Quintessence Models with Exponential Potentials

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Introduction

- The universe today is in a stage of accelerated expansion [1].
- One of the possible explanations is the presence of a scalar field called quintessence (Q) [2].
- One of the simplest models for quintessence is a scalar field with an exponential potential [3]

$$V = V_0 \exp(-\lambda \frac{Q}{M_P})$$

where $M_P = (8\pi G)^{-1/2}$ is the reduced Planck mass, V_0 and λ are constant parameters of the potential.

Fixed Points

- The dynamics of quintessence field is obtained solving the set of equations formed by the scalar field equation of motion and the Friedmann equation for a flat universe with appropriate initial conditions.
- This system has stable fixed points (fp) [4] for

$$\lambda > \sqrt{n} \quad \text{with} \quad \Omega_Q^{fp} = \frac{n}{\lambda^2} \quad \text{and} \quad \omega_Q^{fp} = \omega_n$$

$$\lambda < \sqrt{n} \quad \text{with} \quad \Omega_Q^{fp} = 1 \quad \text{and} \quad \omega_Q^{fp} = \frac{\lambda^2 - 3}{3}$$

where n=3(4) if the system reaches the fixed point during matter (radiation) dominated epoch, Ω is the density parameter and ω is the equation of state of each component .

- The first case **is not able to explain the current observational constraints**, because in this case either the system reaches its fixed point early on in the universe (and the value of Ω_{Q0} is too low) or the quintessence energy density contributes too much to the total energy density in the early universe and spoils big bang nucleosynthesis (BBN) predictions. Furthermore, as long as

  $\omega_Q = \omega_{matter} = 0$, **the universe is not accelerating today**.
- The case of interest is the second, given by $\lambda^2 < 3$, in which the quintessence has not yet reached the fixed point regime today, but will do it in the future.
- Other regimes can not explain all observational constraints when one is using simple exponential potential.
- Our main goal is to obtain the region of parameter space (V_0, λ) able to explain all observational constraints and verify how fine tuned the initial conditions must be in order to have realistic models [5].

Parameters and Initial Conditions

- In **all** quintessence models there is an overall constant ($\mathbf{V_0}$ in our case) that is determined by the fact that the major contribution to the energy of the field today ρ_{Q0} must come from the potential term: $V_0 \approx \rho_{Q0} \approx \rho_{C0}$ (the present critical density).
- λ is the free parameter in this model.
- Because the field has not yet entered the fixed point regime, we have that $Q_0 \approx Q_i$. Besides that, Q_i can be absorbed in the definition of $\mathbf{V_0}$. For this reason, we can take $Q_i = 0$ with no loss of generality.
- Natural initial conditions from equipartition of energy after inflation suggests that $\Omega_{Q,i} \approx 10^{-3}$ [6]. Because the initial energy density is in the form of kinetic energy, we have that $Q'_i \approx 0.05$(in units of $3^{1/2}M_P$) where prime denotes differentiation with respect to u = ln(1+z), z being the redshift.

Constraints

The constraints that the equations have to satisfy are:

- Nucleosynthesis [7]

$$\Omega_Q(1Mev \approx z = 10^{10}) \leq 0.045$$

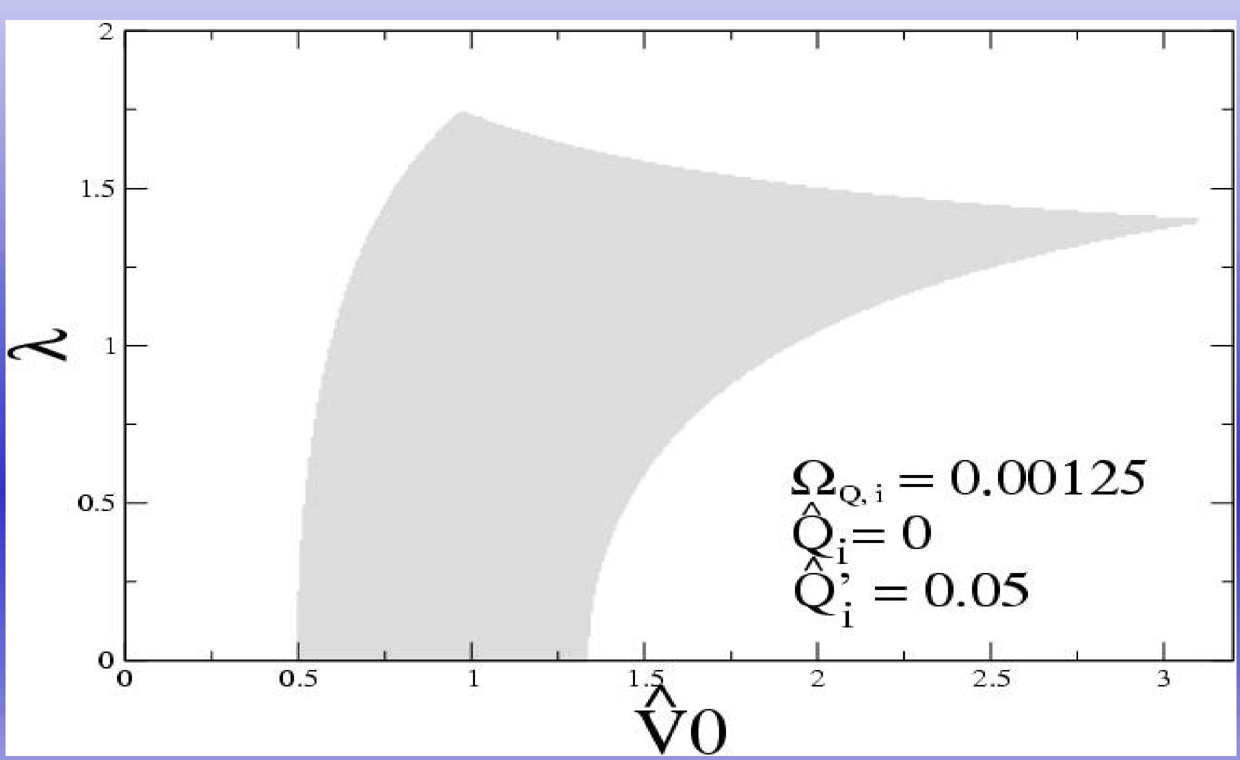
- Present quintessence density parameter [8]

$$\Omega_{Q0} = 0.7 \pm 0.1$$

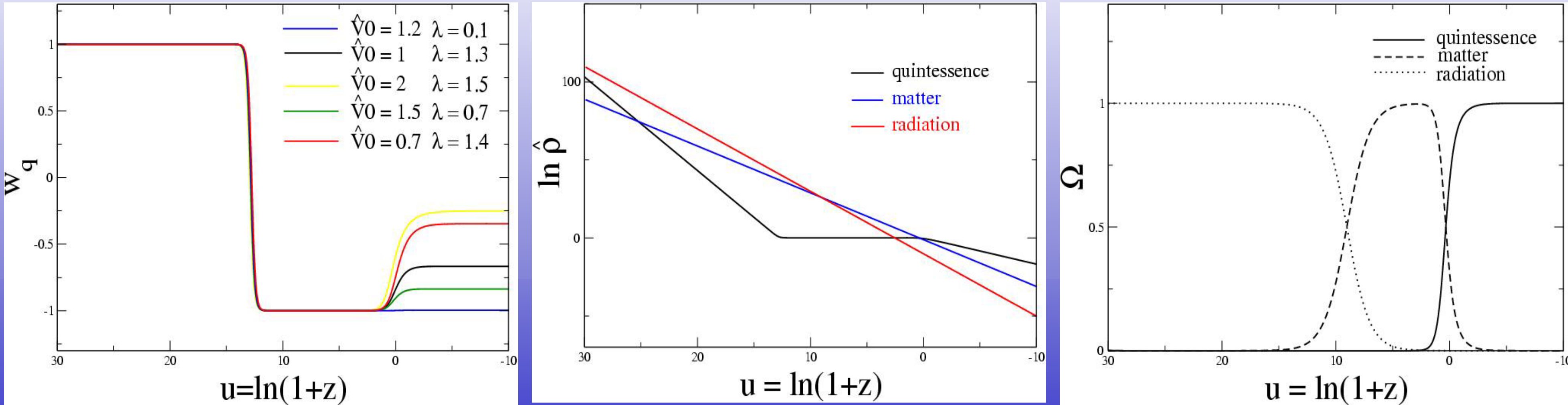
- Current quintessence equation of state [8]

$$-1 \leq \omega_{Q0} \leq -0.6$$

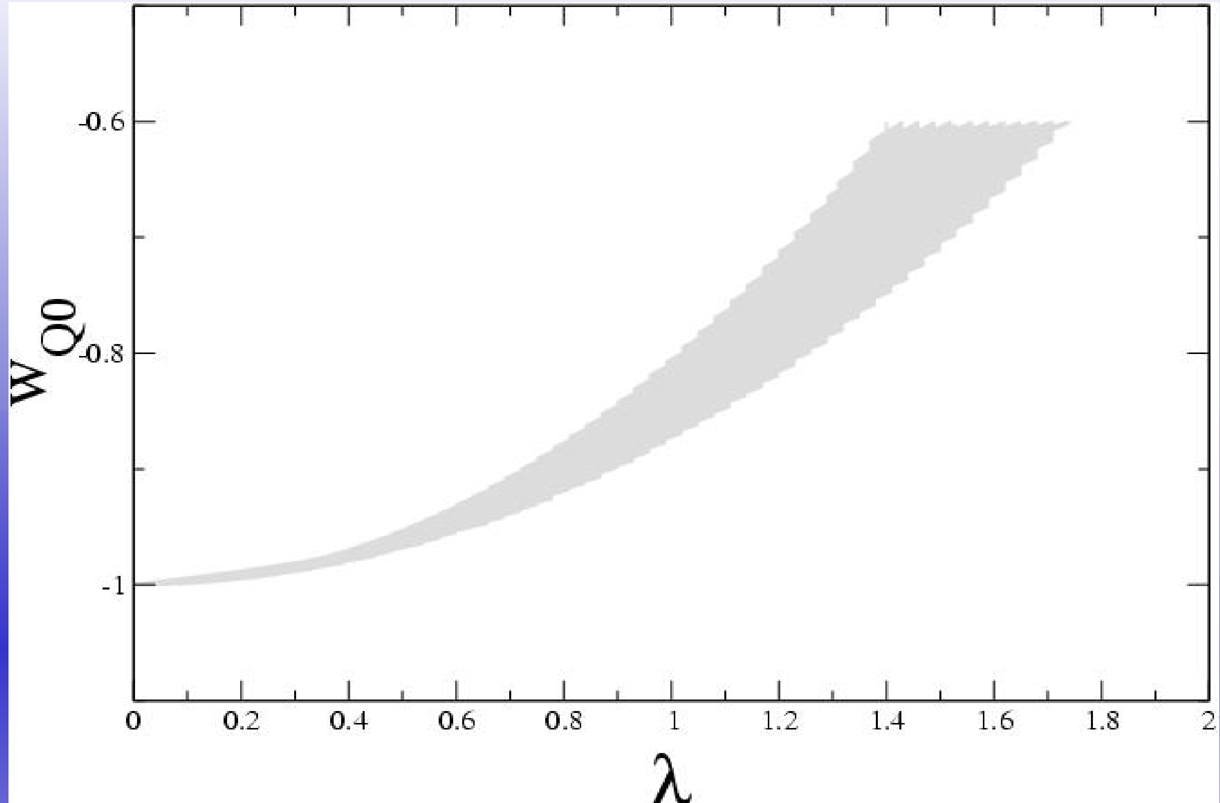
Results



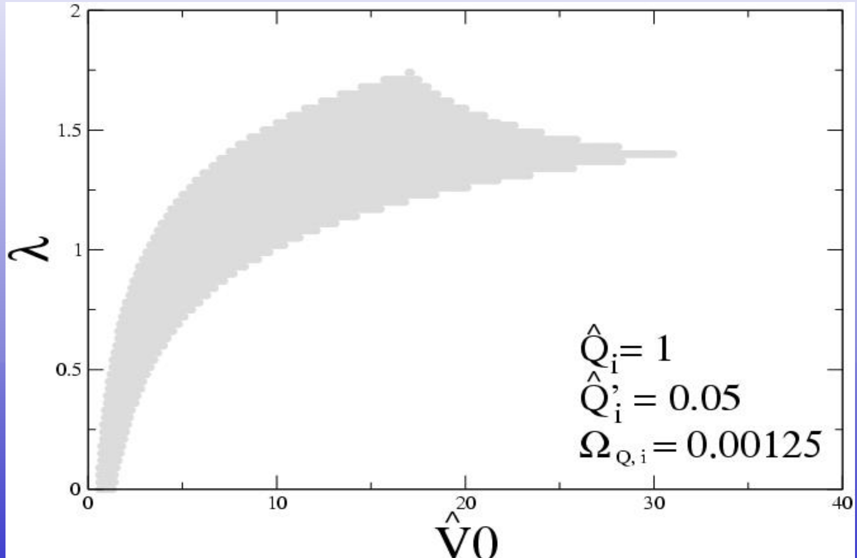
Region of parameter space that satisfies all observational constraints. There is a reasonable region of parameters of the exponential potential that can explain all observations. In fact, all values of λ that produce the tracking solutions satisfy the constraints. The uncertainty on V_0 (in units of ρ_{C0}) comes from the uncertainty on Ω_{matter} today.

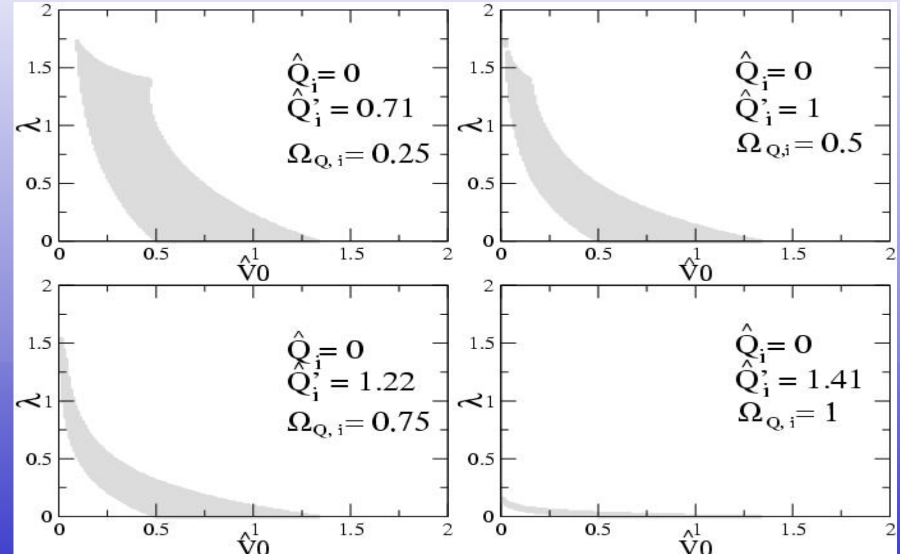


Equation of state of quintessence, energy densities (in units of ρ_{C0}) and density parameters as function of u. Initially, quintessence contributes to a small fraction of the energy of the universe and decreases as R^{-6}, dominated by the kinetic term, faster than matter and radiation densities. When the potential term becomes important, there is a rapid change in the equation of state and the quintessence density freezes until today, when it becomes dominant. Afterwards, quintessence reaches the fixed point regime (FPR), characterised by $\Omega_Q = 1$. In this regime, the energy density decreases as $R^{-3(1+\omega)} = R^{-\lambda^2}$. Notice that a universe that is accelerating today not necessarily will accelerate forever.



Allowed region of (λ, ω_{Q0}) space. This plot is independent on the values of V_0 and Q_i and changes only for "high" values of Q'_i ($Q'_i \gtrsim 1.25$, in units of $3^{1/2} M_P$). The fact that different values of λ give the same ω_{Q0} comes from the fact that today the fixed point regime was not yet reached. With better measurements of ω_{Q0}, one could put severe constraints on the free parameter of the exponential potential, specially if $\omega_{Q0} \lesssim 0.85$.





Regions of parameter space (V_0 in units of ρ_{C0}) for initial conditions different from the most likely set ($Q_i = 0, Q'_i = 0.05$, both in units of $3^{1/2}$ M_P). Notice that using $Q_i \neq 0$ just corresponds to rescale the parameter space in V_0 by a factor of $exp(3^{1/2}\,\lambda Q_i)$. Notice also that for almost all possible values of $|Q'_i|$ a significant region on parameter space stills exist. It only vanishes for $\Omega_{Qi} \gtrsim 0.75$.

Conclusions

- We have studied the exponential quintessence in a regime where the field has not yet reached its fixed point regime today.
- Contrary to common belief, this potential is able to satisfy all observational constraints for a reasonable region of parameter space.
- The resulting parameters and initial conditions are not less natural than that used for all the other models of quintessence.
- The allowed regions are essentialy due to the present experimental uncertainties.
- This potential can not be discarded by any of the constraints discussed here, even if better measurements were made.
- At the moment, there is no reason to discard the exponential potential or to consider it less natural than any other quintessence model.

References

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