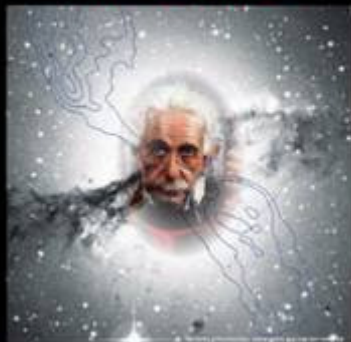


# Strong Gravitational Lensing

## Overview, Status and Opportunities

Antonio C. C. Guimarães  
IAG-USP

IX WORKSHOP  
Nova Física no Espaço

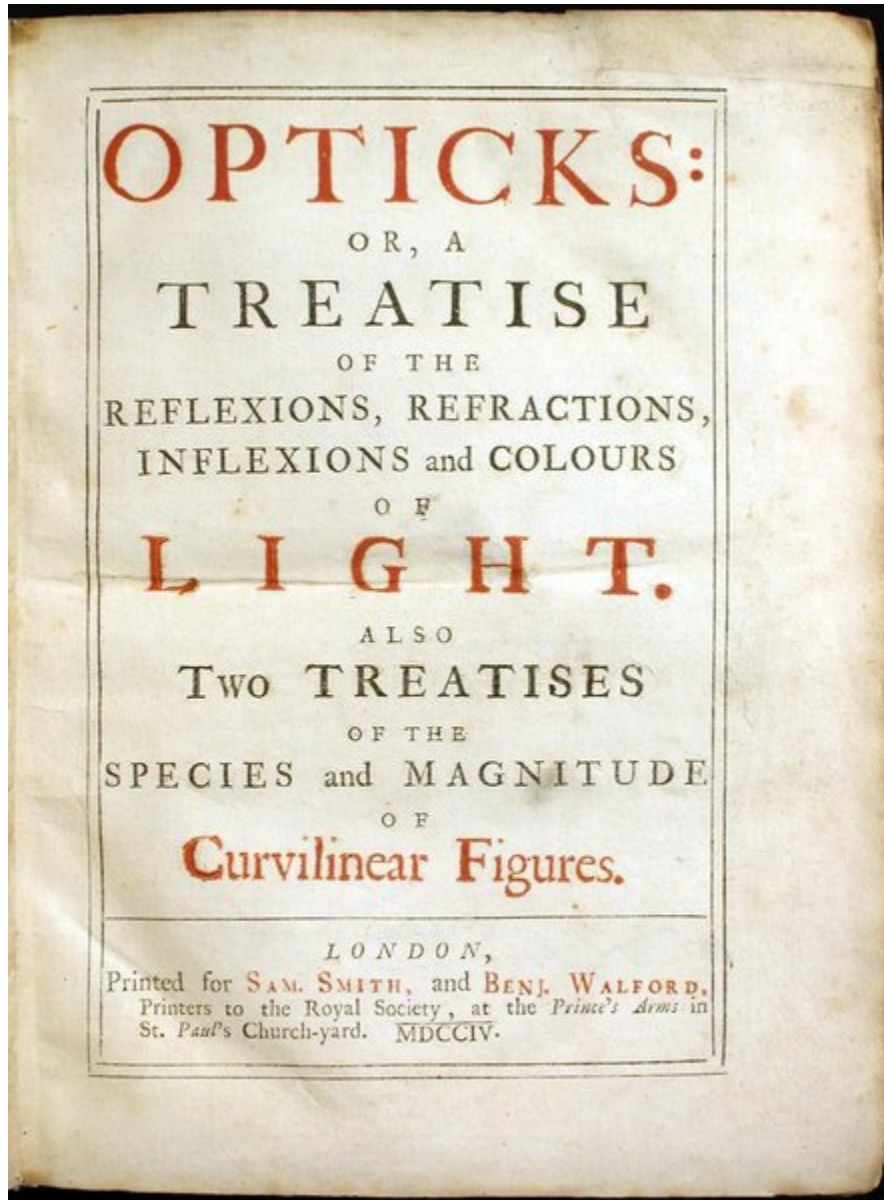


Campos do Jordão, March/2010



# **Roteiro da apresentação**

- fundamentos de lentes fortes**
- observações: passado, presente e futuro**
- fenomenologia e ciência com lentes fortes**

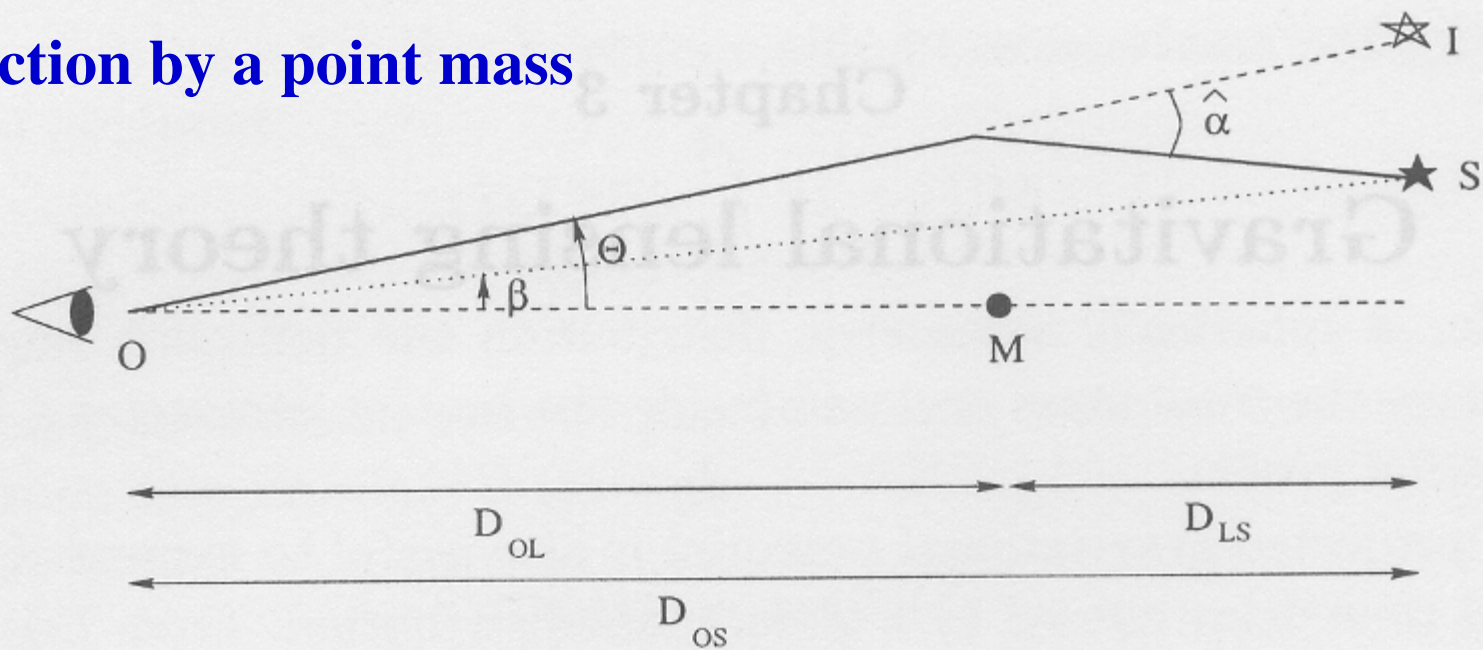


“Do not Bodies act upon Light at a distance, and by their action bend its Rays; and is not this action strongest at the least distance?”

Sir Isaac Newton, *Opticks*, 1704



# light deflection by a point mass



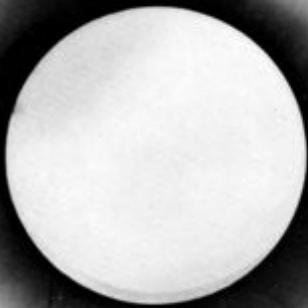
1916: General Theory of Relativity, Einstein derives the **correct** deflection angle:

$$\tilde{\alpha}_{\odot} = \frac{4GM_{\odot}}{c^2} \frac{1}{R_{\odot}} = 1.74 \text{ arcsec}$$

1919: Arthur Eddington and his group confirms the GR prediction for the light deflection by the Sun.

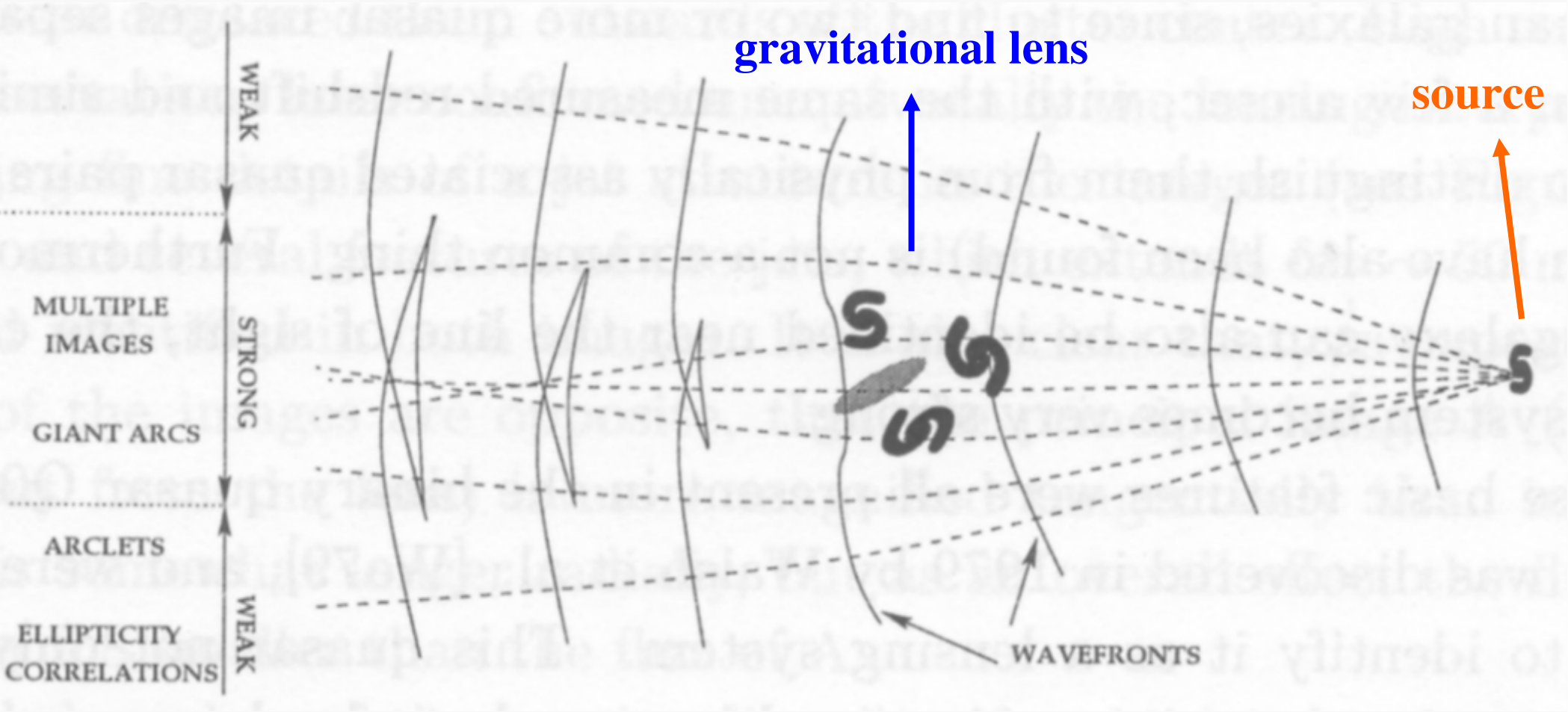
**SOBRAL = 1,98"**

**PRÍNCIPE = 1,60"**

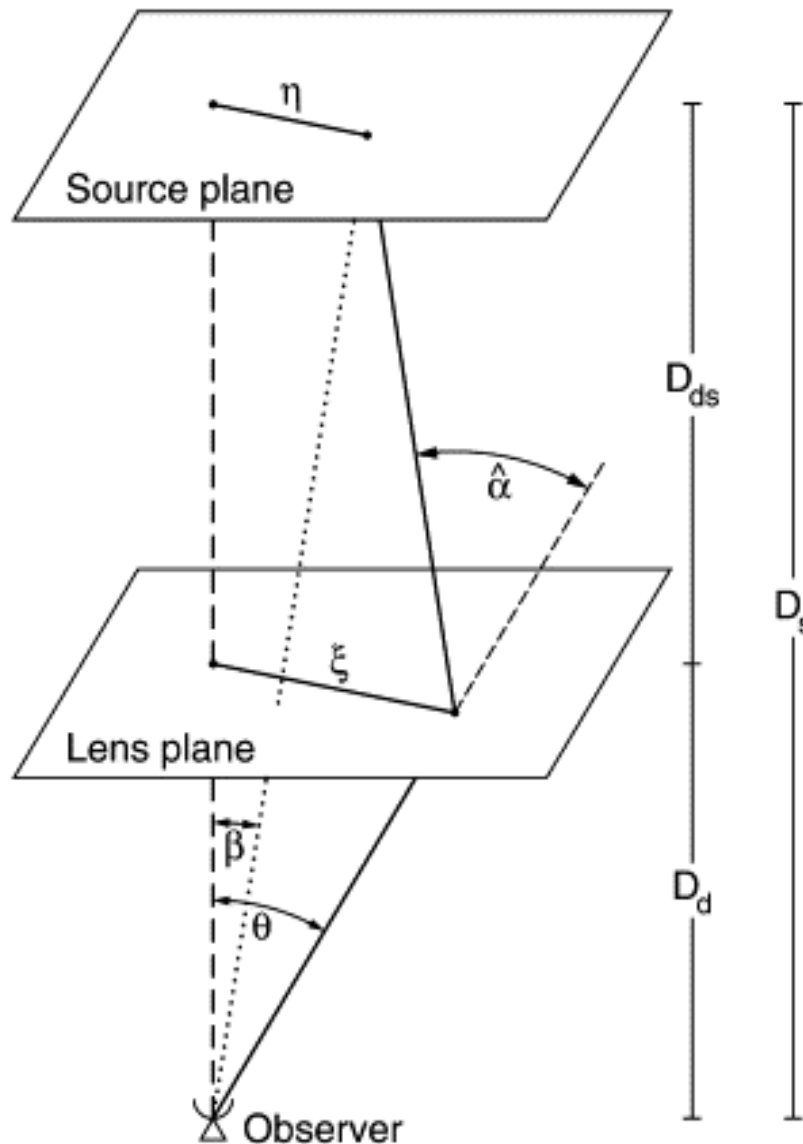


galaxy, galaxy group,  
galaxy cluster

quasar,  
galaxy,  
supernova



## Lens Equation



- coordinates  $\vec{\theta}$  for the image(s) and  $\vec{\beta}$  for the source(s) are related by

$$D_s \vec{\beta} = D_s \vec{\theta} - D_{ls} \hat{\alpha}$$

- *reduced* deflection angle

$$\vec{\alpha} \equiv \frac{D_{ls}}{D_s} \hat{\alpha}$$

- gives *lens equation*

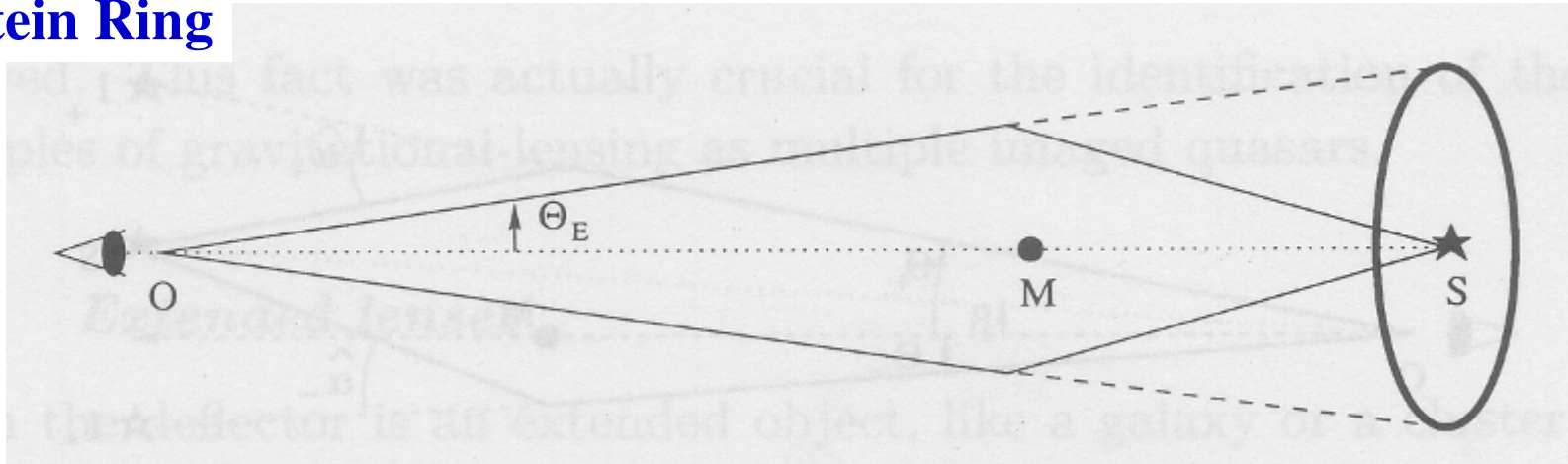
$$\vec{\beta} = \vec{\theta} - \vec{\alpha}$$

**true position = apparent position – deflection**

$$\vec{\alpha}(\vec{\theta}) = \vec{\nabla} \psi$$

$$\psi(\vec{\theta}) = \frac{D_{LS}}{D_L D_S} \frac{2}{c^2} \int \Phi(\vec{\theta}, z) dz$$

# Einstein Ring



point mass

- deflection angle is

$$\hat{\alpha} = \frac{4GM}{c^2 b} = \frac{4GM}{c^2 D_1 \theta}$$

- lens equation becomes

$$\beta = \theta - \frac{4GM}{c^2 D_1 \theta} \frac{D_{ls}}{D_s}$$

- introducing Einstein angle

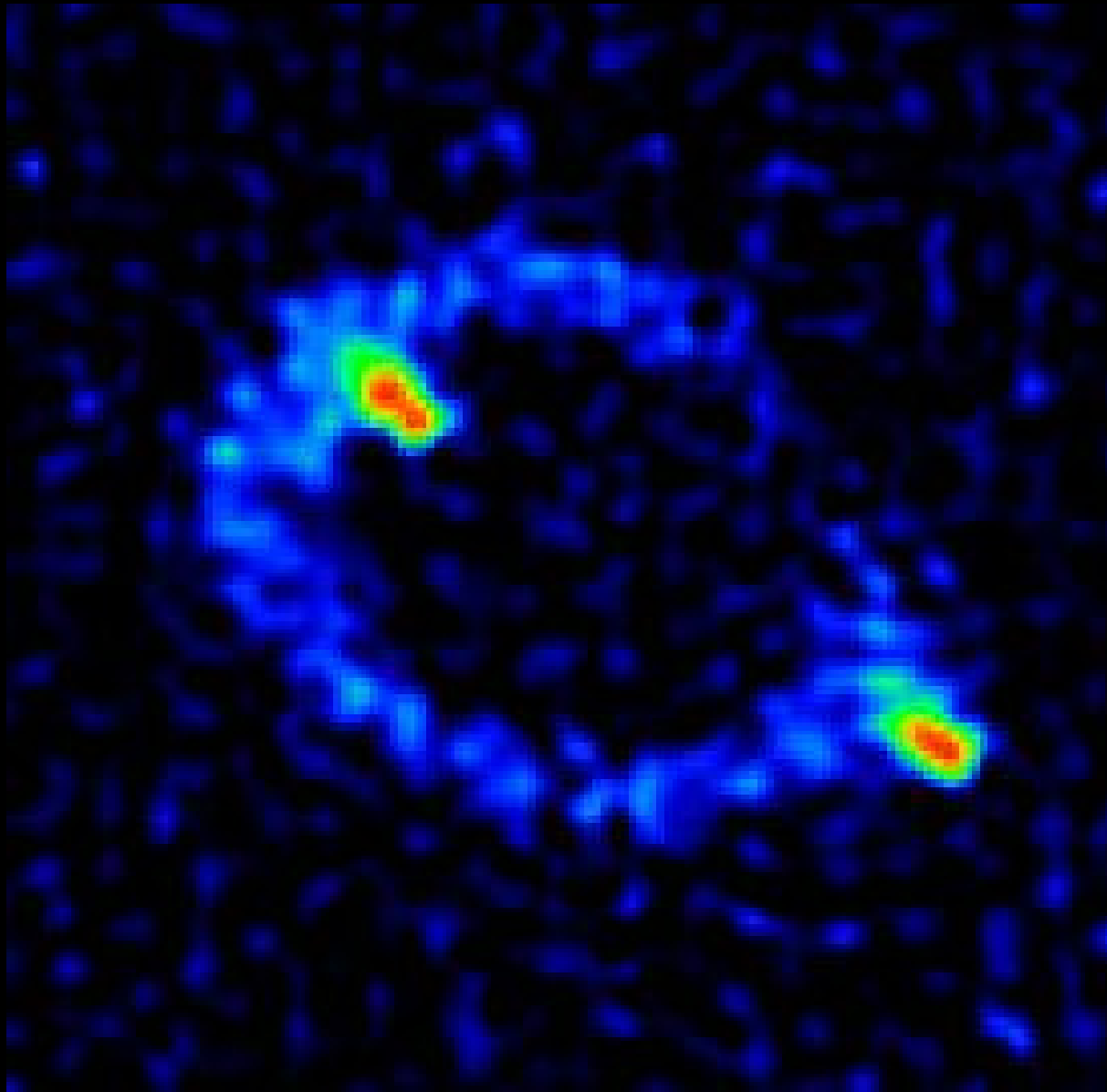
$$\theta_E \equiv \left( \frac{4GM}{c^2} \frac{D_{ls}}{D_1 D_s} \right)^{1/2}$$

- always two solutions of the lens equation:

$$\theta_{\pm} = \frac{1}{2} \left[ \beta \pm \left( \beta^2 + 4\theta_E^2 \right)^{1/2} \right]$$

- if  $\beta = 0$ ,  $\theta_{\pm} = \pm\theta_E$
- let  $D \equiv (D_1 D_s / D_{ls})$ , then

$$\begin{aligned} \theta_E &\approx (10^{-3})'' \left( \frac{M}{M_{\odot}} \right)^{1/2} \left( \frac{D}{\text{kpc}} \right)^{-1/2} \\ &\approx 1'' \left( \frac{M}{10^{12} M_{\odot}} \right)^{1/2} \left( \frac{D}{\text{Gpc}} \right)^{-1/2} \end{aligned}$$

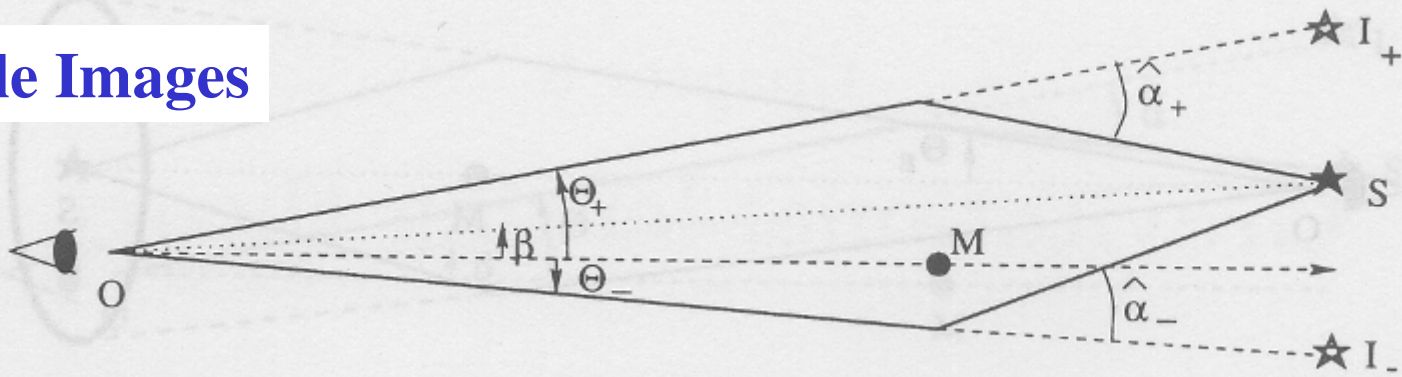


**1987: The First Einstein Ring**

MG1131+0456: VLA, quasar radio emission lensed by a galaxy



# Multiple Images



## 1979: The First Gravitational Lens

0957+561A,B: D. Walsh, R. F. Carswell & R. J. Weymann

$$\theta_{\pm} = \frac{1}{2} \left[ \beta \pm (\beta^2 + 4\theta_E^2)^{1/2} \right]$$

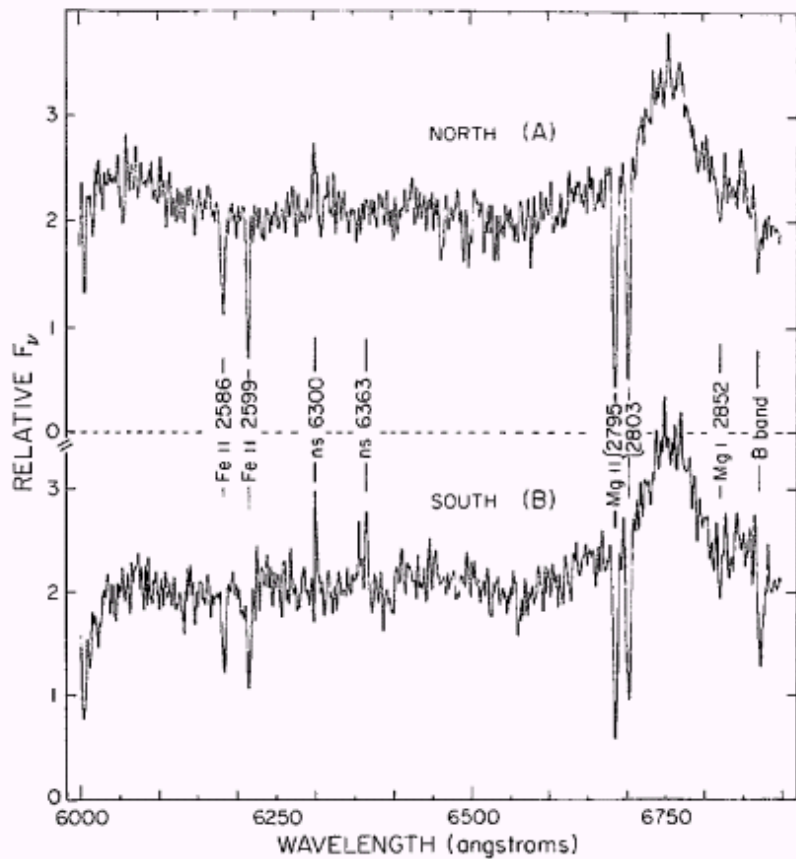
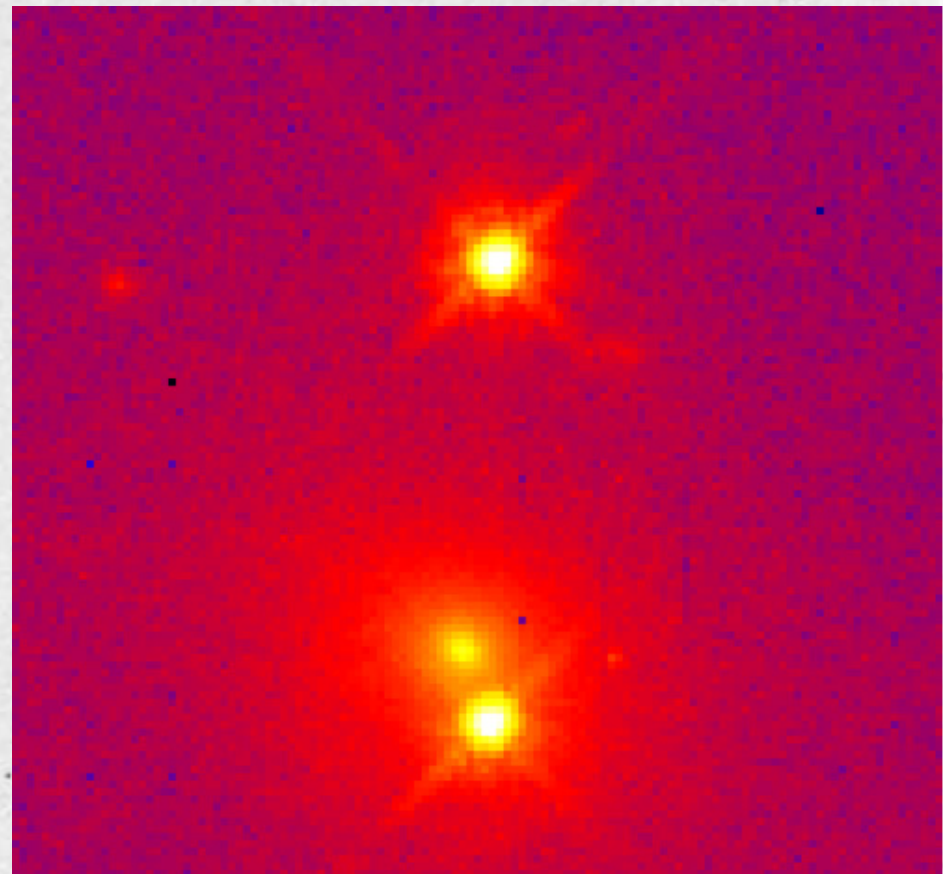
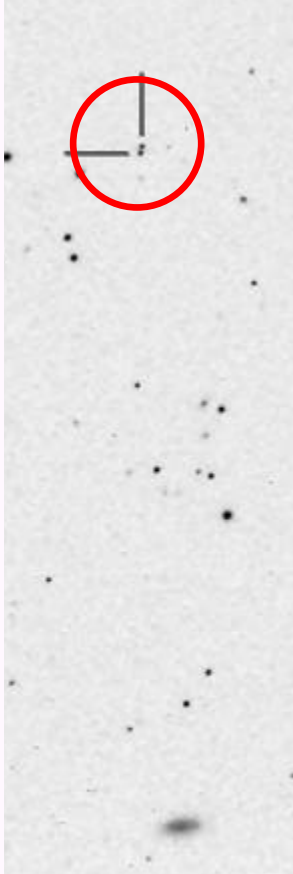
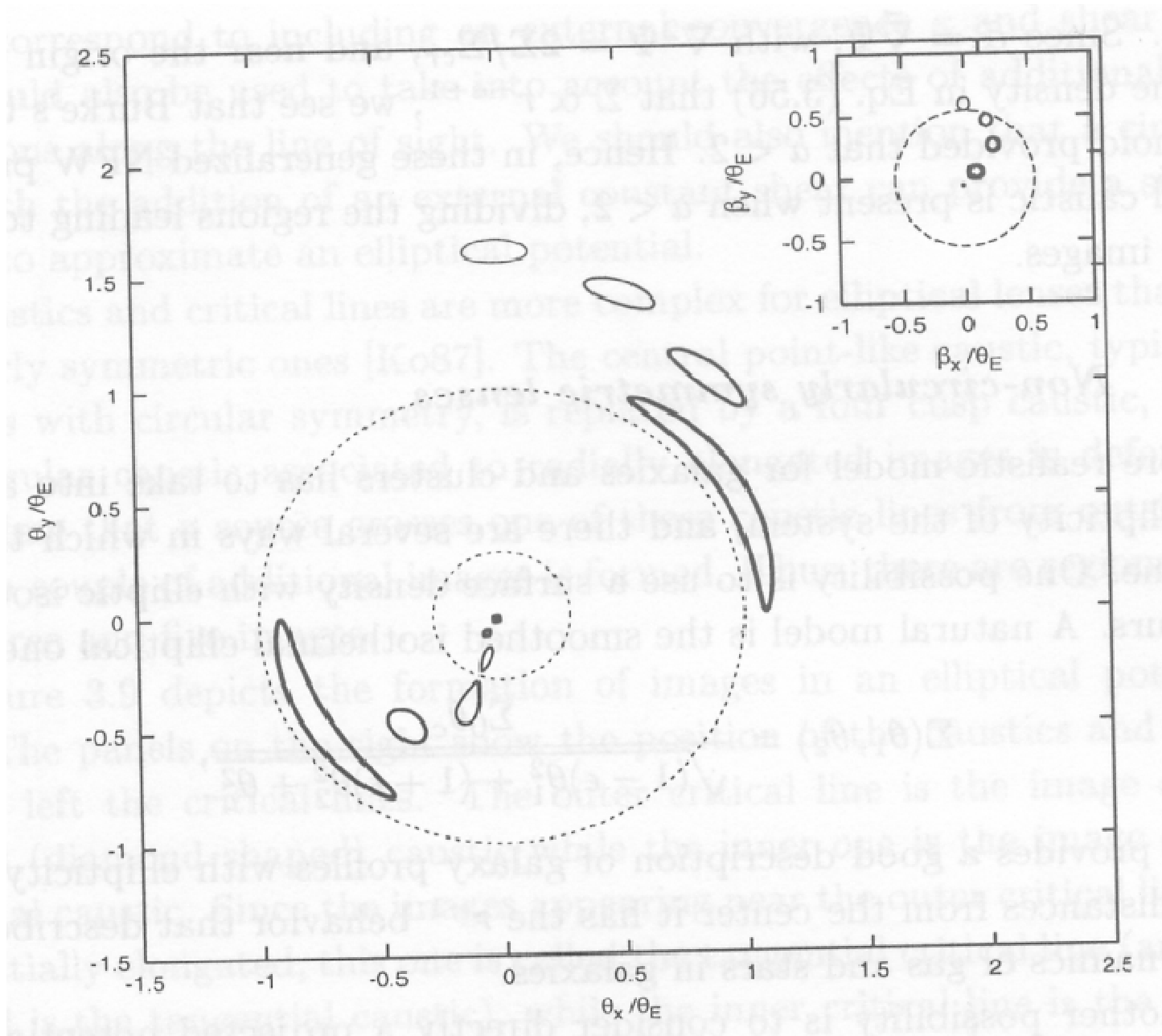


FIG. 1.—Spectra of the two QSOs 0957+561 A,B obtained



E.E. Falco et al. (CASTLE collaboration) and NASA

# Circular Lens



source  
real position

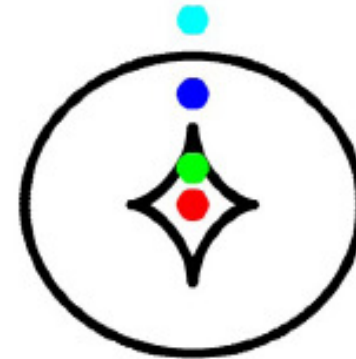
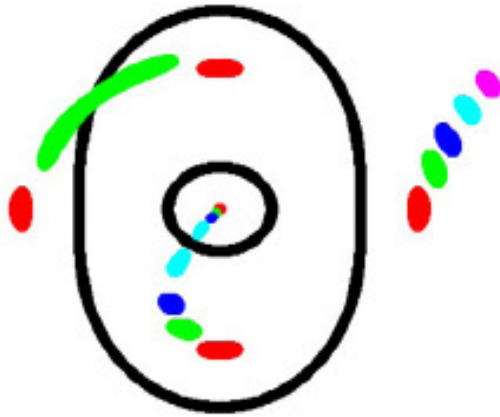
image

# Non-Circular Lens (elliptical)

critical lines

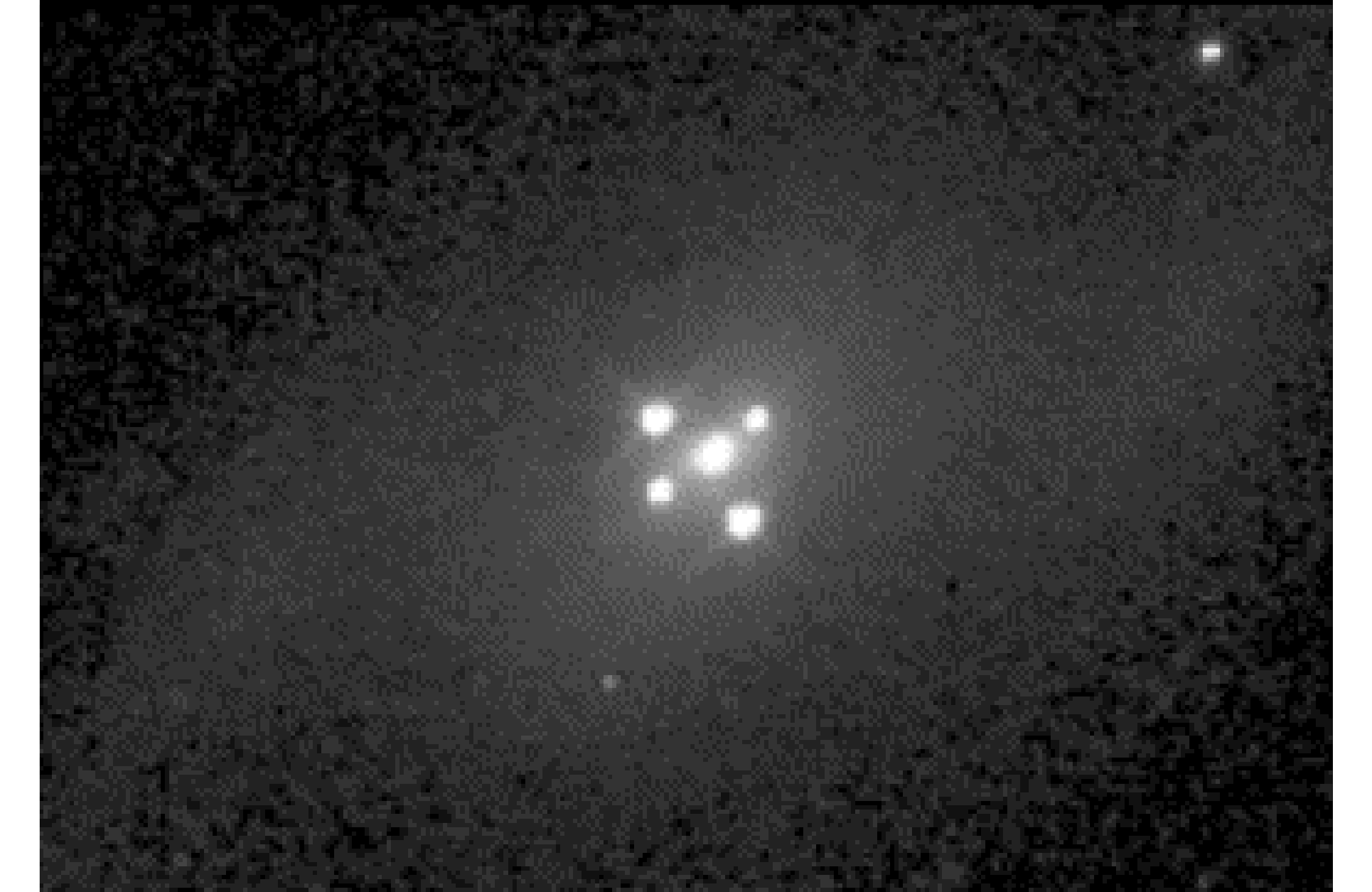
&

caustics



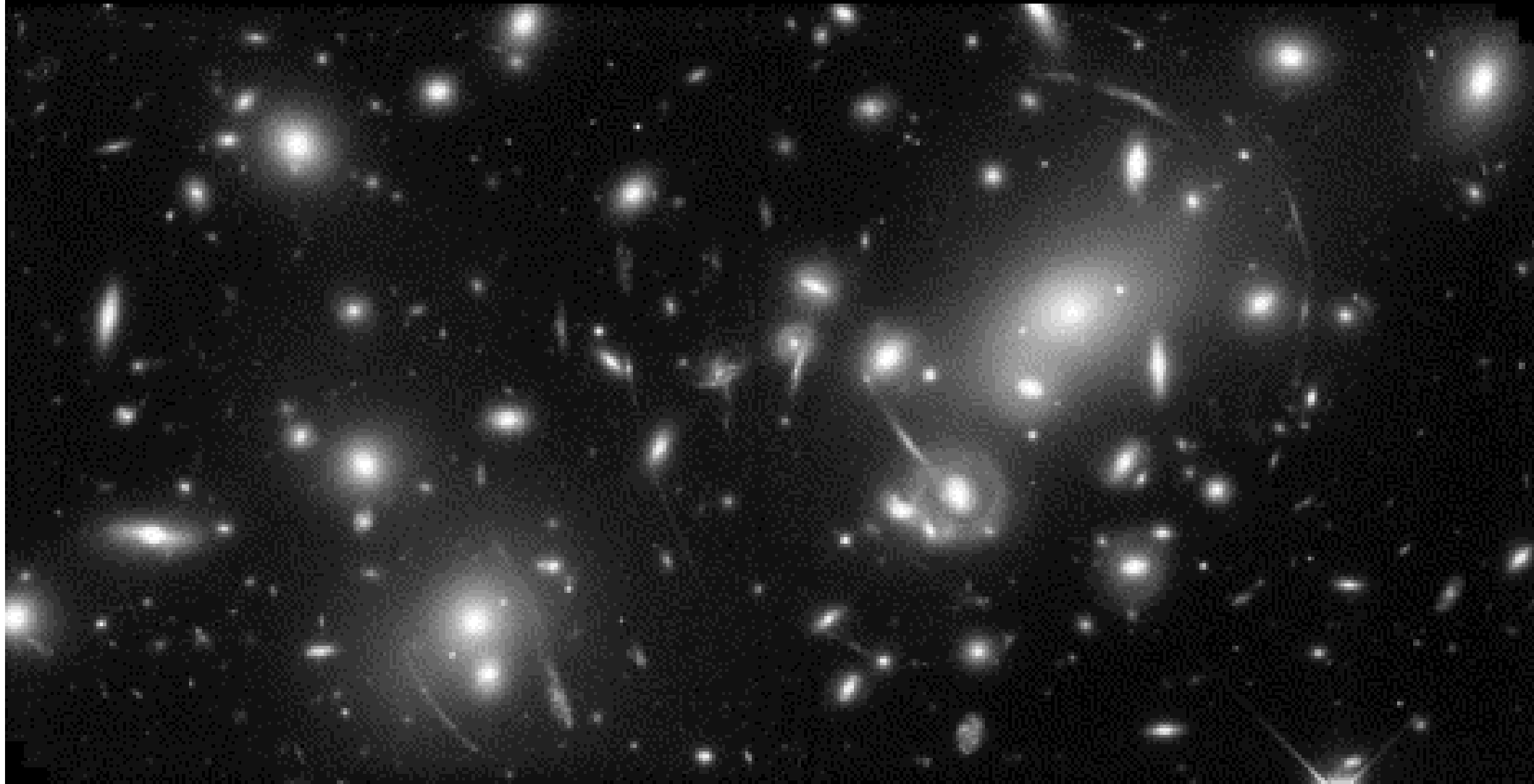
image

source real position



1985: Q2237+0305, Einstein Cross, Huchra et al.

# Giant Arcs



Abell 2218 , HST

# Strong Gravitational Lensing Observational Status

~ 200 galaxy-scale lenses

## Major past and present surveys

- CLASS: 22 lenses, radio source-selected  
(JVAS, Browne et al 2003)
- SLACS: ~100 galaxy lenses, lens-selected, spectroscopy  
(SDSS, Bolton et al 2006)
- SQLS: >25 quasar lenses, source-selected, spectroscopy  
(SDSS, Oguri et al 2007)
- SL2S: >20 galaxy lenses, lens-selected  
(CFHTLS, Cabanac et al 2007)
- COSMOS: ~20 galaxy lenses, lens-selected, optical imaging  
(COSMOS, Faure et al 2008)

# CASTLES Survey (~ 100 lenses)

(CfA-Arizona Space Telescope LENS Survey of gravitational lenses)

Complete survey of all them known galaxy-mass gravitational lens systems (those with image separations of less than 10 arcseconds)



<http://www.cfa.harvard.edu/castles/>

#	Image	Lens Name	G	$z_s$	$q$	RA (J2000)	Dec (J2000)	E(B-V)	$m_s$ (mag)	$m_l$ (mag)	$F_{GHz}$ (mJy)	$N_{im}$	size (")	dt (days)	sigma (km/s)
1		<a href="#">Q0047-2808</a>	A	3.60	0.48	00:49:41.89	-27:52:25.7	0.016		I=20.05		4ER	2.7		229±15
2		<a href="#">HE0047-1756</a>	A	1.66	0.41	00:50:27.83	-17:40:8.8	0.022	I=16.52/2	I=18.96		2	1.44		
3		<a href="#">HST01247+0352</a>	C			01:24:44.4	+03:52:00	0.029	I=24.13/2	I=21.86		2	2.20		
4		<a href="#">HST01248+0351</a>	C			01:24:45.6	+03:51:06	0.029				2	0.74		
5		<a href="#">B0128+437</a>	B	3.124		01:31:13.405	+43:58:13.14	0.082			$F_5=48$	4	0.55		
6		<a href="#">PMNJ0134-0931</a>	A	2.216	0.77	01:34:35.67	-09:31:02.9	0.031	I=18.96/4	I=19.31	$F_5=529$	5R	0.73		
7		<a href="#">Q0142-100</a>	A	2.72	0.49	01:45:16.5	-09:45:17	0.031	I=16.47/2	I=18.72	$F_5 \sim 1$	2	2.24		
8		<a href="#">QJ0158-4325</a>	A	1.29	0.317	01:58:41.44	-43:25:04.20	0.015	I=17.39/2	I=18.91	$F_8 < 0.2$	2	1.22		
9		<a href="#">B0218+357</a>	A	0.96	0.68	02:21:05.483	+35:56:13.78	0.068	I=19.28/2	I=20.06	$F_5=1209$	2ER	0.34	10.5±0.4	

redshift

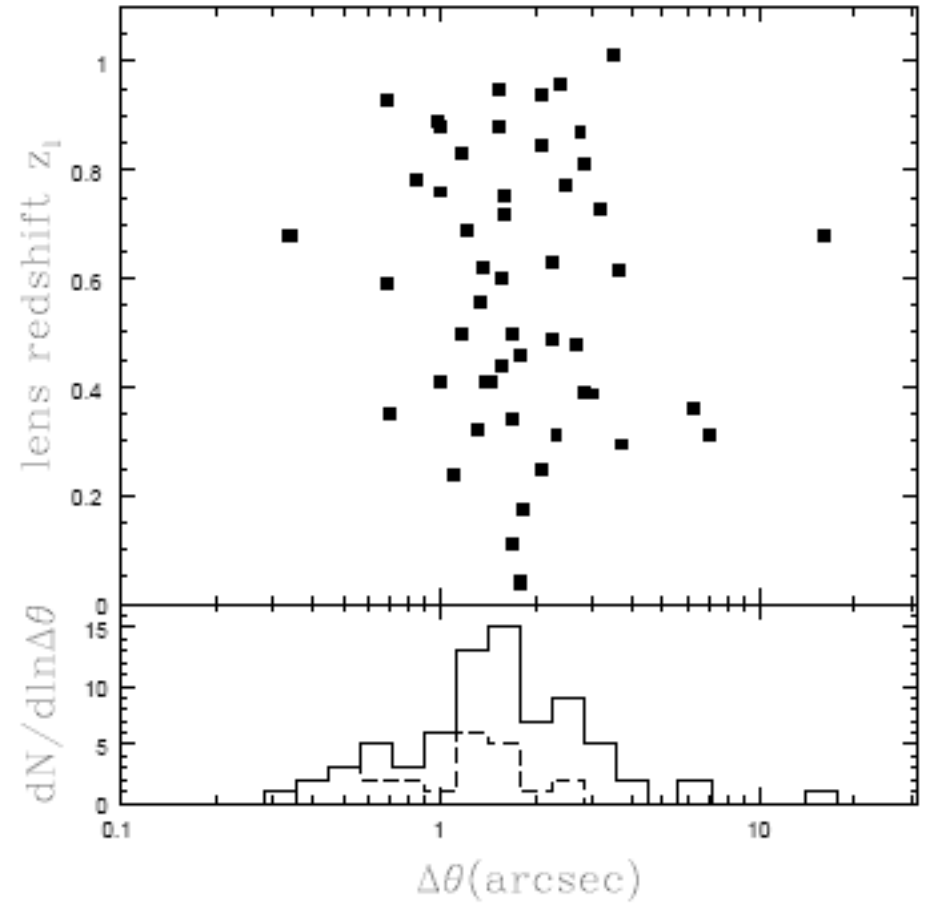
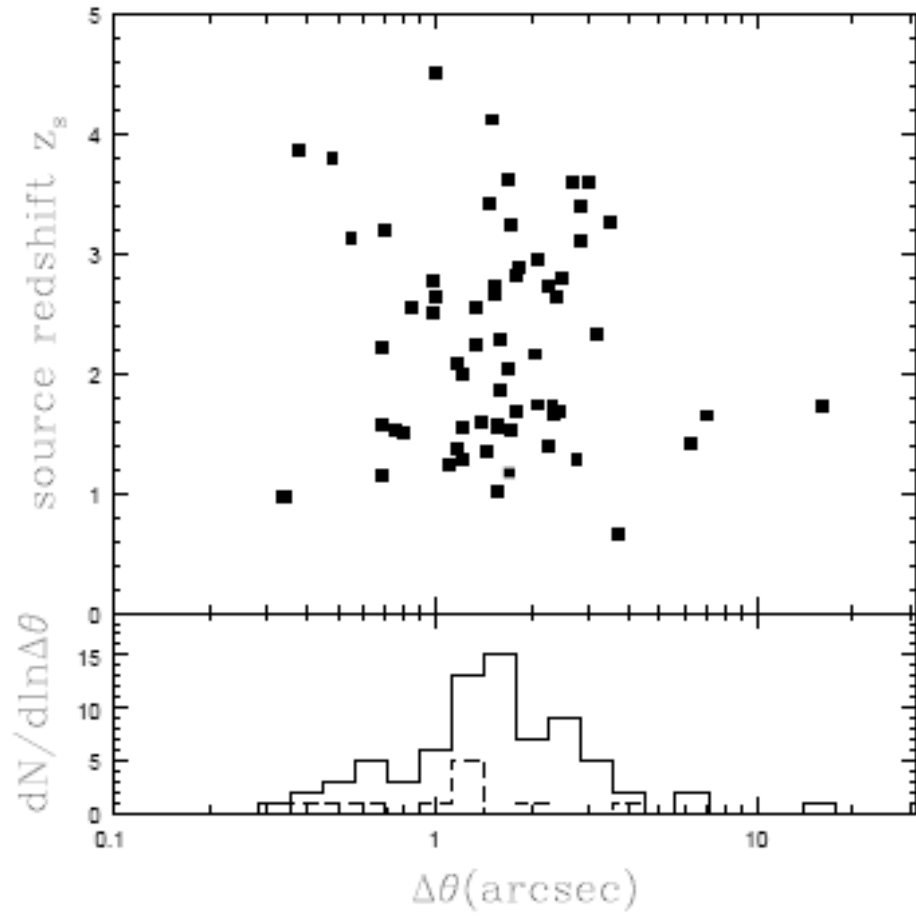
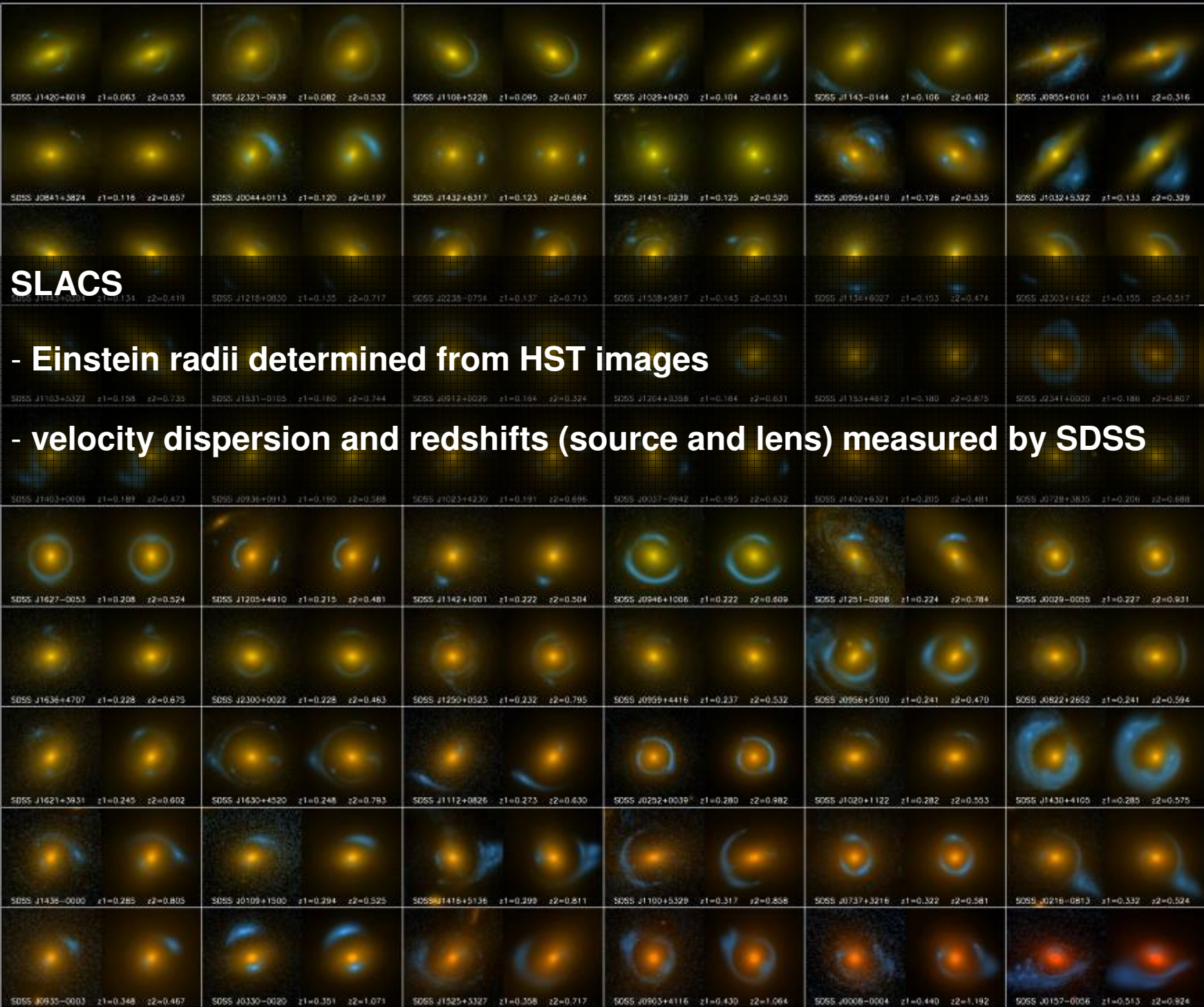


image separation





**SLACS**

- Einstein radii determined from HST images

- velocity dispersion and redshifts (source and lens) measured by SDSS

# Current and Future Surveys

## Plenty of lenses

Survey	Depth	Res	Area	Gal	QSO	Sn
SL2S	25	0.9	130	30	10	-
SDSS	21	1.5	8000	30	100	-
HST	26	0.1	3	30	1	-
PS1	22.5	1.0	30000	300	2000	20
DES	25	0.9	5000	1000	100	1
LSST	24	0.6	20000	10000	10000	600
SNAP-WL	27	0.1	1000	20000	300	-
SNAP-SN	29	0.1	10	300	3	200

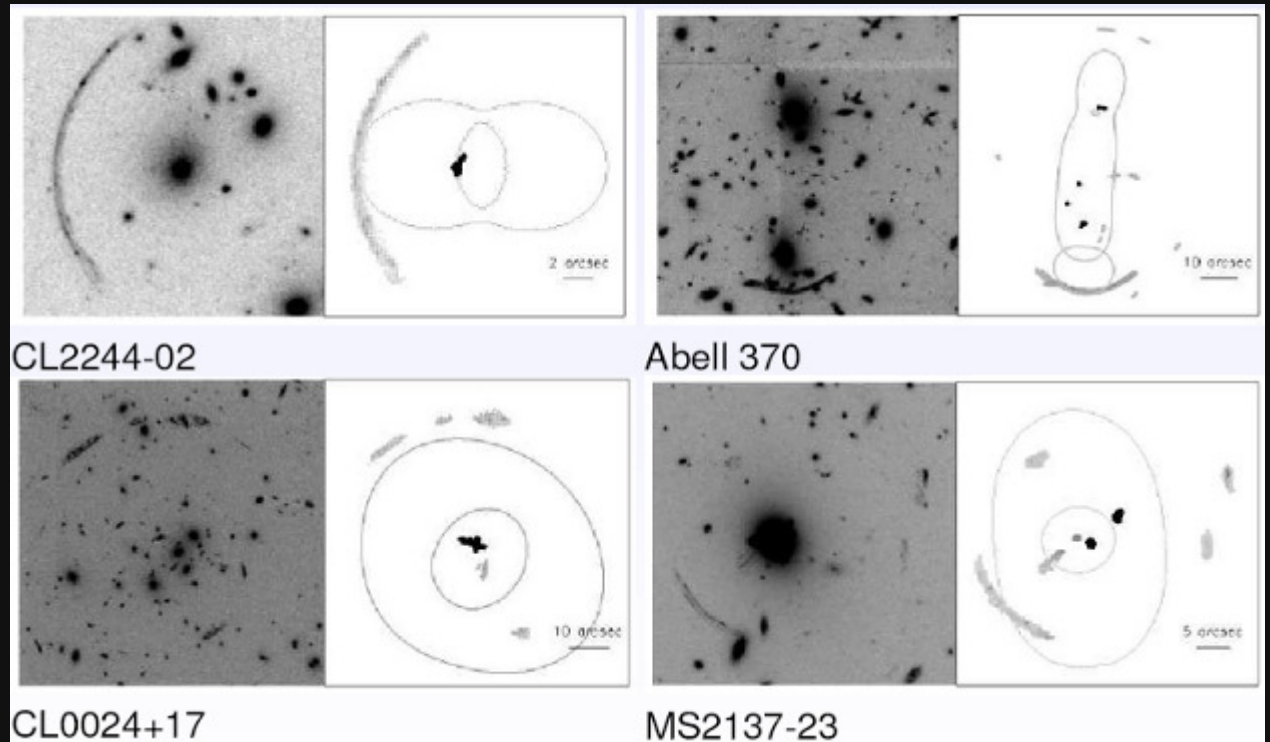
Marshall, 2008, private communication

**AND RADIO WILL PROVIDE A LOT OF LENSES TOO!**

# **Science with Strong Lenses**

# Lens (galaxy, group, cluster) Modeling

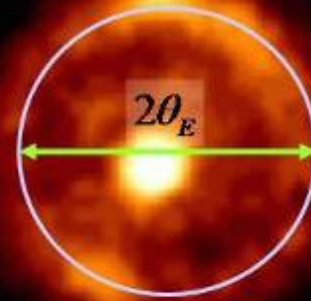
- the image positions and relative fluxes can be used to constrain
  - mass
  - profile
  - shape
  - substructure



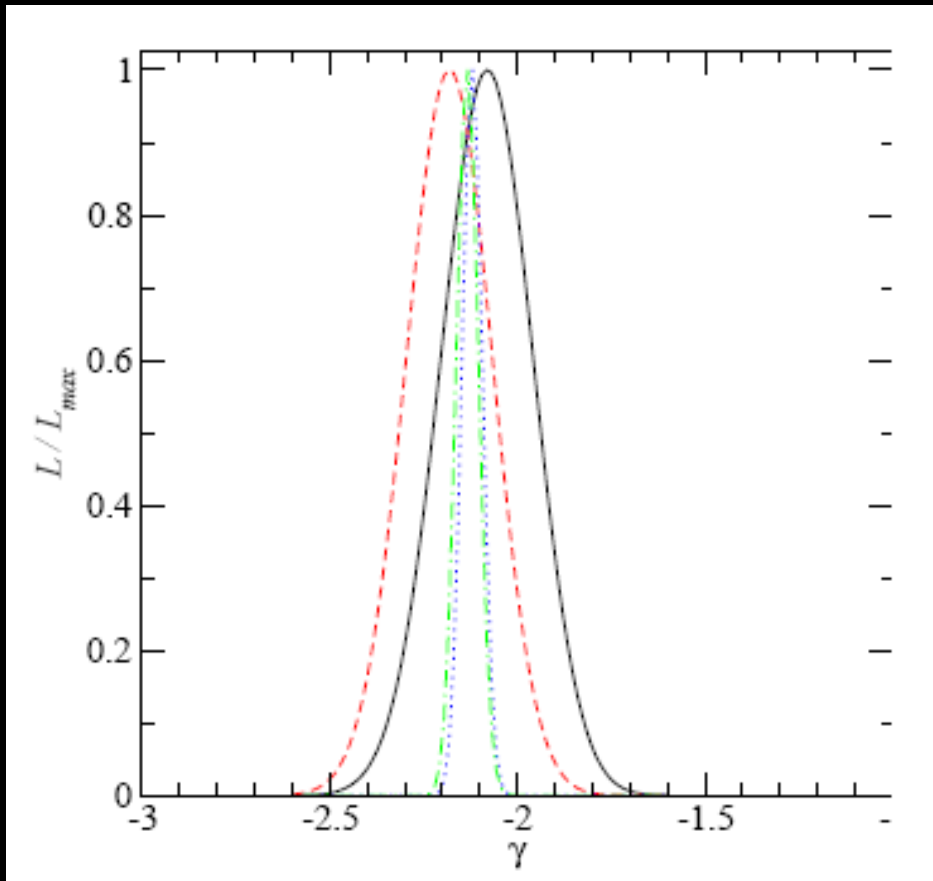
# Stellar Dynamics and Lensing

spherical Jeans equation

$$\frac{1}{\nu} \frac{d\nu\sigma_r^2}{dr} + \frac{2\beta(r)}{r}\sigma_r^2 = -\frac{GM(r)}{r^2}$$



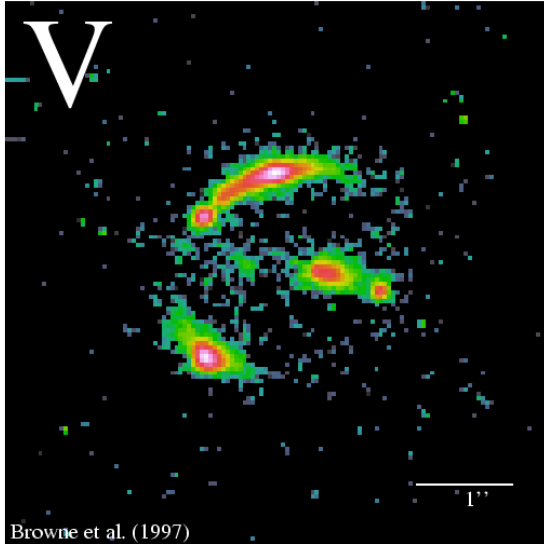
$$M_L = \frac{c^2}{4G} \frac{D_l D_s}{D_{ls}} \theta_E^2$$



$$\rho(r) \propto r^\gamma$$

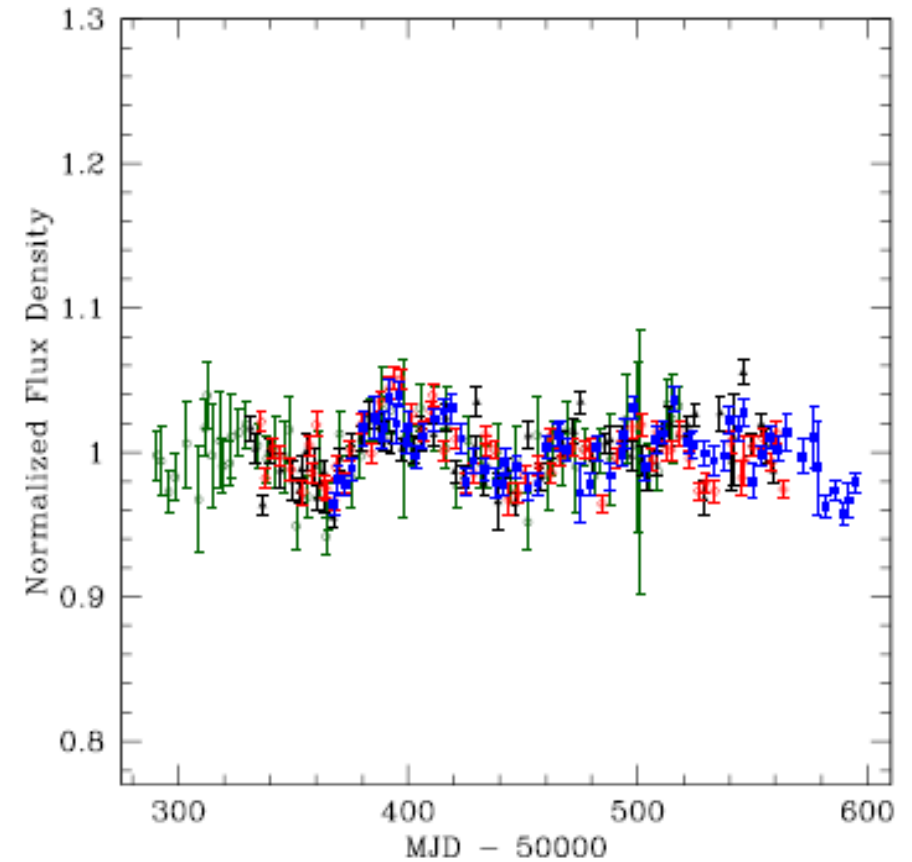
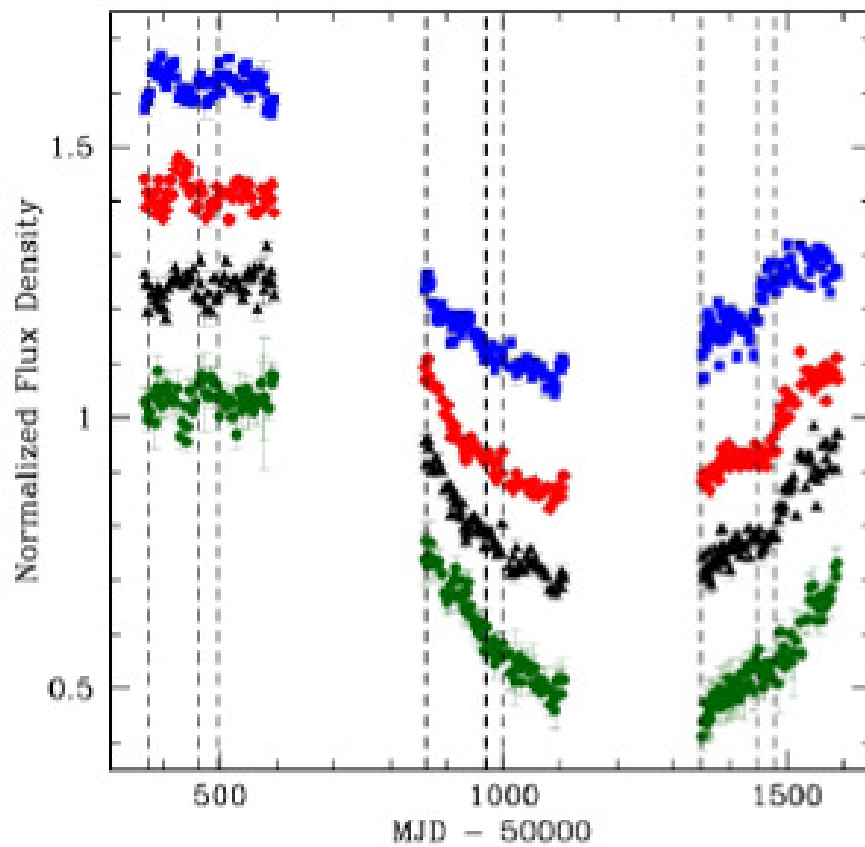
58 strong lensing events from SLACS  
Sloan velocity dispersions

density profile slope



## Time Delay

time delays and magnifications  
adjusted image fluxes



time (days)

# Time Delay

light **travel time** for each image relative to a fiducial unperturbed ray

$$\tau(\theta) = \frac{D_d D_s}{c D_{ds}} \left[ \frac{1}{2} (\theta - \beta)^2 - \Psi(\theta) \right]$$

**time delay** between two images, assuming a lens model (SIS),  $\Psi = b\theta$

$$\Delta t_{SIS} = \tau_B - \tau_A = \frac{1}{2} \frac{D_d D_s}{c D_{ds}} (\theta_A^2 - \theta_B^2)$$

The time scales as  $H_0^{-1}$

because of the scalings of the distances

method advantages:

- measures expansion rate directly at high redshifts (negligible peculiar velocities)
- does not depend on distance ladders nor standard candle

disadvantage: needs lens model

HUBBLE CONSTANT FROM EACH LENS SYSTEM	
Lens Name	$h$ ( $1 \sigma$ Range)
B0218+357.....	0.21 (...)
HE 0435-1223.....	1.02 (0.70-1.39)
RX J0911+0551.....	0.96 (0.75-1.21)
SBS 0909+532.....	0.84 (0.47-)
FBQ 0951+2635.....	0.67 (0.56-0.81)
Q0957+561.....	0.99 (0.82-1.17)
HE 1104-1805.....	1.04 (0.92-1.22)
PG 1115+080.....	0.66 (0.49-0.84)
RX J1131-1231.....	0.79 (0.59-1.03)
B1422+231.....	0.16 (-0.36)
SBS 1520+530.....	0.53 (0.46-0.61)
B1600+434.....	0.65 (0.54-0.77)
B1608+656.....	0.89 (0.77-1.20)
SDSS J1650+4251.....	0.53 (0.44-0.63)
PKS 1830-211.....	0.88 (0.58-)
HE 2149-2745.....	0.69 (0.57-0.82)
All.....	0.70 (0.68-0.73)

Oguri et al. 2007

# Statistical Lensing

## Observables:

- lens number (relates to cosmological volume and structure formation)
- image separation distribution (relates to halo mass function)

## Defining quantities:

- lens cross section

$$\sigma = \pi(\text{Einstein radius})^2$$

- optical depth (probability that a source beam hits a lens)

$$d\tau = n_l \sigma \frac{cdt}{dz_l} dz_l$$

comoving density of lenses

$$\tau = \frac{3}{2} \Omega_L \frac{H_0}{c} \int \frac{D_l D_{ls}}{D_s} \frac{(1-z_l)^2}{H(z_l)} dz_l$$

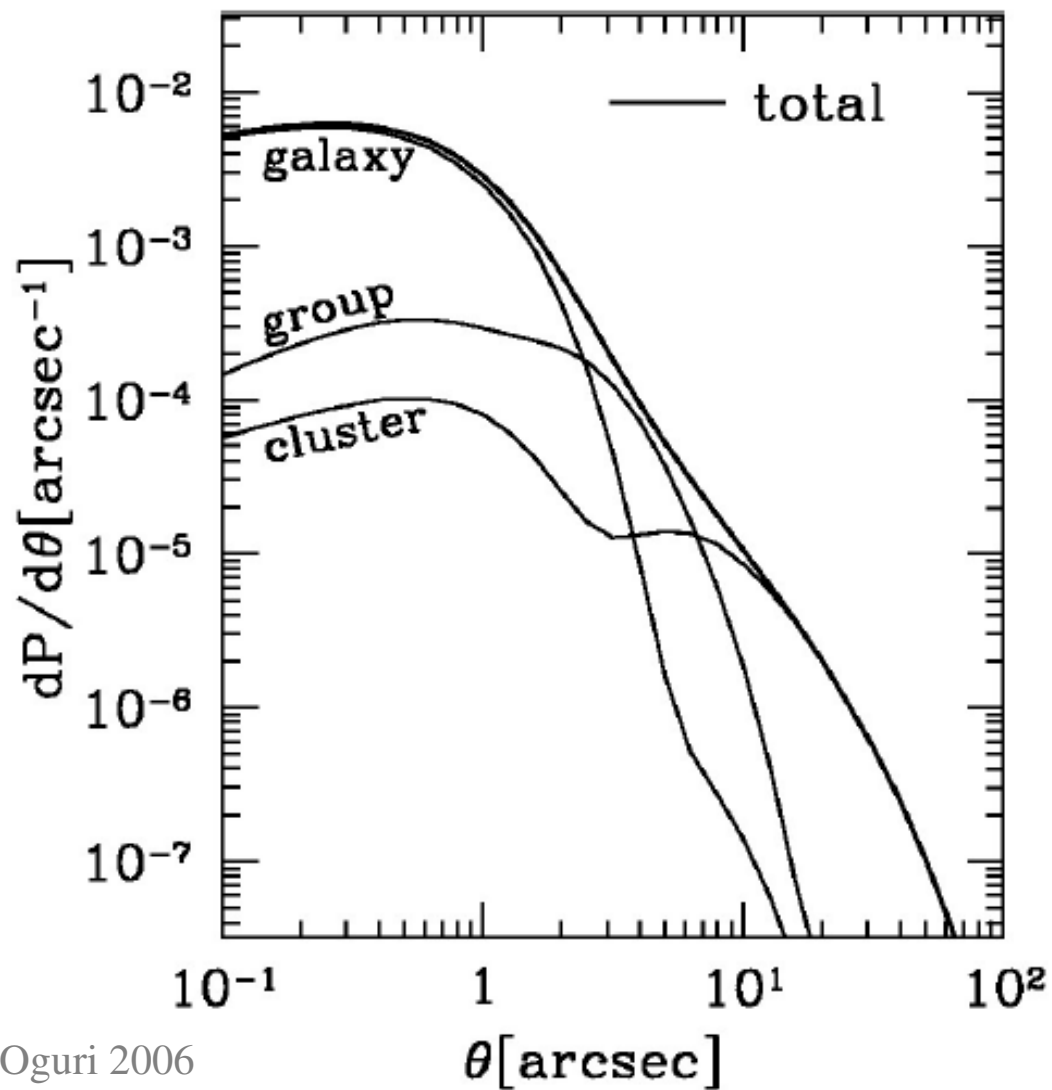
lens density parameter

cosmological  
dependence



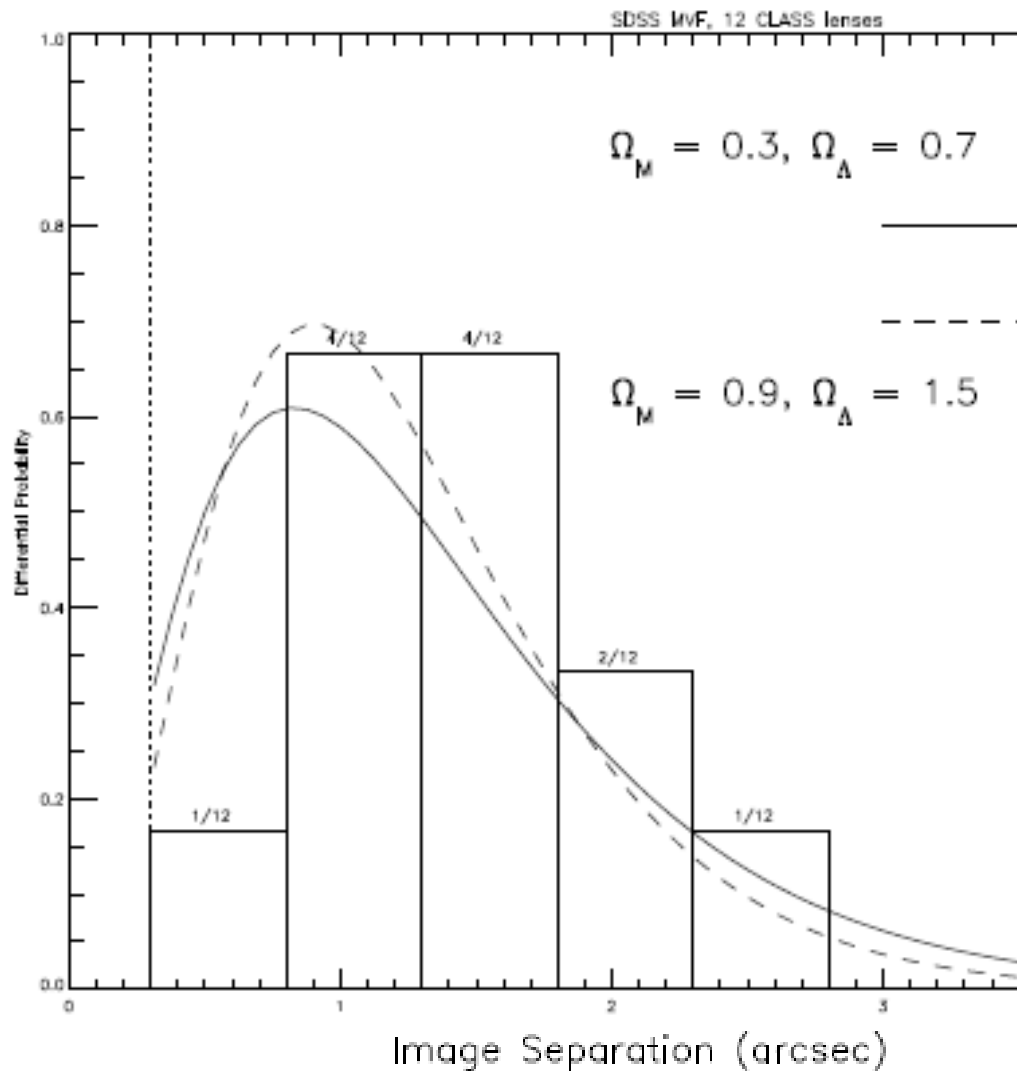
# Distribution of lens image separations

**predicted** (using halo model)



Oguri 2006

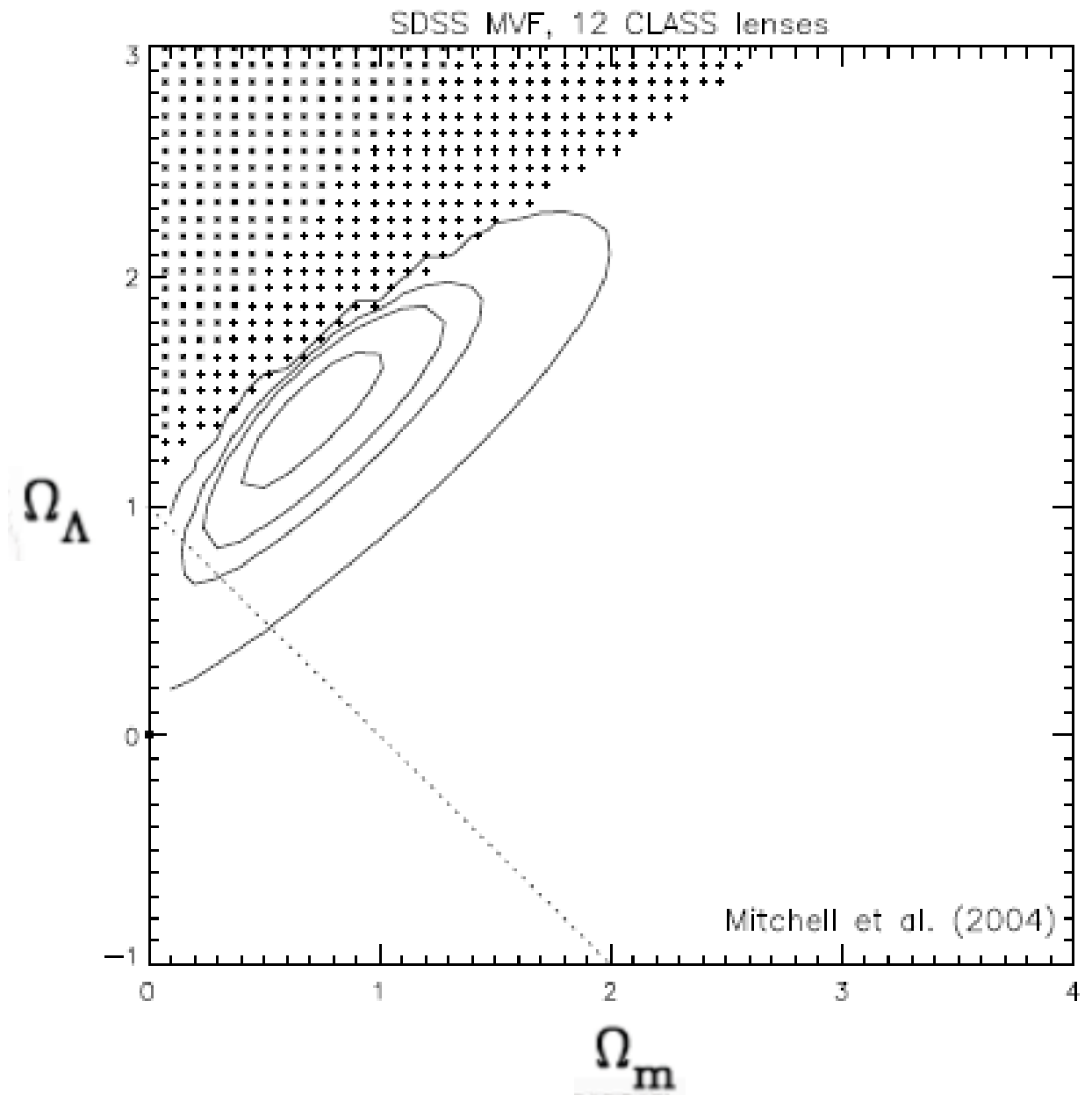
**measured** (12 CLASS lenses)



Mitchell et al. 2004

→ maps halo mass function

# Cosmography

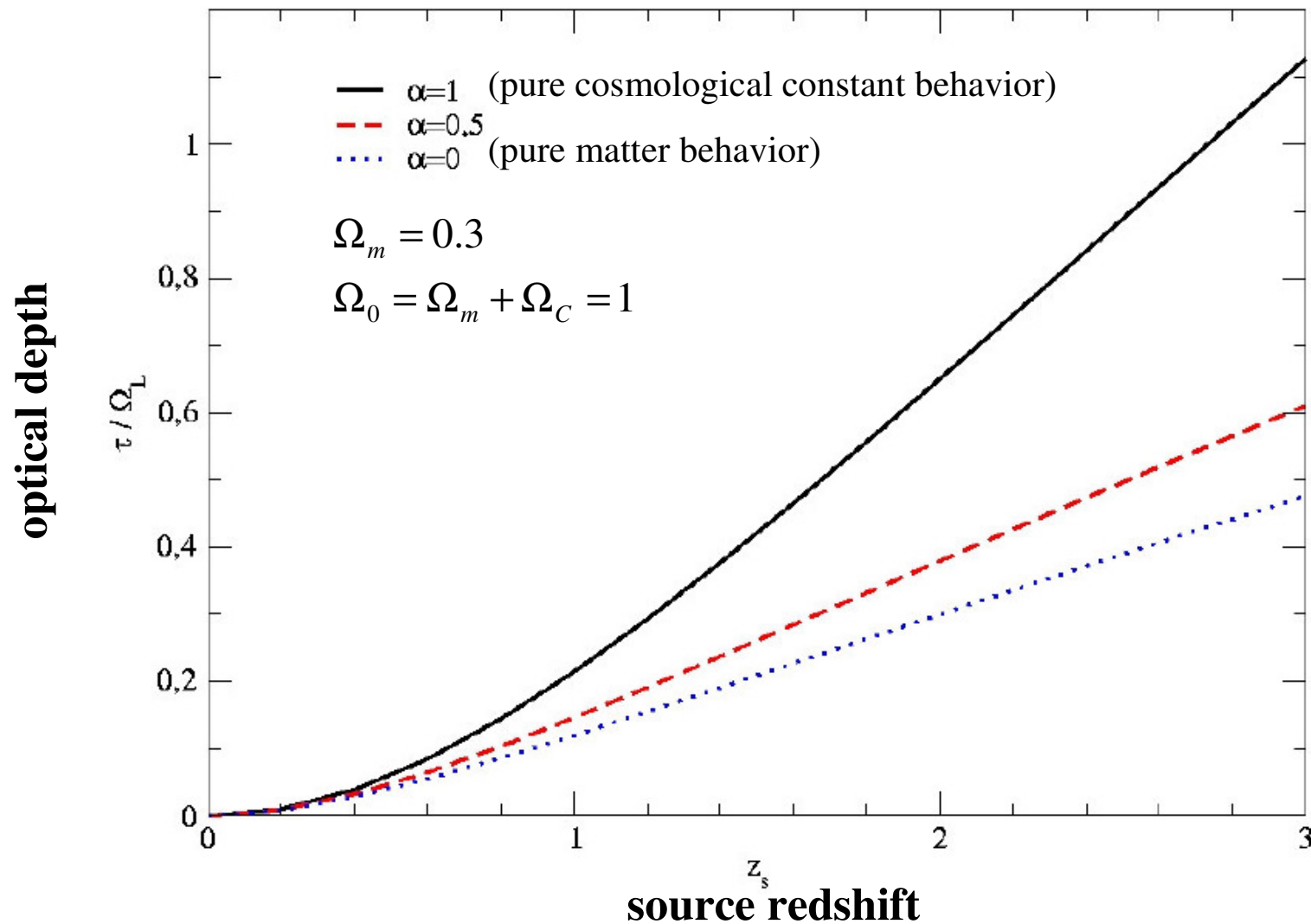


Likelihood functions for the cosmological model using the velocity function of galaxies measured from the SDSS survey and a sample of 12 CLASS lenses. The contours show the 68, 90, 95 and 99% confidence intervals on the cosmological model. In the shaded regions the cosmological distances either become imaginary or there is no big bang

# Optical Depth for a Chaplygin gas model

$$\begin{cases} p_C = -\alpha \rho_{C0} \left( \frac{\rho_{C0}}{\rho_C} \right)^\alpha \\ \rho_C = \rho_{C0} \left[ \alpha + (1-\alpha) a^{-3(1+\alpha)} \right]^{1/(1+\alpha)} \end{cases}$$

$$H(a) = H_0 \left\{ \Omega_m a^{-3} + (1-\Omega_m) \left[ \alpha + (1-\alpha) a^{-3(1+\alpha)} \right]^{1/(1+\alpha)} \right\}^{1/2}$$

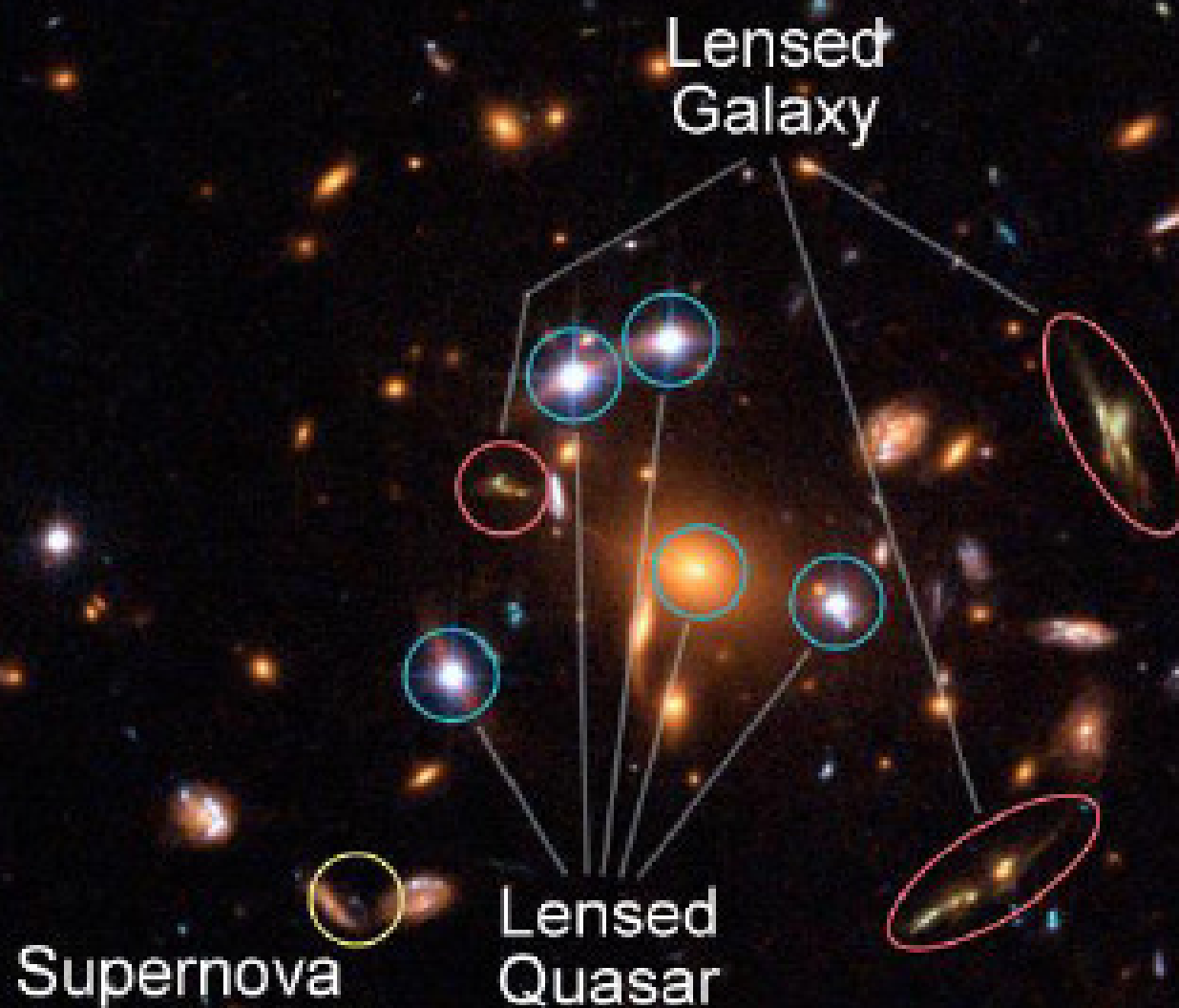


## Other areas of application (non exhaustive)

- cluster lensing (giant arcs statistics, gravitational telescopes)
- supernovae lensing
- microlensing (planetary discovery, galactic substructure, MACHOS)



Galaxy Cluster SDSS J1004+4112  
HST ACS/WFC



10"