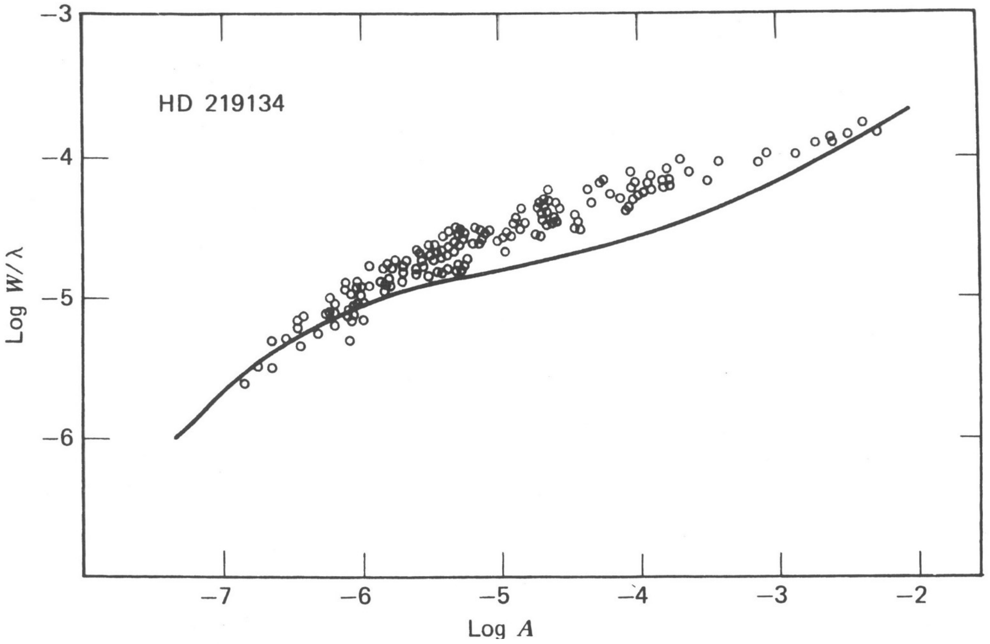
Lecture 17

Stellar Atmospheres
prof. Marcos Diaz



from Gray 1976

treasure map:

H&M: pg. 611
Bohn-Vitense: pg. 128
Gray: pg. 326, 387
Rutten: pg. 127

Model curves of growth

relate $F(\tau=0)_{line} / F(\tau=0)_{cont.}$ *with* $\eta_\nu \equiv k_l / k_c$

basic model assumptions:

i. semi-infinite plane-parallel atmosphere
ii. in radiative equilibrium
iii. without induced emission
iv. without scattering - pure absorption in lines and continuum
v. LTE
vi. $\eta_\nu \equiv k_l / k_c$ = *constant with total optical depth* τ_ν

from Formal Solution with arbitrary source function

$$F_\nu(0) = 2\int_0^\infty E_2(\tau) S_\nu(\tau)\, d\tau_\nu$$

with

$$\tau_\nu = \tau_{cont}(1+\eta_\nu) = \tau_{cont}\left(1+\frac{k_{line}}{k_{cont}}\right)$$

$$S_\nu(\tau) \;=\; B_\nu(\tau) \;\;=\;\; B_0(\tau=0) \;+\; \frac{\partial B}{\partial T}(\tau=0).\,\tau$$

$$F_\nu(0) \;=\; 2\int_0^\infty E_2\left[(1+\eta_\nu)\,\tau_c\right]\left(B_0+B_1\,\tau_c\,(1+\eta_\nu)\right)d\,\tau_c$$

$$S_\nu(\tau) = B_\nu(\tau) = B_0(\tau=0) + \frac{\partial B}{\partial T}(\tau=0).\tau$$

$$F_\nu(0) = 2\int_0^\infty E_2\left[(1+\eta_\nu)\tau_c\right]\left(B_0+B_1\tau_c(1+\eta_\nu)\right)d\tau_c$$

$$\tau' = \tau_c\,(1+\eta_\nu)$$

$$d\tau' = d\tau_c\,(1+\eta_\nu)$$

$$F_\nu(0) = 2B_0\int_0^\infty E_2(\tau')d\tau' + 2B_1\int_0^\infty \frac{\tau'}{(1+\eta_\nu)}E_2(\tau')d\tau'$$

$$S_\nu(\tau) = B_\nu(\tau) = B_0(\tau=0) + \frac{\partial B}{\partial T}(\tau=0).\tau$$

$$F_\nu(0) = 2\int_0^\infty E_2\left[(1+\eta_\nu)\tau_c\right]\left(B_0+B_1\tau_c(1+\eta_\nu)\right)d\tau_c$$

$\tau' = \tau_c\,(1+\eta_\nu)$

$d\tau' = d\tau_c\,(1+\eta_\nu)$

$$F_\nu(0) = 2B_0\int_0^\infty E_2(\tau')d\tau' + 2B_1\int_0^\infty \frac{\tau'}{(1+\eta_\nu)}E_2(\tau')d\tau'$$

$$F_\nu(0) = B_0 + \frac{2}{3}\frac{B_1}{(1+\eta_\nu)} \qquad (1)$$

$\eta_\nu \to 0$ (without line opacity): $F_c(0) = B_0 + \frac{2}{3}B_1 = B_\nu(\tau=2/3)$

(2)

from (1) and (2)

$$R_\nu = \frac{F_c(0) - F_\nu(0)}{F_c(0)} = \frac{\eta_\nu}{(1+\eta_\nu)\left(1+\frac{3}{2}\frac{B_0}{B_1}\right)}$$

if $k_l >> k_c$:

$$\lim_{\eta_\nu \to \infty} R_\nu = \left(1+\frac{3}{2}\frac{B_0}{B_1}\right)^{-1} \equiv R_0$$

$$R_\nu = R_0 \frac{\eta_\nu}{(1+\eta_\nu)}$$

$$EW \equiv \int_0^\infty R_\lambda \, d\lambda = R_0 \int_0^\infty \frac{\eta_\lambda}{(1+\eta_\lambda)} d\lambda$$

$$\frac{v_{profile}}{c} = \frac{\Delta\lambda}{\lambda_0}$$

$$\eta_\lambda = \eta_{\lambda 0} H[a, v(\lambda)]$$

$$v = \frac{v_{prof}}{v_D} = \frac{\Delta\lambda}{\Delta\lambda_D}$$

$$dv = \frac{d\lambda}{\Delta\lambda_D}$$

$$EW = 2 R_0 \Delta\lambda_D \int_0^\infty \frac{\eta_0 H(a,v)}{(1+\eta_0 H(a,v))} dv \qquad (1)$$

a) weak lines: $\eta_0 << 1$ $H(a,v) \propto e^{-v^2}$

$$EW = 2 R_0 \Delta\lambda_D \eta_0 \int_0^\infty \frac{e^{-v^2}}{(1+\eta_0 e^{-v^2})} dv$$

$$\frac{1}{(1+x)} \approx 1 - x$$

for a symmetric profile:

$$EW = 2R_0 \Delta\lambda_D \eta_0 \int_0^{\infty} e^{-v^2}\left(1-\eta_0 e^{-v^2}\right) dv$$

$$EW = 2R_0 \Delta\lambda_D \eta_0 \left[\pi^{1/2} - \left(\frac{\pi}{2}\right)^{1/2} \underset{\searrow\,\sim 0}{\eta_0}\right] = 2R_0 \Delta\lambda_D \eta_0 \pi^{1/2}$$

$$EW = 2R_0\Delta\lambda_D\eta_0\int_0^\infty e^{-v^2}\left(1-\eta_0 e^{-v^2}\right)dv$$

$$EW = 2R_0\Delta\lambda_D\eta_0\left[\pi^{1/2}-\left(\frac{\pi}{2}\right)^{1/2}\underset{\searrow\,\sim 0}{\eta_0}\right] = 2R_0\Delta\lambda_D\eta_0\pi^{1/2}$$

$$\eta_0=\frac{1}{k_c}\frac{\pi^{1/2}e^2}{mc^2}\lambda^2 f_{ij}\frac{N_i}{\Delta\lambda_D}H(a,0)$$

$H(a,0) \sim 1$

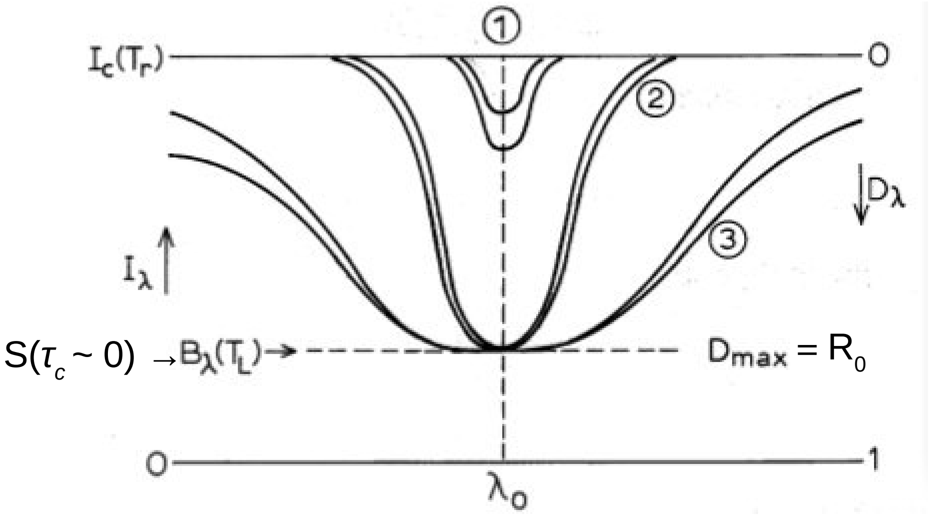
$$\frac{EW}{\Delta\lambda_D} = \frac{R_0}{k_c}\frac{\pi^{1/2}e^2}{mc^2}\lambda^2 f_{ij}\frac{N_i}{\Delta\lambda_D}$$

$$\log(EW) = \log(N_i)+C_\lambda$$

b) saturated lines

$k_l > k_c$; $\eta_0 > 1$

line formed at low τ_c



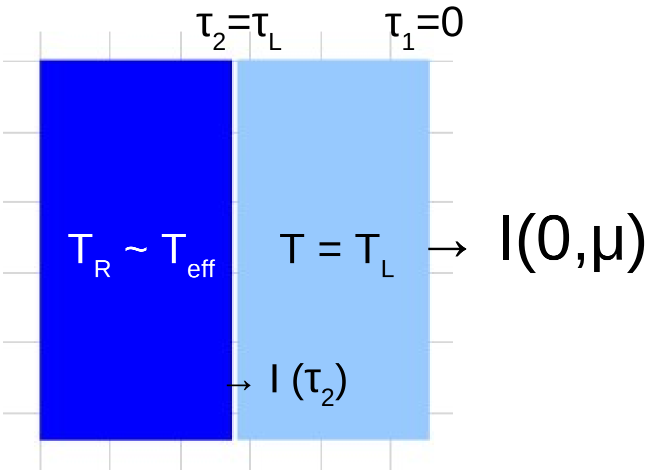
$$S(0) \sim B_\lambda[T(\tau_l \sim 1)] \sim B_\lambda[T(\tau_C << 1)]$$

→ *model dependent saturation*
EW depends on source function S(0)

$$R_0 = \frac{B_\lambda(T_{eff}) - B_\lambda[T(\tau_c \sim 0)]}{B_\lambda(T_{eff})}$$

Schuster-Schwarzschild two region model

upper region: continuum thin, line thick
lower region: continuum thick



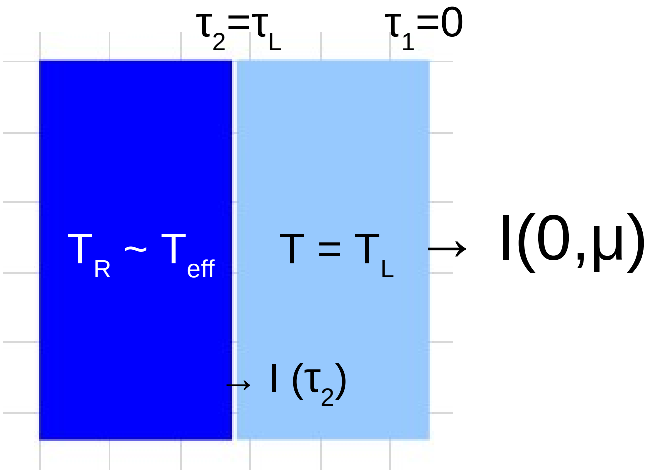
$$I_c(0,\mu) \;=\; B_\lambda(T_R)$$

$$S_\lambda(\tau) \;=\; constant \;=\; B_\lambda(T_L)$$

$$I_\lambda(0,\mu) \;=\; B_\lambda(T_L)(1-e^{-\tau_L/\mu}) \;+\; B_\lambda(T_R)e^{-(\tau_L/\mu)}$$

Schuster-Schwarzschild two region model

upper region: continuum thin, line thick
lower region: continuum thick



$$I_c(0,\mu) = B_\lambda(T_R)$$

$$S_\lambda(\tau) = constant = B_\lambda(T_L)\text{¿}$$

$$I_\lambda(0,\mu) = B_\lambda(T_L)(1-e^{-\tau_L/\mu}) + B_\lambda(T_R)e^{-(\tau_L/\mu)}$$

without limb darkening:

$$R_\lambda = \frac{I_c(0)-I_\lambda(0)}{I_c(0)} = \left[\frac{B_\lambda(T_R)-B_\lambda(T_L)}{B_\lambda(T_R)}\right](1-e^{-\tau_L})$$

$$R_0 = R_\lambda(\tau_L \gg 1) = \left[\frac{B_\lambda(T_R) - B_\lambda(T_L)}{B_\lambda(T_R)}\right]$$

$$EW = \int_0^\infty R_\lambda \, d\lambda = \int_0^\infty R_0 \left(1 - e^{-\tau_L}\right) d\lambda$$

$$EW(\tau_L \gg 1) = R_0 \int_{-\Delta\lambda_D}^{\Delta\lambda_D} d\lambda = 2 R_0 \Delta\lambda_D = constant$$

$$\log(EW) = \log(\Delta\lambda_D) + C'$$

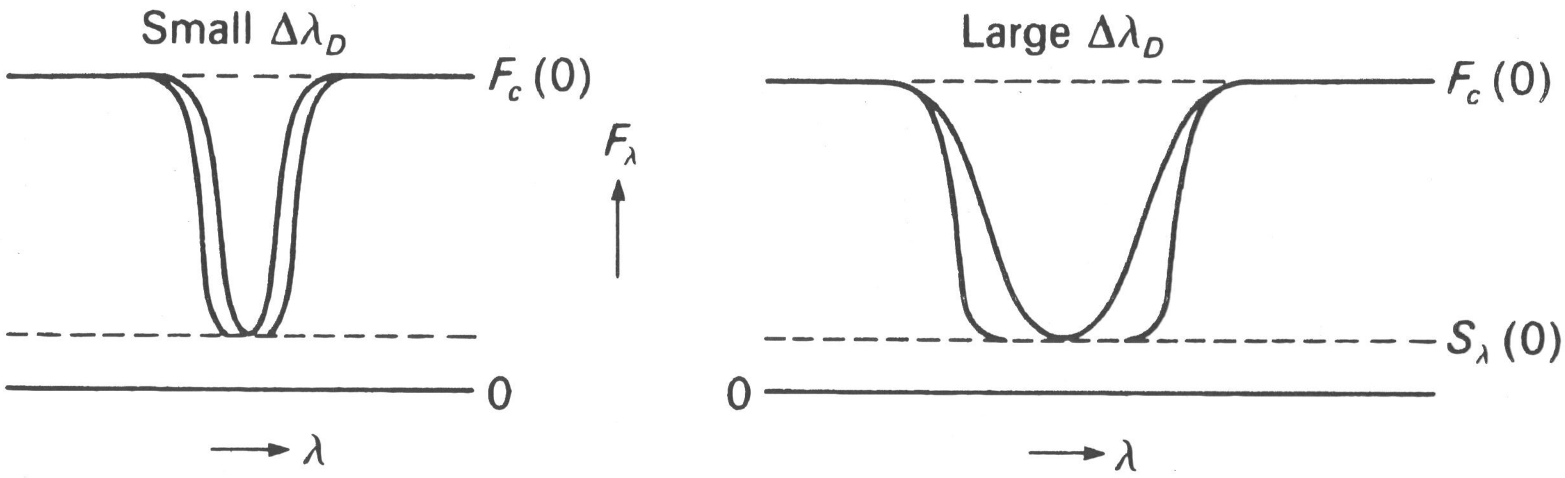


Fig. 10.14. For the same central line depth, the equivalent widths of optically thick lines increase proportionately to the line widths.

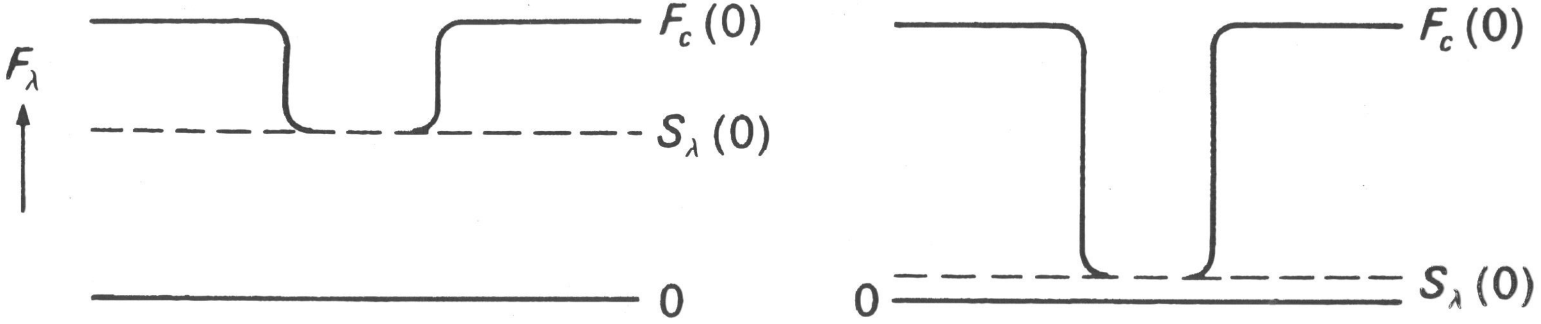
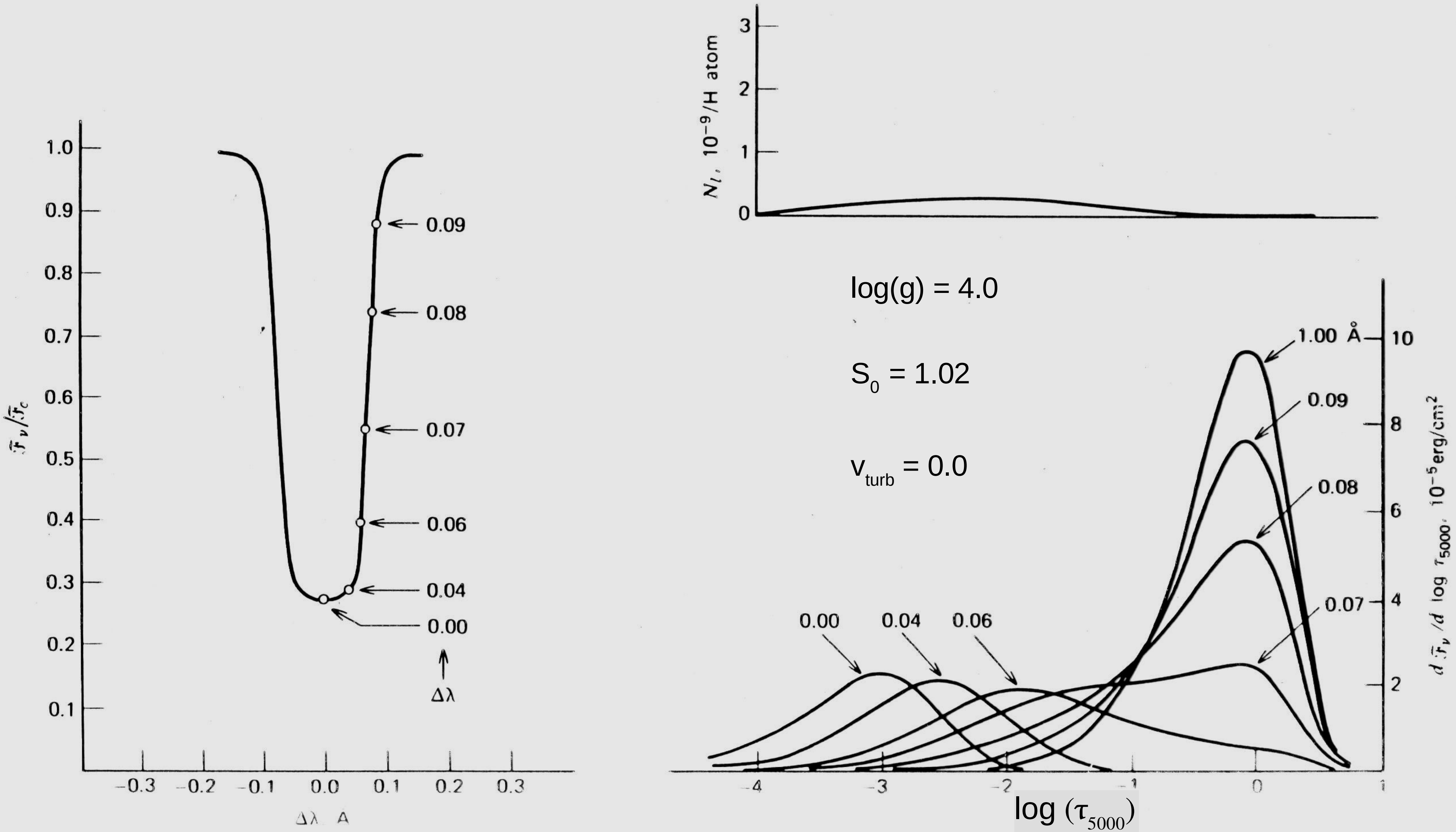


Fig. 10.15. For saturated lines the equivalent width becomes proportional to the central line depth.

from Bohn-Vitense 1989



$\log(g) = 4.0$

$S_0 = 1.02$

$v_{turb} = 0.0$

$\log(\tau_{5000})$

Fig. 13.2 The contribution functions for residual flux are shown for iron I λ6065 with ξ taken to be zero; the model has $S_0 = 1.02$ and $g = 10^4$ cm/s^2. The line profile is shown at the left; the number of line absorbers and the integrands of eq. (13.15) on the right. Each labeled point on the profile has a corresponding contribution function on the right, as labeled.

b) Gamma damped lines

$$H(a,v) \sim \frac{a}{\pi^{1/2} v^2}$$

the high-density line transfer domain

$$EW = 2 R_0 \Delta\lambda_D \int_0^\infty \frac{\eta_0 H(a,v)}{(1+\eta_0 H(a,v))} dv$$

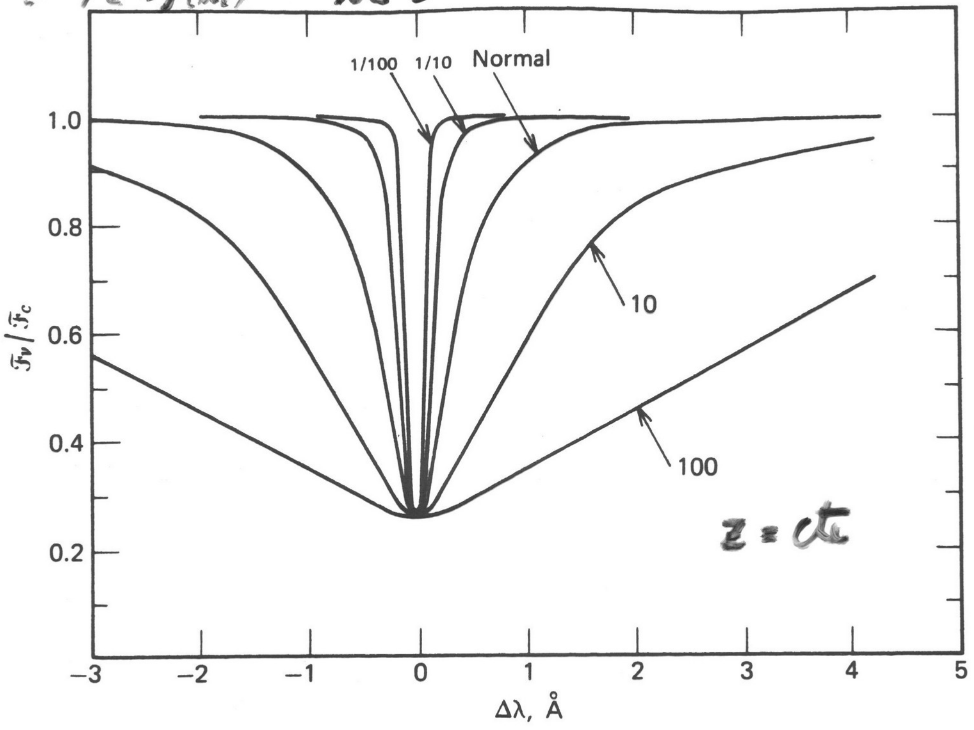
$$EW = 2 R_0 \Delta\lambda_D \int_0^\infty \frac{c/v^2}{(1+c/v^2)} dv$$

$$c = \frac{\eta_0 a}{\pi^{1/2}}$$

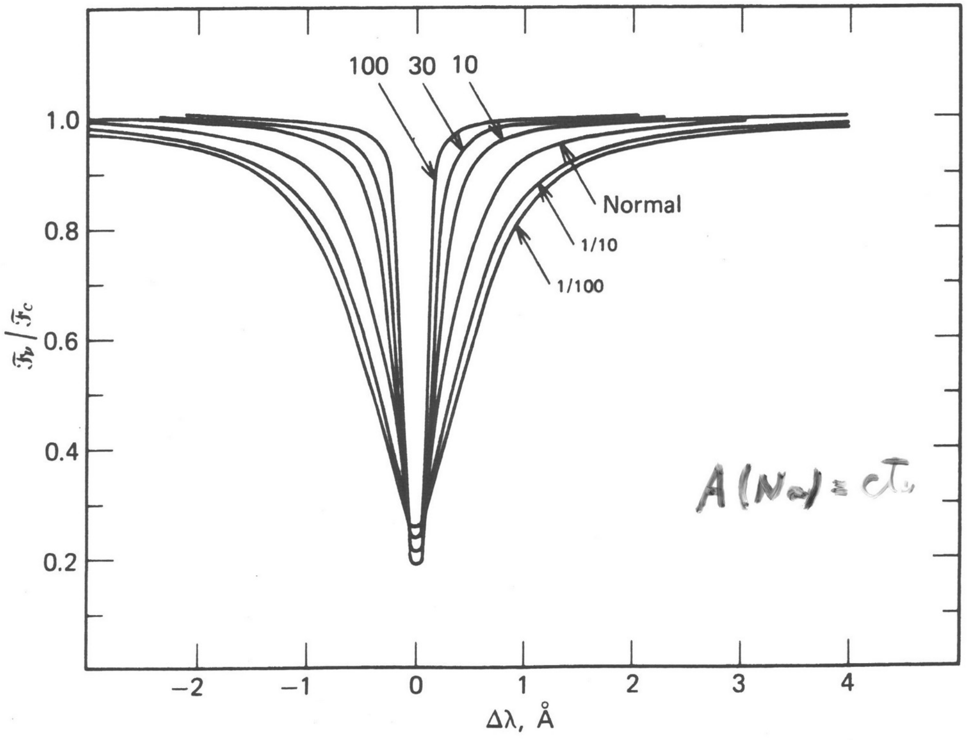
$$EW = \pi^{3/4} R_0 \Delta\lambda_D a \eta_0^{1/2}$$

$$\log\left(\frac{EW}{\Delta\lambda_D}\right) = \frac{1}{2}\log(N_i) + C'$$

s' $P_e = f(m_t)$ Na D



(a)



(b)

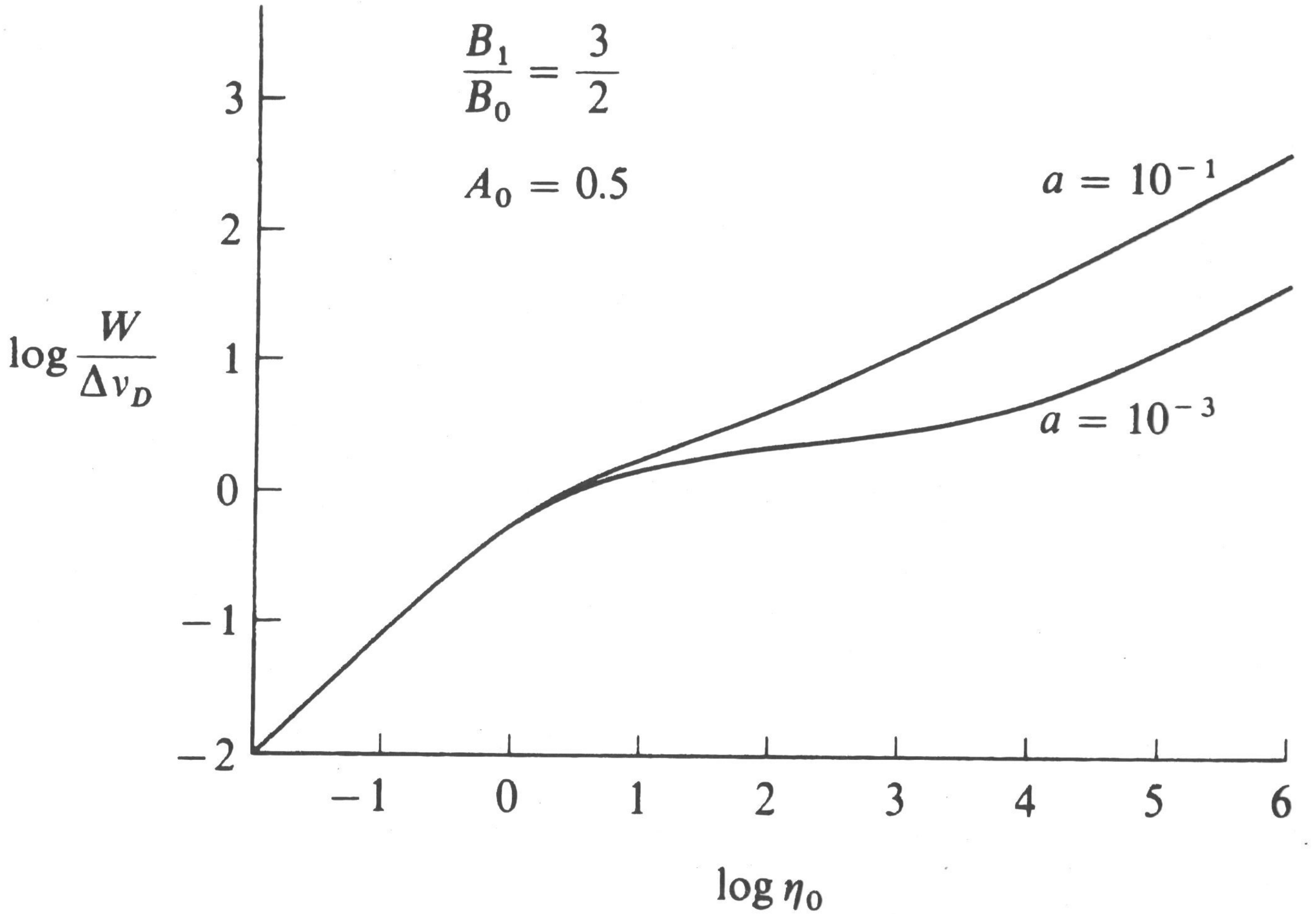


FIG. 11-2. Curves of growth for pure absorption lines. Note that the larger the value of a, the sooner the square root part of the curve rises away from the flat part.

(from Bohn-Vitense 1989)

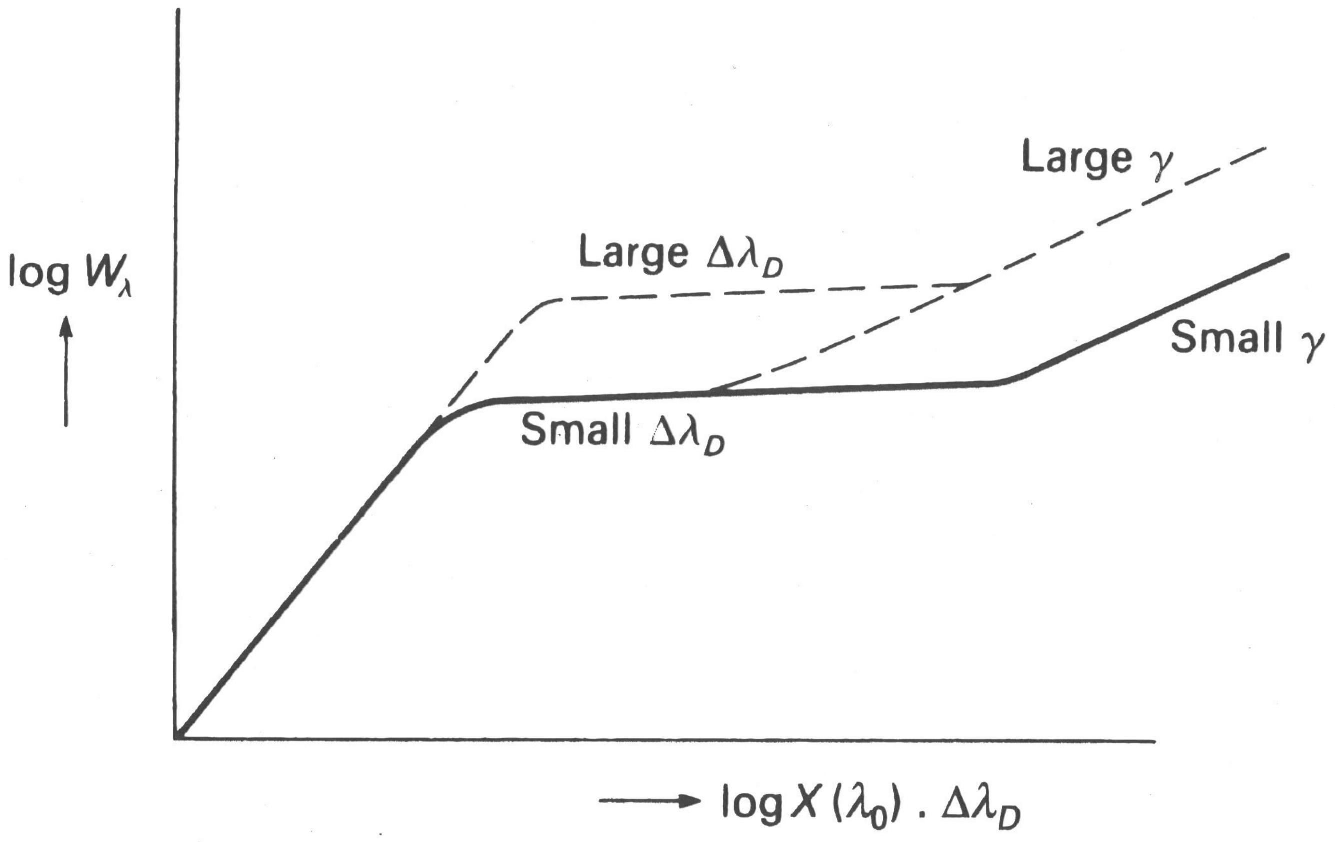


Fig. 10.13. Schematic curve of growth for different values of the Doppler width $\Delta\lambda_D$ and damping constants γ.

(from Bohn-Vitense 1989)

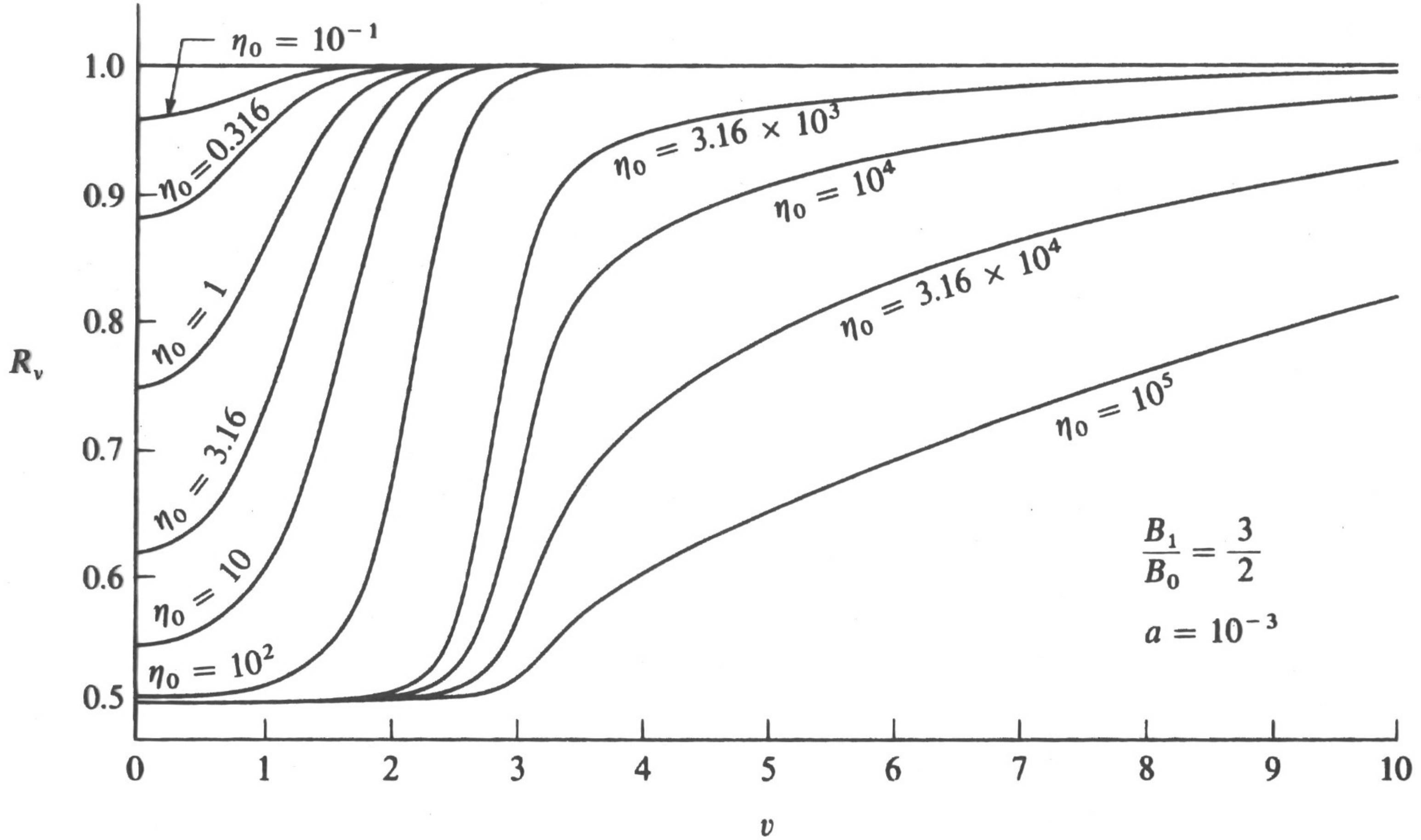


FIG. 11-1. Development of a spectrum line with increasing number of atoms in the line of sight. The line is assumed to be formed in pure absorption. For $\eta_0 \lesssim 1$, the line strength is proportional to the number of absorbers. For $30 \lesssim \eta_0 \lesssim 10^3$ the line is saturated, but the wings have not yet begun to develop. For $\eta_0 \gtrsim 10^4$ the line wings are strong and contribute the dominant part of the equivalent width.

(from Mihalas