Lecture 14

Stellar Atmospheres
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treasure map:

H&M: pg 607
Mihalas, 1978
Rutten: pg 92
Gebbie & Thomas, ApJ, 154, 285, 1968
Israelian & Nikoghossian, J.Q.S.R.T, 56, 509, 1996

Basic line radiation transfer

line photons are subject to:
- pure absorption (ε)
- scattering (^{s} superscript) (1-ε)
- thermal emission (^{t} superscript)

$$l_L = (1-\epsilon) l_L + \epsilon l_L$$

(line scattering and absorption coef.)

line scattering is:
- isotropic
- complete redistribution ($R^S_{\nu,\phi} = \delta_\nu$, $1/4\pi$)

$$\frac{\mu}{\rho}\frac{dI_\nu}{dz} = j_L^t + j_L^s + j_C^t + j_C^s - \left[k_C + \sigma_C + l_L\right] I_\nu$$

$$d\tau_\nu = -(k_C + \sigma_C + l_L)\rho\, dz$$

with $$j_L^s = (1-\epsilon) J_\nu l_L$$ (line scattering emission into the beam)

$$\epsilon \, l_L J_\nu = \epsilon \, l_L B_\nu = j_L^t$$ (line thermal emission in LTE)

$$j_C^s = \sigma_C J_\nu$$ (continuum scattering emission)

$$j_C^t = k_C B_\nu$$ (continuum thermal emission in LTE)

$$\frac{\mu}{\rho} \frac{dI_\nu}{dz} = \epsilon \, l_L B_\nu + (1-\epsilon) l_L J_\nu + k_C B_\nu + \sigma_C J_\nu - \left[k_C + \sigma_C + l_L \right] I_\nu$$

with $$\eta_\nu = \frac{l_L}{(k_C + \sigma_C)}$$ (line-to-continuum total abs. ratio)

and $$\zeta_C = \frac{\sigma_C}{(k_C + \sigma_C)}$$ (scattering-to-total abs. ratio for continuum)

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \frac{(1-\zeta) + \epsilon \, \eta_\nu}{(1+\eta_\nu)} B_\nu + \frac{\zeta + (1-\epsilon) \eta_\nu}{(1+\eta_\nu)} J_\nu$$

with $$\lambda_\nu = \frac{(1-\zeta)+\epsilon\,\eta_\nu}{(1+\eta_\nu)}$$

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \lambda_\nu B_\nu - (1-\lambda_\nu) J_\nu$$ *Milne-Eddington equation*

with $B_\nu(\tau) = B_0 + B_1\tau$ and $B_1 = \frac{C_1}{(1+\eta_\nu)}$

$$J_\nu(\tau) = B_0 + B_1\tau + \left(\frac{B_1 - \sqrt{3}\,B_0}{\sqrt{3}}\right)\frac{1}{(1+\sqrt{\lambda_\nu})} e^{\sqrt{3\lambda_\nu}\,\tau}$$

with $J_\nu(0) = \sqrt{3}\,H_\nu(0)$ (2-stream approx)

$$H_\nu(0) = \frac{1}{3}\left(\frac{B_1 - \sqrt{3\lambda}\,B_0}{(1+\sqrt{\lambda})}\right) \qquad (1)$$

1. continuum

$\eta_\nu = 0; \quad \lambda_\nu = 1 - \zeta$

$$H_{Cont}(0) = \frac{1}{3}\left(\frac{B_1 - \sqrt{3(1-\zeta)}\,B_0}{1+\sqrt{1-\zeta}}\right) \qquad (2)$$

2. lines

$$R_\nu = \frac{H_{Cont} - H_\nu}{H_{Cont}} = \frac{\left(\dfrac{B_1 - \sqrt{3(1-\zeta)}\,B_0}{1+\sqrt{1-\zeta}}\right) - \left(\dfrac{B_1 - \sqrt{3\lambda_\nu}\,B_0}{1+\sqrt{3\lambda_\nu}}\right)}{\left(\dfrac{B_1 - \sqrt{3(1-\zeta)}\,B_0}{1+\sqrt{1-\zeta}}\right)}$$

$$R_\nu = 1 - \frac{\left(C_1/(1+\eta_\nu) + B_0\sqrt{3\lambda_\nu}\right)}{\left(C_1 + B_0\sqrt{3(1-\zeta)}\right)} \cdot \frac{\left(1+\sqrt{1-\zeta}\right)}{\left(1+\sqrt{\lambda_\nu}\right)} \qquad (3)$$

2.i Pure scattering lines & absence of continuum scattering

$\varepsilon = 0$; $\sigma_C = 0$; $\zeta = 0$;

$$\lambda_\nu = \frac{1}{(1+\eta_\nu)}$$

from 1 and 2:

$$\frac{H_\nu}{H_C} = \frac{\frac{2C_1}{(1+\eta_\nu)} + \left(\frac{12}{(1+\eta_\nu)}\right)^{1/2} B_0}{(C_1 + \sqrt{3}\, B_0)}$$

with $\eta(\nu=\nu_0) = \eta_0 >> C_1$ $\quad \left(\frac{H_\nu}{H_C}\right)_{\eta_0} \rightarrow 0$ (dark core line)

2.ii Pure absorption lines & absence of continuum scattering

$\varepsilon = 1; \quad \sigma_C = 0; \qquad \zeta = 0;$

$$\lambda_\nu = 1$$

with $\eta(\nu=\nu_0) = \eta_0 >> C_1$

from 3 ($\nu \approx \nu_0$):

$$R_\nu = 1 - \frac{\sqrt{3}\,B_0}{C_1 + \sqrt{3}\,B_0}$$

$C_1 > 0 \rightarrow \qquad R_\nu > 0$

$$\frac{H_\nu}{H_C} = \frac{\sqrt{3}\,B_0}{\left(C_1 + \sqrt{3}\,B_0\right)}$$

even with $\eta_0 >> C_1$ $\qquad \left(\frac{H_\nu}{H_C}\right)_{\eta_0} \rightarrow \neq 0$ $\qquad$ (non-dark core line)

2.iii Pure absorption line & pure continuum scattering

$\varepsilon = 1$; $k_C = 0$; $\zeta = 1$;

$$\lambda_\nu = \frac{\eta_\nu}{1+\eta_\nu}$$

with $\eta(\nu \sim \nu_0) = \eta_\nu >> C_1$ $\lambda_\nu \rightarrow 1$

from 3 and $\nu \sim \nu_0$

$$R_\nu = 1 - \frac{\sqrt{3}}{2}\frac{B_0}{C_1}$$

$$\frac{B_0}{C_1} > \frac{2}{\sqrt{3}} \rightarrow R_\nu < 0$$

$$C_1 = \frac{\partial B_\nu}{\partial \tau}(\tau = 0)$$

(line center, more often wings or even whole line in emission for normal temperature gradient)

Schuster mechanism