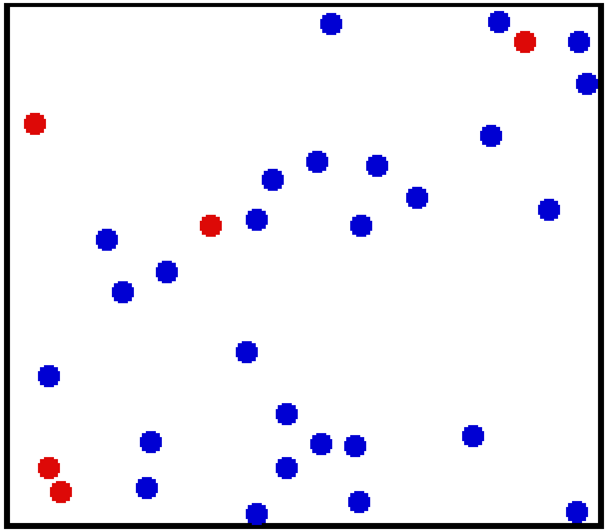
Lecture 9



treasure map:
Gray: pg 236
H&M: pg 228
Bohn-Vitense: pg 117
Rutten: pg 116

Microscopic Line Broadening

I. Natural line profile

$$P_j(t) \;=\; \psi_j^* \psi_j e^{-A_{ji} t} \quad with \quad A_{ji} = \frac{1}{\Delta t_{ji}}$$

j state probability and life time

$$\psi_j(r,t) e^{-i w_{ji} t/\hbar} \;\rightarrow\; u_j e^{-i E_{ji} t/\hbar} e^{-A_{ji} t/2} \quad with \quad E_{ji} = \omega_0/\hbar$$

(similar to classical damped osc.)

$$|\mathscr{F}[\psi(t)]|^2 \;=\; I(\omega) \;=\; \frac{A_{ji}/2\pi}{(\omega-\omega_0)^2 + (A_{ji}/2)^2}$$

from A_{ji} to actual Γ_{ji} :

$$I(\omega) = \frac{\Gamma_{ji}/2\pi}{(\omega-\omega_0)^2 + (\Gamma_{ji}/2)^2}$$

sublevel probabilities summed (see H&M 2015)

$$\Delta\omega = \Gamma_{ji} / 2 \sim (10^{-8} \text{ s})^{-1}$$

absorption cross-section per atom:

$$a_\nu = \frac{\pi e^2}{mc} f_{ij} \frac{\Gamma_{ji}/4\pi^2}{(\nu-\nu_0)^2 + (\Gamma_{ji}/4\pi)^2}$$

II. Thermal Doppler broadening

the Voigt profile

$$a_\nu(\nu) \rightarrow a_\nu\left(\nu - \frac{V_{part}\,\nu_0}{c}\right)$$

where V_{part} = particle thermal velocity
ν = obs. frequency
ν_0 = line frequency

with

$$\frac{\bar{m}\,v_0^2}{2} = \frac{3\,kT}{2}$$

$$V_0^2 = V_x^2 + V_y^2 + V_z^2$$

$$V_x^2 = \frac{kT}{\bar{m}} \rightarrow \frac{\Delta\,\nu_D}{\nu_0} = \frac{1}{c}\left(\frac{kT}{\bar{m}}\right)^{1/2}$$

$$W(T, V_x)\,dV = \left(\frac{2\pi kT}{\bar{m}}\right)^{-1/2} e^{-\frac{V_x^2 \bar{m}}{2kT}}\,dV$$ Maxwell-Boltzmann probability

$$a_\nu(\nu) = \left(\frac{m}{2\pi kT}\right)^{1/2} \left(\frac{\pi c^2}{mc}\right) f_{ij} \int_{-\infty}^{\infty} \frac{\left(\Gamma_{ji}/4\pi^2\right) e^{-\frac{V_x^2}{2 v_D^2}}}{\left(\nu - \nu_0 - \frac{V_x \nu_0}{c}\right)^2 + \left(\Gamma_{ji}/4\pi\right)^2}\,dV$$

$$a = \frac{\Gamma}{4\pi\Delta\nu_D}; \quad b = \frac{\nu - \nu_0}{\Delta\nu_D}; \quad y = \frac{V_x}{\left(kT/\bar{m}\right)^{1/2}};$$

$$a_\nu(\nu) = \frac{\pi^{1/2} e^2}{mc} f_{ij} H \frac{(a, b)}{\Delta \nu_D}$$ with

$$H(a,b) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{a^2 + (b-y)^2} dy$$ Voigt function

i. $b \rightarrow \infty$ - far line wings

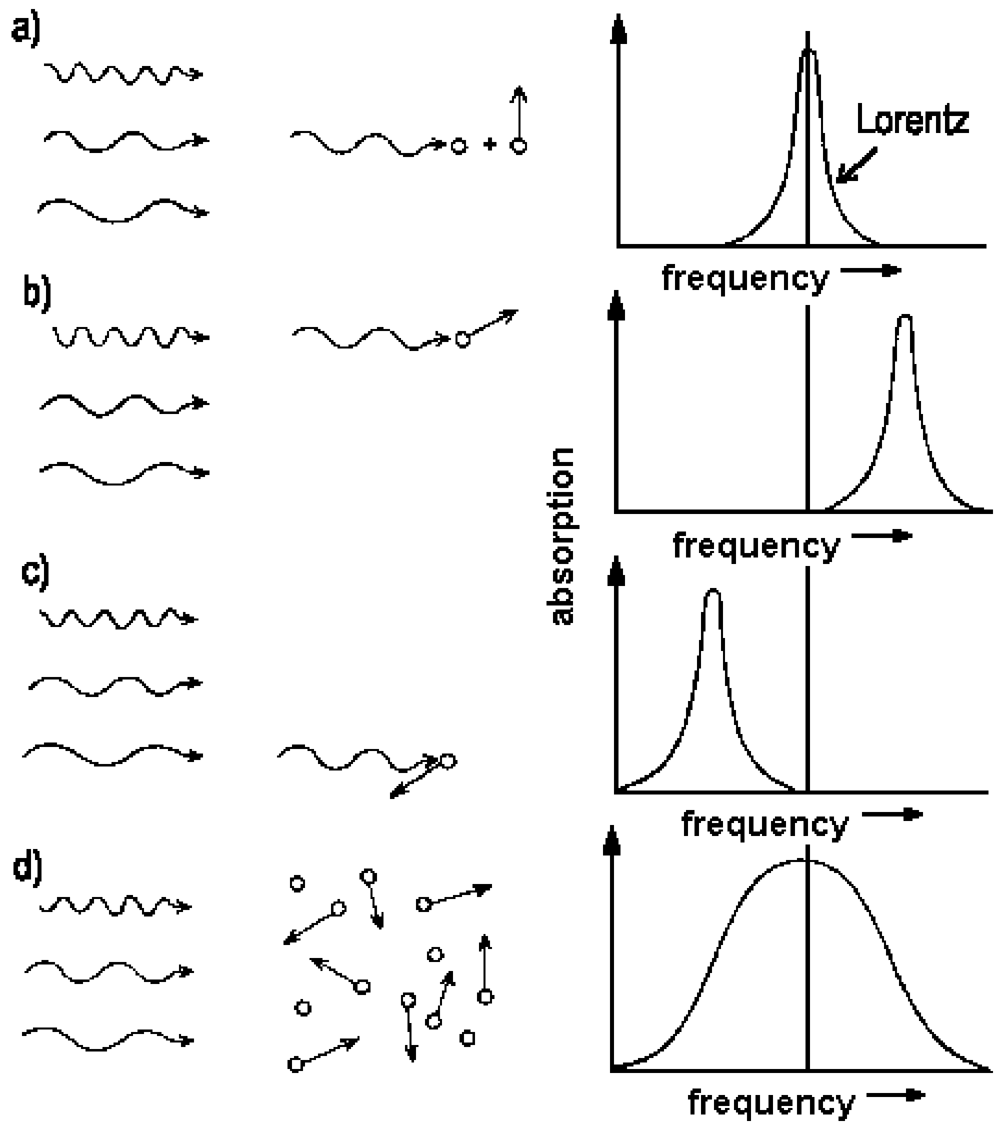
$$H(a,b) \rightarrow \sim \frac{a}{\pi^{1/2} b^2} \propto \frac{a}{(\nu - \nu_0)^2}$$

ii. $\Delta\nu_D >> \Gamma \rightarrow a \rightarrow 0$

all terms with *b* and *y* remain:

$$H(a,b) \rightarrow \sim e^{-b^2} \quad \propto \quad e^{-\left[(\nu-\nu_0)/\Delta\nu_D\right]^2}$$

gaussian core with thermal width



from Gericke 2015

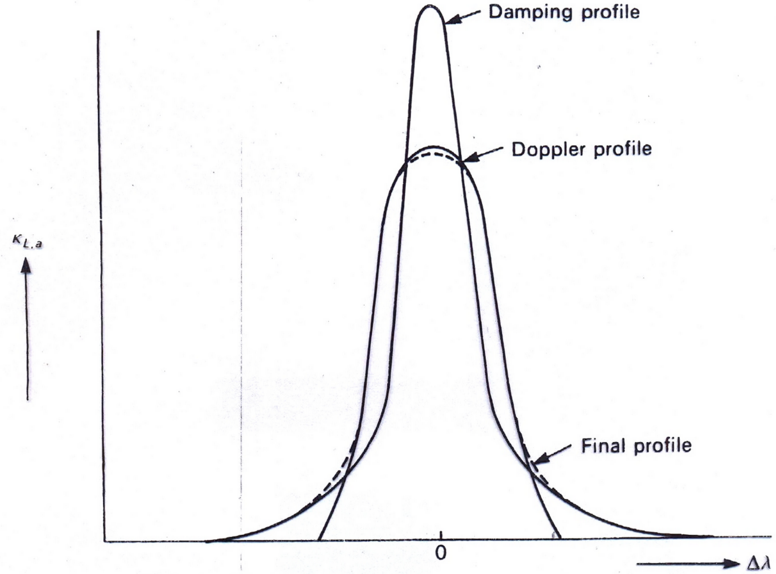


Fig. 10.6. A schematic plot of the different line profiles contributing to the Voigt profile. Generally, the line profile consists of the Doppler core and the damping wings. It is called a Voigt profile.

$$\alpha = \frac{\Gamma}{4\pi\Delta\nu_0} = a$$

$$v = \frac{\nu - \nu_0}{\Delta\nu_0} = b$$

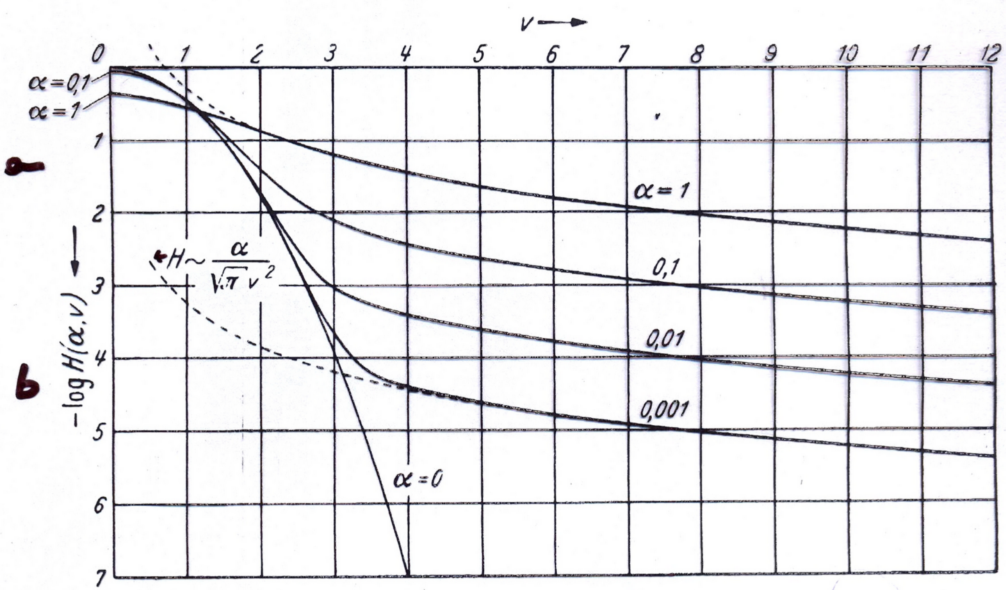


Fig. 10.7. Voigt profiles for different values of α are shown in a logarithmic plot. (From Unsöld, 1955, p. 265.)

from Bohn-Vitense 1989