Atmosferas Estelares

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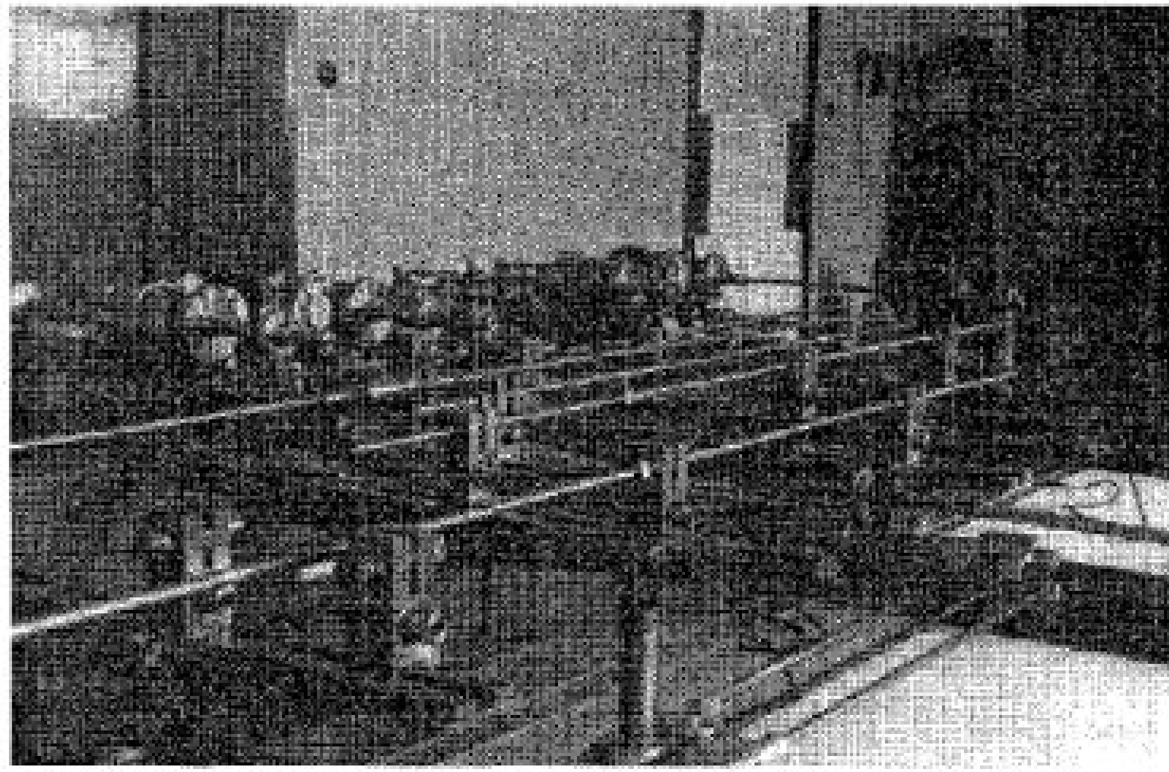

The Oslo analyzer by S. Rosseland (1938)

https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=539912

Lecture 4

treasure map:

H&M: pg 374, 347

Rutten: pg 30

Vitense: pg 46

Gray: pg 139

The Diffusion Approximation

$S_\nu \rightarrow B_\nu$; $\tau_\nu \rightarrow \infty$

expansion around an arbitrary (as good as large) τ_ν :

$$S_\nu(t_\nu) = \sum_{n=0}^{\infty} \frac{\mathrm{d}^n B_\nu}{\mathrm{d}\tau_\nu^n} \frac{(t_\nu - \tau_\nu)^n}{n!}$$

into the formal solution in 2 currents:

$$I_\nu(\tau_\nu, \mu) = \int_{\tau_\nu}^{\infty} S_\nu(t) \mathrm{e}^{-(t-\tau_\nu)/\mu} \, \mathrm{d}t/\mu \, , \qquad \text{for} \qquad \mu \geq 0$$

$$I_\nu(\tau_\nu, \mu) = \int_{0}^{\tau_\nu} S_\nu(t) \mathrm{e}^{-(\tau_\nu - t)/(-\mu)} \, \mathrm{d}t/(-\mu) \, , \qquad \text{for} \qquad \mu < 0$$

The Diffusion Approximation (cont.)

taking the outward ($0 < \mu < 1$) intensity at τ_ν:

$$I_\nu(\tau,\mu) = \int_0^\infty \sum \frac{(t-\tau_\nu)^n}{n!} \frac{d^n B_\nu}{d\tau^n} \; e^{\frac{-(t-\tau_\nu)}{\mu}} \frac{dt}{\mu} \qquad \text{with} \quad x=(t-\tau_\nu)$$

The Diffusion Approximation (cont.)

taking the outward ($0 < \mu < 1$) intensity at τ_ν:

$$I_\nu(\tau,\mu) = \int_0^\infty \sum \frac{(t-\tau_\nu)^n}{n!} \frac{d^n B_\nu}{d\tau^n} \; e^{\frac{-(t-\tau_\nu)}{\mu}} \frac{dt}{\mu} \qquad \text{with} \quad x=(t-\tau_\nu)$$

$$I_\nu(\tau,\mu) = \sum \frac{d^n B_\nu}{d\tau^n} \int_0^\infty \frac{x^n}{n!} \; e^{-\frac{x}{\mu}} \frac{dx}{\mu}$$

The Diffusion Approximation (cont.)

taking the outward ($0 < \mu < 1$) intensity at τ_ν:

$$I_\nu(\tau,\mu) = \int_0^\infty \sum \frac{(t-\tau_\nu)^n}{n!} \frac{d^n B_\nu}{d\tau^n} e^{\frac{-(t-\tau_\nu)}{\mu}} \frac{dt}{\mu} \qquad \text{with} \quad x=(t-\tau_\nu)$$

$$I_\nu(\tau,\mu) = \sum \frac{d^n B_\nu}{d\tau^n} \int_0^\infty \frac{x^n}{n!} e^{-\frac{x}{\mu}} \frac{dx}{\mu}$$

$$I_\nu(\tau,\mu) = \sum \frac{d^n B_\nu}{d\tau^n} \mu^n = B_\nu(\tau) + \mu \frac{dB_{nu}}{d\tau} . \; . \; .$$

The Diffusion Approximation (cont.)

error in diffusive approximation

convergence of the expansion

$$\frac{\delta S_{\nu,n}}{S_\nu} \propto \frac{\dfrac{d^{n+2} B_\nu}{d\tau^{n+2}}}{\dfrac{d^n B_\nu}{d\tau^n}} \sim \frac{\dfrac{B_\nu^{n+2}}{\tau^{n+2}}}{\dfrac{B_\nu^n}{\tau^n}} \sim \tau^{-2}$$

The Diffusion Approximation (cont.)

The diffusion approx. in the higher radiation moments

➡ multiply by μ and integrate over μ.

$$J_\nu(\tau,\mu) = \int_{-1}^{1} \left(B_\nu(\tau) + \mu \frac{dB_\nu}{d\tau} + \mu^2 \frac{d^2 B_\nu}{d\tau^2} + \ . \ . \ . \right) d\mu$$

all odd terms on μ vanish

$$J_\nu(\tau_\nu) = B_\nu(\tau_\nu) + \frac{1}{3}\frac{\mathrm{d}^2 B_\nu}{\mathrm{d}\tau_\nu^2} + \cdots$$

$$H_\nu(\tau_\nu) = \frac{1}{3}\frac{\mathrm{d} B_\nu}{\mathrm{d}\tau_\nu} + \cdots \quad ,$$

$$K_\nu(\tau_\nu) = \frac{1}{3} B_\nu(\tau_\nu) + \frac{1}{5}\frac{\mathrm{d}^2 B_\nu}{\mathrm{d}\tau_\nu^2} + \cdots$$

The Diffusion Approximation (cont.)

The diffusion approx. in the higher radiation moments

deep enough:

as $S_\nu \rightarrow B_\nu$:

$J_\nu \rightarrow B_\nu$

$J_\nu / K_\nu \rightarrow 3$

$$J_\nu(\tau_\nu) = B_\nu(\tau_\nu) + \frac{1}{3}\frac{\mathrm{d}^2 B_\nu}{\mathrm{d}\tau_\nu^2} + \cdots$$

$$H_\nu(\tau_\nu) = \frac{1}{3}\frac{\mathrm{d}B_\nu}{\mathrm{d}\tau_\nu} + \cdots ,$$

$$K_\nu(\tau_\nu) = \frac{1}{3}B_\nu(\tau_\nu) + \frac{1}{5}\frac{\mathrm{d}^2 B_\nu}{\mathrm{d}\tau_\nu^2} + \cdots$$

$$H_\nu = \frac{1}{3}\frac{\mathrm{d}B_\nu}{\mathrm{d}\tau_\nu} = -\frac{1}{3}\frac{1}{\chi_\nu}\frac{\mathrm{d}B_\nu}{\mathrm{d}z} = -\frac{1}{3}\frac{1}{\chi_\nu}\frac{\mathrm{d}B_\nu}{\mathrm{d}T}\frac{\mathrm{d}T}{\mathrm{d}z}$$

where χ_ν is the linear opacity coef.

The Diffusion Approximation (cont.)

The diffusion coefficient for photons is:

$$-\frac{1}{3}\frac{1}{\chi_\nu}\frac{\mathrm{d}B_\nu}{\mathrm{d}T}$$

The bolometric flux is defined in terms of an appropriate opacity coefficient:

$$H = -\left(\frac{1}{3}\frac{1}{\chi_\mathrm{R}}\frac{\mathrm{d}B}{\mathrm{d}T}\right)\frac{\mathrm{d}T}{\mathrm{d}z}$$

where χ_R is the Rosseland opacity coefficient

$$\frac{1}{\chi_\mathrm{R}}\frac{\mathrm{d}B}{\mathrm{d}T} = \int_0^\infty \frac{1}{\chi_\nu}\frac{\mathrm{d}B_\nu}{\mathrm{d}T}\,\mathrm{d}\nu$$

i. weight proportional to the flux
ii. exact flux (and temp.) structure @ diffusion

however...

iii. does not care about your favorite "ν"

Moments of the Transfer Equation

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{n}.\boldsymbol{\nabla}\right) I(\boldsymbol{r},\boldsymbol{n},\nu,t) = \eta(\boldsymbol{r},\boldsymbol{n},\nu,t) - x(\boldsymbol{r},\boldsymbol{n},\nu,t) I(\boldsymbol{r},\boldsymbol{n},\nu,t)$$

μ^j integrated over solid angle:

$$\frac{\partial E_\nu}{\partial t} + \nabla \cdot \mathbf{F}_\nu = \eta_\nu - \chi_\nu c E_\nu \qquad j = 0$$

$$\frac{1}{c}\frac{\partial \mathbf{F}_\nu}{\partial t} + c\nabla \cdot \mathrm{P}_\nu = -\chi_\nu \mathbf{F}_\nu \qquad j = 1$$

With a symmetric emission coef. η

With astrophysical notation, changing to perpendicular optical depth:

$$\frac{\mathrm{d}H_\nu}{\mathrm{d}\tau_\nu} = J_\nu - S_\nu \qquad (1)$$

open equation set

$$\frac{\mathrm{d}K_\nu}{\mathrm{d}\tau_\nu} = H_\nu \quad . \qquad (2)$$

with: $f_\nu^{\mathrm{K}} \equiv K_\nu / J_\nu$ Eddington factor – predictor in iterative solutions

➡ $$\frac{\mathrm{d}^2(f_\nu^{\mathrm{K}} J_\nu)}{\mathrm{d}\tau_\nu^2} = J_\nu - S_\nu$$ angle-averaged quantities - only

The Grey Atmosphere

i. The grey linear opacity coef. is the Rosseland harmonic mean.

ii. The diffusive approximation for J extends over the whole atmosphere (the *Eddington approximation*).

iii. The atmosphere is in strict radiative equilibrium.

from eq. 2 with Eddington factor = ⅓ :

$$H = \frac{1}{3}\frac{dJ}{d\tau}; \qquad with \quad \tau = -\chi_{Rosseland}\, dz$$

$$\frac{dH}{d\tau} = 0 = \frac{1}{3}\frac{d^2 J}{d\tau^2} \qquad \text{(rad. equilibrium)}$$

$$J = 3H\tau + q; \qquad with \quad q = Hopf\ function \qquad (1)$$

The Grey Atmosphere (cont.)

The observed H(0) boundary condition fix the Hopf function

$$H(0) = \frac{1}{2}\int_0^1 \mu I(0,\mu)\, d\mu = \frac{1}{2}\bar{\mu}\int_0^1 I(0,\mu)\, d\mu = \frac{1}{2} J(0)$$

$$\left(with \quad \bar{\mu} = 1/2\right)$$

$$J(0) = q = 2H(0) \;;$$

in 1: $$J(\tau) = 3H(0)[\tau + 2/3]$$ (2) the grey τ structure

The Grey Atmosphere (cont.)

i. the temperature structure

$$H(0) = F(0)/4\pi = \frac{\sigma}{4\pi} T_{eff}^4$$

$$J(\tau) = \frac{3\sigma}{4\pi} T_{eff}^4 \left[\tau + \frac{2}{3}\right] \; ; \qquad \textit{with}$$

$$J(\tau) \cong \frac{\sigma}{\pi} \; T^4(\tau) \qquad \text{(Eddington approx.)}$$

$$T^4(\tau) = \frac{3}{4} T_{eff}^4 \left[\tau + \frac{2}{3}\right]$$

The Grey Atmosphere (cont.)

ii. the angle dependence – a limb darkened atmosphere

the formal solution for semi-infinite atmosphere
with $S(\tau) = J(\tau)$

$$I(0,\mu) = \frac{1}{\mu}\int_0^\infty S(\tau')\,e^{\frac{-\tau'}{\mu}}\,d\tau'$$

$$I(0,\mu) = \frac{1}{\mu}\int_0^\infty 3H(0)[\tau'+2/3]\,e^{\frac{-\tau'}{\mu}}\,d\tau'$$

The Grey Atmosphere (cont.)

ii. the angle dependence – a limb darkened atmosphere

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with $S(\tau) = J(\tau)$

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$$I(0,\mu) = \frac{1}{\mu}\int_0^\infty 3H(0)[\tau'+2/3]\,e^{\frac{-\tau'}{\mu}}\,d\tau'$$

$$I(0,\mu) = \frac{3H(0)}{\mu}\left[\int_0^\infty \tau' e^{\frac{-\tau'}{\mu}}\,d\tau' + \frac{2}{3}\int_0^\infty e^{\frac{-\tau'}{\mu}}\,d\tau'\right]$$

The Grey Atmosphere (cont.)

ii. the angle dependence – a limb darkened atmosphere

the formal solution for semi-infinite atmosphere
with S(τ) = J(τ)

$$I(0,\mu) = \frac{1}{\mu}\int_0^\infty S(\tau')e^{\frac{-\tau'}{\mu}}d\tau'$$

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$$I(0,\mu) = \frac{3H(0)}{\mu}\left[\int_0^\infty \tau' e^{\frac{-\tau'}{\mu}}d\tau' + \frac{2}{3}\int_0^\infty e^{\frac{-\tau'}{\mu}}d\tau'\right]$$

$$I(0,\mu) = 3H(0)\left[\mu + \frac{2}{3}\right] = \frac{3\sigma T_{eff}^4}{4\pi}\left[\mu + \frac{2}{3}\right]$$

The Grey Atmosphere (cont.)

ii. the angle dependence – a linear limb darkened atmosphere

$$I(0,\mu) = 3H(0)\left[\mu + \frac{2}{3}\right]; \quad \text{with (2)} \quad J(\tau) = 3H(0)\left[\tau + \frac{2}{3}\right]$$

$$J(\tau) = I(0, \mu{=}\tau)$$

Eddington-Barbier relation with $S(\tau) = J(\tau)$

(linear LD and E-B relation found for non-arbitrary linear $S(\tau)$*)*