Dark Sector Candidates from Bi-gravity

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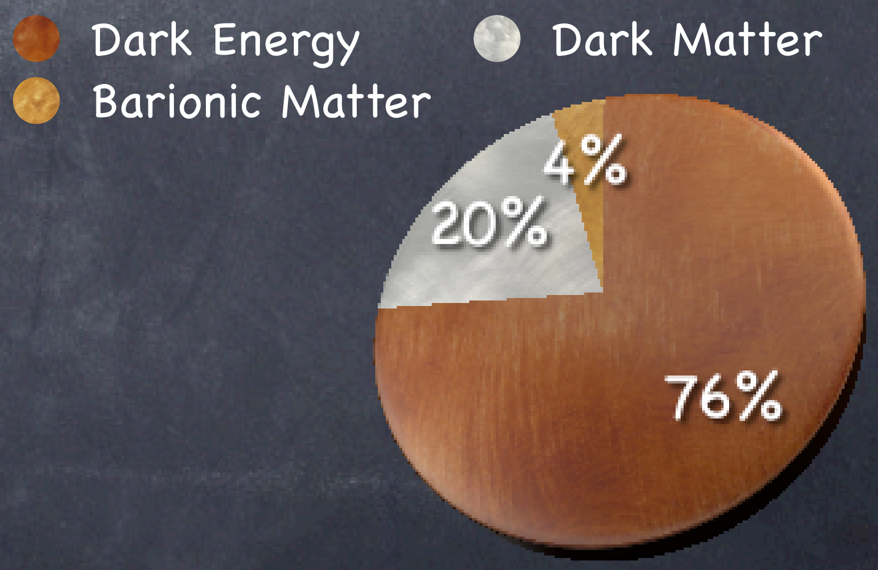
I’ll mainly talk on the results found in

- D.C. Rodrigues, “Evolution of Anisotropies in Eddington-Born-Infeld Cosmology”, Phys.Rev.D78:063013,2008.

- M. Bañados, A. Gomberoff, D.C. Rodrigues, C. Skordis, “ A note on Bi-gravity and Dark Matter”, arXiv:0811.1270 [gr-qc], to appear in PRD.



Introduction: Bi-gravity

- Bi-gravity has a long history, it dates back to 1971 [C.J. Isham, A. Salam, J.A. Strathdee, Phys.Rev.D3:867-873,1971].

- It describes two types of interacting spin-2 particles (one massive and the other massless).

- Recent works revived the original idea but with focus on cosmology [I.I. Kogan, G.G. Ross, Phys.Lett.B485:255-262,2000; T. Damour, I.I. Kogan, A. Papazoglou, Phys.Rev.D66:104025,2002, Phys.Rev.D66: 104024, 2002; N. Arkani-Hamed, H. Georgi, M.D. Schwartz, Annals Phys.305:96-118,2003, etc].

Among other previously known applications...

- It provides an alternative explanation to the current universe expansion.
- It appears as an effective 4D model for higher dimensional models (e.g. DGP).

- Our main goal is to extend the cosmological possibilities for bi-gravity. In particular, its connection to Dark Matter.

Introduction: EBI

- Last year a model with a different approach to the Dark Sector was proposed, Eddington-Born-Infeld (EBI) [M. Bañados, Phys.Rev.D77: 123534, 2008].

- Isotropy perturbations in EBI are not trivial and open new physical possibilities [D.C. Rodrigues, Phys.Rev.D78:063013,2008].

- In [M. Bañados, A. Gomberoff, D. C. Rodrigues, C. Skordis, 0811.1270], it's shown that the EBI model is a limiting case of bi-gravity, and its dark matter connection is presented.

I'll start by introducing EBI and then moving to bi-gravity.

Eddington-Born-Infeld

- Its action was proposed as an effective 4D extension of General Relativity for (almost) singular metrics. Schematically,

Einstein-Hilbert (g) + Dynamical, Lorentz Inv. and Metric Independent (C) + Interaction (g,C)

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↓ Eddington term: $\sqrt{|K_{\mu\nu}(C)|}$

↓ Born-Infeld

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Ricci tensor — new affine connections.

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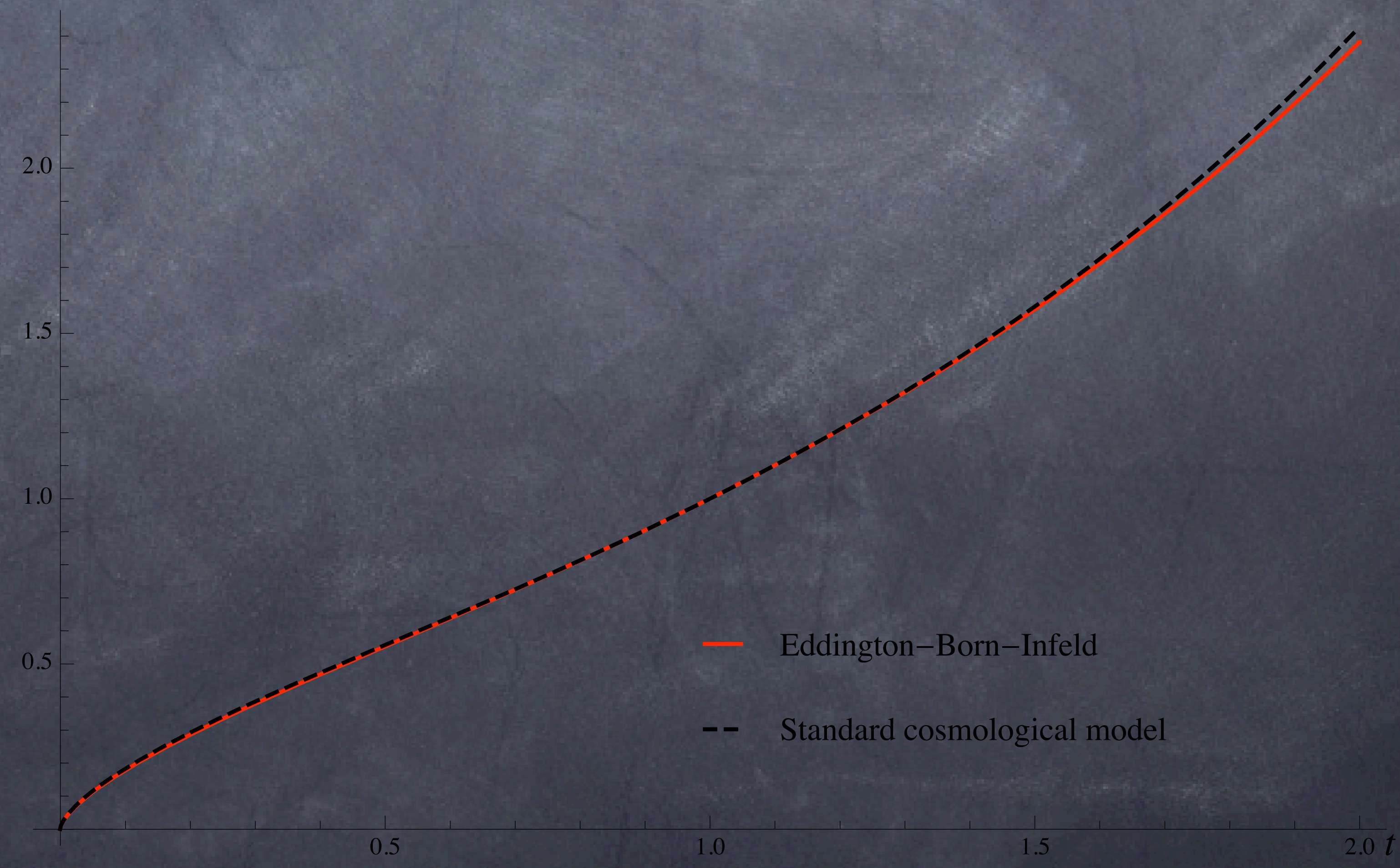
↓ Born-Infeld

$$S_{\rm EBI}[g,C] = \frac{1}{16\pi G}\int\left[\sqrt{-g}\left(R-\frac{2\alpha_g}{\ell^2}\right)+\frac{2}{\ell^2\alpha_q}\sqrt{|g_{\mu\nu}-\ell^2 K_{\mu\nu}|}\right]d^4x,$$

EBI as a Unified Dark Sector ($\alpha_g = 0$)

(background solution)

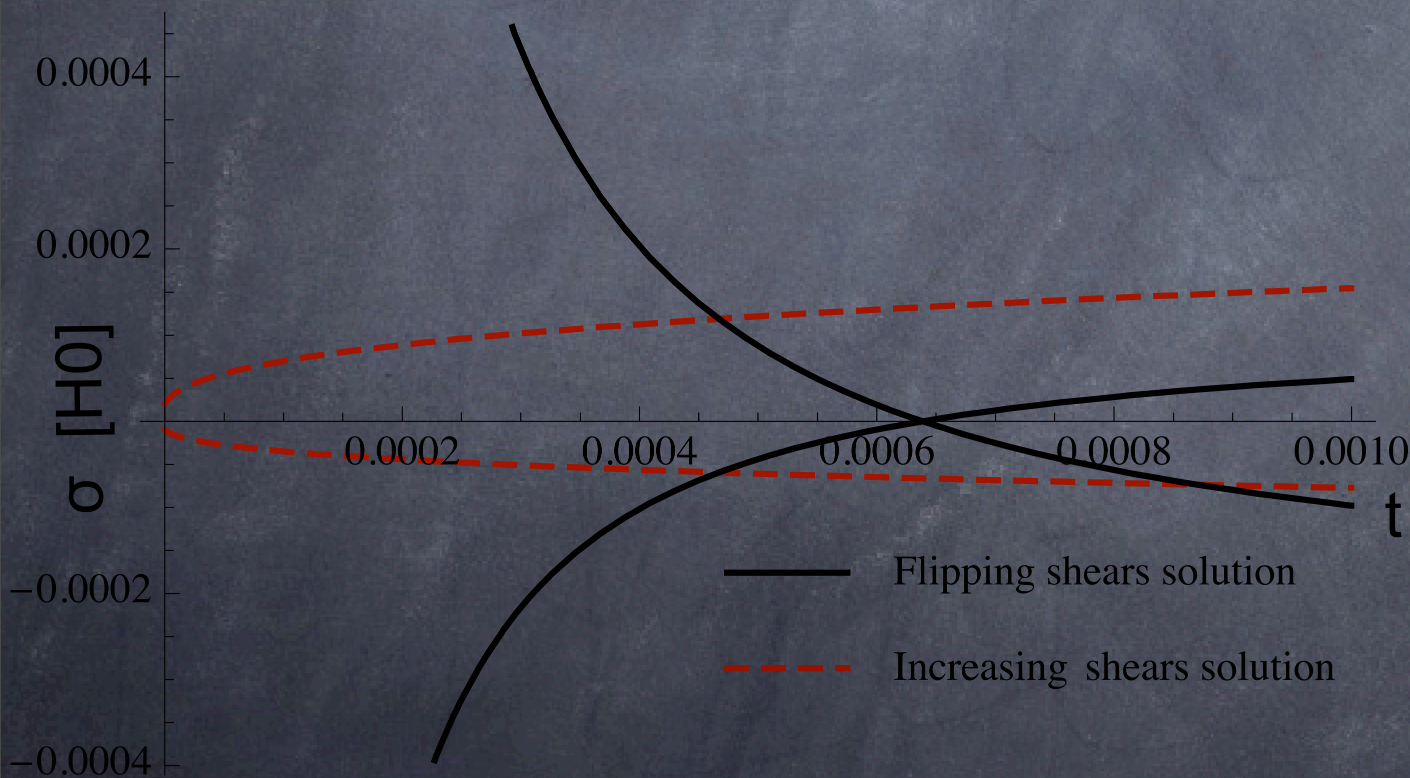


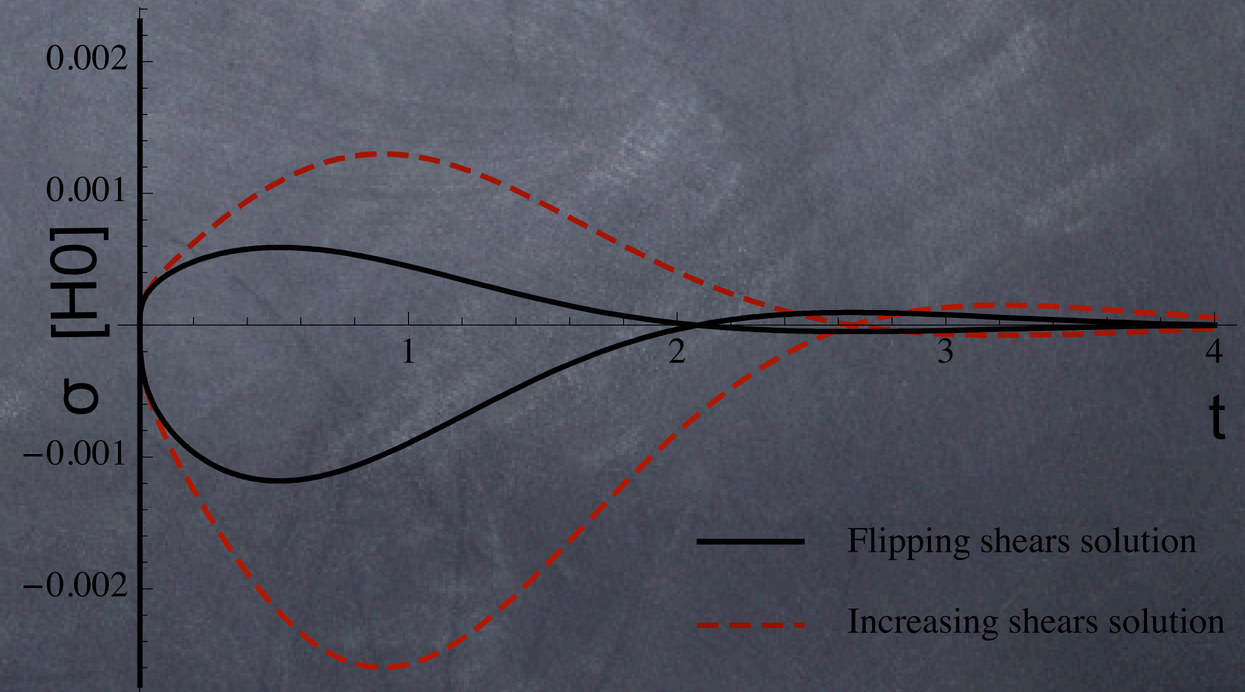
Eddington-Born-Infeld: Isotropy Perturbations

- Since EBI's fundamental fields are not scalars, isotropy issues are raised.

- In [D.C. Rodrigues, Phys.Rev.D78:063013,2008] it was shown that (fortunately) the isotropic solution is an atractor.

- However, the shear's evolution can be quite involving,





Recombination at t=0
Present time at t=1

Eddington-Born-Infeld and the LSS (A Comment)

- The Unified EBI does not work well for the CMB angular power spectrum.

- But it do work well as an alternative to standard Dark Matter. This is the ΛEBI model. [M. Bañados, P.G. Ferreira, C. Skordis, 0811.1272]

Bi-gravity action

- The Bi-gravity action reads [C.J. Isham, A. Salam, J.A. Strathdee, Phys.Rev.D3:867-873,1971]

Einstein-Hilbert(g) + Einstein-Hilbert(q) + Interaction(q,g)

- The interaction is generalization of massive gravity that does not depends on fixed backgrounds. – It has a Fierz-Pauli potential form for massive spin-2 fields.

- It can be set in the form

$$S[g,q] = \frac{1}{16\pi G}\int\left[\left(R_g - 2\frac{\alpha_g}{\ell^2}\right)\sqrt{|g|} + \left(R_q - 2\frac{\alpha_q}{\ell^2}\right)\sqrt{|q|} + \mathcal{V}\right] d^4x$$

$$\mathcal{V} = \sqrt{|q|}\frac{1}{\ell^2}\left[-q^{\mu\nu}g_{\mu\nu} + \kappa\left((q^{\mu\nu}g_{\mu\nu})^2 - q^{\mu\nu}g_{\nu\alpha}q^{\alpha\beta}g_{\beta\mu}\right)\right].$$

-

Nonlinear Massive Gravity

"Strong Gravity"

A nonlinear extension that removes the reference to a fixed background was proposed in [2].

Let $\gamma_{\mu\nu}$ be the background metric (Minkowski or de Sitter) and $\gamma^{\mu\nu}$ its inverse, so

$$S_{\text{MG}} = S_{\text{EH}}[g] + \frac{M^2}{8k} \int \sqrt{-\gamma} \left((\gamma^{\mu\nu} h_{\mu\nu})^2 - h_{\mu\nu}\gamma^{\nu\alpha} h_{\alpha\lambda}\gamma^{\lambda\mu} \right) d^4x.$$

$\gamma_{\mu\nu}$ (fixed) $\longrightarrow f_{\mu\nu}$ (dynamical) $\quad \downarrow \quad$ $h_{\mu\nu} \, (= g_{\mu\nu} - \gamma_{\mu\nu}) \;\rightarrow\; g_{\mu\nu} - f_{\mu\nu}$

$$S_{\text{SG}}[g,f] = S_{\text{EH}}[g] + S_{\text{EH}}[f] + \frac{M^2}{8k} \int \sqrt{-f} \left[12 - 6 f^{\mu\nu} g_{\mu\nu} + (f^{\mu\nu} g_{\mu\nu})^2 - f^{\mu\nu} g_{\nu\alpha} f^{\alpha\beta} g_{\beta\mu} \right] d^4x$$

[2] C. J. Isham, A. Salam and J. A. Strathdee, "F-dominance of gravity," Phys. Rev. D **3**, 867 (1971).

Bi-gravity and EBI

- EBI can be found from Bi-gravity when the product (g q^{-1}) is small [M. Bañados, A. Gomberoff, D. C. Rodrigues, C. Skordis, 0811.1270],

$$\mathcal{V} \approx \sqrt{|q|}\,\frac{1}{\ell^2}\left(-q^{\mu\nu} g_{\mu\nu}\right).$$

$$S[g,q] = \frac{1}{16\pi G}\int \left[\left(R_g - 2\frac{\alpha_g}{\ell^2}\right)\sqrt{|g|} + \left(R_q - 2\frac{\alpha_q}{\ell^2}\right)\sqrt{|q|} + \mathcal{V}\right] d^4x$$

↓ After a Polyakov-like duality transf., the Bi-gravity action becomes

$$S_{\rm EBI}[g,C] = \frac{1}{16\pi G}\int \left[\sqrt{-g}\left(R - \frac{2\alpha_g}{\ell^2}\right) + \frac{2}{\ell^2\alpha_q}\sqrt{|g_{\mu\nu} - \ell^2 R^q_{\mu\nu}|}\right] d^4x,$$

- Since EBI can work as Dark Matter (at least for the LSS), this points the same answer should hold for bi-gravity.

- And... Indeed that's true. In particular, for the background,

$$3\frac{\dot{a}^2}{a^2} \approx \frac{\mathrm{const}(q_0)}{a^3} + O(a^{-2})$$

Hence,

$$\frac{1}{a^2}\left[-2\Delta\Psi+6\frac{a'}{a}\Psi'+6\Phi\left(\frac{a'}{a}\right)^2\right]=\frac{Y^3}{a^3X\ell^2}\left[-\Phi+\Phi^{\,q}-3\Psi+3\Psi^q+\frac{Y^3}{aX}\left(\frac{aX}{Y^3}\Xi\right)'+\Delta\xi\right]+\frac{6\kappa Y}{aX\ell^2}\left[\Phi-\Phi^q+\Psi-\Psi^q-\frac{Y}{aX}\left(\frac{aX}{Y}\Xi\right)'-\frac{1}{3}\Delta\xi\right],$$

$$\frac{1}{a^2}\left[(\Psi-\Phi)_{,ij}+\delta_{ij}\left(\Delta\Phi-\Delta\Psi+2\frac{a'}{a}\Phi'+4\frac{a'}{a}\Psi'+2\Psi''-2\left(\frac{a'^2}{a^2}-2\frac{a''}{a}\right)\Phi\right)\right]=\frac{XY}{a\ell^2}\left[\delta_{ij}\left(\Phi-\Phi^q-\Psi+\Psi^{\,q}-\frac{1}{aXY}(aXY\Xi)'+\Delta\xi\right)-2\xi_{,ij}\right]+$$

$$+4\kappa\frac{aX}{Y\ell^2}\left[\delta_{ij}\left(-\Phi+\Phi^{\,q}-\Psi+\Psi^{\,q}+\frac{Y}{aX}\left(\frac{aX}{Y}\Xi\right)'\right)+\xi_{,ij}\right]+2\kappa\frac{Y}{aX\ell^2}\left[\delta_{ij}\left(\Phi-\Phi^q+\Psi-\Psi^q-\frac{Y}{aX}\left(\frac{aX}{Y}\Xi\right)'-\Delta\xi\right)+2\xi_{,ij}\right],$$

$$\frac{2}{a^2}\left(\Psi'_{,i}+\frac{a'}{a}\Phi_{,i}\right)=\frac{1}{\ell^2}\left(-\frac{XY}{a}+4\kappa\frac{aX}{Y}\right)\left(\Xi_{,i}+\frac{Y^2}{a^2X^2}\xi'_{,i}\right).$$

For $\kappa=0$ and in the Newtonian gauge, the above equations are the same found in[2] [2]. And the corresponding equations for q are

$$\frac{1}{Y^2}\left[-2\Delta\Psi^q+6\frac{Y'}{Y}\Psi^{q\prime}\frac{Y^2}{a^2X^2}+6\Phi^q\left(\frac{Y'}{Y}\right)^2\frac{Y^2}{a^2X^2}\right]=\frac{1}{\ell^2X^2}\left[\Phi-\Phi^q-\frac{(\Xi a)'}{a}\right]+\frac{3a^2}{\ell^2Y^2}\left[\Psi-\Psi^q+\Xi\frac{a'}{a}-\Delta\xi\right]+$$

$$+6\kappa\frac{a^2}{X^2Y^2\ell^2}\left[-\Phi+\Phi^{\,q}+\Psi-\Psi^q+\frac{(a^2\Xi)'}{a^2}-\Delta\xi\right]+12\kappa\frac{a^4}{\ell^2Y^4}\left[-\Psi+\Psi^{\,q}-\Xi\frac{a'}{a}+\frac{1}{3}\Delta\xi\right], \tag{14}$$

$$\frac{1}{Y^2}\left[\left(\Psi^q_{,ij}-\Phi^q_{,ij}\right)+\delta_{ij}\left(\Delta\Phi^q-\Delta\Psi^q-\frac{2}{3}Y^2\bar{Q}_k^{\,k}\Phi^q+2\frac{Y'}{Y}\frac{Y^2}{a^2X^2}\Phi^{q\prime}+\frac{2}{aXY}\left(\frac{Y^3}{aX}\Psi^{q\prime}\right)'\right)\right]= \tag{15}$$

$$=\frac{1}{\ell^2X^2}\delta_{ij}\left(-\Phi+\Phi^{\,q}+\frac{(\Xi a)'}{a}\right)+\frac{a^2}{\ell^2Y^2}\left[\left(\Psi-\Psi^q+\Xi\frac{a'}{a}-\Delta\xi\right)\delta_{ij}+2\xi_{,ij}\right]+$$

$$+2\kappa\frac{a^2}{X^2Y^2\ell^2}\left[\delta_{ij}\left(\Phi-\Phi^q-\Psi+\Psi^{\,q}-\frac{(\Xi a^2)'}{a^2}+\Delta\xi\right)-2\xi_{,ij}\right]+4\kappa\frac{a^4}{\ell^2Y^4}\left[\delta_{ij}\left(\Psi-\Psi^q+\Xi\frac{a'}{a}\right)-\xi_{,ij}\right],$$

$$\frac{2}{Y^2}\left(\Psi^{q\prime}{}_{,i}+\frac{Y'}{Y}\Phi^q{}_{,i}\right)=\frac{a^2}{\ell^2Y^2}\left(1-4\kappa\frac{a^2}{Y^2}\right)\left(\Xi_{,i}+\xi'_{,i}\right), \tag{16}$$

where $\bar{Q}_k^{\,k}$ in (15) is the trace of the spacial components of the q-metric background Einstein tensor,

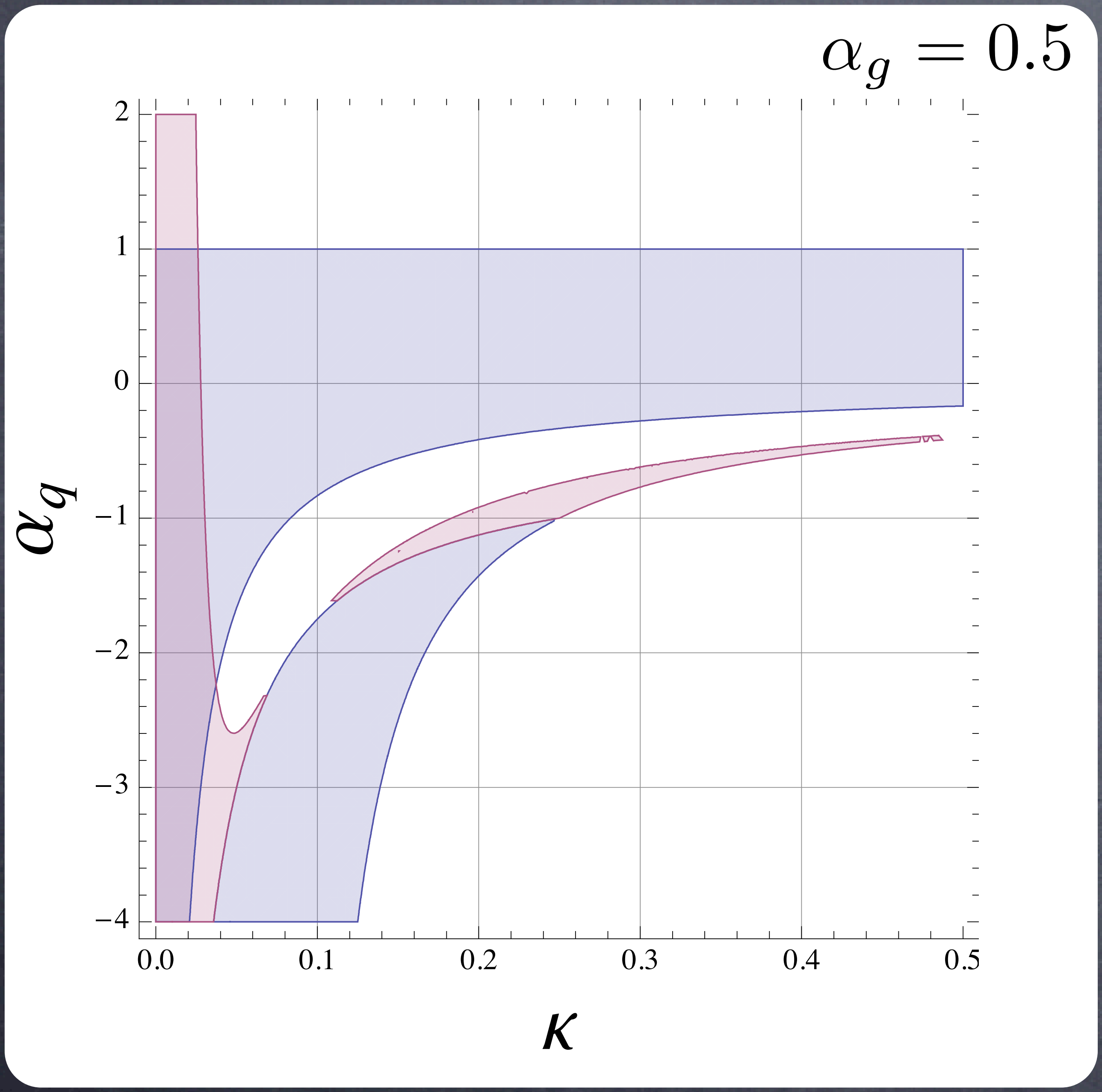
Bi-gravity and the Dark Sector

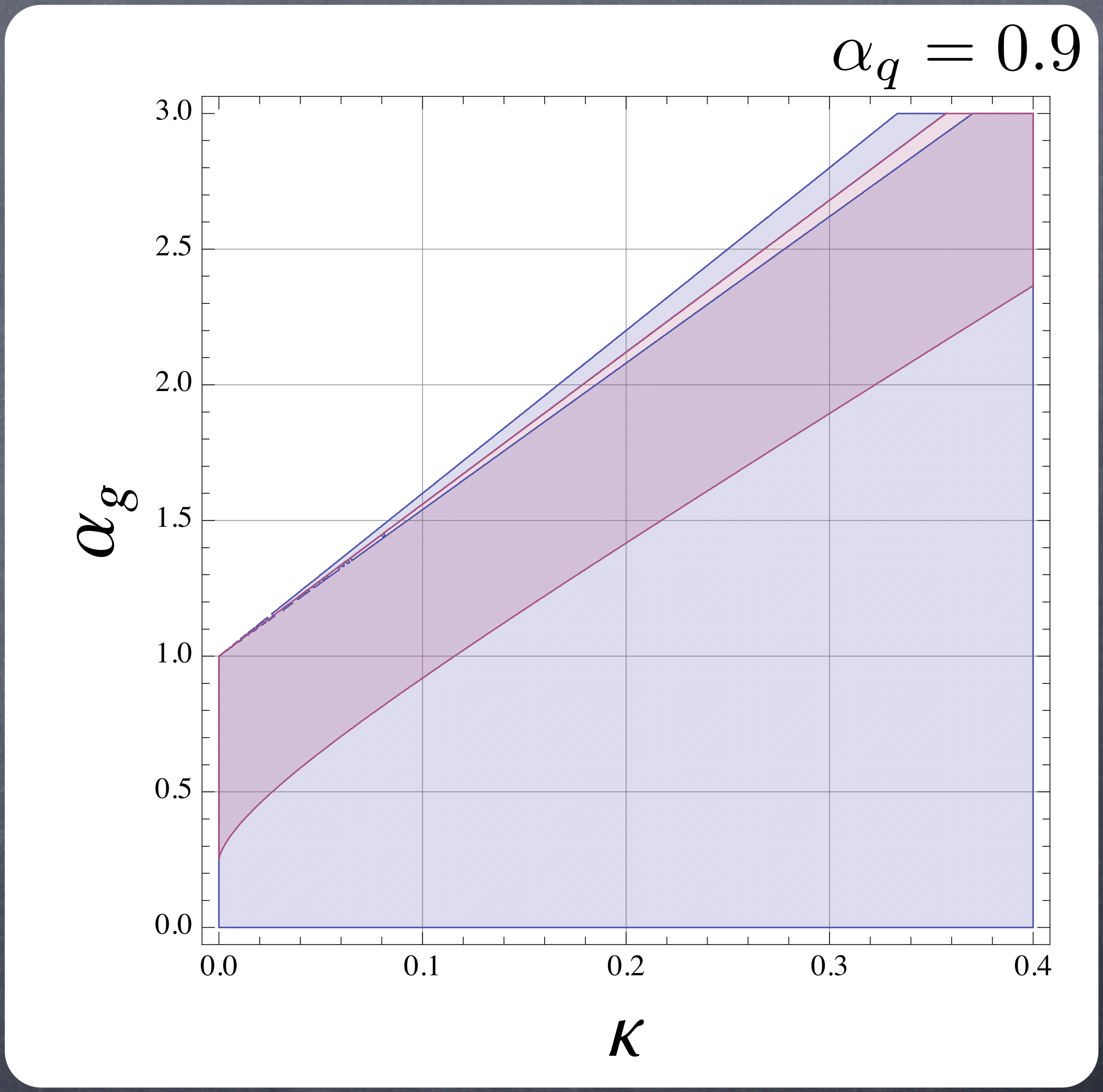
- EBI and Bi-gravity have the same LSS Dark Matter behavior.
- The background evolution of Bi-gravity is as good as ΛCDM.
- On the other hand, Bi-gravity potentials have more de Sitter solutions, with different perturbations. Hence...
- The issue of an Unified Dark Sector returns in Bi-gravity.
- Bi-gravity’s unstable de Sitter solutions opens inflationary possibilities of the Hilltop type.

de Sitter Solutions as Attractors

- Bi-gravity parameters: κ , α_g, α_q , ℓ. Only the first 3 are relevant.

- Under a reduced set of perturbations (preserving the FRW structure):





- Blue: $g \propto q$ de Sitter Sol; Red: $g \not\propto q$ de Sitter Sol.

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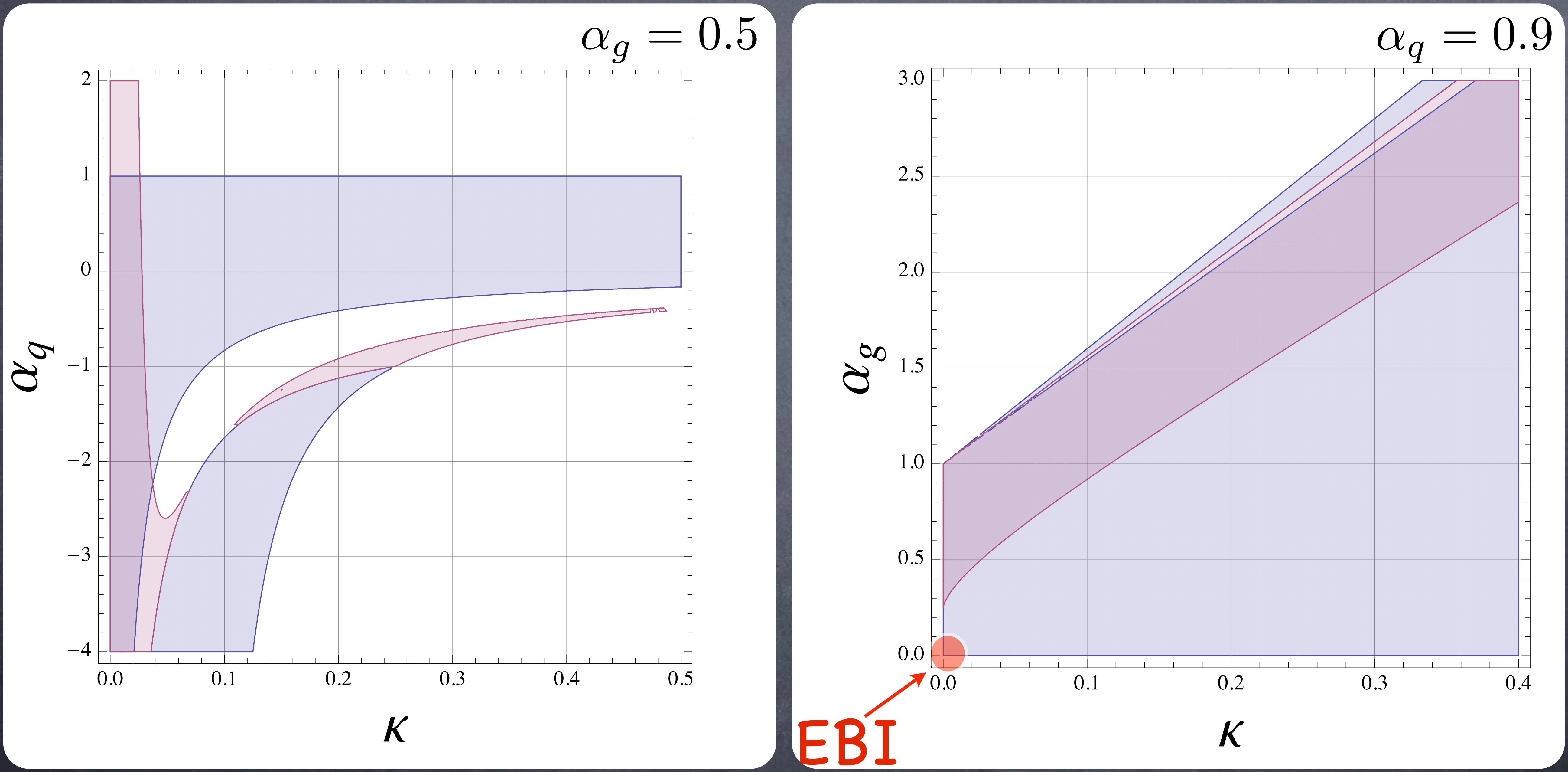

- Blue: $g \propto q$ de Sitter Sol; Red: $g \not\propto q$ de Sitter Sol.

Conclusions

- Bi-gravity has a long history, but now its cosmological implications are being evaluated in detail and in face of the current cosmological issues.

- It has ties with other models (Brane worlds, pure massive gravity, Connes' noncommutative geometry, etc); and a not-so-explored mathematical structure, which make of it (at least) a nice laboratory on effective physics.

Our results:

- Duality: EBI can be seen as the simplest Bi-gravity model.

- Dark Matter: we pointed that Bi-gravity can generate a Dark Matter behavior (up to linear perturbations). Progress is needed on structure formation, rotation curves etc.

- Unified dark sector: it's a possibility on bi-gravity models. With dark matter decaying into dark energy.

Other perspectives

- EBI galaxy rotation curves are already under work at PUC-Chile. Preliminary results are positive.

- Inflation?

- Bianchi Cosmologies and Bi-gravity.

- Can it be associated with some of the recent experimental anomalies attributed to Dark Matter (Dama, Pamela...)?

Linear Massive Gravity

A simple demonstration on the FP elimination of ghosts

First note there are 2 terms in S_{h^2} with the "wrong" kinetic sign. These are $(\partial_\alpha h^{\alpha\beta})^2$ and $(\partial_\alpha h)^2$.

In standard GR, the above terms are pure gauge, but once diffeomorphism invariance is broken they can become physical.

We show that this does not happens for the FP mass.

The Eq. of motion for the perturbations without mass read

$$X_{\mu\nu} \equiv \frac{1}{2}\left(\Box h_{\mu\nu} - \partial_\mu \partial_\alpha h^\alpha_\nu - \partial_\nu \partial_\alpha h^\alpha_\mu + \partial_\mu \partial_\nu h\right) + \frac{1}{2} g_{\mu\nu}\left(\partial_\alpha \partial_\beta h^{\alpha\beta} - \Box h\right) = 0.$$

One can check that the identity $\partial^\mu X_{\mu\nu} = 0$ holds ("background Biachi identity").

Once the mass terms are included, the EOM become,

$$X_{\mu\nu} - \frac{m_1^2}{2} h_{\mu\nu} + \frac{m_2^2}{2} h g_{\mu\nu} = 0 \text{ Ê } \Rightarrow \text{ Ê}\partial_\mu h^{\mu\nu} = \frac{m_2^2}{m_1^2} \partial^\nu h.$$

Using the latter, the trace of the EOM imply $h = 0$ iff $m_1^2 = m_2^2$. And hence, no ghosts.