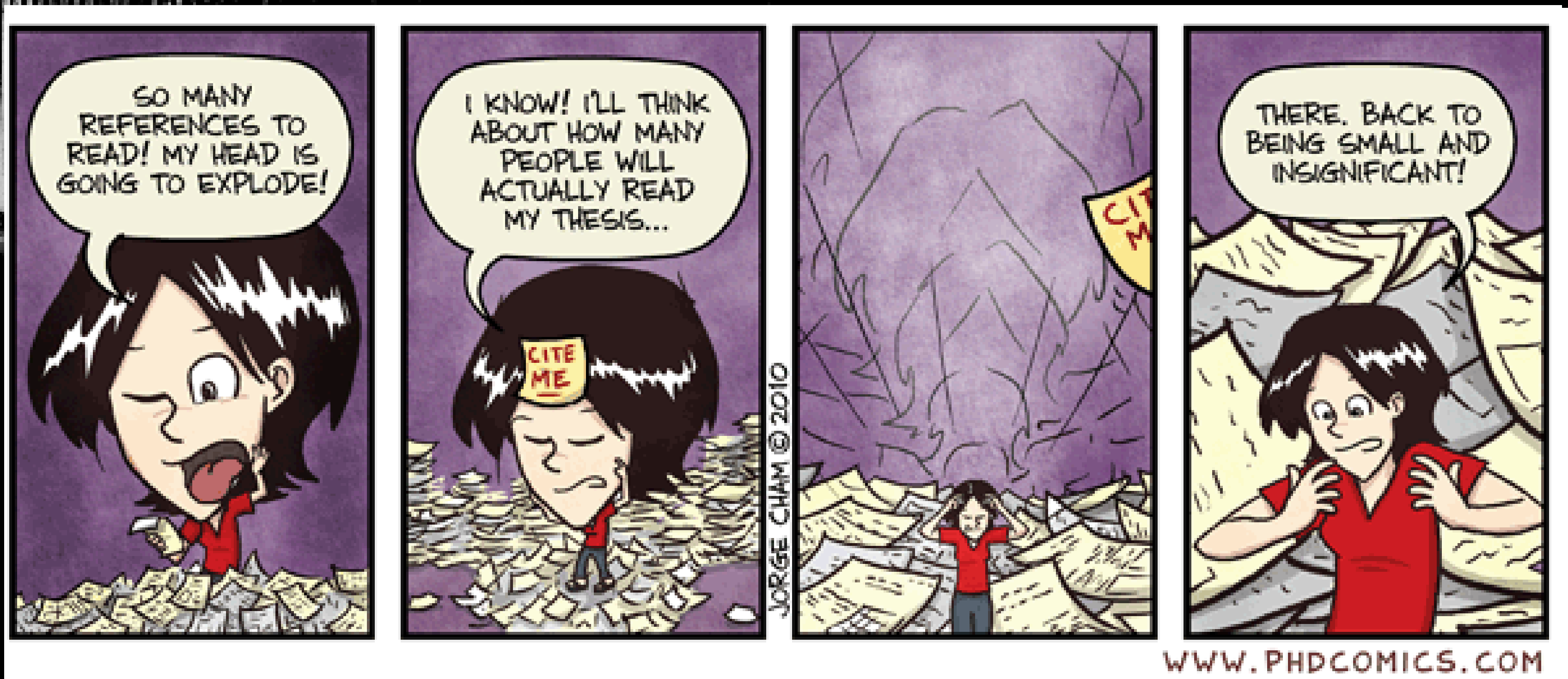
Roland de Putter and Eric V. Linder

arXiv:0812.1794v1

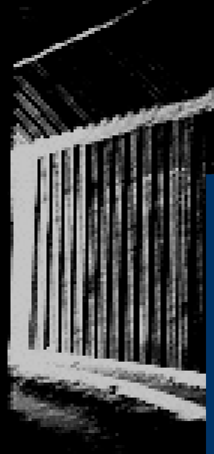
Fisher Matrix Decomposition for Dark Energy Prediction

MNRAS, Volume 398, Issue 4, pp. 2134-2142

T. D. Kitching[1*], A. Amara[2]



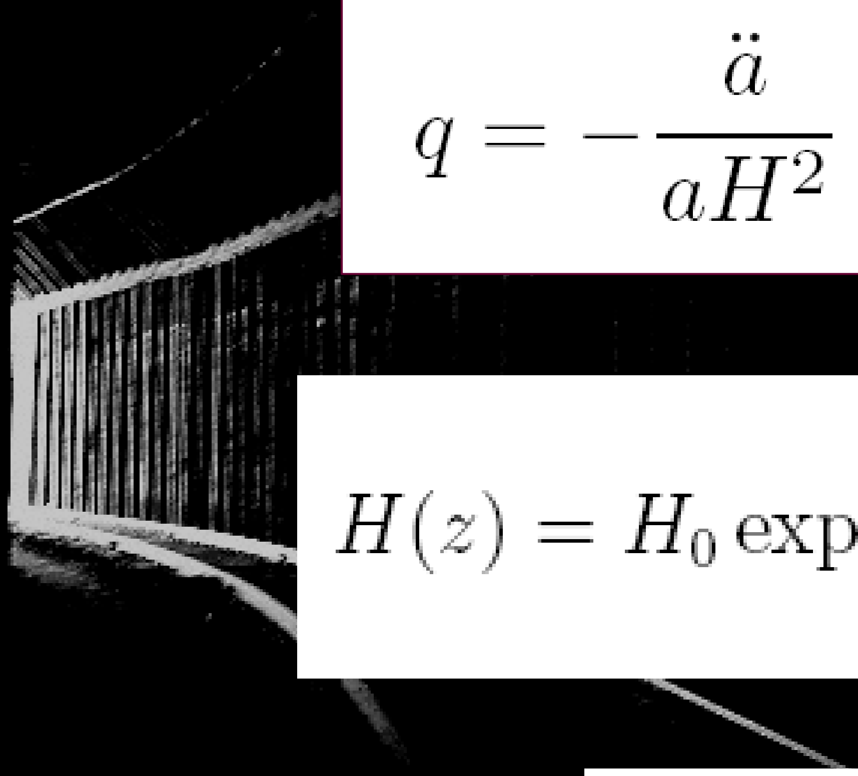
Diferentes maneiras de determinar a matriz de Covariância...



Matriz de Fisher

$$F_{ij} = \left(C^{-1}\right)_{ij} = \left\langle -\frac{\partial^2 \ln L}{\partial \beta_i \partial \beta_j} \right\rangle$$

Foco no Parâmetro de Desaceleração



$$q = -\frac{\ddot{a}}{aH^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1.$$

$$H(z) = H_0 \exp\left[\int_0^z \frac{(q(v)+1)}{(1+v)} dv\right]$$

$$D_C \quad = \quad \int_{t_0}^{t} \frac{dz}{H(z)}$$

Coisas que eu já falei...

Shapiro & Turner (2006)...

$$q(z)=\sum_{i=1}^{N}\beta_i\, c_i(z) \qquad \text{onde} \qquad \begin{cases} c_i(z)=1, & z\in\Delta z_i \\ c_i(z)=0, & z\notin\Delta z_i \end{cases}$$

- ✓ $\Delta z = 0.10$;
- ✓ Dados do *SDSS* (Kessler *et al. 2009*)

Probabilidade gaussiana para cada evento...

$$f\left(\mu_i;\sigma_i,\mu(\bar{\theta})\right)=\frac{1}{\sigma_i\sqrt{2\pi}}\exp\left[-\frac{\left(\mu_i-\mu(\bar{\beta})\right)^2}{2\sigma_i^{\,2}}\right]$$

Distância Módulo

$$\mu(z)=25+5\log\left[\frac{1+z}{H_0 Mpc}\int_0^z du\,\exp\left(-\int_0^u [1+q(u)]\,d\ln(1+v)\right)\right]$$

Coisas que eu já falei...

Forma analítica para a Matriz de Fisher

(está no apêndice... mas é bastante importante!)

$$F_{kl} = \frac{25}{\ln(10)^2}\left\{\frac{1}{C}\sum_{i=1}^{N}\left[\frac{1}{D_L(z_i;\bar\beta)}\frac{\partial D_L(z_i;\bar\beta)}{\partial\beta_k}\right]\times\right.$$
$$\times\sum_{j=1}^{N}\left[\frac{1}{\sigma_j^2}\frac{1}{D_L(z_j;\bar\beta)}\frac{\partial D_L(z_j;\bar\beta)}{\partial\beta_l}\right]+$$
$$\left.-\sum_{i=1}^{N}\frac{1}{\sigma_i^2}\frac{1}{D_L(z_i;\bar\beta)^2}\frac{\partial D_L(z_i;\bar\beta)}{\partial\beta_k}\frac{\partial D_L(z_i;\bar\beta)}{\partial\beta_l}\right\},$$

onde

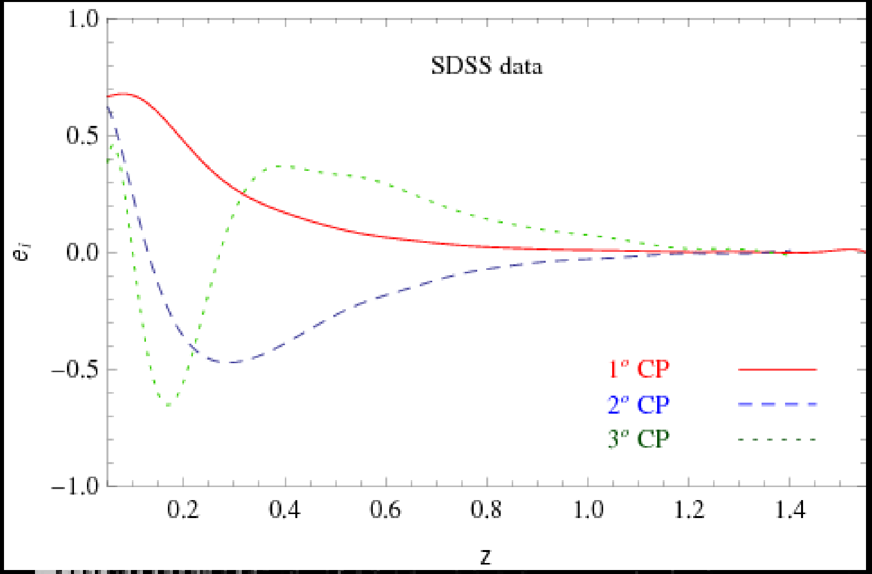
$$\frac{D_L(z)}{H_0} = (1+z)\int_0^z\frac{du}{H(u)}$$

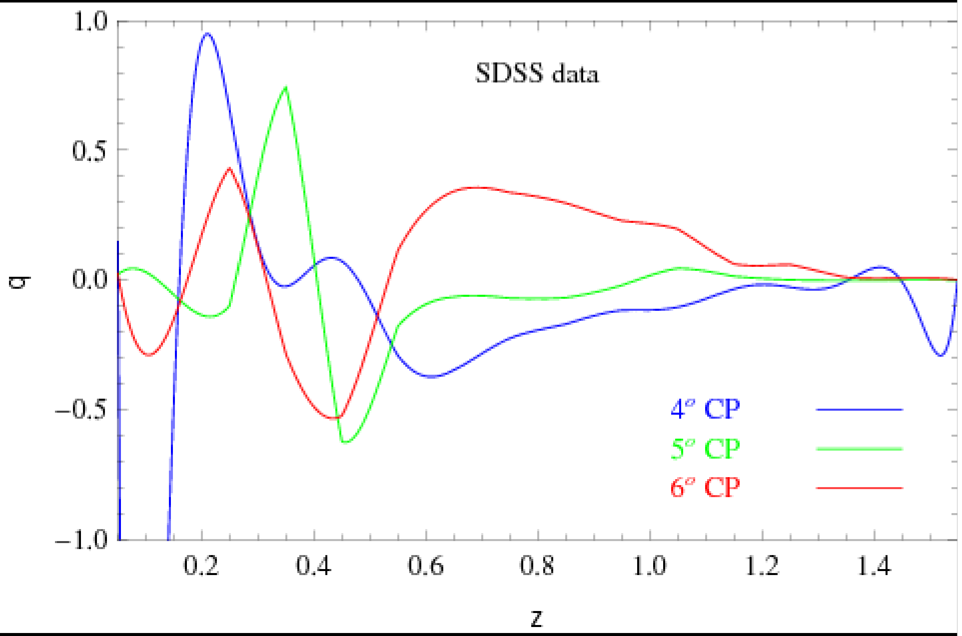
$$\frac{\partial D_L(z,\bar\beta)}{\partial\beta_l} = (1+z)\left[\frac{\partial f_1(z;\bar\beta)}{\partial\beta_l}+\frac{\partial f_2(z;\bar\beta)}{\partial\beta_l}\right]$$

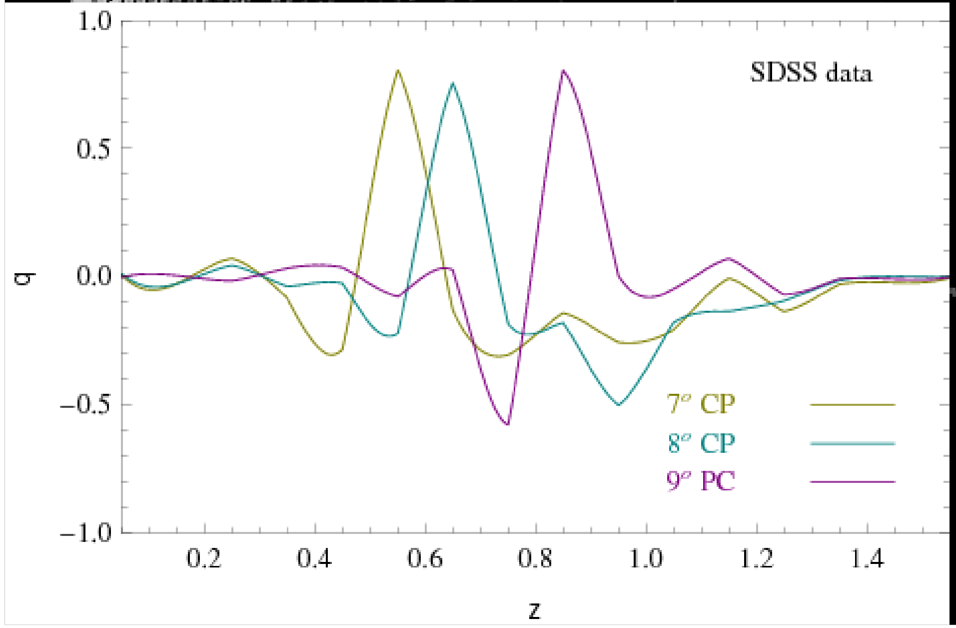
$$\frac{\partial f_1}{\partial\beta_l} = \sum_{k=1}^{J}\left\{\left[\prod_{i=1}^{k-1}\left(\frac{1+z_i}{1+z_{i-1}}\right)^{-\beta_i}\right]\times\right.$$
$$\times\left\{\left(\frac{\delta_{kl}}{\beta_k}\right)\left[\left(\frac{1+z_k}{1+z_{k-1}}\right)^{-\beta_k}\ln\left(\frac{1+z_k}{1+z_{k-1}}\right)+\right.\right.$$
$$\left.-\frac{1}{\beta_k}\left(1-\left(\frac{1+z_k}{1+z_{k-1}}\right)^{-\beta_k}\right)\right]+$$
$$-\Theta(k-1-l)\left[\ln\left(\frac{1+z_l}{1+z_{l-1}}\right)\left(\frac{1}{\beta_k}\right)\times\right.$$
$$\left.\left.\left.\times\left[1-\left(\frac{1+z_k}{1+z_{k-1}}\right)^{-\beta_k}\right]\right]\right\}\right\},$$

$$\frac{\partial f_2}{\partial\beta_l} = \prod_{i=1}^{J}\left(\frac{1+z_i}{1+z_{i-1}}\right)^{-\beta_i}\times$$
$$\times\left\{\frac{\delta_{(J)l}}{\beta_{J+1}}\left[\left(\frac{1+z}{1+z_J}\right)^{-\beta_{J+1}}\ln\left(\frac{1+z}{1+z_J}\right)+\right.\right.$$
$$\left.-\frac{1}{\beta_{J+1}}\left[1-\left(\frac{1+z}{1+z_J}\right)^{-\beta_{J+1}}\right]\right]+$$
$$\left.-\left[\frac{\Theta(J-l)}{\beta_{J+1}}\ln\left(\frac{1+z_l}{1+z_{l-1}}\right)\left[1-\left(\frac{1+z}{1+z_J}\right)^{-\beta_{J+1}}\right]\right]\right\}.$$

Componentes encontrados com dados de SNIa do SDSS



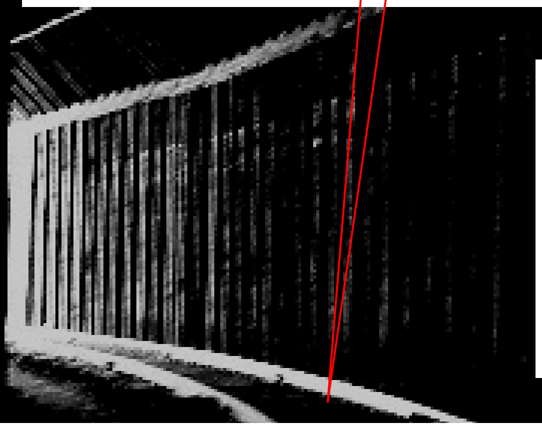




Reconstrução de q(z)

$$q(z;\bar{\alpha}) = \sum_{i=1}^{M'} \alpha_i e_i(z)$$

$$\sigma^2_{rec}(z) = \sum_{j=1}^{M'} \left(\sigma_{\alpha_j} e_j(z)\right)^2 + (\alpha_j \sigma_j)^2 .$$



$$\chi^2 = \sum_{i=1}^{N} \frac{(\mu_i - \mu_{th}(z_i;\bar{\alpha}))^2}{\sigma_i^2}$$

Critérios de Seleção:

Quantos componentes usar na reconstrução?

Porcentagem cumulativa da variância total

Porcentagem cumulativa da variância total ($t_{M'}$)

Os autovalores representam a porcentagem da variância total que está incluída na construção do autovetor correspondente...

$$t_{M'} = 100 \frac{\sum_{i=1}^{M'} \lambda_i}{\sum_{j=1}^{M} \lambda_j}.$$

... desta forma para um determinado conjunto de dados, podemos definir um valor mínimo dessa variância que julgamos necessário para representar as características presente nos dados.

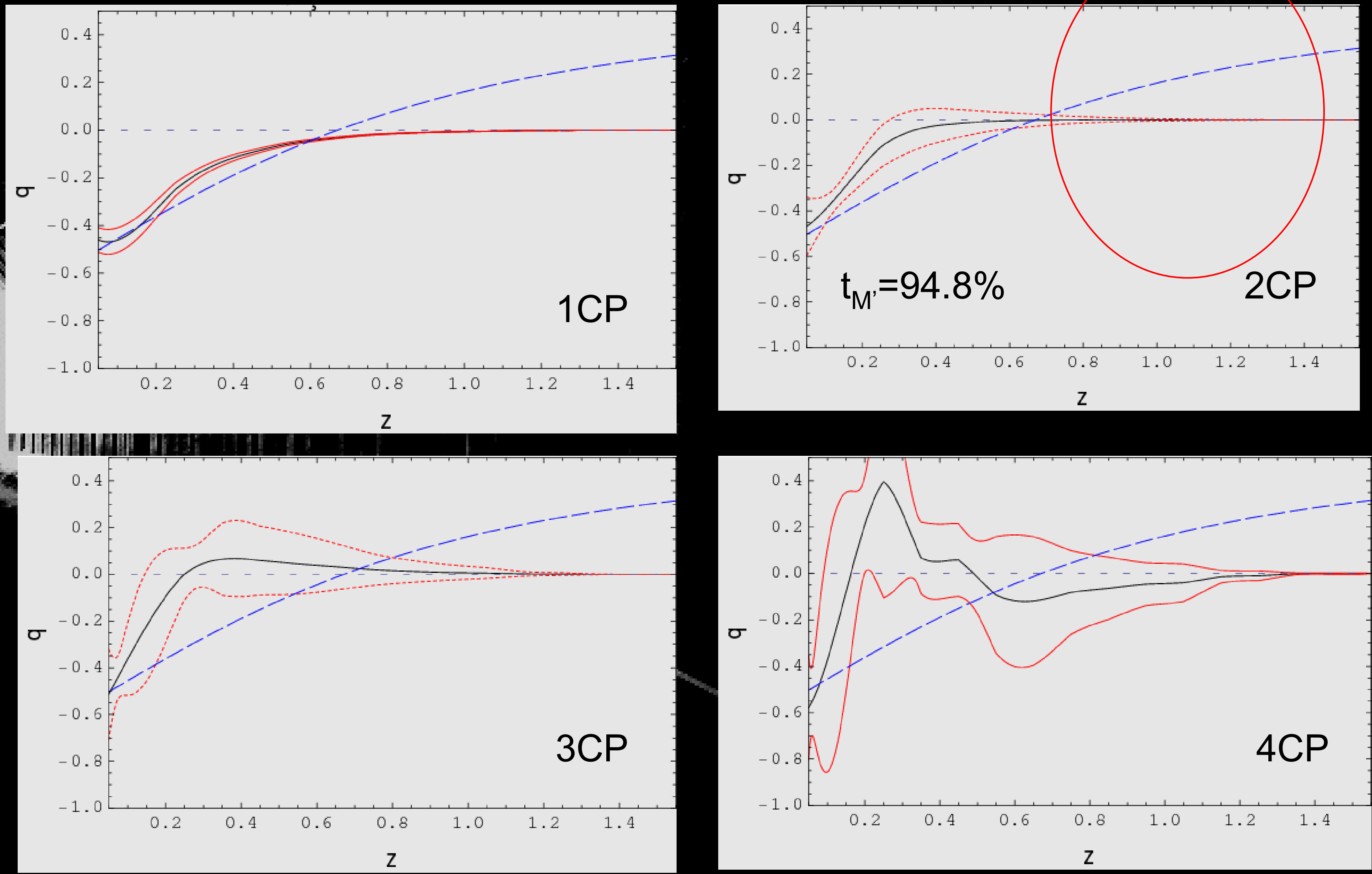
CP	$t_{M'}$
1	81.5
2	94.8
3	98.1
4	99.5
5	99.8

Data set	$t_{M'}$
Current data	$\geq 90,0\%$
Simulation - stage 1	$\geq 95,0\%$
Simulation - stage 2	$\geq 99,0\%$
Simulation - stage 3	$\geq 99,9\%$

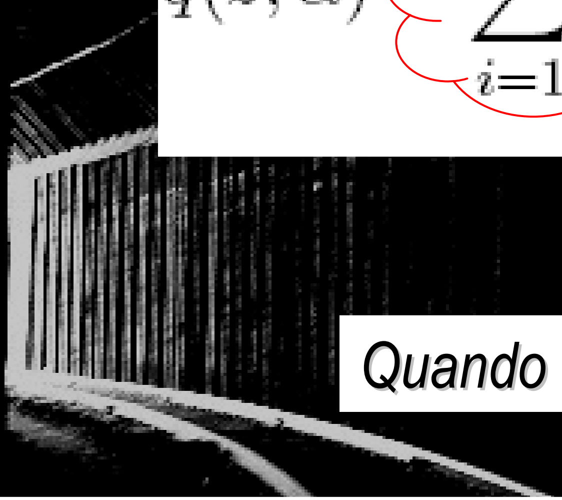
Table 1: Values of minimum $t_{M'}$ required for determining the number of components to use in the deceleration parameter reconstruction for data set of luminosity distances and angular diameter distances measurements.

Reconstrução de q(z)

Por que tudo sempre vai a zero?



O que ainda não falei sobre a Reconstrução de q(z)

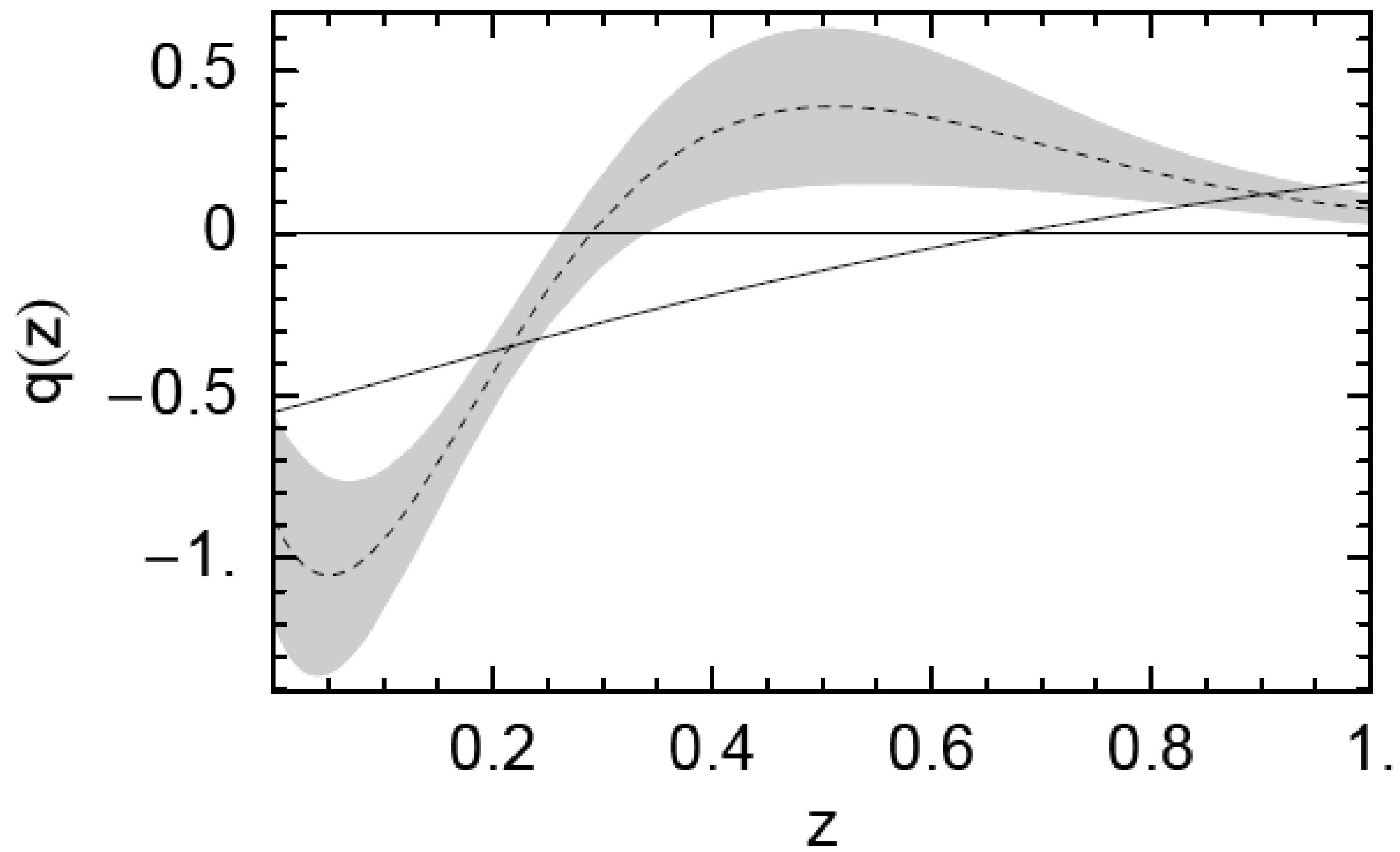


$$q(z;\bar{\alpha}) = \sum_{i=1}^{M'} \alpha_i e_i(z) + q_{fid} = 0$$

... quando não existe informação nos dados

Quando existe informação...

Definimos um limite superior para z, até o qual consideramos a reconstrução livre de características que não estãp presentes nos dados. Este limite é definido de acordo com o valor de z onde a derivada da reconstrução é nula e posteriormente o comportamento tende ao valor imposto para q(z).

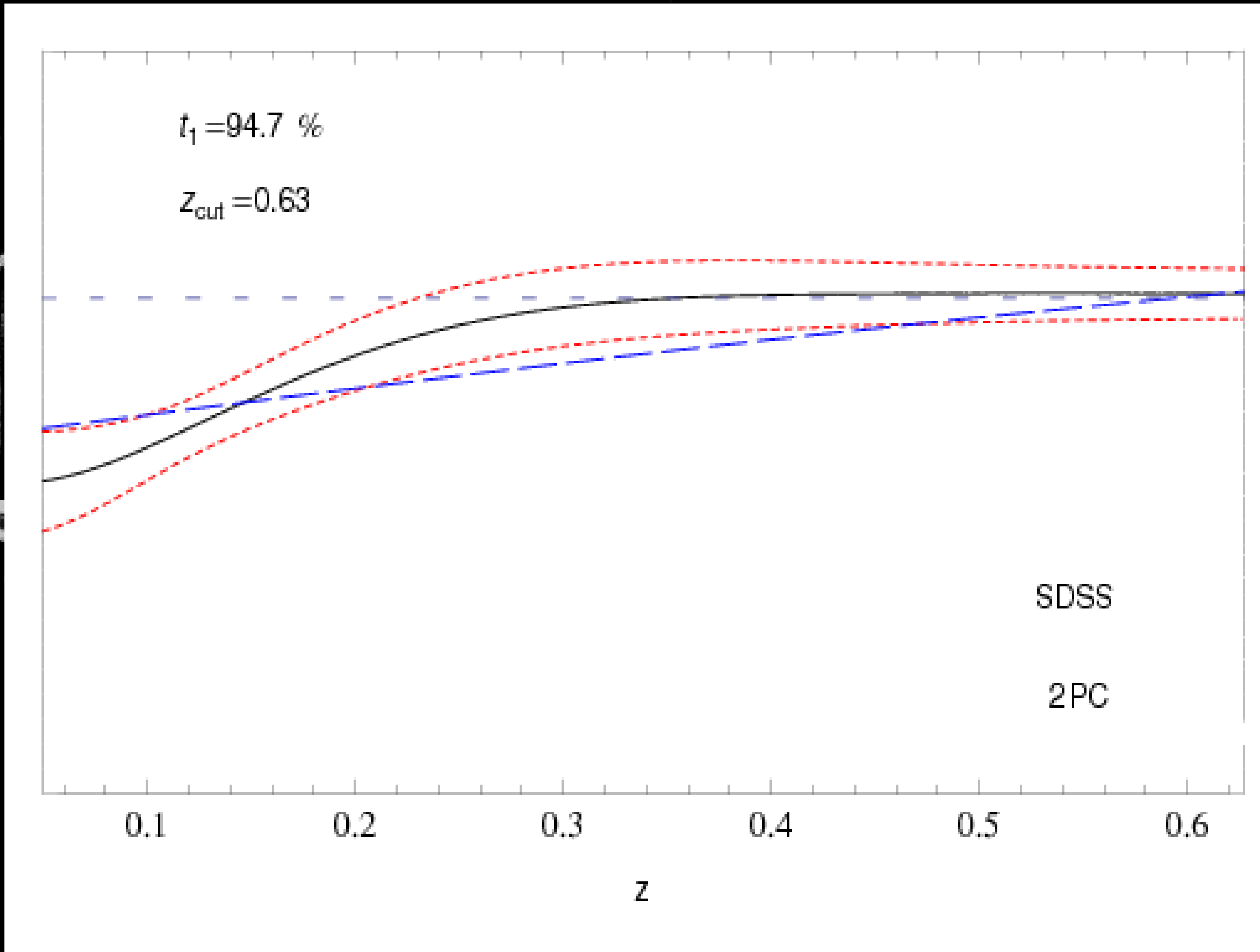


The Astrophysical Journal, 649:563–569, 2006 October 1

WHAT DO WE REALLY KNOW ABOUT COSMIC ACCELERATION?

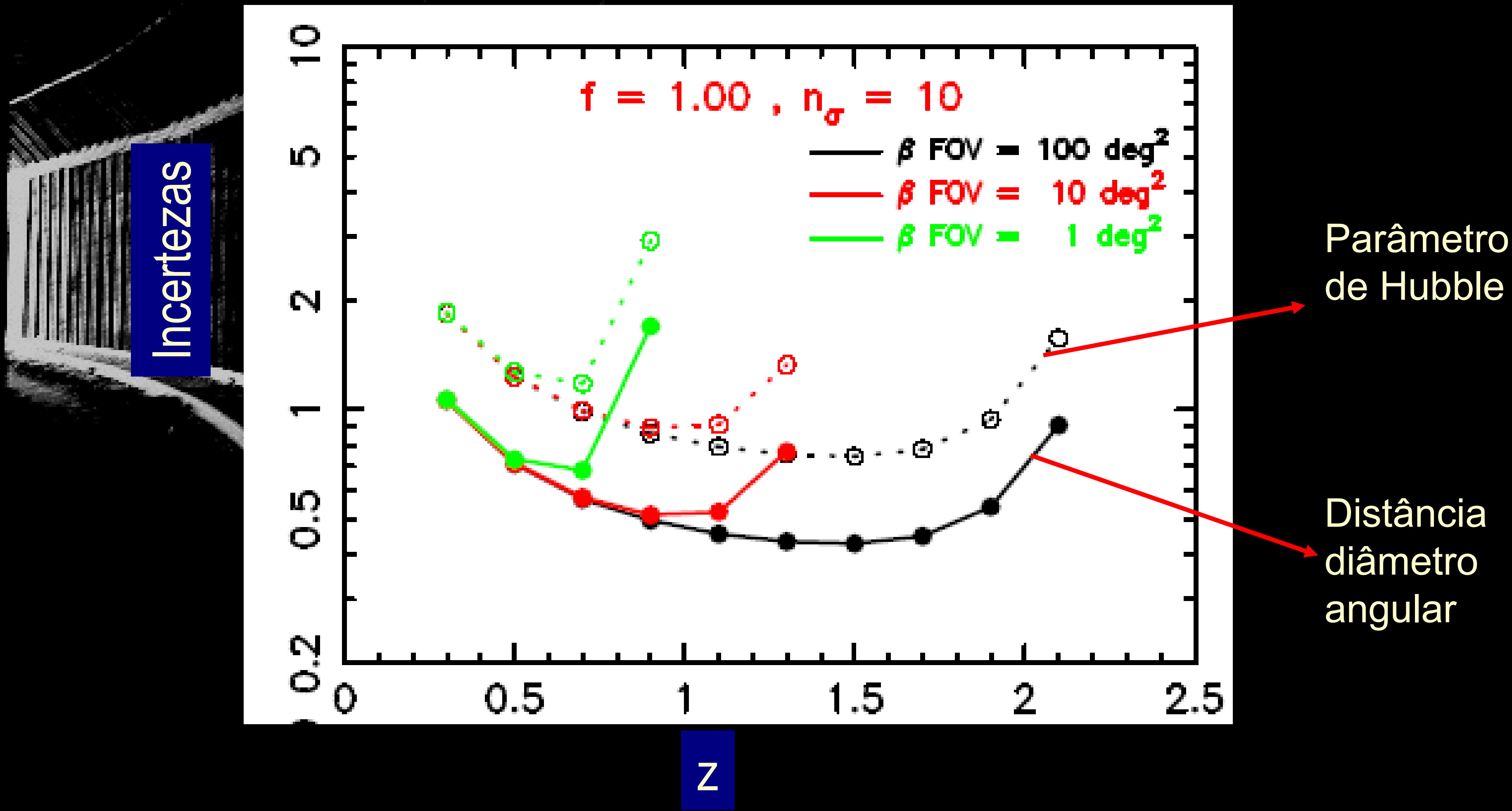
Charles Shapiro[1,2] and Michael S. Turner[1,2,3]

Reconstrução final para dados do SDSS



Podemos fazer a mesma coisa para BAO

De acordo com *Abdalla et al., 2009*, um experimento como o SKA resultará em:



Matriz de Fisher analítica para $D_A(z)$

$$d_A(z) = \frac{1}{(1+z)^2} d_L(z) = \frac{1}{(1+z)} \int_0^z \frac{du}{H(u)}.$$

$$E \equiv \sum_{i=1}^{N} \frac{d_{A_i}^2}{\sigma_{data_i}^2},$$

$$F \equiv \sum_{i=1}^{N} \frac{d_{A_i} D_A(z_i;\bar{\beta})}{\sigma_{data_i}^2},$$

$$G \equiv \sum_{i=1}^{N} \frac{D_A(z_i;\bar{\beta})^2}{\sigma_{data_i}^2}.$$

$$F_{kl}^{D_A} = \left\langle -\frac{\partial^2 \ln L_{D_A}}{\partial\beta_k \partial\beta_l} \right\rangle = -\frac{1}{G^2}\frac{\partial G}{\partial\beta_k}\frac{\partial G}{\beta_l}\left(\frac{F^2}{G} - \frac{1}{2}\right) - \frac{1}{2G}\frac{\partial^2 G}{\partial\beta_k\partial\beta_l}\left(1 - \frac{F^2}{G}\right) +$$

$$-\frac{1}{G}\frac{\partial F}{\partial\beta_k}\frac{\partial F}{\partial\beta_l} + \frac{F}{G^2}\frac{\partial G}{\partial\beta_k}\frac{\partial F}{\partial\beta_l} - \frac{F}{G}\frac{\partial^2 F}{\partial\beta_k\partial\beta_l} + \frac{F}{G^2}\frac{\partial F}{\partial\beta_k}\frac{\partial G}{\partial\beta_l}.$$

Matriz de Fisher analítica para H(z)

$$g(z;\bar{\beta}) = (1+z)\left[\prod_{i=1}^{J}\left(\frac{1+z_i}{1+z_{i-1}}\right)^{\beta_i}\right]\left(\frac{1+z}{1+z_J}\right)^{\beta_{J+1}},$$

$$K \equiv \sum_{i=1}^{N}\frac{h_i g(z_i;\bar{\beta})}{\sigma^2_{data_i}},$$

$$L \equiv \sum_{i=1}^{N}\frac{h_i^2}{\sigma^2_{data_i}},$$

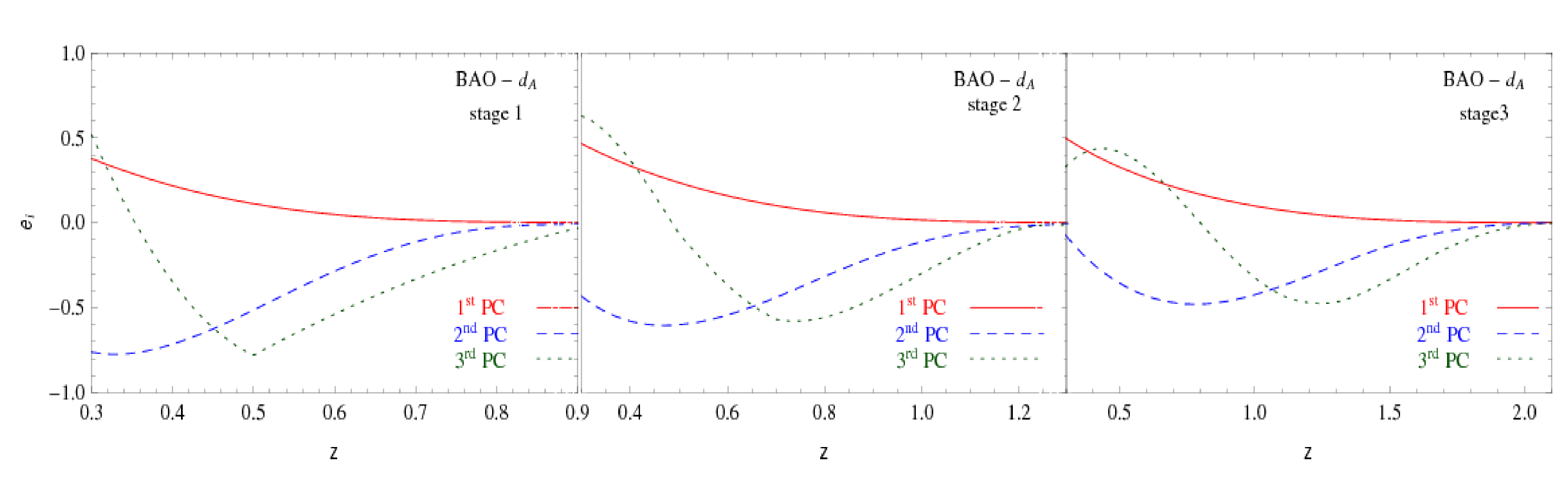
$$M \equiv \sum_{i=1}^{N}\frac{g(z_i;\bar{\beta})^2}{\sigma^2_{data_i}},$$

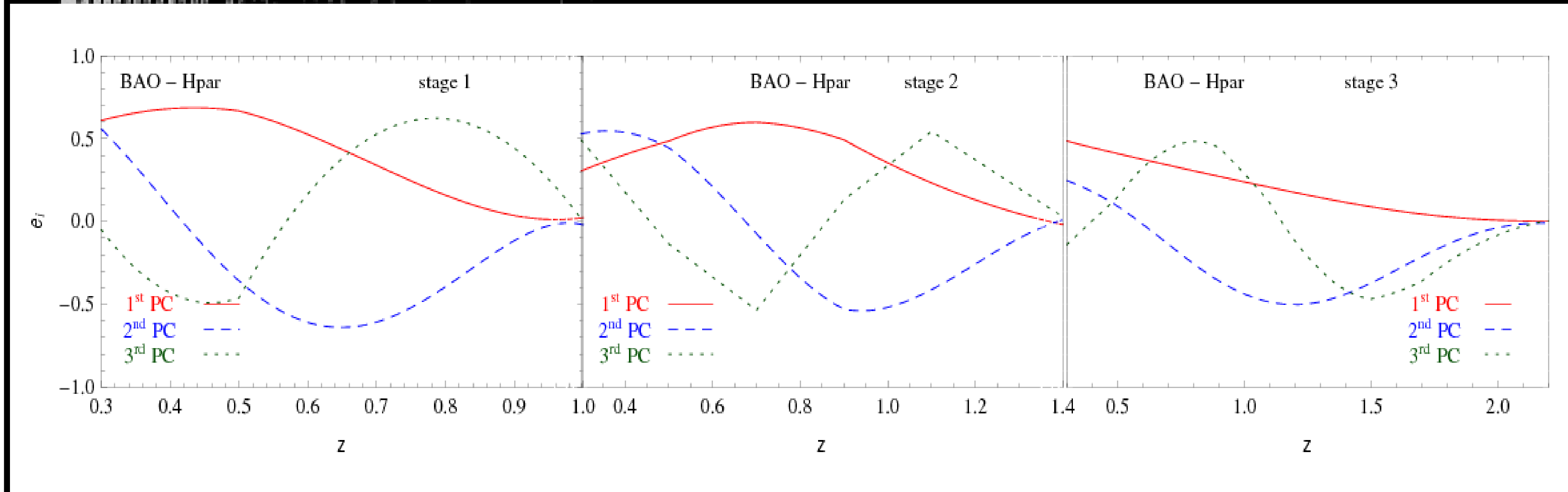
$$\frac{\partial g(z;\bar{\beta})}{\partial \beta_l} = (1+z)\left[\prod_{i=1}^{J}\left(\frac{1+z_i}{1+z_{i-1}}\right)^{\beta_i}\right]\left(\frac{1+z}{1+z_J}\right)^{\beta_{J+1}}\times$$
$$\times\left\{\Theta(J+1-l)\ln\left(\frac{1+z_l}{1+z_{l-1}}\right)+\delta_{J+1,l}\ln\left(\frac{1+z}{1+z_J}\right)\right\}$$

$$\frac{\partial^2 g(z;\bar{\beta})}{\partial \beta_k \partial \beta_l} = (1+z)\left[\prod_{i=1}^{J}\left(\frac{1+z_i}{1+z_{i-1}}\right)^{\beta_i}\right]\left(\frac{1+z}{1+z_J}\right)^{\beta_{J+1}}\times$$
$$\times\left\{\Theta(J+1-k)\ln\left(\frac{1+z_k}{1+z_{k-1}}\right)+\delta_{J+1,k}\ln\left(\frac{1+z}{1+z_J}\right)\right\}\times$$
$$\times\left\{\Theta(J+1-l)\ln\left(\frac{1+z_l}{1+z_{l-1}}\right)+\delta_{J+1,l}\ln\left(\frac{1+z}{1+z_J}\right)\right\}$$

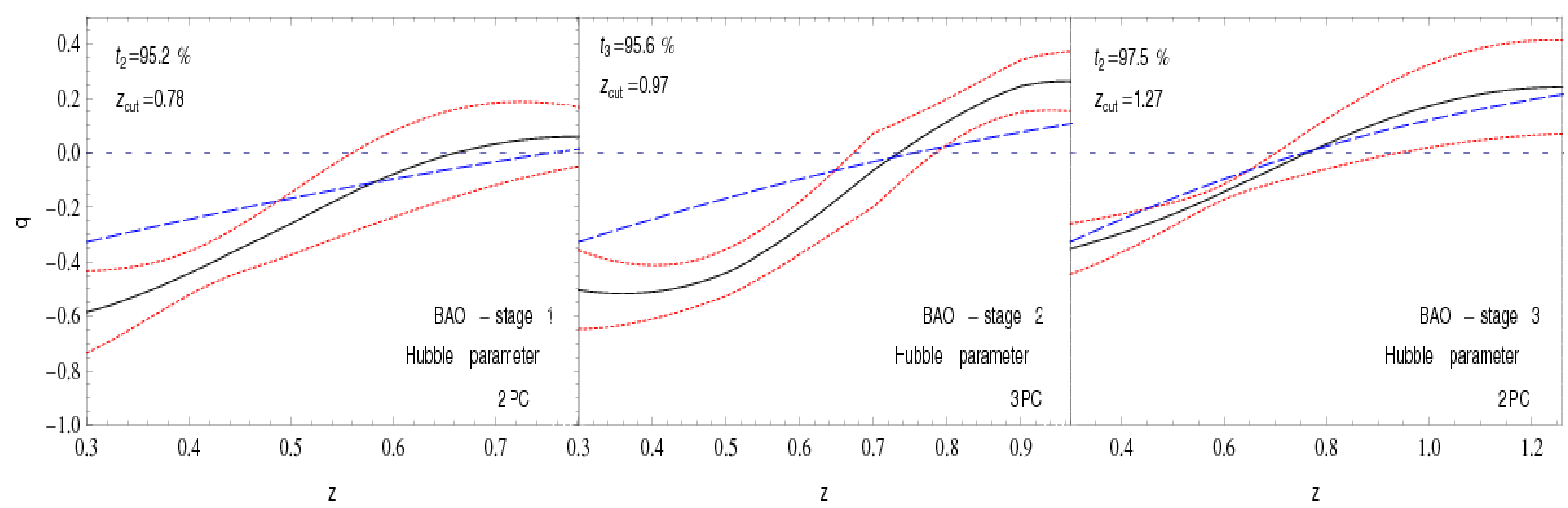
$$F^H_{kl} = \left\langle -\frac{\partial^2 \ln L_H}{\partial\beta_k\beta_l}\right\rangle = -\frac{1}{2M^2}\frac{\partial M}{\partial\beta_k}\frac{\partial M}{\partial\beta_l}\left(1+\frac{K^2}{M}\right)+\frac{1}{2M}\frac{\partial^2 M}{\partial\beta_k\beta_l}\left(1+\frac{K^2}{M}\right)+\frac{K}{M^2}\frac{\partial K}{\partial\beta_k}\frac{\partial M}{\partial\beta_l}+$$
$$-\frac{K^2}{2M^3}\frac{\partial M}{\partial\beta_k}\frac{\partial M}{\partial\beta_l}-\frac{1}{M}\frac{\partial K}{\partial\beta_k}\frac{\partial K}{\partial\beta_l}+\frac{K}{M^2}\frac{\partial M}{\partial\beta_k}\frac{\partial K}{\partial\beta_l}-\frac{K}{M}\frac{\partial^2 K}{\partial\beta_k\partial\beta_l}. \qquad (2.20)$$

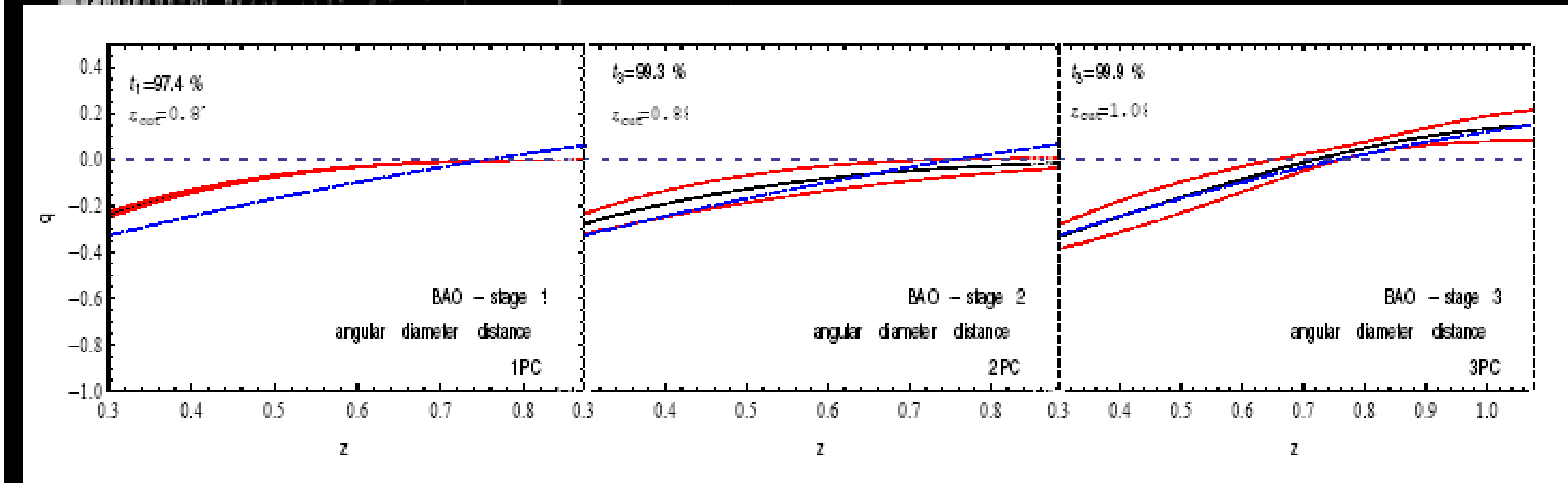
Componentes diferentes exigem critérios diferentes



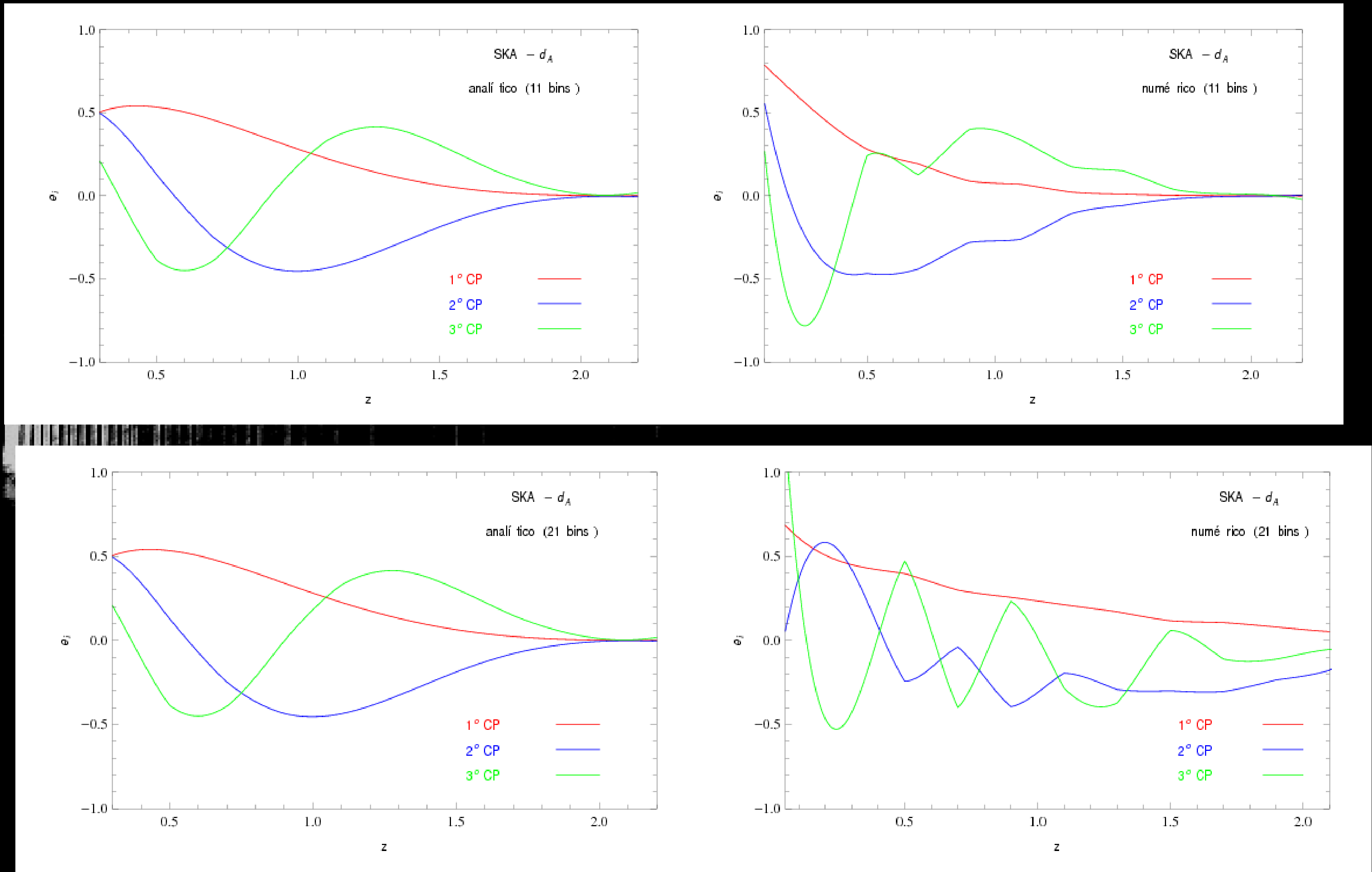


Reconstruções Finais





FAQ, parte 2: Por que tanta conta?



Conclusões:

Vale a pena usar as contas analíticas;

Resultados independentes de modelo cosmológico ou teoria de gravitação

Existem pontos importantes em relação à reconstrução que muitas vezes não é sequer comentada na literatura

O procedimento apresentado aqui pode vir a ser um boa maneira de fugir aos erros numéricos e as formais funcionais impostas por parametrizações.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...ISN'T THAT WONDERFUL!?
IT MEANS MY THESIS ADDRESSES A PROBLEM NO ONE'S FIGURED OUT YET!
IF EVERYTHING ALREADY MADE SENSE, IT WOULDN'T BE CALLED RESEARCH!