DM particles: How warm they can be?

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Brief Contents

Our purpose is to consider a universe filled by a WDM, using ideal relativistic gas as a model. For this end we construct a simple approximation for relativistic gas of massive particles (RRG model).

The equation of state is given by an elementary function and admits analytic solution of the Friedmann equation.

We analyze density pertubartions in the RRG model and impose an upper bound for the velocities of the WDM by using the LSS data.

Assuming that DM is formed by noninteracting particles forming ideal gas, the density perturbation analysis tell us to which extent the DM content can be hot or at least warm, independent on the physical nature of this content.

Relativistic Gas

Consider the Universe filled out by a gas of relativistic particles. The equation of state for this gas is known since 1911 (F. Juttner).

Starting from the Maxwell distribution we arrive at

$$\rho_M(P) = \frac{K_3(\rho_d/P)}{K_2(\rho_d/P)}\,\rho_d - P\,.$$

Here $\rho_d = Nmc^2/V$ is the density of the rest mass and

$$K_n(\zeta) = \left(\frac{\zeta}{2}\right)^n \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(n+\frac{1}{2}\right)} \int_o^\infty e^{-\zeta y}(y^2-1)^{n-\frac{1}{2}} dy$$

is the modified Bessel function.

Not really useful to resolve de Friedmann equation !!

Model of reduced relativistic gas (RRG)

Instead of using the Maxwell distribution we assume that all particles have equal kinetic energies.

This is what we call the "Reduced Relativistic Gas Model" (RRG).

The simplified state equation is

$$P = \frac{\rho}{3}\left[1 - \frac{\rho_d^2}{\rho^2}\right].$$

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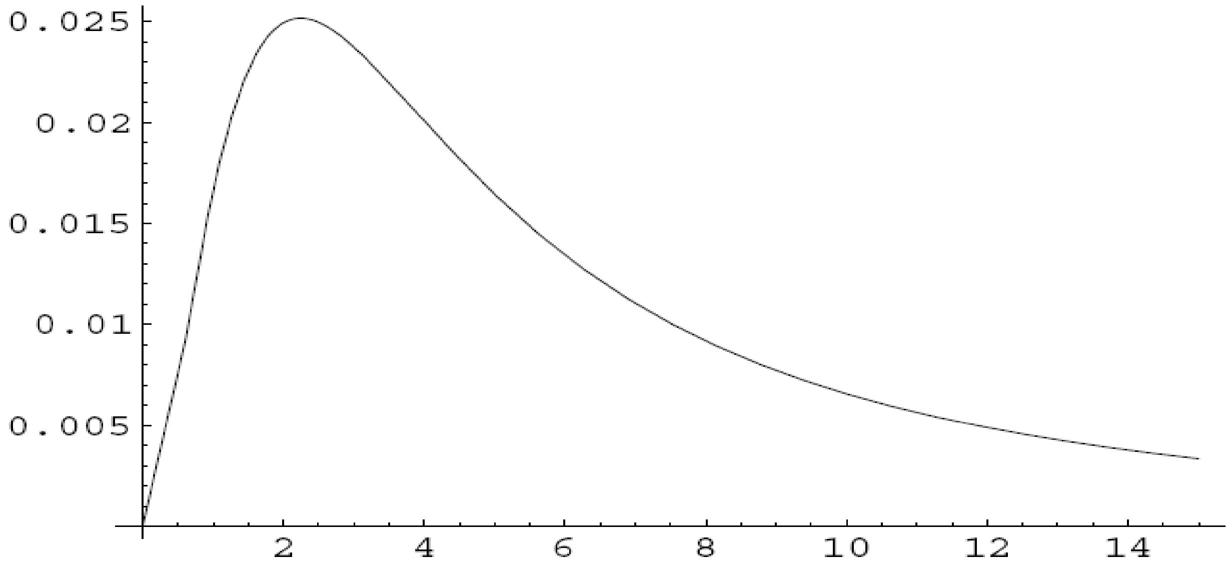
Maxwell x RRG

What is the difference between RRG and the maxwell case?

Comparing the two distributions numerically, one can obtain the plot of

$\delta_\rho = \frac{|\rho - \rho_M|}{\rho_M}$ x ρ .



The difference with the Maxwell distribution is always below 2.5% .

Dependence on the scale factor

From the relation $$3\frac{da}{a} = -\frac{d\rho}{\rho + P},$$

we can easily find

$$\rho(a) = \left[\rho_1^2 \left(\frac{a_0}{a}\right)^6 + \rho_2^2 \left(\frac{a_0}{a}\right)^8\right]^{1/2}.$$

Very important parameter!!

where ρ_1 and ρ_2 depend on the initial data.

We can write that formula as:

$$\rho(z) = \frac{\rho_c^0 \Omega_M^0}{\sqrt{1+b^2}}(1+z)^3\sqrt{1+b^2(1+z)^2}$$

$$\Omega_M^0 = \Omega_{DM}^0 + \Omega_{BM}^0 \quad , \quad b = (\rho_2/\rho_1)$$

and ρ_c^0 is the present day critical density.

Note that in the extreme cases:

$\rho_2 = 0, \quad \rho \approx a^{-3}$ dust

$\rho_1 = 0, \quad \rho \approx a^{-4}$ radiation

Solving the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{k}{a^2} = \frac{8\pi G}{3}\rho\,.$$

For simplicity, we consider $k = 0$. The solution has the form:

$$\left(a^2 + b^2\right)^{3/4} = \sqrt{6\pi G\rho_1} \cdot t$$

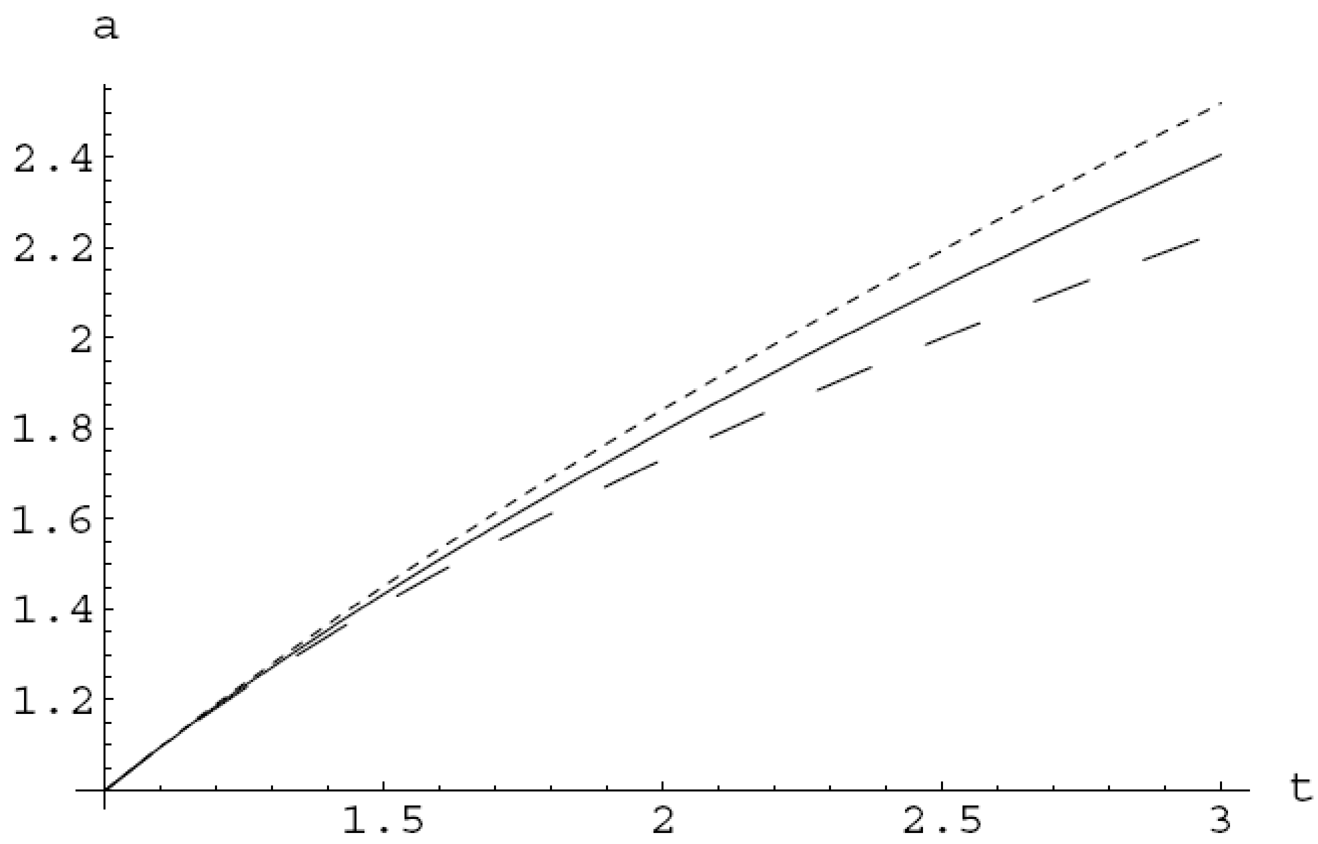
The RRG model interpolates between the era dominated by the radiation and the era dominated by the dust.

$$\rho_1 \ll \rho_2 \implies a \approx t^{1/2}$$ radiation

$$\rho_1 \gg \rho_2 \implies a \approx t^{2/3}$$ dust

Graphic

The plot of conformal factor versus time.



The presence of the RRG accelerates the expansion of the universe compared to the pure radiation case

b - parameter

The **dimensionless parameter *b*** shows whether the velocity of RRG particles is large or small or,
in other words, whether the matter is "cold", or "warm", or hot.

In order to better understand the physical sense of this parameter, let's express it in the form:

$$b = \frac{\rho_d^o}{\rho^o} = \frac{\beta}{\sqrt{1-\beta^2}} \quad .$$

$b \approx 0$ Means that the particles are norelativistic and RRG is nothing but the dust, but for small velocites one can just set $b = \beta$.

$b \to \infty$ Particles are ultra-relativistic.

Perturbations spectrum

Let us consider the simultaneous perturbations of metric, energy density and 4-velocity in the co-moving coordinates

$$\rho \to \rho(1+\delta)\,,\; g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}\,,\; U^{\alpha} \to U^{\alpha} + \delta U^{\alpha}$$

In the synchronous coordinates we have $h_{0\mu} = 0$. The perturbation of the pressure is given by

$$\delta P = \frac{\delta\rho(1-r)}{3}$$

where $r = r(z) = \rho_d^2(z)/\rho^2(z)$.

In this way we arrive at the 00-component of the Einstein equation

$$h' - \frac{2h}{(1+z)} = -\frac{f_1(2-r)}{g}\delta\,,$$

where $h = \partial_t\left(h_{kk}/a^2\right)$.

Here the functions $g(z)$ and $f_1(z)$ are defined as

$$g(z) = \frac{(1+z)H}{3[H^2 - \Omega_k^0 H_0^2(1+z)^2]}\,,$$

$$f_1(z) = \frac{\rho(z)}{\rho_t(z)} = \frac{(1+z)(H^2)' - 2\Omega_k^0 H_0^2(1+z)^2}{[H^2 - \Omega_k^0 H_0^2(1+z)^2](4-r)}\,.$$

Other equation follow from the variation of the conservation law $\delta(\nabla_\mu T^\mu_\nu) = 0$

$$\delta' - \frac{1}{(1+z)}[4 - r - \frac{(1+z)\rho'}{\rho}]\,\delta + \frac{4-r}{3H(1+z)}(\frac{h}{2} - \frac{v}{f_1}) = 0\,,$$

and

$$v' + (\frac{\rho'}{\rho} - \frac{r'}{4-r} - \frac{5}{1+z} - \frac{f_1'}{f_1})\,v + \frac{k^2(1+z)f_1}{H}\frac{1-r}{4-r}\delta = 0,$$

where $v = f_1(\nabla_k \delta U^k)$ and we used Fourier expansions

$$v(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3}\, v(\vec{k},t)\, e^{i\vec{k}\cdot\vec{x}}\,,$$

$$\delta(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3}\, \delta(\vec{k},t)\, e^{i\vec{k}\cdot\vec{x}}\,,$$

$$k = \left|\vec{k}\right|.$$

Numerical analysis

The numerical analysis was performed using a transfer function that assume a scale invariant primordial spectrum, and determine the spectrum today considering th Universe with the cosmological constant and filled by DM.

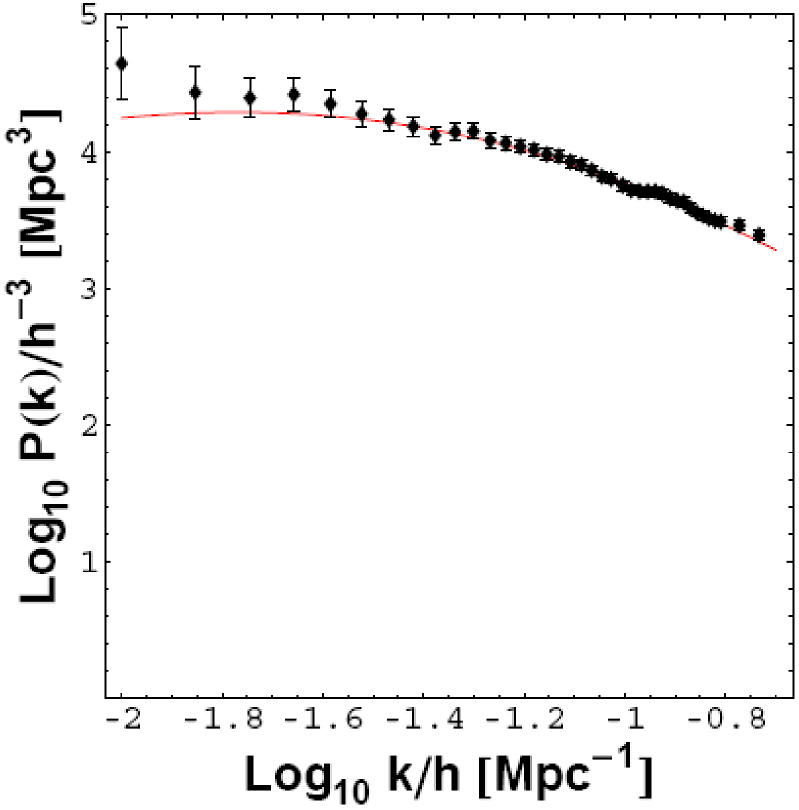
Using the transfer function we can fix the initial conditions at the redshift

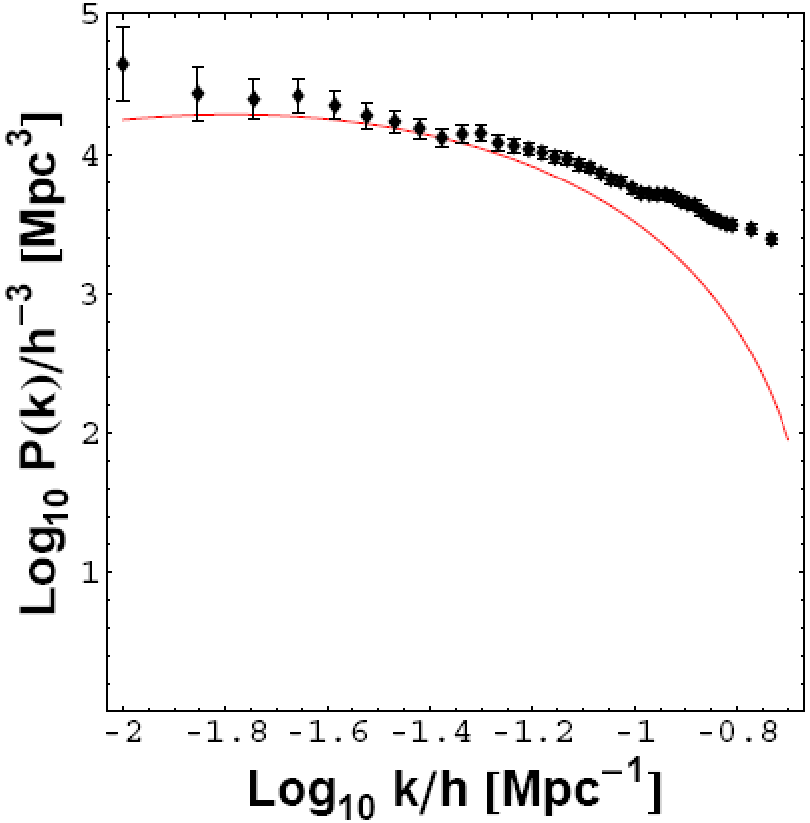
after the recombination epoch.

The relevant quantity to be compared with the 2dFGRS observational data is the power spectrum parameter defined by $\mathcal{P}_k = \delta_k^2$, where δ_k is the component of the Fourier transform of the density contrast $\delta(t)$, which is computed by integrating the equation for the cosmic perturbations for a given value of K and with a given initial conditions.

Results

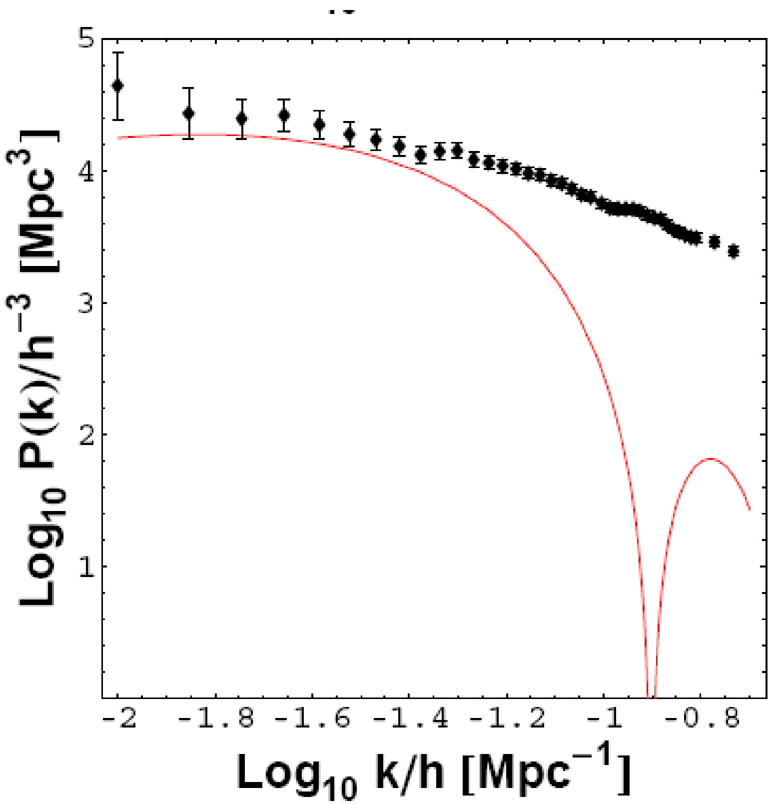
Power spectrum for the values b = 10^{-5} and for b = 10^{-4}

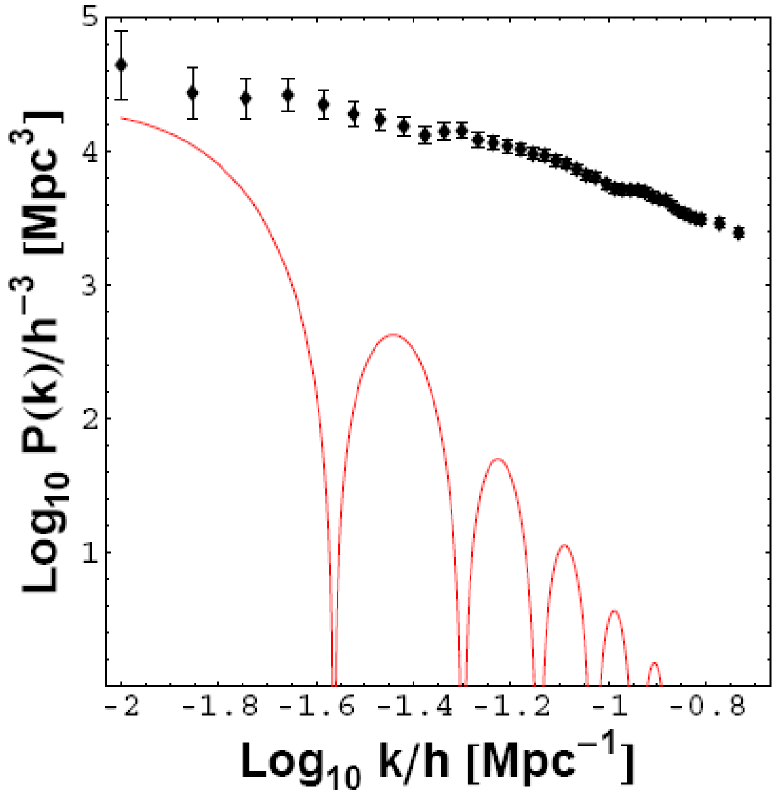


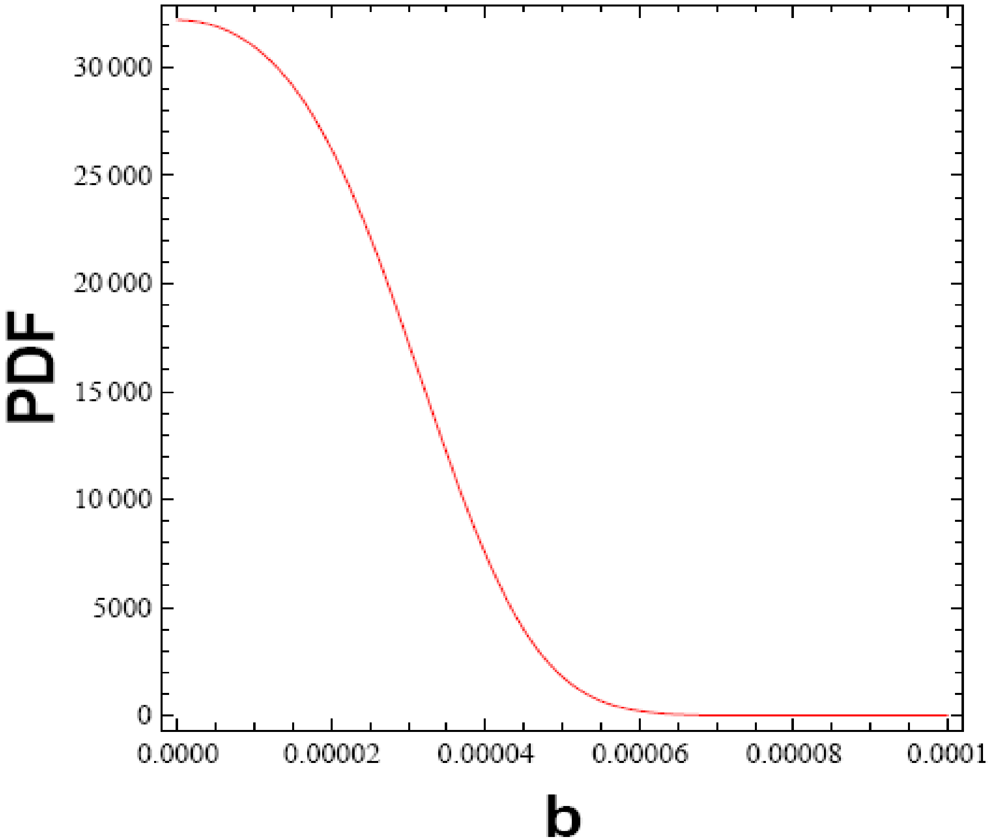


Grafics

Power spectrum for the values b = 2 x 10^{-4} and for b = 10^{-3}







Probability distribution for the parameter ***b***. The probability becomes essentially zero for ***b*** $\geq 5 \times 10^{-5}$

Discussions and conclusions

We have considered the structure formation in the model where DM is described by the ideal relativistic gas of identical massive particles. As a result we arrive at the strong limit on the parameter b, which should satisfy the upper $b \leq 3-4\times 10^{-5}$. This is equivalent to the upper bound on the velocities of the $v \leq v_o = 3-4\times 10^{-5} c = 10-12 km/s.$

The red-shift behavior of the DM velocities has a standard form,

V(z) ~ $T_{CMB.}$

This result does not mean that the actual velocities of DM particles can not be greater than the v_O Both DM and baryonic matter can acquire an extra kinetic energy after the galaxy starts to form and the linear regime of the cosmological perturbations can not be applied. The result is valid only in the linear regime. However, the model presented above enables one to consider a number of interesting generalizations, including interacting BM or/and DM.