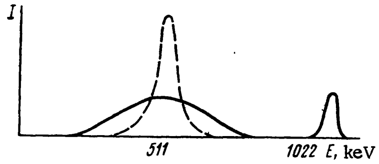

FIG. 2. Schematic spectrum for annihilation of positrons. The dashed curve represents the case $H = 0$; the solid curve, $H \approx H^*$.

these quantities will not necessarily be conserved.

It is evident from the expression (8) that the decay probability will be finite in the case where the transverse momenta are confined to angles such that $\theta^2 \approx mc^2\omega_H/h\omega^2$. Knowing the angular dependence, we can apply the conservation laws for the momentum and energy longitudinal with respect to the magnetic field to determine the width of the line representing two-photon annihilation of an electron–positron pair. Since the resultant projection of the momentum on the direction of the field vanishes in the system of the center of inertia:

$$\omega_1 \cos\theta_1 = \omega_2 \cos\theta_2,$$

we will have

$$2mc^2 = \omega_1 + \omega_2 = \omega\left(1 + \frac{\cos\theta_1}{\cos\theta_2}\right) \approx 2\omega + \left(\frac{\theta_1^2}{2} - \frac{\theta_2^2}{2}\right)\omega.$$

The line width will therefore be given by

$$\frac{\Delta\omega}{\omega} \approx \frac{h\omega_H}{mc^2}.$$

We may now write a simple condition for the magnetic field, such that it will determine the line width:

$$H \gtrsim \frac{m^2c^3}{eh}\sqrt{\frac{kT}{m}} = H^* \frac{v_T}{c}.$$

Figure 2 illustrates the final spectrum for the annihilation.

In closing I wish to thank R. A. Syunyaev, D. G. Yakovlev, and Ya. B. Zel'dovich for valuable comments.

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Formation of absorption lines in a gray atmosphere

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The advantages of describing the radiation field by means of partial quantities are illustrated for the case of a Milne–Eddington gray atmosphere. A curve of growth is derived which accurately takes into account the variation in the source function with optical depth.

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One of the most laborious steps involved in analyzing observed stellar spectra by the model atmosphere method is the calculation of a synthetic spectrum. In the case of Milne–Eddington models, space and frequency variables are separated in the absorption coefficient, and it becomes much simpler to compute the emergent radiation. For these models the radiation field may conveniently be described by using the partial radiant intensity or flux.[1] Although this quantity depends on a larger number of variables than the ordinary radiant intensity (along with the frequency it also depends on the value of the absorption coefficient), it has the important advantage of depending primarily on its own variables. This advantage arises because if the partial radiant intensity is used the frequency enters only as a measure of the energy of a photon, while in the case of the ordinary intensity the frequency also serves to measure the abundant strong fluctuations in the absorption coefficient.

To illustrate how partial quantities can be applied, let us consider the formation of lines in a gray atmosphere that is in a state of local thermodynamic equilibrium. Even though a gray model atmosphere presupposes that the absorption coefficient is independent of frequency, lines may be regarded as being formed in a gray atmosphere, but their overall contribution to the transfer of radiant energy is negligible. It is also noteworthy that the radiation field of a gray atmosphere may be adopted as a natural standard for analysis of nongray atmospheres.

In a gray atmosphere the temperature $T(\tau)$ varies with the optical depth τ in the continuous spectrum according to the familiar law

$$T(\tau) = T_e\,[3/4(\tau + q(\tau))]^{1/4} \equiv T_e t(\tau), \tag{1}$$

where T_e is the effective temperature of the star and $q(\tau)$ represents the Hopf function. The natural variable to use for measuring the frequency in the problem at hand is the dimensionless quantity

$$y = \frac{h\nu}{kT_e}, \tag{2}$$

where ν is the ordinary frequency.

Now let $r(y, \alpha)$ denote the residual partial flux at frequency y and for a ratio α of the line absorption coefficient

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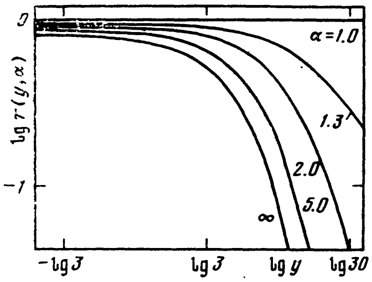

FIG. 1. Residual partial flux.

to the continuous absorption coefficient. Then

$$r(y,\alpha)=\frac{F_0(y,1/\alpha)}{F_0(y,1)}, \tag{3}$$

where F_0, a two-parameter functional of the temperature profile $t(\tau)$, has the form

$$F_0(y,z)=2\int_0^\infty \frac{\exp[y/t(0)-y/t(sz)]}{1-\exp[-y/t(sz)]}E_2(s)\,ds,$$
$$E_2(s)=\int_0^1 e^{-s/\mu}d\mu. \tag{4}$$

Here $E_2(s)$ is the second exponential integral.

The spectrum can be broken down in a natural way into short-wavelength [$y \gg t(0)$] and long-wavelength [$y \ll t(0)$] regions. For the latter region we may write

$$F_0(y,z)\sim\frac{2}{y}\int_0^\infty t(sz)E_2(s)\,ds, \tag{5}$$

which implies, in particular, that the residual flux in the long-wave region does not depend on the dimensionless frequency y. This behavior is clearly demonstrated by Fig. 1. In the short-wave region, the residual flux varies over a much wider range than can be accounted for by the larger central line depths (for equal total intensities) compared with the long-wave spectral region. As Fig. 1 indicates, in a gray atmosphere saturation,

$$r(y,\alpha)\sim r(y,\infty), \tag{6}$$

sets in when α rises to about 10.

The residual partial flux (Fig. 1) describes the emergent radiation so completely that the calculation of line profiles reduces to the following simple procedure. Suppose that the profile of the absorption coefficient is specified by the function $f(x)$ with the condition $f(0)=1$, where the dimensionless frequency x is measured from the center of the line in some appropriate units, and let η

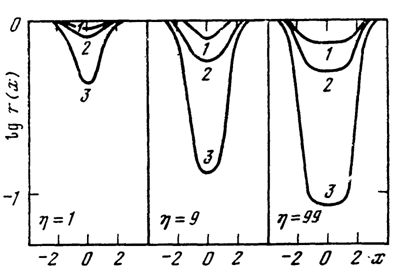

FIG. 2. Doppler absorption line profiles for: 1) y = 0.949; 2) y = 3.000; 3) y = 9.487.

denote the ratio of the absorption coefficient at the center of the line to the absorption coefficient in the continuum. Then the profile $r(x)$ of an absorption line at the large-scale frequency y will be expressed as follows in terms of the residual partial flux:

$$r(x)=r(y,1+\eta f(x)). \tag{7}$$

Figure 2 displays several Doppler line profiles $f(x)=\exp(-x^2)$ as functions of the line intensity and the position in the spectrum. Notice that as y increases, lines of equal intensity (that is, with the same value of η) gradually broaden and their central depth becomes greater, due to the difference in the amplitude by which the source function varies in the atmospheric layers where the line originates. For $\eta>10$, the saturation of the line will result in a saturated core, which will be formed in surface layers whose temperature is close to T(0).

Theoretical curves of growth in the case of pure absorption are generally computed for Milne–Eddington atmospheres[2] by assuming that the source function is a linear function of the optical depth. This is a rather crude approximation, whose use is warranted only by the major simplification of the expressions for the residual intensity and flux. Equations (3) and (7) enable us to construct a curve of growth in which the variation in the source function with depth in an LTE atmosphere is taken exactly into account:

$$\frac{W}{\Delta\nu_D}=\int_{-\infty}^{\infty}[1-r(x)]\,dx. \tag{8}$$

Here W denotes the equivalent width of the line. The distance from the center of the line is measured in units of the Doppler halfwidth $\Delta\nu_D$, and $r(x)$ is given by Eq. (7), in which the line profile is specified by the Voigt function

$$f(x)=\frac{1}{\pi V_0}\int_{-\infty}^{\infty}\frac{e^{-(ay+x)^2}}{1+y^2}dy. \tag{9}$$

In view of the condition $f(0)=1$, the constant V_0 is expressed by

$$V_0=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{e^{-(ay)^2}}{1+y^2}dy. \tag{10}$$

The series of transformations (8) together with Eqs. (3) and (4) enable us to reduce the curve of growth to the more convenient form

$$\frac{W}{\Delta\nu_D}=\frac{2}{F_0(y,1)}\int_{\frac{1}{1+\eta}}^{1}x\left(\frac{1/s-1}{\eta}\right)F_1(y,s)\,ds, \tag{11}$$

where the function x(s) is the inverse of the Voigt function (9), and

$$F_1(y,z)=\frac{d}{dz}F_0(y,z).$$

Equation (11) for the curve of growth has the noteworthy property that it explicitly shows the separation of parameters; the function in the integrand is the product of two functions, of which x(s) conveys information on the pro-

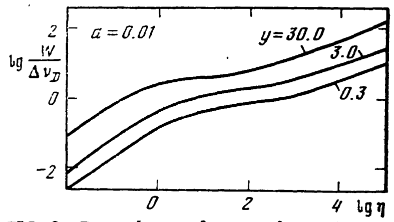

FIG. 3. Dependence of curve of growth on the position of the line in the spectrum.

file of the absorption coefficient, while $F_1(y, z)$ describes the variation of the source function with depth. From Eq. (11) we can also infer the following asymptotic properties for the curve of growth:

a) unsaturated lines:

$$\frac{W}{\Delta\nu_D} \sim \eta \frac{\sqrt{\pi}}{V_0} \frac{F_1(y,1)}{F_0(y,1)}, \tag{12}$$

b) lines with saturated cores:

$$\frac{W}{\Delta\nu_D} \sim (\ln\eta)^{1/2} \frac{2}{F_0(y,1)} \int_0^1 F_1(y,s)\,ds + \text{const}, \tag{13}$$

c) lines with saturated wings:

$$\frac{W}{\Delta\nu_D} \sim \eta^{1/2} \frac{2}{F_0(y,1)} \left(\frac{a}{\sqrt{\pi}V_0}\right)^{1/2} \int_0^1 \left(\frac{s}{1-s}\right)^{1/2} F_1(y,s)\,ds. \tag{14}$$

As the asymptotic relations (12)-(14) indicate, the curve of growth (11) depends on the parameter a of the Voigt profile (as in the case of other curves of growth) only for lines with saturated wings. The dependence of the curve of growth on the position of the line in the spectrum (Fig. 3) is of special interest. In the long-wavelength spectral region, the curve of growth (like the residual flux) is independent of the dimensionless frequency y, but for the remainder of the spectrum a change in y serves to shift the curve of growth. The shift is nonlinear for the curve of growth as a whole, but in each of the asymptotic regions it may be considered approximately linear. According to the definition (2), a variation in y may include both a frequency variation and a dependence of the effective temperature of the star. Hence in order to incorporate lines observed in the short-wavelength region into a single curve of growth, appropriate corrections should be applied. Furthermore, if the method of differential curves of growth is used for stars of differing effective temperature, a more accurate correction should be made for the shift of the curve of growth; it is customary to consider only a linear shift along the horizontal axis.

We should like to point out that, except for Eq. (1), the equations given above are suitable for calculating line profiles and curves of growth not only for a gray atmosphere but also for an arbitrary LTE atmosphere of the Milne–Eddington type for which the temperature profile $t(\tau)$ is specified. These equations therefore may also be used to study effects associated with nongray conditions in stellar atmospheres.

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Mass estimates for M giant atmospheres

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Analysis of observed U–B color indices shows that the mass of the atmospheres of cool stars rises sharply from subtype M2 to M8.

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It is customary to attribute the large amplitudes of the B and V light curves of cool stars to variability of the number of titanium oxide molecules absorbing radiation at these temperatures. We shall endeavor to estimate the effect quantitatively, taking as a basis the Schuster–Schwarzschild model.

Assuming that the photosphere emits Planck radiation of effective temperature T_e, we shall adopt the effective-temperature scale obtained by Flower.[1] The temperature of the reversing layer will be taken equal to the temperature T_k describing the Boltzmann population of the vibrational levels of the α-system of TiO bands. We shall use the relation found by Shavrina[2] between T_k/T_e and spectral subtype. The number of TiO molecules above 1 cm^2 of the photosphere can then be estimated from the dependence of $(U-B)_{Pl} - (U-B)_{SS}$ on T_e, where $(U-B)_{Pl}$ denotes the Planck color index of the photospheric radiation, while $(U-B)_{SS}$ is the color index of the radiation of the photosphere and the reversing layer. The magnitude B_{SS} will be affected primarily by absorption in the α-system of TiO bands, and the magnitude U_{SS}, by absorption by lines of neutral metals.

Cross sections for absorption in the α-system of TiO bands will be computed on the basis of our approximate

