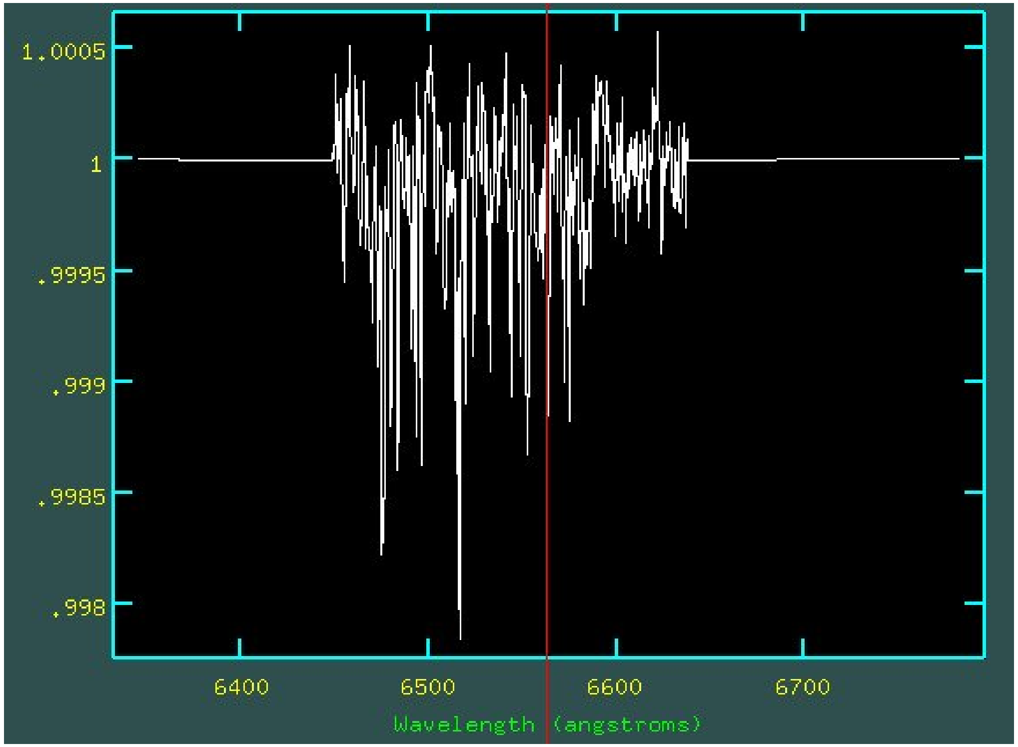
Lecture 8

stellar atmospheres
prof. Marcos Diaz



treasure map:
Williams, R. ("The Analysis of Emission lines", 2009)
Gray: pg 232
H&M: pg 144, 181, 186
Bohn-Vitense: pg 117,167
Rutten: pg 83

The line absorption and emission

I. the classical radiation driven damped oscillator:

$$m(\ddot{\mathbf{d}} + \omega_0^2) = e\mathbf{E}_0(\omega) - m\,\gamma\,\dot{\mathbf{d}}$$

with the classical damping constant:

$$\gamma \equiv \frac{2e^2\omega_0^2}{3mc^3}$$

solved for charged particle acceleration provide:

$$\ddot{\mathbf{d}} = \dot{\mathbf{v}} = \frac{e\,\omega^2}{m}\Re\mathrm{e}\left[\frac{\mathbf{E}_0(\omega)e^{i\omega t}}{(\omega^2 - \omega_0^2) + i\,\gamma\,\omega}\right]$$

with Poynting flux:

$$P(t) = \frac{2e^2\ddot{d}}{3c^3}$$

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with Poynting intensity: $P(t) = \frac{2e^2\ddot{d}}{3c^3}$

EM radiation orthogonal E and H fields with amplitude E_0 and frequency $\omega = 2\pi\nu$ yields a randomly directed Poynting vector power:

$$\langle P(\omega)\rangle_T = \left(\frac{e^4\omega^4}{3m^2c^3}\right)\frac{E_0^2}{(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2}.$$

with $I_0 \equiv E_0^2 / 8\pi$ and $\langle P(\omega) \rangle_{T,4\pi} = \oint \sigma(\omega) I_0 \, d\Omega$ (Poynting flux)

$$\sigma(\omega) = \frac{(8\pi e^4 \omega^4 / 3m^2 c^4)}{[(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2]}$$

for a free electron $\omega_0 = 0$:

$$\sigma(\omega) \rightarrow \sigma_{\mathrm{T}} \equiv \frac{8\pi e^4}{3m_e^2 c^4} = 6.65 \times 10^{-25} \ \mathrm{cm}^2$$

$$\sigma_{class} = \int_0^\infty \sigma(\omega) \, d\omega = \frac{\pi e^2}{mc} \rightarrow \frac{\pi e^2}{mc} f \quad (\sigma_{\mathrm{quantum}})$$

f = fraction of quantum oscillators allowed to participate

II. Equilibrium LTE *gf*

$$N_u A_{ul} + N_u B_{ul} I_\nu + C_{ul} = N_l B_{lu} I_\nu + C_{lu} \qquad \text{(rate equation)}$$

With detailed balance of collisional transitions:

$$I_\nu = \frac{A_{ul}}{B_{lu}(N_l/N_u) - B_{ul}} \qquad \text{LTE} \rightarrow \qquad I_\nu = \frac{A_{ul}}{(g_l/g_u)B_{lu}e^{h\nu/kT} - B_{ul}}$$

Wien ($h\nu >> kT$)

$$I_\nu \ \alpha \ \nu^3 e^{-\frac{h\nu}{kT}} \qquad \rightarrow \quad B_{ul} = B_{lu} g_l / g_u$$

Rayleigh-Jeans ($h\nu << kT$) $\rightarrow$

$$I_\nu \approx \frac{A_{ul}}{(g_l/g_u)B_{lu} - B_{ul} + (g_l/g_u)B_{lu}h\nu/kT}$$

$$I_\nu \approx \frac{2kT\nu^2}{c^2} \qquad \rightarrow \quad A_{ul} = \frac{2h\nu^3}{c^2} B_{ul}$$

$$\sigma_{tot} = \frac{\pi e^2}{mc} f_{lu} = B_{lu} h \nu = \frac{mc^3}{2\pi e^2 \nu^2} \frac{g_u}{g_l} A_{ul}$$

$$g_l f_{lu} = g_u f_{ul} = gf$$

with $g_{l,u} = 2J_{l,u} + 1$

III. Bound state quantum numbers & selection rules

$$-\frac{\hbar^2}{2\mathrm{m}}\nabla^2\psi + V(x,y,z)\psi = E\,\psi \qquad \text{with} \qquad \frac{1}{\mathrm{m}} \equiv \frac{1}{m_N} + \frac{1}{m_e} = \frac{m_N + m_e}{m_N m_e}.$$

$$-\frac{\hbar^2}{2\mathrm{m}}\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}\right] - \frac{Ze^2}{r}\psi = E\,\psi.$$

with $\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$

$$\frac{d^2\Phi}{d\phi^2} + m^2\Phi = 0,$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) + \left[l(l+1) - \frac{m^2}{\sin^2\theta}\right]\Theta = 0, \qquad \text{Legendre } (l,m)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \frac{l(l+1)}{r^2}R + \frac{2\mathrm{m}}{\hbar^2}\left(E + \frac{Ze^2}{r}\right)R = 0. \qquad \text{Laguerre } (n,l)$$

Solution for single electron systems:

$$\psi_{n,l,m}(r,\theta,\phi) = A_{n,l,m} L_{n,l}(r)\, P_{l,m}(\cos\theta)\, e^{im\phi}$$

For many electron systems the energy levels are given by the quantum numbers of each electron:

n_i: orbit of electron i
l_i: orbital angular momentum of electron i; with $l_i < n_i$ ($l_i = 0$ or "s"; $l_i = 1$ or "p"; ...)
L: orbital angular momentum the state: vector sum of l_i ($L = 0$ or "S"; $L = 1$ or "P"; ...)
S: spin angular momentum of the state: vector sum of s_i ($s_i = 1/2$)
J: total angular momentum of the state: vector sum of $L + S$

Spectroscopic Notation:

e.g. for a 12 electron atom: $(n_i, l_i) \quad {}^{2S+1}L_J$

$$[1s^2 2s^2 2p^6]\ 3s^2\ {}^1S_0$$

for 2 different Helium states:

$$1s^2\ {}^1S_0;\ 1s2p\ {}^3P_2$$

from R. Williams 2017

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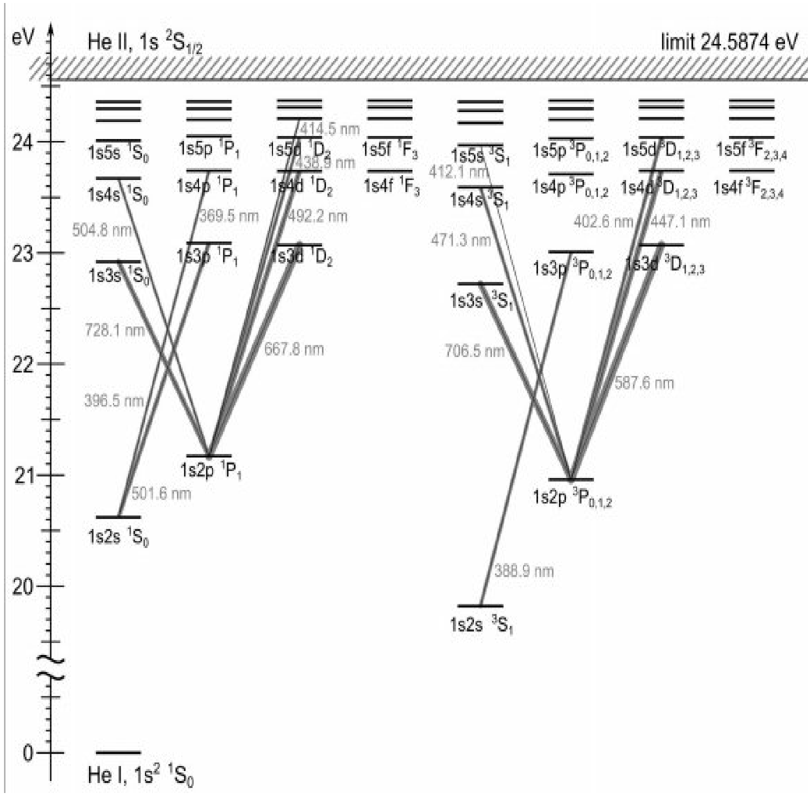

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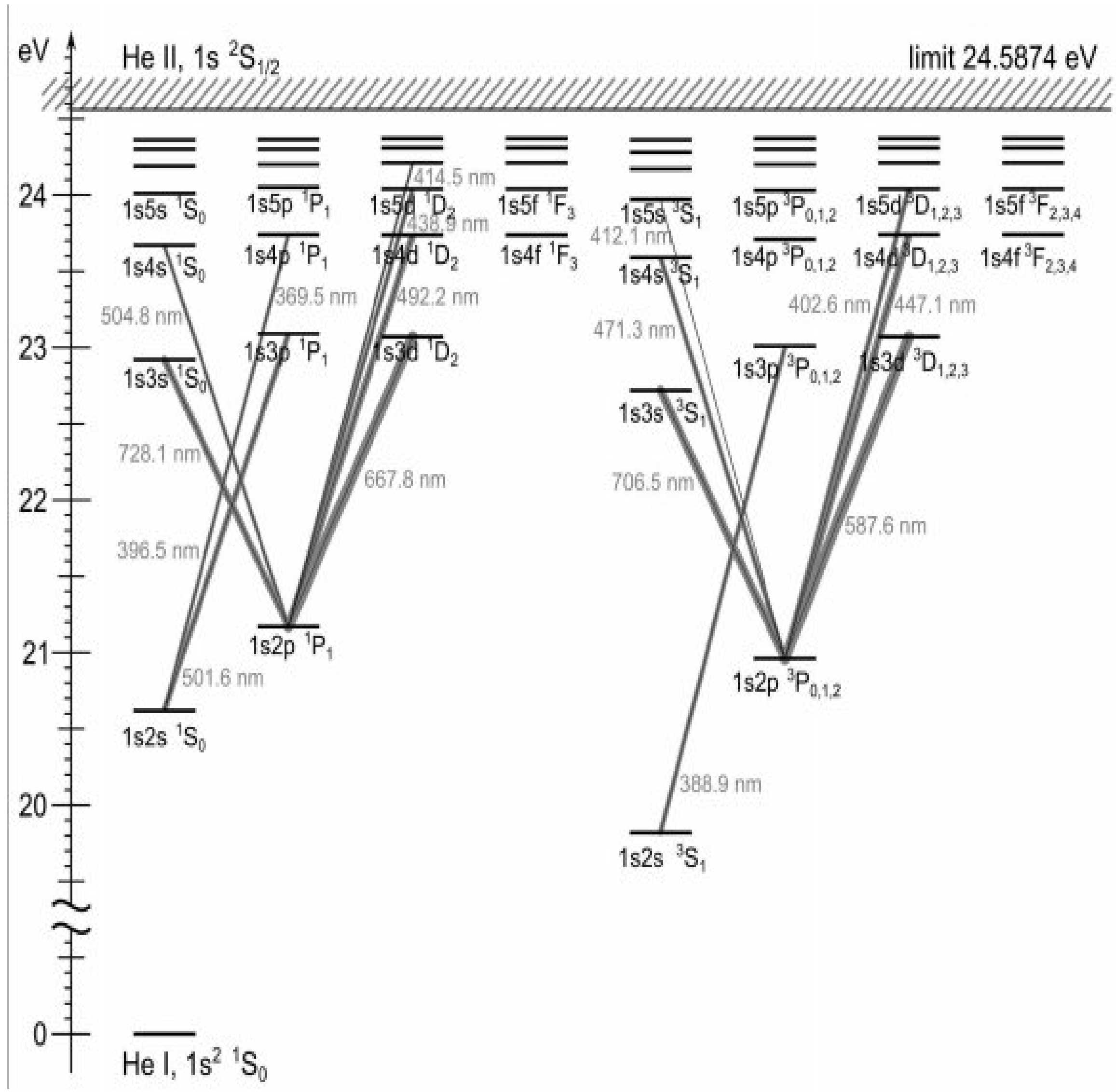

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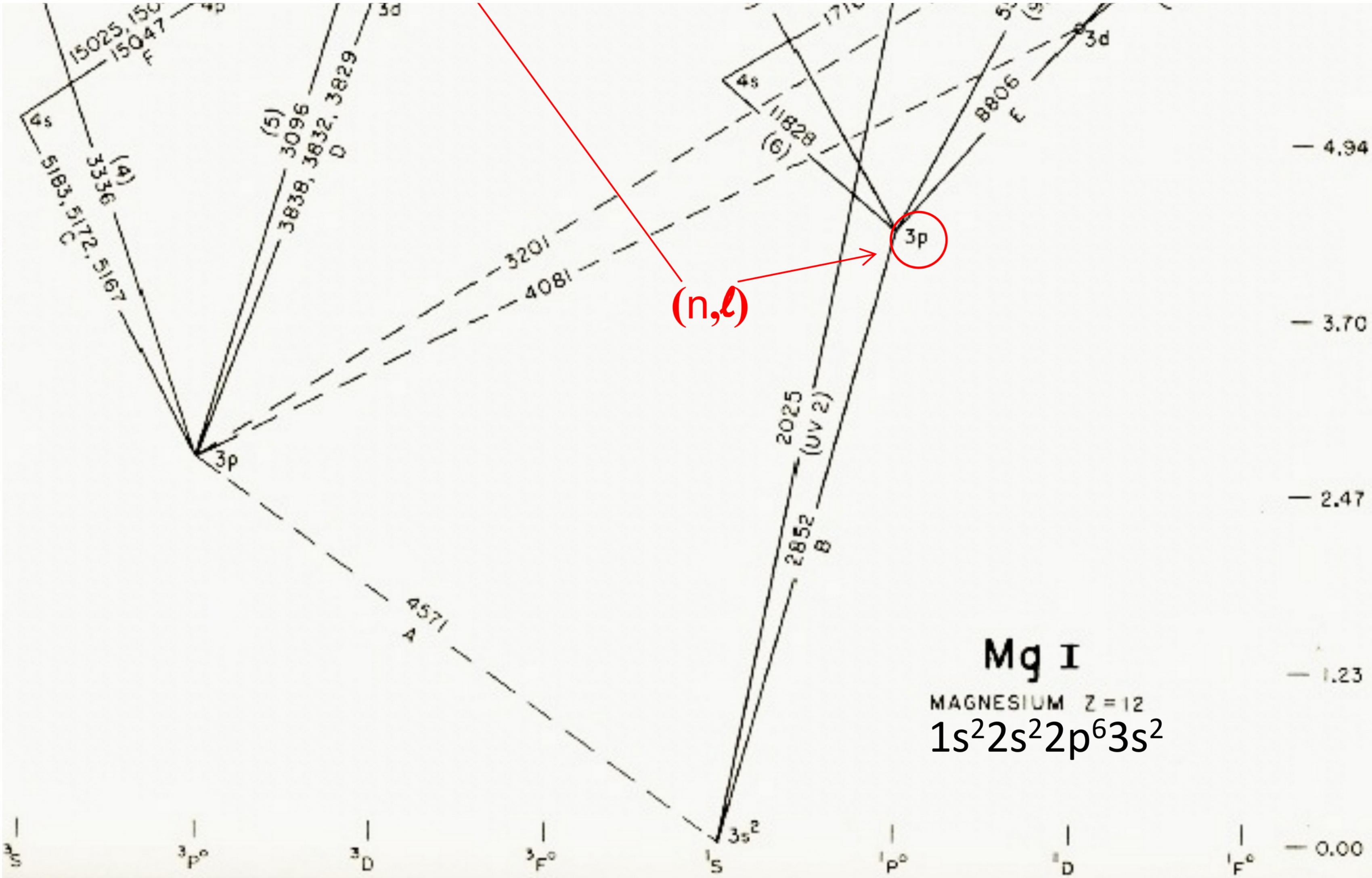

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Selection rules

$$\Delta S = (S' - S) = 0*$$
$$\Delta L = (L'_l - L_l) = 0, \pm 1, \pm 2 (E2)$$
$$\Delta J = (J'_l - J_l) = 0, \pm 1, \pm 2 (E2)$$

i. for atom **electric dipole** E1(permitted) transitions:

→ change in parity of the wave function that describes the state

→ $\Delta l_i = \mp 1$; $\Delta n =$ *any*

ii. for atom **electric quadrupole E2 / mag dipole** M1(forbidden "[X]") transitions:

→ no change in parity of the wave function that describes the state

→ $\Delta l_i = 0$; $\Delta n = 0$ *(same eletronic config) OR one electron change n (M1)*

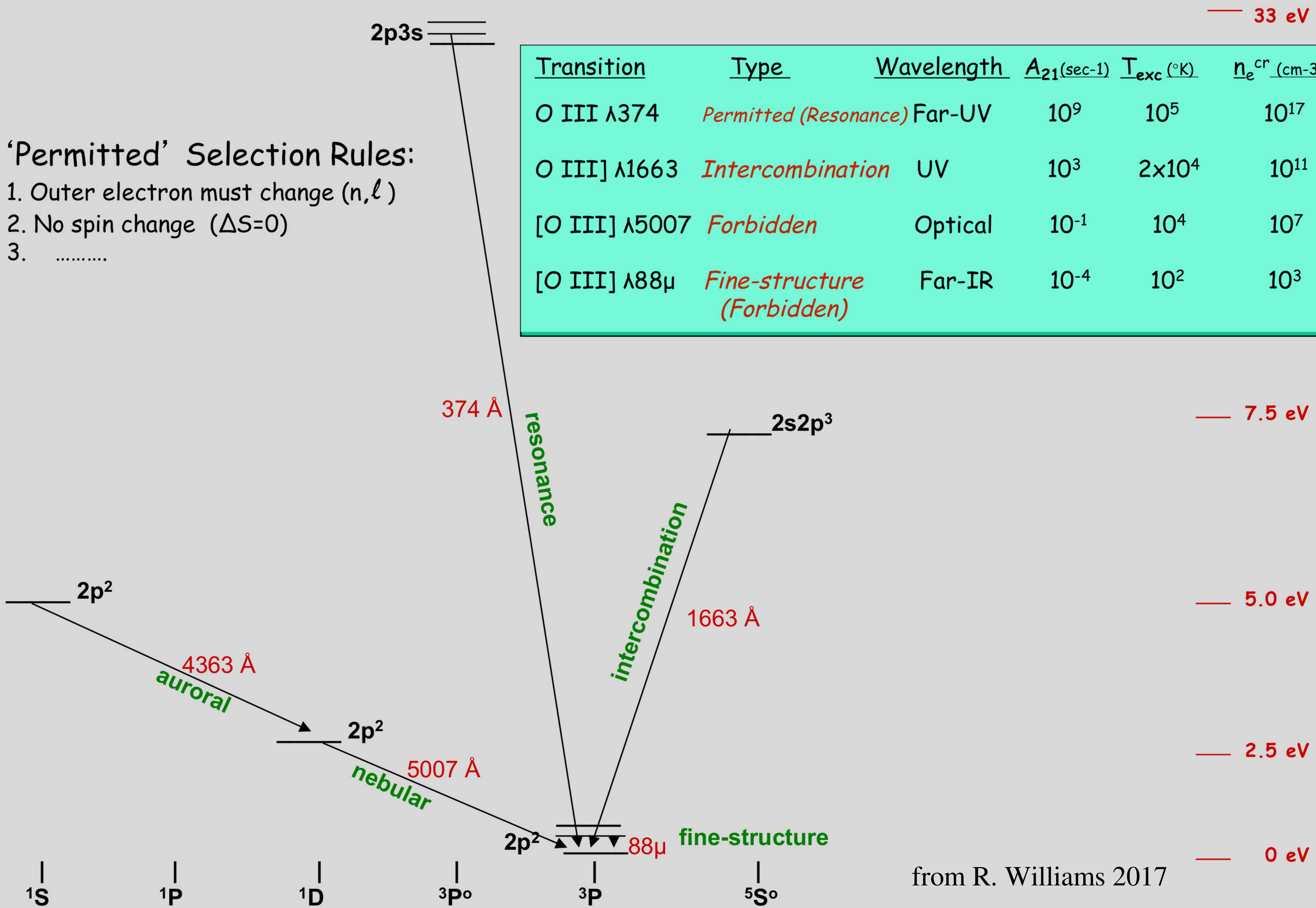
* LS coupling violation $\Delta S \neq 0$ → intercombination (or semi-forbidden "[X") transitions.

Atomic Energy Levels: O III $(1s^2 2s^2 2p^2)$

'Permitted' Selection Rules:

1. Outer electron must change (n, ℓ)
2. No spin change $(\Delta S=0)$
3. ……….

Transition	Type	Wavelength	A_{21} (sec-1)	T_{exc} (°K)	n_e^{cr} (cm-3)
O III λ374	Permitted (Resonance)	Far-UV	10^9	10^5	10^{17}
O III] λ1663	Intercombination	UV	10^3	2×10^4	10^{11}
[O III] λ5007	Forbidden	Optical	10^{-1}	10^4	10^7
[O III] λ88μ	Fine-structure (Forbidden)	Far-IR	10^{-4}	10^2	10^3



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Main link for Atomic data:

https://www.nist.gov/pml/atomic-spectra-database

Grotrian Diagrams:

https://ned.ipac.caltech.edu/level5/Ewald/Grotrian/frames.html