INSTITUTO DE ASTRONOMIA, GEOFÍSICA E CIÊNCIAS ATMOSFÉRICAS, USP

AGA0506 - Transporte de Energia em Astrofísica

Lista 1

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QUESTION 1

(`Rybicky & Lightman Problem 1.3`) X-Ray photons are produced in a cloud of radius R at the uniform rate Γ (photons per unit volume per unit time). The cloud is a distance d away. Neglect absorption of these photons (optically thin medium). A detector at earth has an angular acceptance beam of half-angle $\Delta\theta$ and it has an effective area of ΔA.

(a) Assume that the source is completely resolved. What is the observed intensity (photons per unit time per unit area per steradian) toward the center of the cloud.

(b) Assume that the source is completely unresolved. What is the observed average intensity when the source is in the beam of the detector?

QUESTION 2

(`Rybicky & Lightman Problem 1.5`) A supernova remnant has an angular diameter $\theta = 4.3$ arc minutes and a flux at 100 MHz of $F_{100} = 1.6 \times 10^{-19}$ erg cm^{-2} s^{-1} Hz^{-1}. Assume that the emission is thermal.

(a) What is the brightness temperature T_b? What energy regime of the blackbody curve does this correspond to?

(b) The emitting region is actually more compact than indicated by the observed angular diameter. What effect does this have on the value of T_b?

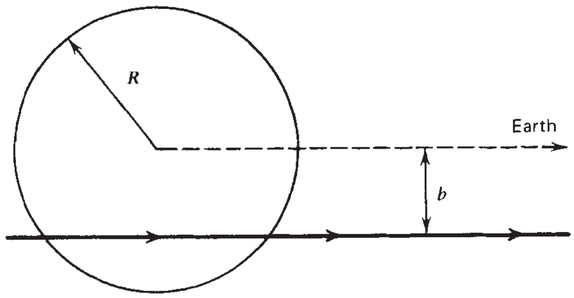

Figura 0.1: Detection of rays from a spherical emitting cloud of radius R.

(c) At what frequency will this object's radiation be maximum, if the emission is black-body?

(d) What can you say about the temperature of the material from the above results?

QUESTION 3

(`Rybicky & Lightman Problem 1.8`) A certain gas emits thermally at the rate $P(\nu)$ (power per unit volume and frequency range). A spherical cloud of this gas has radius R, temperature T and is a distance d from earth ($d \gg R$).

(a) Assume that the cloud is optically *thin*. What is the brightness of the cloud as measured on earth? Give your answer as a function of the distance b away from the cloud center, assuming the cloud may be viewed along parallel rays as shown in Figure 0.1.

(b) What is the effective temperature of the cloud?

(c) What is the flux F_ν, measured at earth coming from the entire cloud?

(d) WHow do the measured brightness temperatures compare with the cloud's temperature?

(e) Answer parts (a)-(d) for an optically thick cloud.

QUESTION 4

Using the root-mean-square speed, v_{rms}, estimate the mean free path of the nitrogen molecules in your classroom at room temperature (300 K). What is the average time between collisions? Take the radius of a nitrogen molecule to be 0.1 nm and the density of air to be 1.2 kg m^{-3}. A nitrogen molecule contains 28 nucleons (protons and neutrons).

QUESTION 5

According to one model of the Sun, the central density is 1.53×10^5 kg m^{-3} and the Rosseland mean opacity at the center is 0.217 m^2 kg^{-1}.

(a) Calculate the mean free path of a photon at the center of the Sun.

(b) Calculate the average time it would take for the photon to escape from the Sun if this mean free path remained constant for the photon's journey to the surface. (Ignore the fact that identifiable photons are constantly destroyed and created through absorption, scattering, and emission.)

QUESTION 6

Practice with j_ν, α_ν, S_ν, B_ν, I_ν

(a) A plane-parallel slab of uniformly dense gas is known to be in LTE (local thermodynamic equilibrium) at a uniform temperature T. Its thickness normal to its surface is s. Its absorption coefficient is $\alpha_{\nu,\mathrm{gas}}$. Write down the specific intensity, I_ν, viewed normal to the slab, in terms of the variables given.

(b) The same slab is now filled uniformly with non-emissive dust having absorption coefficient $\alpha_{\nu,\mathrm{dust}}$. The dust is non-emissive, so its emissivity $j_{\nu,\mathrm{dust}} = 0$. Write down I_ν viewed normal to the slab, in terms of all variables given so far.

(c) The slab of gas and dust is further mixed with a third component: an emissive, non-absorptive uniform medium having emissivity $j_{\nu,\mathrm{med}}$, and absorption coefficient $\alpha_{\nu,\mathrm{med}} = 0$. Write down I_ν viewed normal to the slab, in terms of all variables given[1]

QUESTION 7

Consider a horizontal plane-parallel slab of gas of thickness L that is maintained at a constant temperature T. Assume that the gas has optical depth $\tau_{\lambda,0}$, with $\tau_\lambda = 0$ at the top surface of the slab. Assume further that no radiation enters the gas from outside. Use the general solution of the transfer equation, which is given by:

$$I_\lambda(0) = I_{\lambda,0} e^{-\tau_{\lambda,0}} - \int_{\tau_{\lambda,0}}^{0} S_\lambda e^{-\tau_\lambda} d\tau_\lambda,$$

to show that when looking at the slab from above, you see blackbody radiation if $\tau_{\lambda,0} \gg 1$ and emission lines (where j_λ is large) if $\tau_{\lambda,0} \ll 1$. You may assume that the source function, S_λ,

[1] A physical realization of this problem might be an HII region surrounding an ionizing O star. The material in LTE would be the fully ionized plasma, emitting thermal bremsstrahlung radiation. The dust would be dust. The emissive, non-absorptive medium would be the same ionized plasma emitting recombination (line) radiation. For the assumptions stated in the problem to be valid, we would have to evaluate ν at, say, an optical recombination line like H_α.

does not vary with position inside the gas. You may also assume thermodynamic equilibrium when $\tau_{\lambda,0} \gg 1$.