Lecture 6 stellar atmospheres prof. Marcos Diaz

treasure map:

H&M: pg 379, 421 Rutten: pg 50



Pierre-Simon Laplace

1749-1827

The Lambda operator

again:

$$k_v = k_{abs} + \sigma_{sct}$$

and thermal emission + for coherent and isotropic scattering:

$$\eta_{v} = k_{abs} B_{v} + \sigma_{sct} J_{v}$$

$$\Rightarrow S_{v} = (k_{abs} B_{v} + \sigma_{sct} J_{v}) / (k_{abs} + \sigma_{sct})$$
with
$$\epsilon_{v} = \frac{k_{absorption}}{(k_{absorption} + \sigma_{sct})}$$

$$S_{v} = \epsilon_{v} B_{v} + (1 - \epsilon_{v}) J_{v}$$

thermal coupling parameter

 $(1 - \varepsilon_v)$ is the scattering albedo

with corresponding solution for J_{v} at a given frequency:

$$J_{\nu}(\tau_{\nu}) = \Lambda_{\tau_{\nu}}[S_{\nu}] = \Lambda_{\tau_{\nu}}[(1 - \epsilon_{\nu})J_{\nu}] + \Lambda_{\tau_{\nu}}[\epsilon_{\nu}B_{\nu}]$$

discrete grid in τ_v for B_v , ε_v , J_v

i. simple solution (sum) for pure absorption $\varepsilon_v = 1$ ii. matrix (NDxND) inversion for $\varepsilon_v < 1$ (number of operations $\propto ND^3$)

where ND is the number of τ_v depths

still need to solve the coupling on $\varepsilon_{v}(\rho,T)$

$$J_{\nu}^{(1)} = \Lambda_{\tau_{\nu}} \Big[(1 - \epsilon_{\nu}) J_{\nu}^{(0)} \Big] + \Lambda_{\tau_{\nu}} [\epsilon_{\nu} B_{\nu}] \qquad \text{with } J_{\nu}^{(0)} = B_{\nu}, \ \varepsilon_{\nu} = \varepsilon_{grey}$$

$$J_{\nu}^{(n+1)} = \Lambda_{\tau_{\nu}} \left[(1 - \epsilon_{\nu}) J_{\nu}^{(n)} \right] + \Lambda_{\tau_{\nu}} [\epsilon_{\nu} B_{\nu}]$$

i. no inversion ii. coupling on ε_{v} *is part of iteration*

scattering + absorption → kernel resolution & coverage scattering mix distant parts of the atmosphere

$$J_{\nu}^{(1)} = \Lambda_{\tau_{\nu}} \left[(1 - \epsilon_{\nu}) J_{\nu}^{(0)} \right] + \Lambda_{\tau_{\nu}} [\epsilon_{\nu} B_{\nu}]$$

with
$$J_{v}^{(0)} = B_{v}$$
, $\varepsilon_{v} = \varepsilon_{grey}$



scattering + absorption \rightarrow kernel resolution & coverage

scattering mix distant parts of the atmosphere

ii. coupling on ε_{v} is part of iteration

i. no inversion

iii. indirect method



ALI – Accelerated Lambda Iteration

(Hubeny & Lanz, 1992, A&A, 262)

 $\Lambda \rightarrow \Lambda^{*} + (\Lambda - \Lambda^{*})$ $J^{new} = \Lambda S^{old} (n^{old}, T^{old}) + \Lambda^{*} S^{new} (n^{new}, T^{new}) - \Lambda^{*} S^{old} (n^{old}, T^{old})$ $\Lambda^{*} = optimum \ lambda \ operator$ $used \ with \ convergence \ accelerators$

Model atmosphere coupled equations

- i. radiative transfer $\mu \frac{\mathrm{d}I_{\nu}}{\mathrm{d}\tau_{\nu}} = I_{\nu} S_{\nu}$
- ii. hydrostatic equilibrium

$$\frac{\mathrm{d}P}{\mathrm{d}z} = -\rho \, g$$

with

$$\mathrm{d}m = -\rho\,\mathrm{d}z$$

(m = column density)

 $\frac{\mathrm{d}P}{\mathrm{d}m} = g \qquad \qquad P(m) = mg$

$$P = P_{\text{gas}} + P_{\text{rad}} + P_{\text{turb}} = NkT + \frac{4\pi}{c} \int_0^\infty K_\nu d\nu + \frac{1}{2} \rho v_{\text{turb}}^2$$

iii. radiative equilibrium

$$\int_0^\infty H_\nu \,\mathrm{d}\nu = \mathrm{const} = \frac{\sigma}{4\pi} \,T_{\mathrm{eff}}^4$$

$$\int_0 \kappa_{\nu} \left(J_{\nu} - S_{\nu} \right) \mathrm{d}\nu = 0$$

iv. statistical equilibrium

$$n_i \sum_{j \neq i} (R_{ij} + C_{ij}) = \sum_{j \neq i} n_j (R_{ji} + C_{ji})$$

with

$$\sum_{i} n_i = N_{\text{atom}}$$

v. charge conservation

$$\sum_{i} n_i Z_i - n_e = 0$$

Equation	Corresponding state parameter
Radiative transfer	Mean intensities, J_{ν}
Radiative equilibrium	Temperature, T
Hydrostatic equilibrium	Total particle density, N
Statistical equilibrium	Populations, n_i
Charge conservation	Electron density, $n_{\rm e}$

The structure-synthesis approach

concept of:

- i. non-structural elements
 ii. non-structural (b-f and b-b) opacities
 iii. optimal frequency sampling
- → Accurate enough description of structure and continuum radiation field @ v_n :

a) $\rho(\tau)$, $N_{H}(\tau)$, $n_{e}(\tau)$, $T(\tau)$, $\tau(z)$, $F_{v}(0)$

b) NLTE: $n_{i,j,k}(\tau)$

→ Detailed spectral synthesis (fast)

 $k_v(\tau_v), \eta_v(\tau_v)$ + radiative transfer equation

high resolution; precise line broadening; precise limb darkening; custom line list (broadening coef, gf, broadening coefs., E, (2J_{ii}+1);

The non-gray generic case

The Ψ matrix

sample for H, He in NLTE, high temperature structure

$$\Psi_{d} = \left[P_{1}, \dots, P_{N}; N_{1}, \dots, N_{NLH}; N_{1}, \dots, N_{NLHe}; n_{H}; n_{He}; n_{e}; n; T_{e}; D_{1}, \dots, D_{ND}\right]$$

where:

 $P_i = \mu$ averaged intensity $= \frac{1}{2} [I(v_i, \mu_i) + I(v_i, -\mu_i)]$ (Feautrier variable); with $\mu = \cos(i_{orb})$ with i = [1, NJ] points in a predefined μ, ν grid.

 N_i = Population of level i (in LTE \Leftarrow Boltzmann equation).

 $n_{\rm H, He, e}$ = hydrogen, helium, electron, and total density.

 T_{e} = electron temperature.

 $D_i = depth index.$

Number of variables (NN): NJ + NLH + NLHe + 5.

Number of equations:NLH statistical equilibrium equations for hydrogen.
NLHe statistical equilibrium equations for helium.
NJ radiation transfer equations.
Radiative equilibrium condition (convection is ignored).
Hydrostatic equilibrium equation.
Charge conservation.
Particle conservation in statistical eq. eq. for hydrogen.
Particle " " for helium.

The elements of the matrix are linearized:

The starting solution Ψ^0 is the gray solution and is perturbed by $\delta \Psi$.

 $\Psi^{*}(D_{i}) \equiv \Psi^{*-1}(D_{i}) + \delta \Psi(D_{i})^{*}$

for a particular iteration:

 $-A(D_i)\delta\Psi(D_{i-1})+B(D_i)\delta\Psi(D_i)-C(D_i)\delta\Psi(D_{i+1})=error(D_i)$

The structure-synthesis approach

concept of:

- i. non-structural elements
- ii. non-structural (b-f and b-b) opacities
- iii. optimal frequency sampling
- → Model atmosphere provide accurate enough description of structure and continuum radiation field @ v_i :

a) $\rho(\tau)$, $N_{H}(\tau)$, $n_{e}(\tau)$, $T(\tau)$, $\tau(z)$, $F_{v}(\tau=0)$

b) optional NLTE: $n_{i,j,k}(\tau)$

c) optional: $k_{cont}(\tau)$, $\eta_{cont}(\tau)$

The structure-synthesis approach (cont.)

→ Detailed spectral synthesis (fast) provide $I(v,\mu) \rightarrow^* F(v)$ * (geometry and velocity field)

 $k_v(\tau_v)$, $\eta_v(\tau_v)$ + radiative transfer equation (direct solution)

with

- custom (observer's) frequency grid;
- precise line broadening (natural, Stark, vdW, microturbulence, macroturbulence, rotation);
- limb darkening;
- custom non-structural abundances;
- custom line list (gf, broadening coefs., E_{i,j}, (2J_{i,j}+1);
- polarization and Zeeman

Types of model atmosphere structures

- i. Grey
- ii. LTÉ non-grey continuum
- *iii. LTE* + *line blancketed*
- *iv. NLTE H*+*He*, *LTE line blanketed*
- v. full NLTE line blanketed models (superlevels)

Features and processes that may be relevant to model structures:

- presence of convection (3D), advection, conduction.
- non-plane-parallel (spherical, flared disk, or 3D geometry).
- lack of radiative equilibrium (sources).
- variable gravity, magnetic fields.
- mass loss (winds, 3D).
- relativistic effects. tide effects.
- irradiation, condensation.
- chemical composition stratification.
- time-dependence (in one or more of the above).



Figure 2 The influence of the molecular and atomic opacities and convection upon the atmospheric structure of a typical model atmosphere; here the $T_{eff} = 2800$ K, log g = 5.0, and solar metallicity model of Allard & Hauschildt (1995b). A corresponding gray structure without convection (bold dot-dashed) is also shown for comparison. While the complete neglect of H₂O opacities causes a dramatic cooling (by CO) of the atmosphere (long-dashed curve), uncertainties by a factor of two in the H₂O opacity cross sections cause only negligible changes in the atmospheric structure (thin dot-dashed relative to dotted curve). A similar drop in the opacity cross section of TiO, however (thin short-dashed relative to dotted curve), causes a much more significant cooling of the atmosphere.

'19. 111,10: The LTE temperature stratification for a typical 1: V-star model atmosphere with $T_{eff} = 25000$ K, log g = 4.0 and solar abundances. a) grey case, b) unblanketed continuum mod-1 (Munich code), c) line blanketed model including one hundred of the strongest lines (Munich code), d) Kurucz model. For dismeson see text.