Lecture 6

stellar atmospheres

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treasure map:

H&M: pg 379, 421

Rutten: pg 50

The Lambda operator

again:

$$k_\nu = k_{abs} + \sigma_{sct}$$

and thermal emission + for coherent and isotropic scattering:

$$\eta_\nu = k_{abs} B_\nu + \sigma_{sct} J_\nu$$

$$\Rightarrow \; S_\nu = (k_{abs} B_\nu + \sigma_{sct} J_\nu) \;/\; (k_{abs} + \sigma_{sct})$$

with $\epsilon_\nu = \dfrac{k_{absorption}}{(k_{absorption} + \sigma_{sct})}$ thermal coupling parameter

$S_\nu = \varepsilon_\nu B_\nu + (1 - \varepsilon_\nu) J_\nu$ $(1 - \varepsilon_\nu)$ is the scattering albedo with corresponding solution for J_ν at a given frequency:

$$J_\nu(\tau_\nu) = \Lambda_{\tau_\nu}[S_\nu] = \Lambda_{\tau_\nu}[(1-\epsilon_\nu)\,J_\nu] + \Lambda_{\tau_\nu}[\epsilon_\nu\,B_\nu]$$

discrete grid in τ_ν for B_ν, ε_ν, J_ν

i. simple solution (sum) for pure absorption $\varepsilon_\nu = 1$
ii. matrix (NDxND) inversion for $\varepsilon_\nu < 1$
(number of operations $\propto$ *ND*^{3}*)*

where ND is the number of τ_ν *depths*

still need to solve the coupling on $\varepsilon_\nu(\rho,T)$

$$J_\nu^{(1)} = \Lambda_{\tau_\nu}\left[(1-\epsilon_\nu)J_\nu^{(0)}\right] + \Lambda_{\tau_\nu}[\epsilon_\nu B_\nu] \qquad \textit{with } J_\nu^{(0)} = B_\nu,\ \varepsilon_\nu = \varepsilon_{grey}$$

$$J_\nu^{(n+1)} = \Lambda_{\tau_\nu}\left[(1-\epsilon_\nu)J_\nu^{(n)}\right] + \Lambda_{\tau_\nu}[\epsilon_\nu B_\nu].$$

i. no inversion
ii. coupling on ε_ν is part of iteration

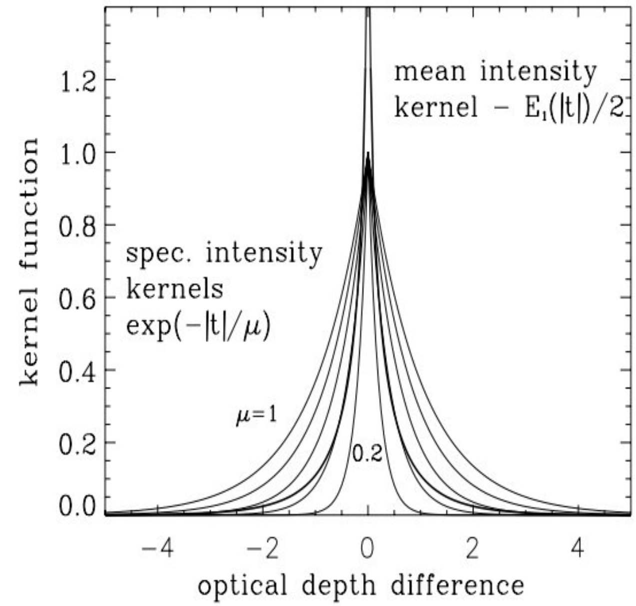
scattering + absorption ➡ *kernel resolution & coverage*

scattering mix distant parts of the atmosphere

$$J_\nu^{(1)} = \Lambda_{\tau_\nu}\left[(1-\epsilon_\nu)J_\nu^{(0)}\right] + \Lambda_{\tau_\nu}[\epsilon_\nu B_\nu]$$

with $J_\nu^{(0)} = B_\nu, \;\; \varepsilon_\nu = \varepsilon_{grey}$

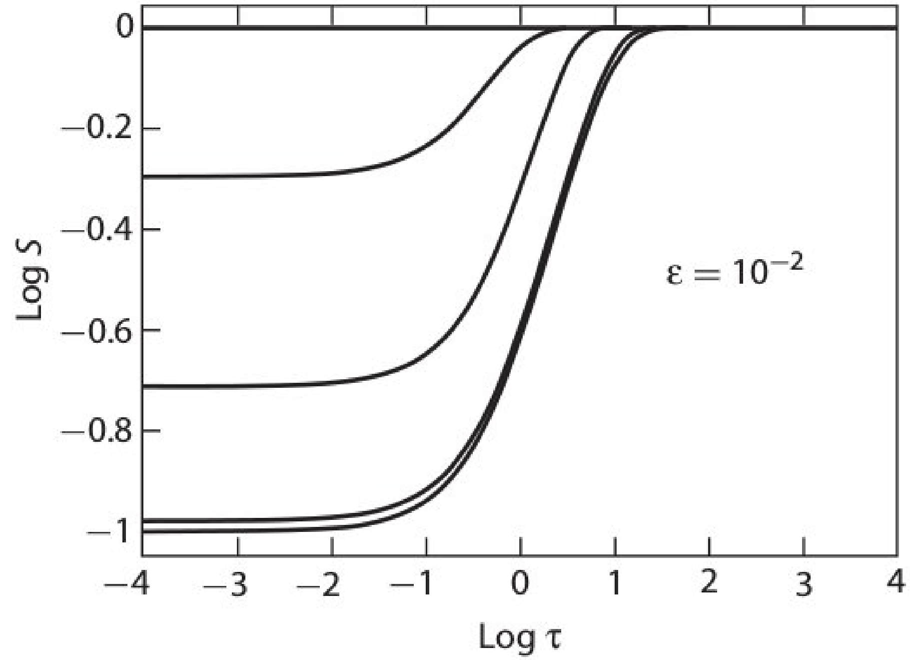
$$J_\nu^{(n+1)} = \Lambda_{\tau_\nu}\left[(1-\epsilon_\nu)J_\nu^{(n)}\right] + \Lambda_{\tau_\nu}[\epsilon_\nu B_\nu].$$



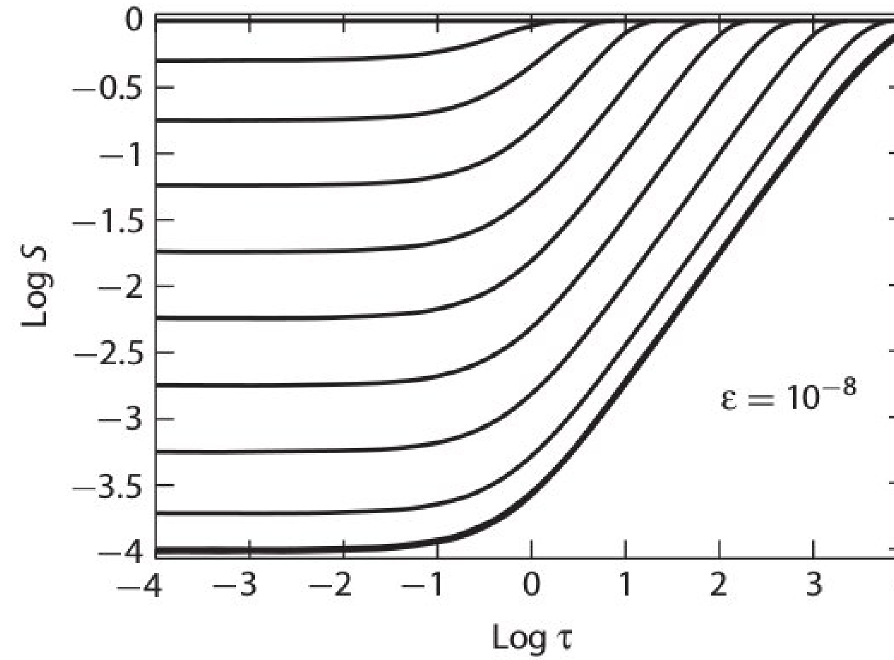
i. no inversion
ii. coupling on ε_ν *is part of iteration*
iii. indirect method

scattering + absorption ➡ *kernel resolution & coverage*

scattering mix distant parts of the atmosphere



$S_n(\tau)$ for $\epsilon = 10^{-2}$ with $n = 1, 10, 100, 1000$ Λ-iterations.



$S_n(\tau)$ for $\epsilon = 10^{-8}$ with $n = 1, 10, \ldots, 10^9$ Λ-iterations.

from Hubeny & Mihalas 2015

ALI – Accelerated Lambda Iteration (Hubeny & Lanz, 1992, A&A, 262)

$$\Lambda \rightarrow \Lambda^* + (\Lambda - \Lambda^*)$$

$$J^{new} = \Lambda S^{old}(n^{old}, T^{old}) + \Lambda^* S^{new}(n^{new}, T^{new}) - \Lambda^* S^{old}(n^{old}, T^{old})$$

Λ^* = *optimum lambda operator*

used with convergence accelerators

Model atmosphere coupled equations

i. radiative transfer

$$\mu \frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = I_\nu - S_\nu$$

ii. hydrostatic equilibrium

$$\frac{\mathrm{d}P}{\mathrm{d}z} = -\rho\, g$$

with (m = column density)

$$\mathrm{d}m = -\rho\,\mathrm{d}z$$

$$\frac{\mathrm{d}P}{\mathrm{d}m} = g \qquad P(m) = mg$$

$$P = P_{\mathrm{gas}} + P_{\mathrm{rad}} + P_{\mathrm{turb}} = NkT + \frac{4\pi}{c}\int_0^\infty K_\nu\,\mathrm{d}\nu + \frac{1}{2}\,\rho\, v_{\mathrm{turb}}^2$$

iii. radiative equilibrium

$$\int_0^\infty H_\nu \, \mathrm{d}\nu = \mathrm{const} = \frac{\sigma}{4\pi} T_{\mathrm{eff}}^4$$

$$\int_0^\infty \kappa_\nu \left(J_\nu - S_\nu \right) \mathrm{d}\nu = 0$$

iv. statistical equilibrium

$$n_i \sum_{j \neq i} \left(R_{ij} + C_{ij} \right) = \sum_{j \neq i} n_j \left(R_{ji} + C_{ji} \right)$$

with

$$\sum_i n_i = N_{\mathrm{atom}}$$

v. charge conservation

$$\sum_i n_i Z_i - n_\mathrm{e} = 0$$

Equation	Corresponding state parameter
Radiative transfer	Mean intensities, J_ν
Radiative equilibrium	Temperature, T
Hydrostatic equilibrium	Total particle density, N
Statistical equilibrium	Populations, n_i
Charge conservation	Electron density, n_e

The structure-synthesis approach

concept of:

i. *non-structural elements*
ii. *non-structural (b-f and b-b) opacities*
iii. *optimal frequency sampling*

➡ Accurate enough description of structure and continuum radiation field @ ν_n :

a) $\rho(\tau)$, $N_H(\tau)$, $n_e(\tau)$, $T(\tau)$, $\tau(z)$, $F_v(0)$

b) NLTE: $n_{i,j,k}(\tau)$

➡ Detailed spectral synthesis (fast)

$k_v(\tau_\nu)$, $\eta_v(\tau_\nu)$ + radiative transfer equation

high resolution;
precise line broadening;
precise limb darkening;
custom line list (broadening coef, gf, broadening coefs., E, $(2J_{i,j}+1)$;

The non-gray generic case

The Ψ matrix

sample for H, He in NLTE, high temperature structure

$$\Psi_d = \left[P_1,\ldots,P_N; N_1,\ldots,N_{NLH}; N_1,\ldots,N_{NLHe}; n_H; n_{He}; n_e; n; T_e; D_1,\ldots,D_{ND}\right]$$

where:

P_i = μ averaged intensity = $\frac{1}{2}\left[I(\nu_i,\mu_i)+I(\nu_i,-\mu_i)\right]$ (Feautrier variable); with $\mu \equiv \cos(i_{orb})$ with i = [1,NJ] points in a predefined μ,ν grid.

N_i = Population of level i (in LTE $\Leftarrow$ Boltzmann equation).

$n_{H,He,e}$ = hydrogen, helium, electron, and total density.

T_e = electron temperature.

D_i = depth index.

Number of variables (NN): NJ + NLH + NLHe + 5.

Number of equations: NLH statistical equilibrium equations for hydrogen.
NLHe statistical equilibrium equations for helium.
NJ radiation transfer equations.
Radiative equilibrium condition (convection is ignored).
Hydrostatic equilibrium equation.
Charge conservation.
Particle conservation in statistical eq. eq. for hydrogen.
Particle " " for helium.

The elements of the matrix are linearized:

The starting solution Ψ^0 is the gray solution and is perturbed by $\delta\Psi$.

$$\Psi^n(D_i) \equiv \Psi^{n-1}(D_i) + \delta\Psi(D_i)^n$$

for a particular iteration:

$$-A(D_i)\delta\Psi(D_{i-1}) + B(D_i)\delta\Psi(D_i) - C(D_i)\delta\Psi(D_{i+1}) = error(D_i)$$

The structure-synthesis approach

concept of:

i. *non-structural elements*
ii. *non-structural (b-f and b-b) opacities*
iii. *optimal frequency sampling*

➡ Model atmosphere provide accurate enough description of structure and continuum radiation field @ ν_i :

a) $\rho(\tau)$, $N_H(\tau)$, $n_e(\tau)$, $T(\tau)$, $\tau(z)$, $F_\nu(\tau=0)$

b) optional NLTE: $n_{i,j,k}(\tau)$

c) optional: $k_{cont}(\tau)$, $\eta_{cont}(\tau)$

The structure-synthesis approach (cont.)

➡ Detailed spectral synthesis (fast) provide $I(\nu,\mu) \rightarrow^{*} F(\nu)$
* *(geometry and velocity field)*

$k_{\nu}(\tau_{\nu})$, $\eta_{\nu}(\tau_{\nu})$ + radiative transfer equation (direct solution)

with

- custom (observer´s) frequency grid;
- precise line broadening (natural, Stark, vdW, microturbulence, macroturbulence, rotation);
- limb darkening;
- custom non-structural abundances;
- custom line list (gf, broadening coefs., $E_{i,j}$, $(2J_{i,j}+1)$);
- polarization and Zeeman

Types of model atmosphere structures

i. Grey
ii. LTE non-grey continuum
iii. LTE + line blancketed
iv. NLTE H+He, LTE line blanketed
v. full NLTE line blanketed models (superlevels)

Features and processes that may be relevant to model structures:

- presence of convection (3D), advection, conduction.
- non-plane-parallel (spherical, flared disk, or 3D geometry).
- lack of radiative equilibrium (sources).
- variable gravity, magnetic fields.
- mass loss (winds, 3D).
- relativistic effects. tide effects.
- irradiation, condensation.
- chemical composition stratification.

- time-dependence (in one or more of the above).

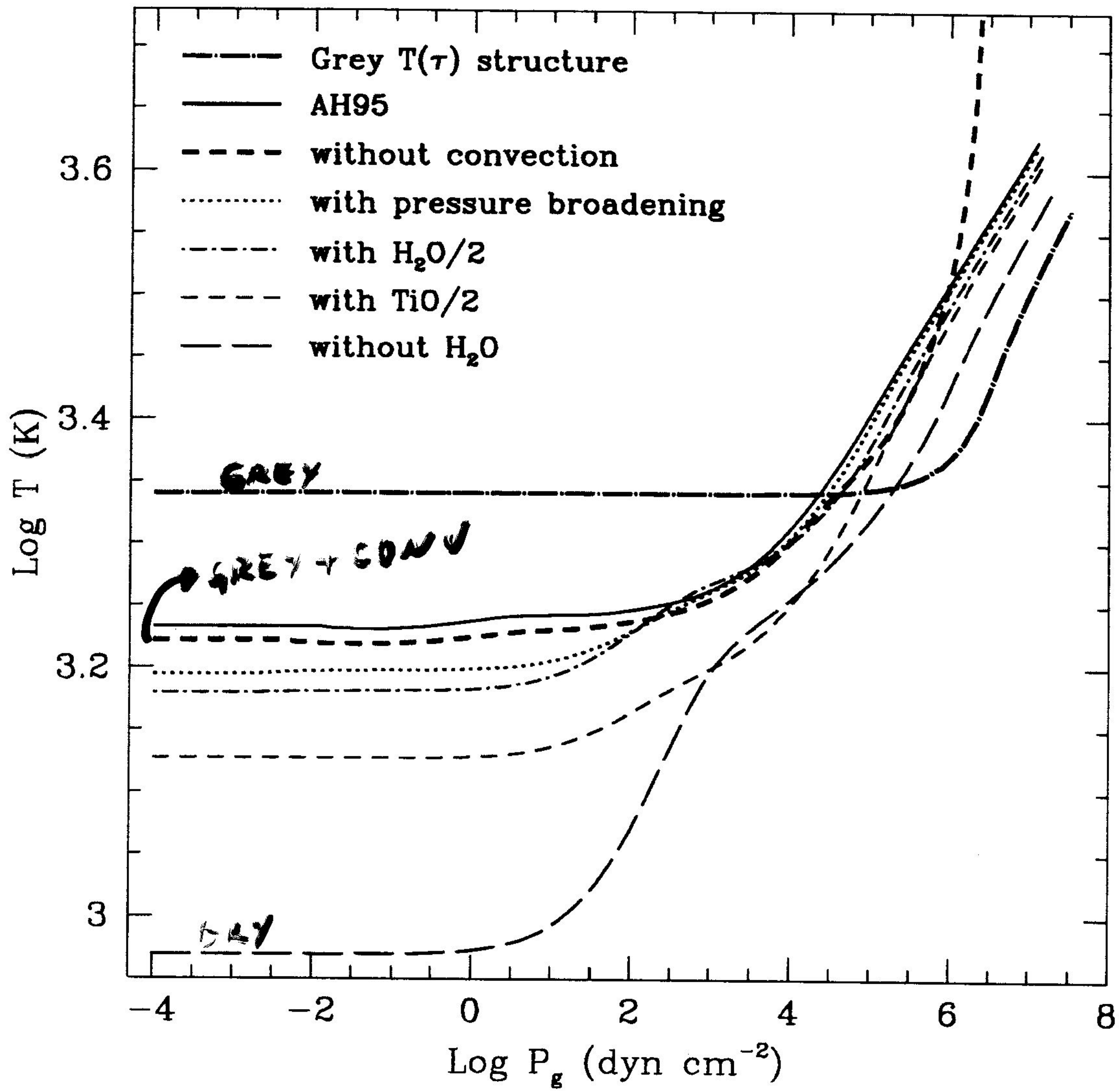


Figure 2 The influence of the molecular and atomic opacities and convection upon the atmospheric structure of a typical model atmosphere; here the T_{eff} = 2800 K, log g = 5.0, and solar metallicity model of Allard & Hauschildt (1995b). A corresponding gray structure without convection (*bold dot-dashed*) is also shown for comparison. While the complete neglect of H_2O opacities causes a dramatic cooling (by CO) of the atmosphere (*long-dashed curve*), uncertainties by a factor of two in the H_2O opacity cross sections cause only negligible changes in the atmospheric structure (*thin dot-dashed* relative to *dotted curve*). A similar drop in the opacity cross section of TiO, however (*thin short-dashed* relative to *dotted curve*), causes a much more significant cooling of the atmosphere.

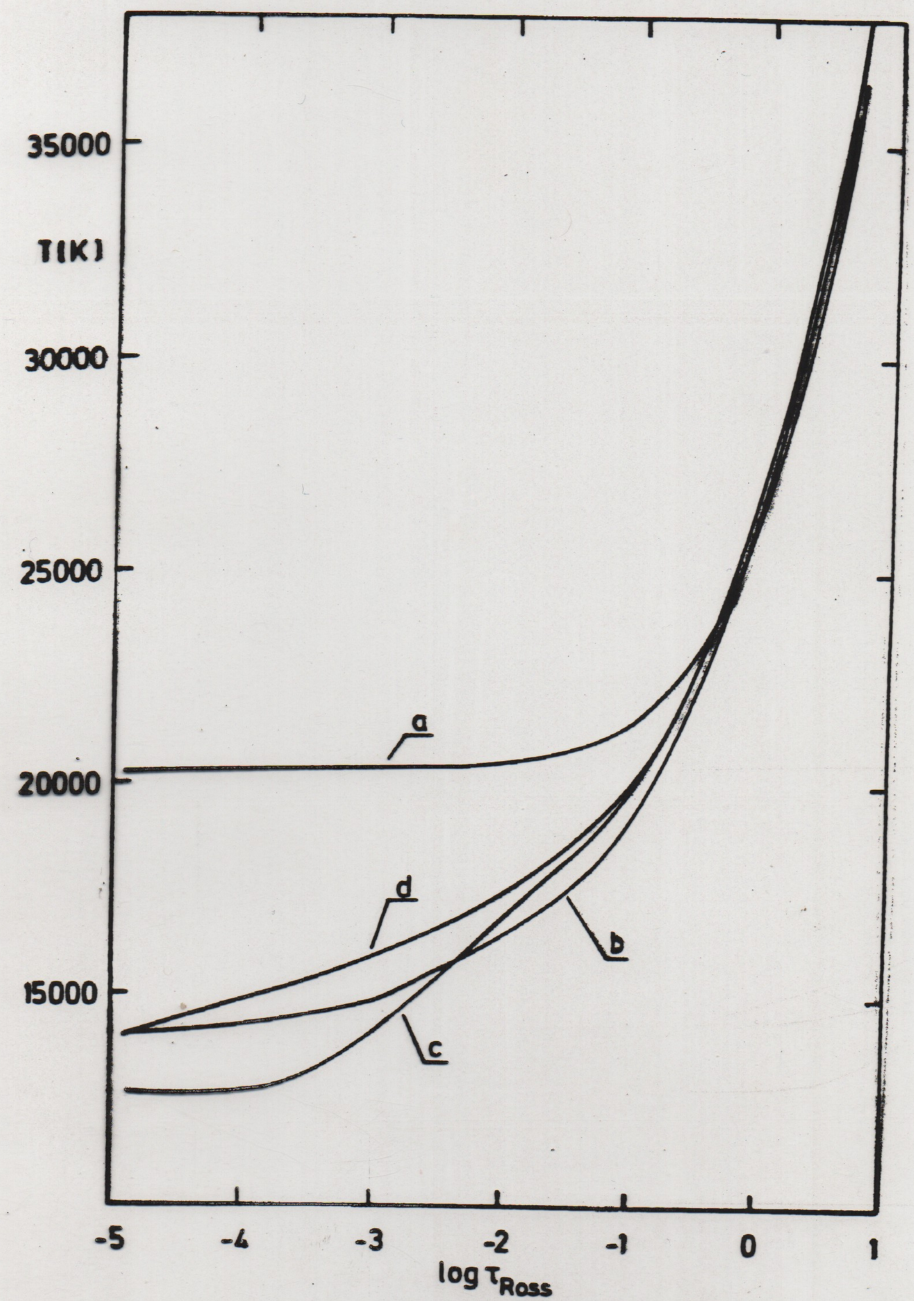


Fig. III,10: The LTE temperature stratification for a typical B V-star model atmosphere with T_{eff} = 25000 K, log g = 4.0 and solar abundances. a) grey case, b) unblanketed continuum model (Munich code), c) line blanketed model including one hundred of the strongest lines (Munich code), d) Kurucz model. For discusson see text.