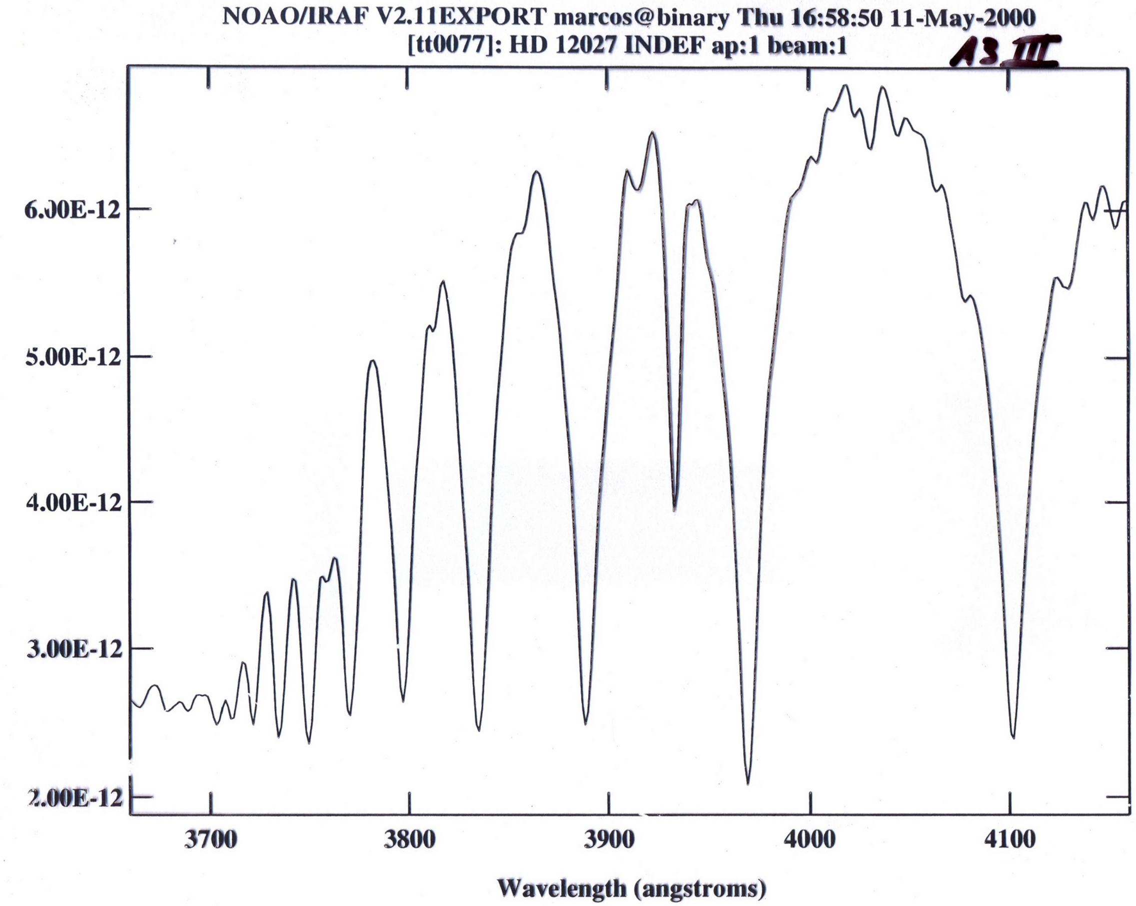
Lecture 11

Stellar atmospheres
prof. Marcos Diaz



treasure map:

H&M: pg 233
Bohn-Vitense: pg 139
Rutten: pg 119

Microscopic Line Broadening

IV. Application: the Inglis-Teller diagnostic

Hydrogen level splitting by external E effective potential into $2n^2$ sublevels

$$\Delta E_k = C_k E \qquad h\Delta\nu_k = C_k / r^p$$

$$\frac{E}{E_0} = \beta; \qquad \frac{E_0}{E} = \left(\frac{\bar{r}}{r_0}\right)^2 \rightarrow \Delta\nu_k = C_k \beta E_0 \tag{1}$$

with $p=2$ and

$$r_0 = 0.62\, N^{-1/3}$$

k = index of Stark component defined by n_1 and n_2

$$\bar{r} = 0.55\, N^{-1/3}$$

$$\therefore$$

$$1.0 \lesssim \beta \lesssim 1.5$$

Each Stark component k is defined by n_1 and n_2, where:

n	=	upper level
n'	=	lower level
n_1	=	unperturbed sublevel
n_2	=	perturbed (splitted) sublevel

$n_1 \geq 0$ and $n_2 \leq (n-1)$

$$C_k = \frac{3 h^7 c}{32 \pi^6 m^3 e^9 Z^5} \frac{n'^4 n^4}{(n^2 - n'^2)^2} \left[n(n_2 - n_1) - n'(n'_2 - n'_1) \right]$$

$$x_k = \left[n(n_2 - n_1) - n'(n'_2 - n'_1) \right] \in \mathbb{Z}$$

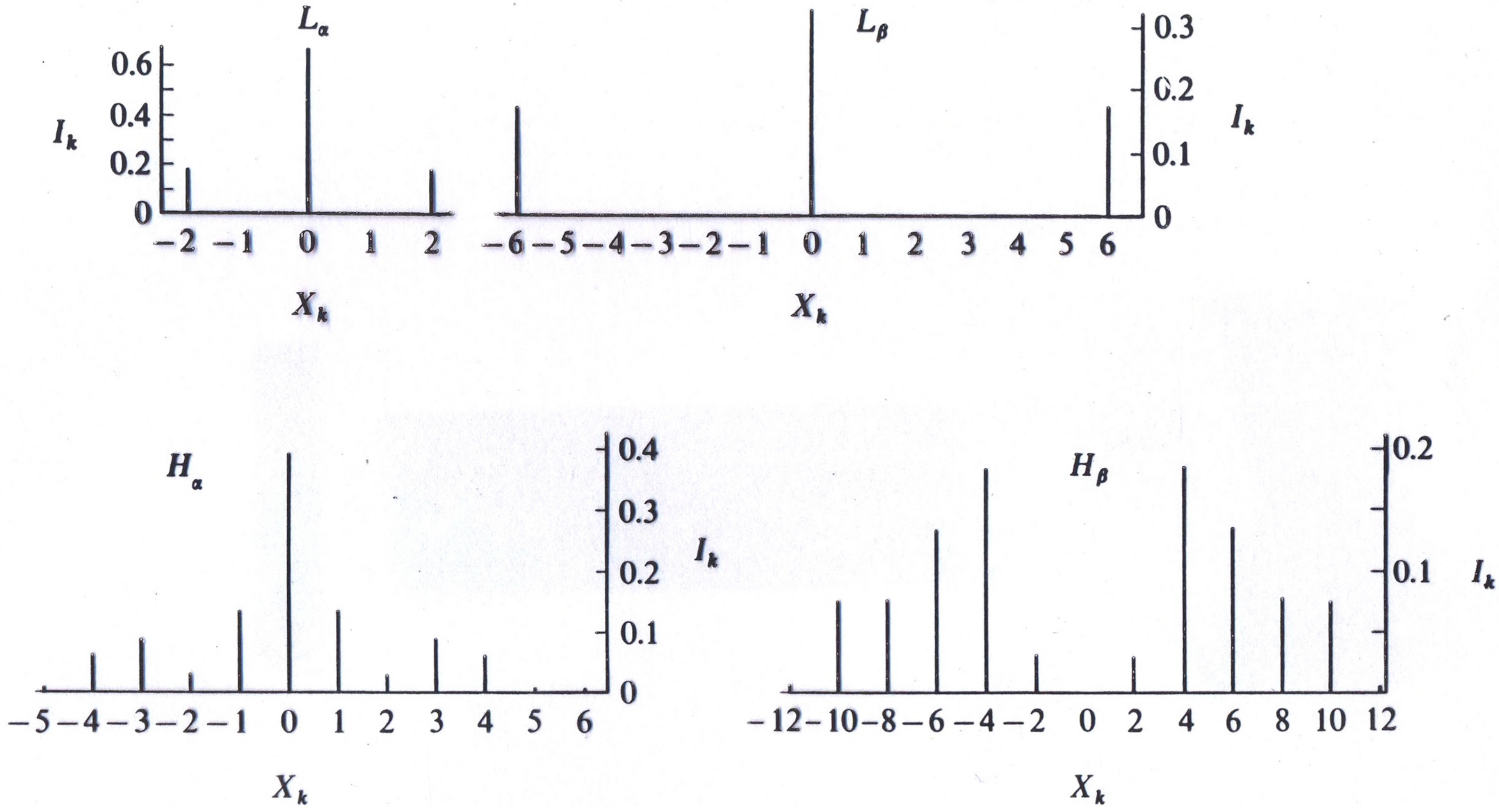
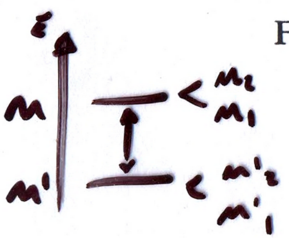


FIG. 9-3. Stark patterns for $L\alpha$, $L\beta$, $H\alpha$, and $H\beta$. Note that $H\beta$ lacks a central unshifted component.



$m_1 \geq 0$

$m_2 \leq n-1$

$$X_K = n(m_2 - m_1) - n'(m'_2 - m'_1)$$

1 NO PERT.
2 PERT.

from Mihalas 1978

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for the high terms in the series: $n >> n'$ $\quad x_k \rightarrow n(n_2 - n_1)$

for Z = 1:

$$C_k = \frac{3h^7 c}{32\pi^6 m_e^3 e^9} n'^4 n\left(n_2 - n_1\right)$$

for $n' = 2$ (Balmer series) in eq. 1:

$$\Delta\nu_k = A\beta E_0 \; n\left(n_2 - n_1\right)$$

$k_{max} \rightarrow$ $n_1 = 0$ and $n_2 = n - 1 \approx n$

$$\Delta\nu_{max} = A\beta E_0 \; n^2 \quad \propto \; HWZI_\nu \tag{2}$$

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$$\Delta\nu_{max} = A\beta E_0 \; n^2 \quad \propto \; HWZI_\nu \qquad (2)$$

$$E_n = -\frac{e^2}{2a_0 n^2}; \quad \frac{dE_n}{dn} = \frac{e^2}{a_0 n^3}; \qquad \Delta\nu(n+1, n) = Bn^{-3} \qquad (3)$$

Bohr energy levels for H

indistinguishable line criteria - overlapping of Stark line wings

$$\Delta\nu_{max} = \frac{\Delta\nu(n+1,n)}{2}$$

maximum width of a "visible" line close to series limit

from 2 and 3

$$\frac{B}{n_{max}^3} = A\beta E_0 n_{max}^2$$

with:

$$E_0 = \frac{e^2}{r_0^2} = \frac{e^2}{\left[3/(4\pi N_e)\right]^{2/3}}$$

$$C N_e^{-2/3} = n_{max}^5 \beta$$

Inglis-Teller relation

$$C^{3/2} N_e = n_{max}^{15/2} \beta^{3/2} \quad \rightarrow \quad \log(N_e) + 1.5 \log(\beta) = C' - 7.5 \log n_{max}$$

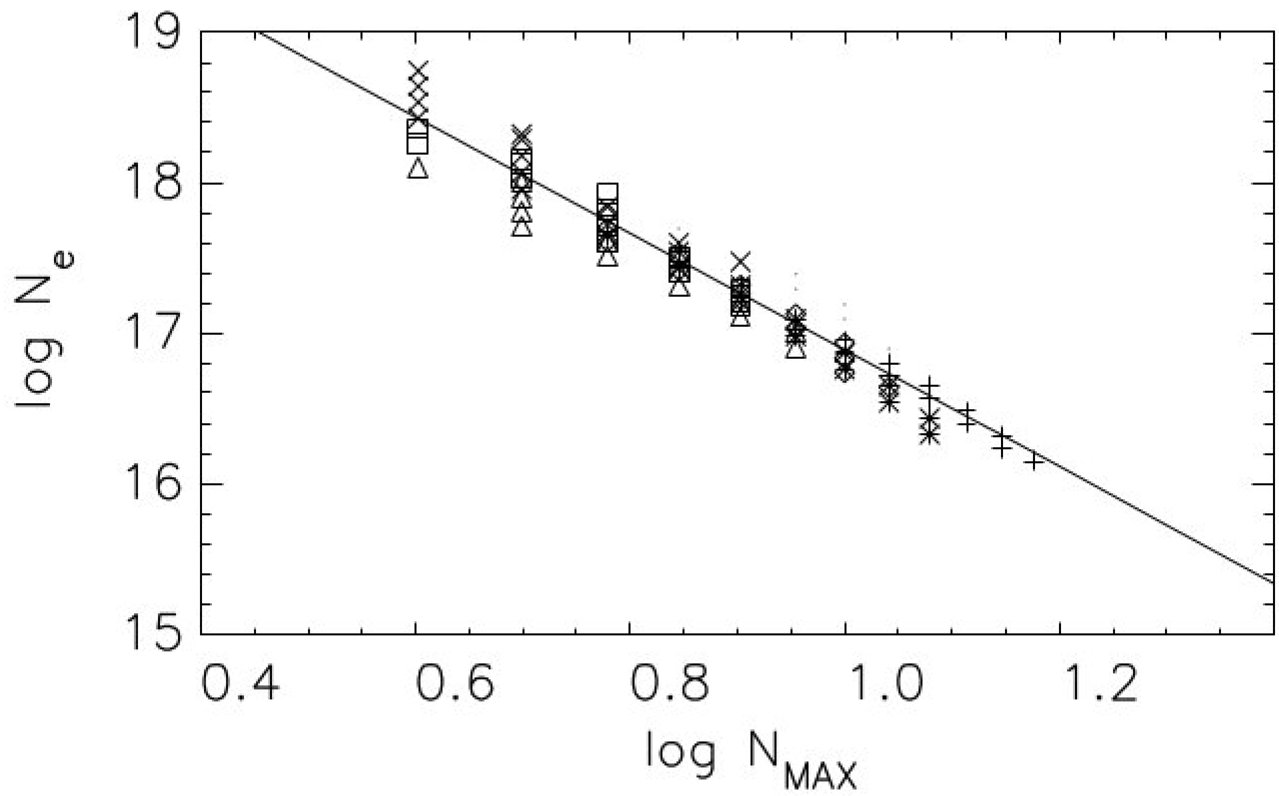
$$\log(N_e) - 23.3 = -7.5 \log n_{max}$$

IV. Quasi-molecular absorption

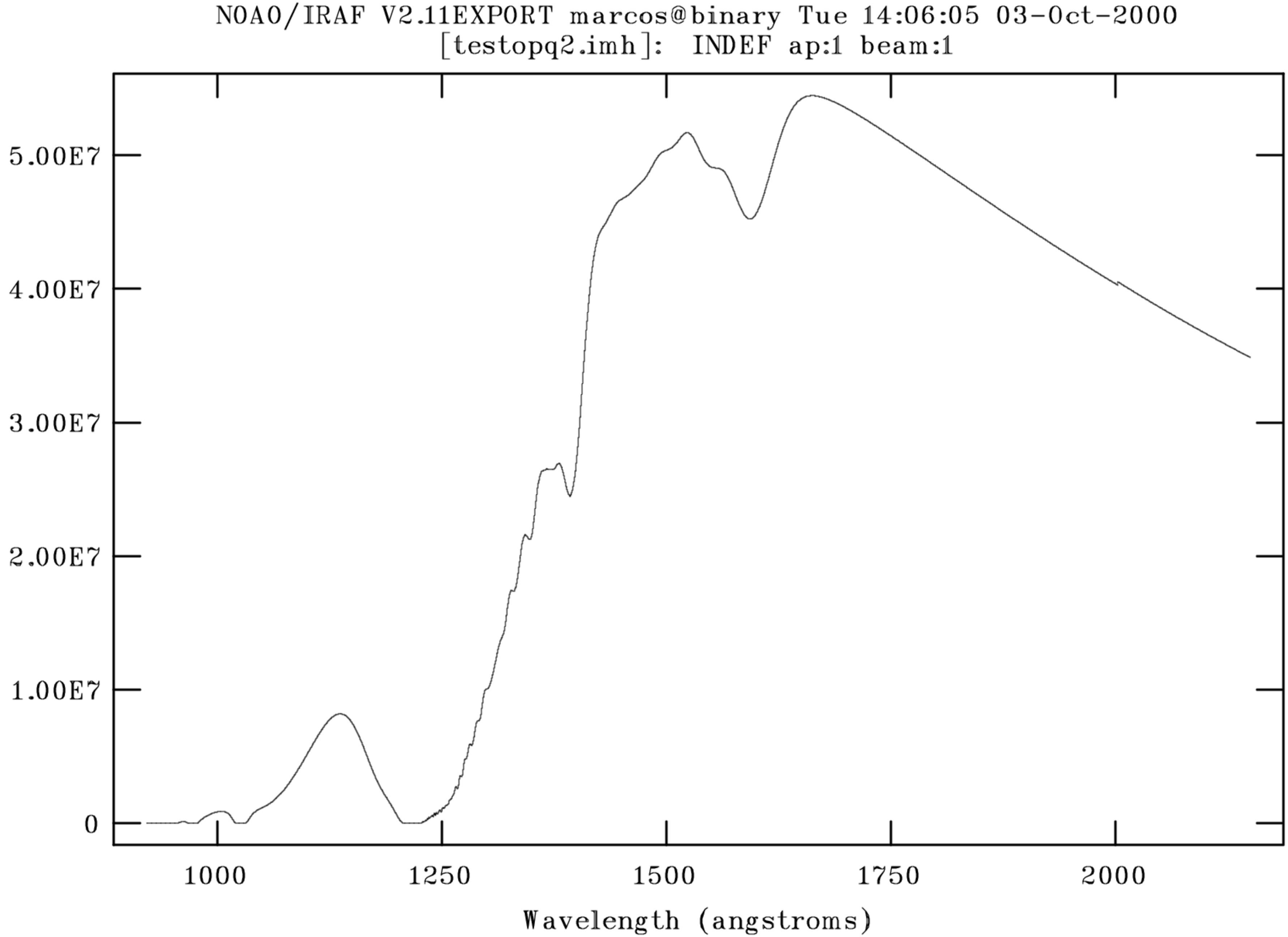
slow collisions form transient (molecular) bound states

High density moderately hot atmospheres

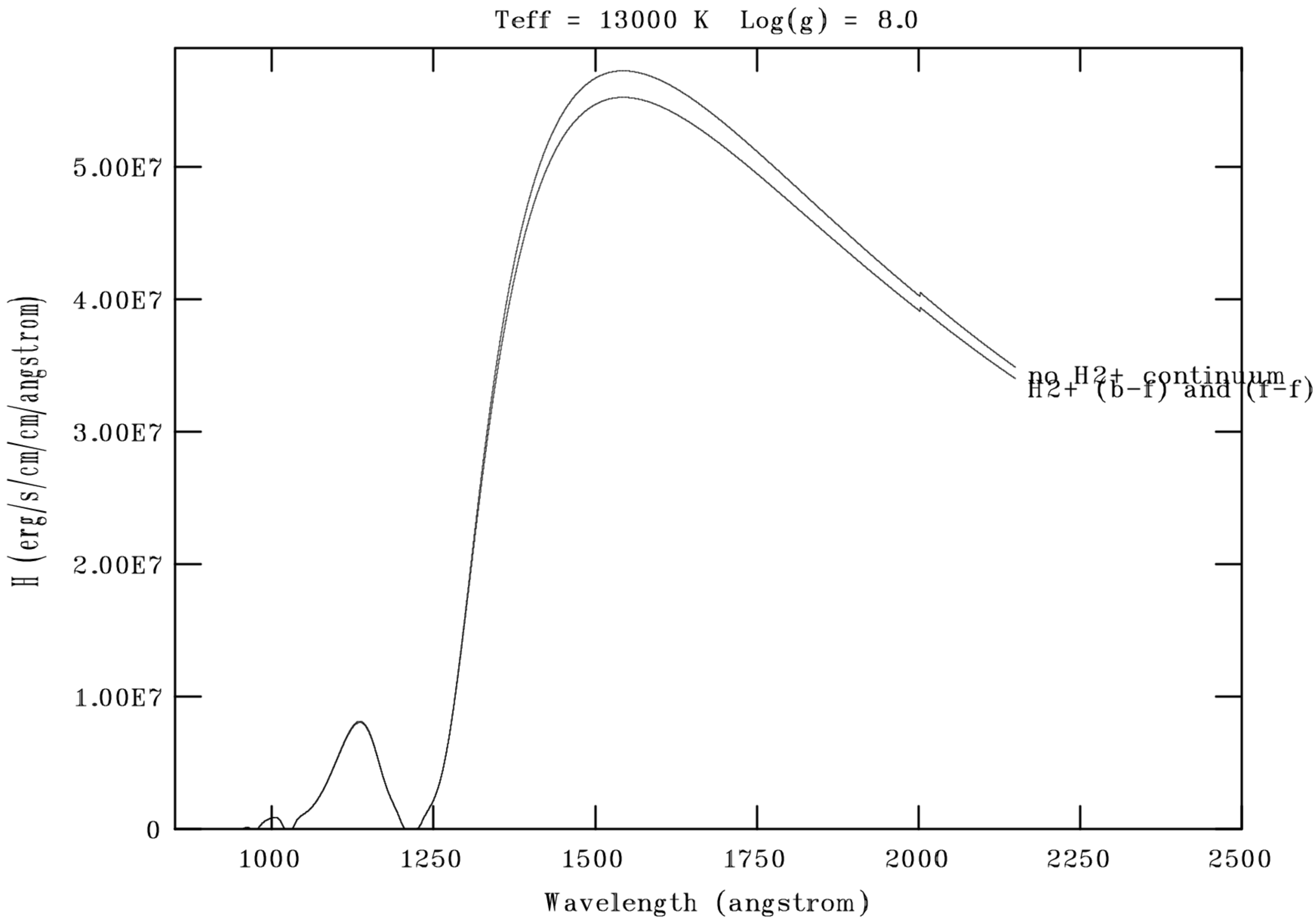
e.g. H-H^+ (quasi-H_2^+) and H-H (quasi-H_2)



Diagnostic for DA white dwarfs with 17,000 < T_{eff} < 100,000 K at $\tau_{Rosseland}$ = 1.0 (from Levenhagen et al. 2017).



Model WD DA spectrum showing quasi-molecular Ly-alpha satellites



H-H⁺ quasi-molecular b-f and f-f continua