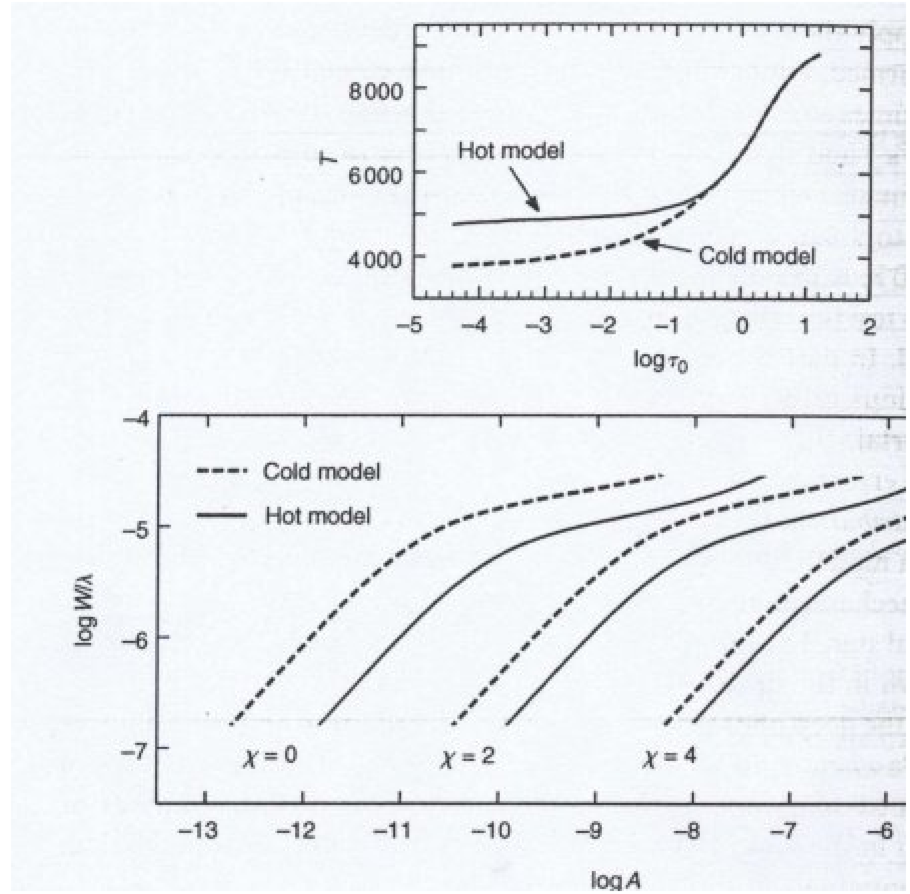
# Lecture 18

Stellar Atmospheres
prof. Marcos Diaz

## treasure map:

H&M: pg. 611
Bohn-Vitense: pg. 128
Gray: pg. 326, 387
Rutten: pg. 127




from Gray 2005

# Curves of growth in LTE

*scaling and differential relations*

$$N_{i,r}=\frac{N_r}{u_r(T)}g_i e^{-\chi_i/kT}$$

$$\eta_0=\frac{1}{k_c(T)}\frac{(\pi)^{1/2}e^2}{mc^2}\lambda_{ij}^2 g_i\frac{f_{ij}}{\Delta\lambda_D}\frac{N_r}{u_r(T)}e^{-\chi_i/kT}*\frac{N_z}{N_z}*\frac{N_H}{N_H}$$

$$C(T,N_H)=\frac{(\pi)^{1/2}e^2}{mc^2}\frac{N_H}{u_r(T)}\left(\frac{N_r}{N_z}\right)$$

with

*r = ionization state*

*i = lower excitation state*

*j = upper excitation state*

$N_z$ *= total element density*

$u_r$ *= excitation partition function*

*T = excitation temperature*

$$\log(\eta_0) = \log(A) + \log(g_i f_{ij} \lambda^2) - \log(\Delta\lambda_D) - \log\left(e^{\chi_i/kT}\right)$$
$$+ \log(C(T)) - \log(k_c)$$

$$\log(\eta_0) = \log(A) + \log(g_i f_{ij} \lambda^2) - \log(\Delta\lambda_D) - \theta\chi_i + \log(C(T)) - \log(k_c);$$

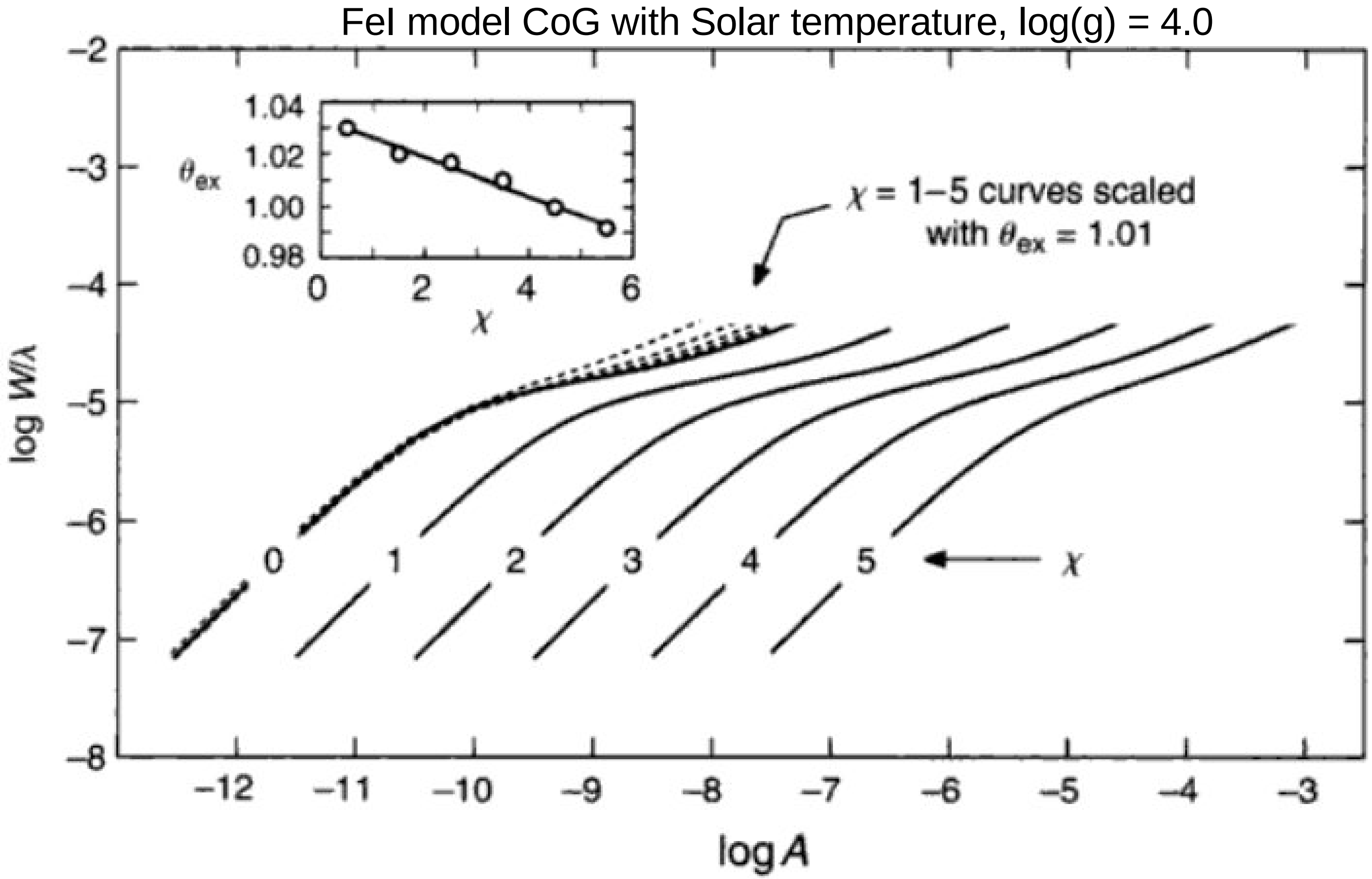
$$with \quad \theta = 5040/T$$

$$C'(T, N_H) = C(T, N_H)\pi^{1/2} R_0 \qquad W = 2 R_0 \Delta\lambda_D \eta_0 \pi^{1/2}$$

$$\log\left(\frac{W}{\lambda}\right) = \log(A) + \log(g_i f_{ij} \lambda) - \theta\chi_i + \log(C') - \log\left(k_c(T, \lambda)\right)$$

*C’ is constant, for a given atmosphere and ion*

Curves of growth of lines from the same ion differ in abscissa by: $\log(gf\lambda)$, $\theta\chi$ and $\log(k_c)$




from Gray 2005

Curves of growth of lines from the same ion differ in abscissa by: $\log(gf\lambda)$, $\theta\chi$ and $\log(k_c)$

Empirical CoG with LTE model

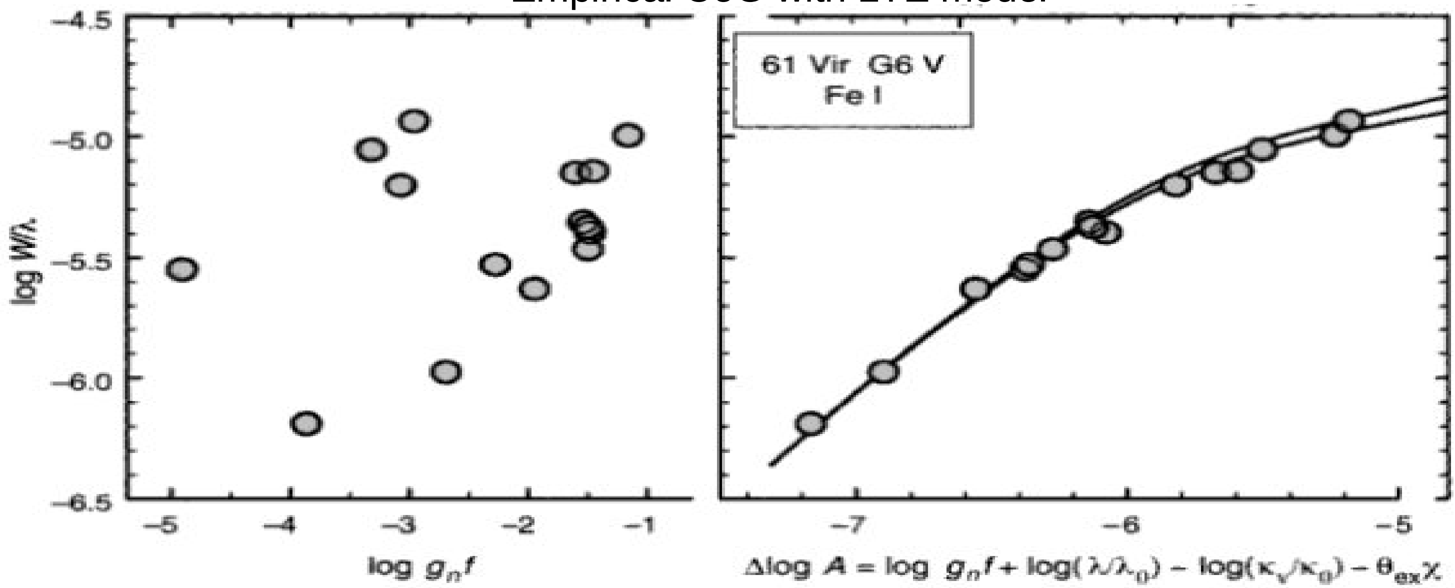
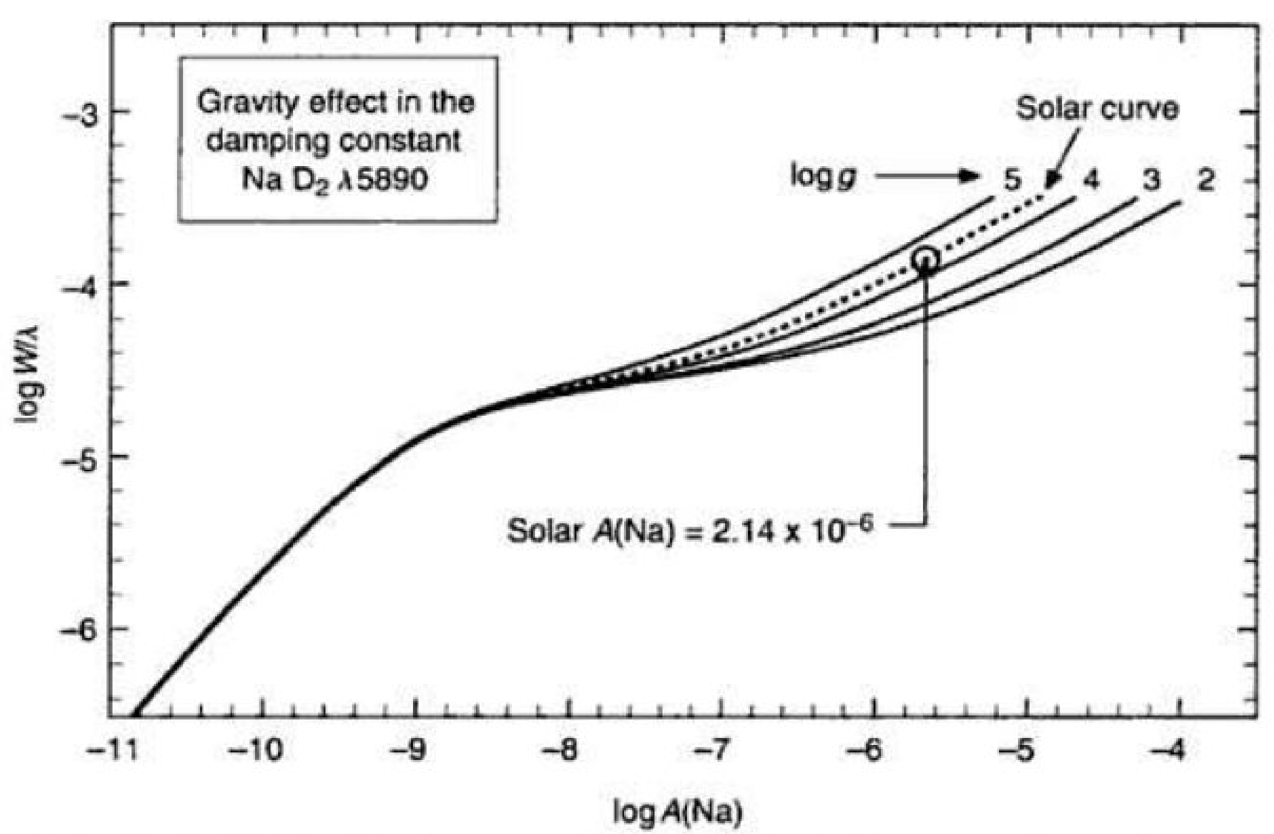



Fig. 16.7. An empirical curve of growth of Fe I is constructed from the equivalent widths of Perrin *et al.* (1988) and the oscillator strengths of May *et al.* (1974) and Rutten and van der Zalm (1984). The raw data in the left-hand panel scale into the empirical curve of growth in the right-hand panel, and comparing this to theoretical curves of growth computed from a model photosphere yields an abundance of $\log A(\mathrm{Fe}) = -4.38$. A value of $\theta_{ex} = 1.00$ gives minimum scatter in the empirical curve. The two theoretical curves have $\xi$ values of 0.6 and 1.0 km/s.

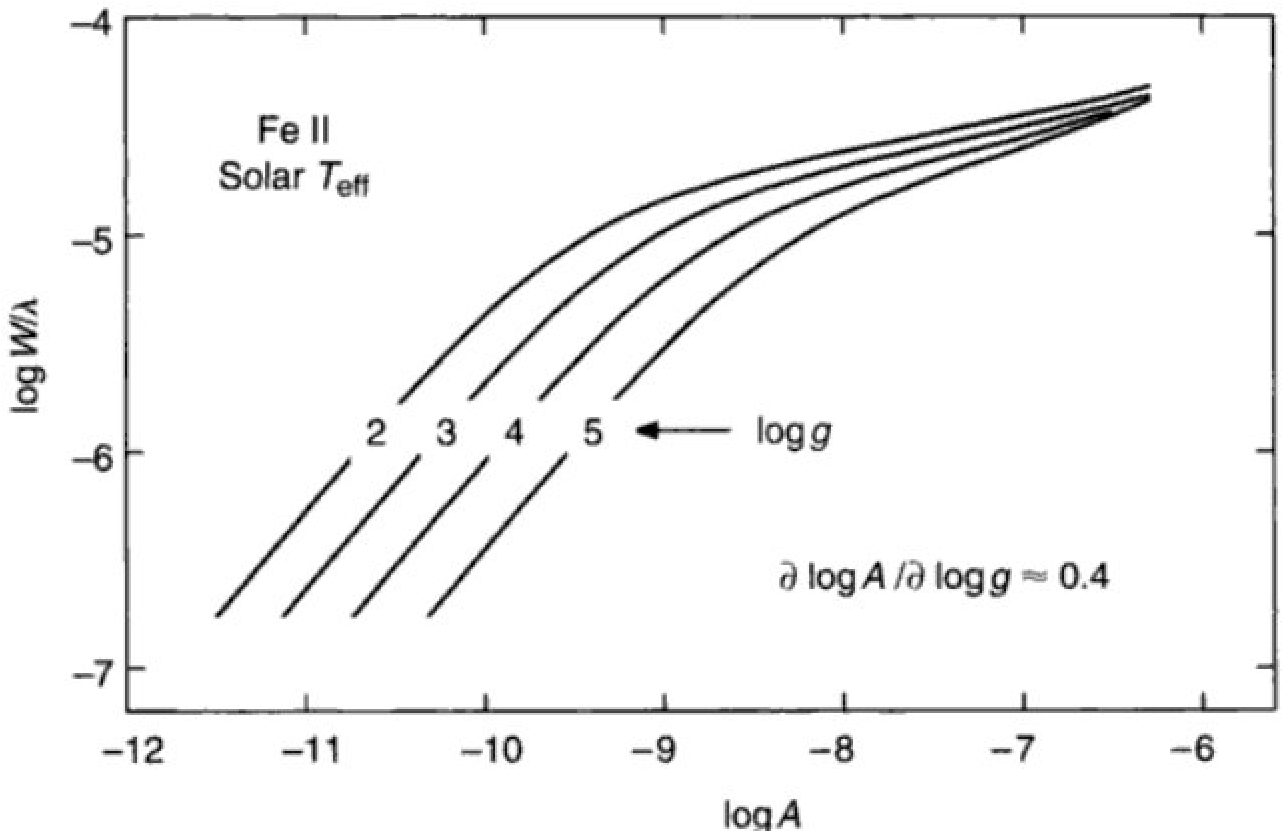
from Gray 2005

# Gravity dependence in damping region by pressure




from Gray 2005

FeII model CoG with Solar temperature, log(g) = 2 - 5




from Gray 2005

Gravity dependence of CoG by ionization (C') and $k_c$

For weak lines (linear part)

i. cool atmospheres ($H^-$ continuum opacity):

$$\frac{N_H}{N_{H^-}} = \frac{f(T)}{P_e} \quad with \quad N_H \gg N_{H^-} \quad N_H \approx N_{total} \quad \therefore \quad N_{H^-} \propto P_e \propto k_c$$

i.a) most abundant ionization state *r* and **lines from *r*+1**

$$\frac{N_{r+1}}{N_r} = \frac{f(T)}{P_e} \quad with \quad N_r \gg N_{r+1} \quad N_r \approx N_{total} \quad \therefore \quad N_{r+1} \propto P_e^{-1} \propto k_{line}$$

$$\frac{k_{line}}{k_c} \propto \frac{N_{r+1}}{P_e^2}$$

(e.g. FeIII lines in solar type stars)

i.b) most abundant ionization state *r* and **lines from *r***

$$\frac{N_{r+1}}{N_r} = \frac{f(T)}{P_e} \quad with\, N_r \gg N_{r+1} \quad N_r \approx N_{total} \quad \therefore \quad N_r \propto k_{line} \approx constant$$

$$\frac{k_{line}}{k_c} \propto \frac{N_r}{P_e}$$

(e.g. FeII lines in solar type stars)

i.c) most abundant ionization state *r+1* and **lines from *r***

$$\frac{N_{r+1}}{N_r} = \frac{f(T)}{P_e} \quad with\, N_{r+1} \gg N_r \quad N_{r+1} \approx N_{total} \quad \therefore \quad N_r \propto P_e \propto k_{line}$$

$$\frac{k_{line}}{k_c} \approx constant$$

*provide better chemical diagnostic*

(e.g. FeI lines in the Sun)

ii. moderately hotter atmospheres

- $k_c = f(T)$, b-f opacity
- negligible Thomson scattering

→ Most CoGs are pressure (gravity) dependent in their linear section
Exceptions occur when:

ii.a) most abundant ionization state *r* and **lines from *r***

$$\frac{N_{r+1}}{N_r} = \frac{f(T)}{P_e} \quad with\, N_r \gg N_{r+1} \quad N_r \approx N_{total} \quad \therefore \quad N_r \propto k_{line} \approx constant$$

$$\frac{k_{line}}{k_c} \approx constant$$

**Differential abundance analysis:**

$$\log\left(\frac{A}{A_{ref}}\right)=\log\left(\frac{W}{W_{ref}}\right)-\log\left(\frac{k_c}{k_c^{ref}}\right)-\left[\theta(T)-\theta(T_{ref})\right]\chi_i$$

Even differential abundances are strictly model-dependent:

a) $\Delta k_c$ depends on abundances themselves
b) Excitation temperatures T may differ from different contribution functions
c) C' contains second order differences on $N_H$ and partition function