Atmosferas Estelares

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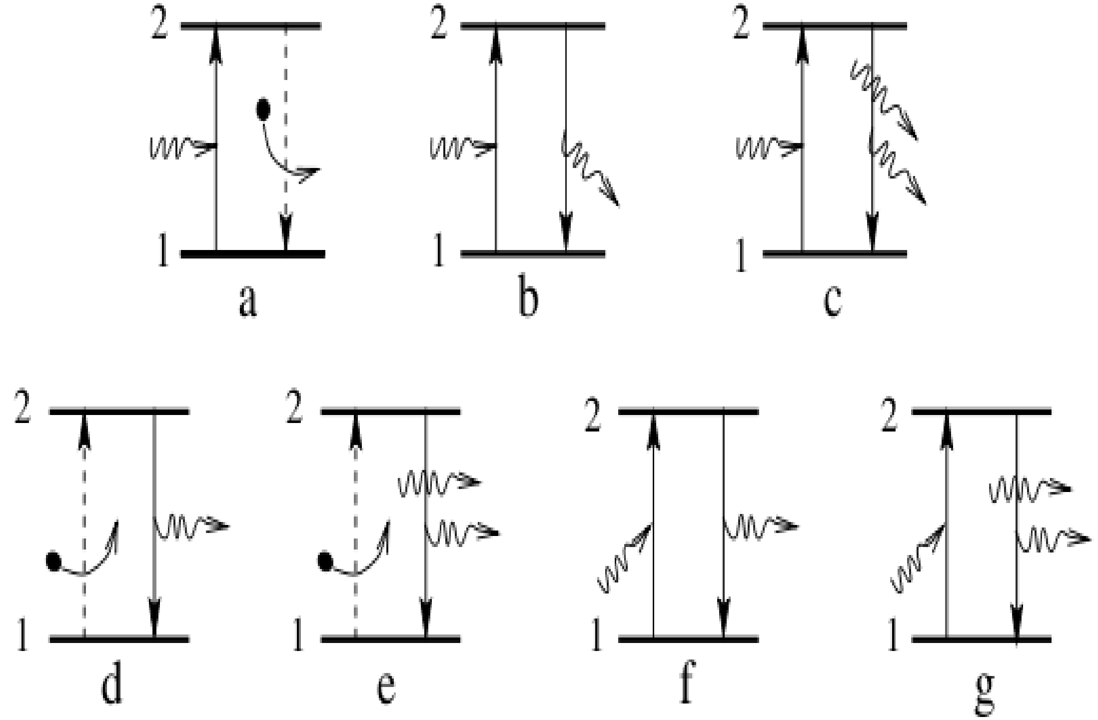
IAG-USP 2023



Contributions to beam intensity from two-level bound-bound sequences.

from Rutten 2004

Lecture 2

treasure map:

H&M: pg 334-361

Rutten: pg 4

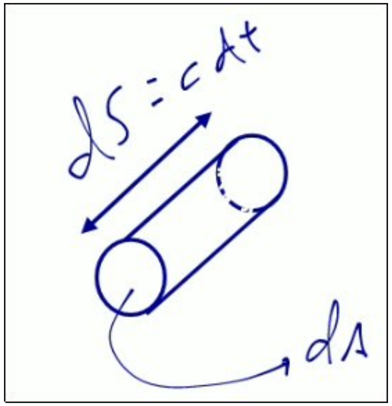
Vitense: pg 39

Gray: pg 127

Keep it small. keep it *simple*:

local linear approximation to beam contributions:

$$k(\boldsymbol{r},\boldsymbol{n},\nu,t)\,I(\boldsymbol{r},\boldsymbol{n},\nu,t) \;+\; \eta(\boldsymbol{r},\boldsymbol{n},\nu,t)$$



$$dN_{total\ ph} \;=\; \frac{1}{h\nu} I(\boldsymbol{r},\boldsymbol{n},\nu,t)\,dA\,d\omega\,d\nu\,dt$$

i. "k_ν" the linear absorption coefficient

$$P=\frac{\Delta a}{\Delta A}; \quad P \;=\; \frac{da}{dA} \qquad (1) \qquad \textit{interaction probability per photon}$$

$$da = \sigma N_{particles} \, dA \, dS$$

is the total unsaturated cross-section, in (1):

per unit length S: $P = \sigma N_{part} = x$ with mean free path: $1/x$

$$dN_{ph} = dN_{tot\,ph} P$$

$$dN_{ph} = \frac{x}{h\nu} I(\boldsymbol{r}, \boldsymbol{n}, \nu, t) \, dA \, d\omega \, d\nu \, dt$$

$$dE = x I(\boldsymbol{r}, \boldsymbol{n}, \nu, t) \, dA \, d\omega \, d\nu \, dt$$

with x = sct. + true abs. - induced emission

ii. η_ν (the local emission coefficient)

$$dE_{ph} = \eta(\boldsymbol{r}, \boldsymbol{n}, \nu, t)\, dA\, d\omega\, d\nu\, dt$$

with η = all spontaneous emission

With (i) and (ii), along *dS* the radiative energy change by:

$$dE_{ph} = \left[I(\boldsymbol{r+dS}, \boldsymbol{n}, \nu, t+dt) - I(\boldsymbol{r}, \boldsymbol{n}, \nu, t) \right] dA\, d\omega\, d\nu\, dt$$

$$= \eta(\boldsymbol{r}, \boldsymbol{n}, \nu, t) - x(\boldsymbol{r}, \boldsymbol{n}, \nu, t) I(\boldsymbol{r}, \boldsymbol{n}, \nu, t)$$

$$diff\big|_{r,t} = \frac{\partial I}{\partial t} dt + \frac{\partial I}{\partial S} dS = \left(\frac{\partial I}{\partial S} + \frac{1}{c} \frac{\partial I}{\partial t} \right) dS$$

$$(with \quad dS = c\, dt)$$

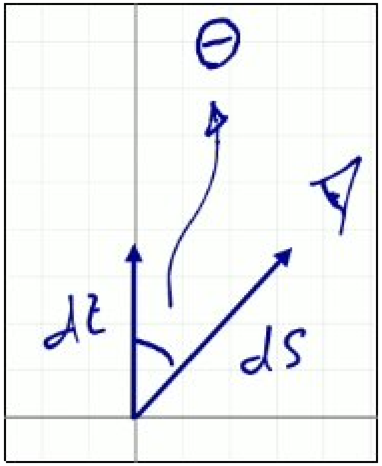
$$with \quad \left(\frac{\partial I}{\partial S}\right) dS = \boldsymbol{n} . \nabla I \quad :$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{n}.\nabla\right) I(\boldsymbol{r},\boldsymbol{n},\nu,t) = \eta(\boldsymbol{r},\boldsymbol{n},\nu,t) - x(\boldsymbol{r},\boldsymbol{n},\nu,t) I(\boldsymbol{r},\boldsymbol{n},\nu,t)$$

time-dependent transfer equation

$$steady\,state: \quad \frac{\partial I}{\partial t} = 0$$

$$plane-par.: \quad \frac{dz}{dS} = \cos(\theta) = \mu$$



$$\Rightarrow \quad \mu \frac{dI(\nu,\mu,z)}{dz} = \eta(\nu,\mu,z) - x(\nu,\mu,z) I(\nu,\mu,z)$$

optical depth $d\tau_\nu \equiv -x_\nu dS = -x_\nu \frac{dz}{\mu}$

"lagrangian coordinate" for radiation

$$x_\nu \frac{dI(\nu,\mu,z)}{d\tau_\nu} = -\eta(\nu,\mu,z) + x(\nu,\mu,z)\,I(\nu,\mu,z)$$

$$\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu \qquad \textit{with} \quad S_\nu \equiv \frac{\eta_\nu}{x_\nu}$$

with $\tau_{\nu\perp}$ in the normal direction: $\mu \frac{dI_\nu}{d\tau_{\nu\perp}} = I_\nu - S_\nu$

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$$dN_{ph} = \frac{\eta_\nu}{h\nu} \, d\nu \, d\omega \, dt \, dA \, ds \quad (\textit{number of photons emitted at } dV = dA\, dS)$$

$$dN_{ph} = \frac{\eta_\nu}{x_\nu}\frac{1}{h\nu} \, d\nu \, d\omega \, dt \, dA \, x_\nu \, ds = S_\nu \frac{4\pi}{h\nu} d\nu \, dt \, dA \, d\tau_\nu \quad (S_\nu \, \textit{isotropic})$$

S_ν is the energy added within $\Delta\tau_\nu = 1$; J_ν is the energy removed within $\Delta\tau_\nu = 1$

The Formal Solution

$$\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$

the standard integrating factor solution of a first-order differential equation:

$$\frac{dI_\nu}{d\tau_\nu} + p(\tau) I_\nu = q(\tau)$$

@ an arbitrary depth τ:

$$I(\tau) = e^{-\int_0^{\tau} p(\tau')d\tau'} \left[\int_0^{\tau} e^{\int_0^{\tau'} p(\tau'')d\tau''} q(\tau') d\tau' + c \right]$$

The Formal Solution (cont.)

with: $p(\tau) = -1; \quad q(\tau) = -S(\tau)$

$$I(\tau) = -e^{\tau}\left[\int_0^{\tau} e^{-\tau'} S(\tau')\, d\tau' + c\right]$$

then relate the intensities at 2 arbitrary depths τ_1 and τ_2 as follows:

$$I(\tau_2)e^{-\tau_2} - I(\tau_1)e^{-\tau_1}$$

assuming p.p. geometry, using $\tau_{1,2}(\mu=1) \equiv \tau_{1\perp,2\perp}$:
$\tau_1 = \tau_{1\perp} / \mu$ and $\tau_2 = \tau_{2\perp} / \mu$

The Formal Solution (cont.)

$\tau_{\perp} = \bar{\tau}$ (change of notation to avoid double indices)

$$I(\bar{\tau}_1, \mu) = I(\bar{\tau}_2) e^{\frac{\bar{\tau}_1 - \bar{\tau}_2}{\mu}} + \frac{1}{\mu} \int_{\bar{\tau}_1}^{\bar{\tau}_2} S(\tau') e^{\frac{\bar{\tau}_1 - \tau'}{\mu}} d\tau'$$

The formal solution above is particularly useful when the intensity is known at a given depth and a recipe for the depth dependence of the source function can be found.