Planetary Nebulae as a Chemical Evolution Tool: Abundance Gradients

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Abstract. We have studied the time variation of the radial abundance gradients using samples of planetary nebulae, open clusters, cepheids and other young objects. Based on the analysis of O/H and S/H abundances for planetary nebulae and metallicities of the remaining objects, we concluded that the gradients have been flattening out in the last 8 Gyr with an average rate of the order of $0.005 - 0.010 \text{ dex kpc}^{-1} \text{ Gyr}^{-1}$. We have estimated the errors involved in the determination of the gradients, and concluded that the existence of systematic abundance variations is more likely than a simple statistical dispersion around a mean value.

Keywords: planetary nebulae, chemical evolution, abundance gradients

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1. INTRODUCTION

Radial abundance gradients in the Milky Way disk are among the main constraints of models of the chemical evolution of the Galaxy. The study of the gradients comprises the determination of their magnitudes along the disk, space variations and their time evolution (see for example Henry & Worthey 1999; Maciel & Costa 2003). Probably the most interesting property of the gradients is their time evolution, which is a distinctive constraint of recent chemical evolution models. Maciel et al. (2003) suggested that the O/H gradient has been flattening during the last few Gyr, on the basis of a large sample of planetary nebulae (PNe) for which accurate abundances are available, and for which the ages of the progenitor stars have been individually estimated. This work has been recently extended (Maciel et al. 2005) to include the S/H ratio in planetary nebulae, [Fe/H] metallicities from open clusters and cepheid variables, as well as some young objects, such as OB associations and HII regions.

In this work, we review the main characteristics of the work by Maciel et al. (2005) and analyze the uncertainties involved in the determination of the gradients. In particular, we investigate whether the derived uncertainties support either a systematic variation of the abundances with the galactocentric distance, as assumed by our work, or simply a dispersion of the abundances around some average value.

2. ABUNDANCE GRADIENTS

The main results for the time variation of the gradients as derived from planetary nebulae, open clusters, and cepheids are shown in tables 1 and 2. Adopting average linear gradients, which can be taken as representative of the whole galactic disk, the abundances can be written in the form

$$y = a + b R$$

where $y = \log(O/H) + 12$ or $y = \log(S/H) + 12$ for PNe, HII regions and OB stars, and $y = [Fe/H]$ for open clusters and cepheids. For planetary nebulae, we have taken into account both O/H and S/H determinations and evaluated the gradient in the galactic disk according to the ages of the progenitor stars. For comparison purposes, we can also derive the [Fe/H] metallicities from the O/H abundances, on the basis of a [Fe/H] $\times$ O/H correlation derived for disk stars (see Maciel 2002 and Maciel et al. 2005 for details). The ages follow from the age-metallicity relation by Edvardsson et al. (1993), which also depends on the galactocentric distance. In this way, we can divide the sample of PNe into different age groups, each one having a characteristic gradient.

Table 1 shows representative examples of 3 age groups for O/H and 2 age groups for S/H. The table gives the gradient $b$ (dex/kpc) as defined by equation (1). All gradients in this paper have been calculated assuming $R_0 = 8.0 \text{ kpc}$ for the galactocentric distance of the LSR. For detailed references on the PN data the reader is referred to Maciel et al. (2003, 2005). It should be mentioned that the PN age groups shown in Table 1 are typical groups, arbitrarily defined. In fact, we have extended this procedure by taking into account a variety of definitions of the age groups, with similar results.

For open clusters we have adopted a similar procedure, except that we have used two different samples: (i) the smaller but homogeneous sample by Friel et al. (2002) and (ii) the larger compilation by Chen et al. (2003). As we will see, there are no important differences in...
the results from both samples. We have also divided the samples into age groups, of which the ones shown in Table 2 are representative. In this case, the given slopes \( b \) refer directly to the \([\text{Fe/H}]\) gradients.

Also shown in Table 2 is the derived gradient from cepheid variables. Here we have adopted the recent detailed work by Andrievsky and coworkers (see Andrievsky et al. 2002 and 2004 for the first and last papers in the series), and considered the \([\text{Fe/H}]\) abundances, which are the best determined. The ages given in Table 2 have been derived by us on the basis of metallicity dependent period-luminosity relationships.

Finally, we have also included some data on young objects, so that we can have a complete view of the time variation of the abundance gradients. We have considered HII regions, for which we have adopted an average gradient from the recent literature, namely \( b = d \log(O/H)/dR = -0.055 \pm 0.015 \) (see for example Henry & Worthey 1999; Deharveng et al. 2000; Pilyugin et al. 2003). Also, a sample of OB stars and associations recently studied by Dafon & Cunha (2004) have been taken into account, with a gradient \( b = d \log(O/H)/dR = -0.031 \pm 0.012 \). These are very young objects indeed, and from the point of view of the present work, their ages can be considered as essentially zero, in the case of HII regions, and about 0 – 50 Myr for OB stars. In both cases we can also convert the O/H gradients into \([\text{Fe/H}]\) using the same relation adopted for planetary nebulae.

As an illustration, Figure 1 shows the derived gradients, converted to \([\text{Fe/H}]\) gradients, for all objects considered in this work. It can be seen that the gradients are clearly flattening out in the last 6 to 8 Gyr, assuming an age of 13.6 Gyr for the galactic disk.

3. STATISTICAL ANALYSIS

3.1 Correlation coefficients and sample size

The simplest way to analyze the uncertainties of the linear fits is to consider the derived correlation coefficients. We have \( 0.64 < |r| < 0.94 \) for planetary nebulae, \( 0.33 < |r| < 0.77 \) for open clusters (Friel et al. sample), \( 0.22 < |r| < 0.71 \) for open clusters (Chen et al. sample), and \( |r| = 0.82 \) for cepheid variables. This suggests that the PN and cepheid gradients are well determined, while the analysis of the open cluster data is more complex. In fact, the correlation coefficient depends on the age group considered, being larger for groups II and III and smaller for the youngest group. Since Group I is the largest in both samples, there may be some contamination from older objects, or some effect derived from the space distribution of the clusters. In particular, young clusters seem to concentrate in the inner parts of the galactic disk, so that their distribution presents a more limited range of galactocentric distances, contributing to a flatter gradient.

The derived correlation coefficients along with the sample sizes can be used in order to compare these values with the probability distribution of a parent population which is completely uncorrelated. In this way we can have an indication on whether or not it is probable...
that our data points could be used to represent a sample derived from an uncorrelated population. Naturally, if this probability is small, we can conclude that our data points represent a sample derived from a parent population in which the quantities involved, that is, abundances and galactocentric distances, are correlated.

We have then estimated the probability \( P(r; n) \) that a given sample containing \( n \) objects, for which a linear correlation coefficient \( r \) was derived, could have come from an uncorrelated parent population, assuming gaussian distributions. The results are shown in Table 3 for PN, open clusters and cepheid variables.

It can be seen that the probability \( P(r; n) \) is exceedingly small in all cases, in fact several orders of magnitude lower than the indicated value of 0.001. Again, the open clusters are exceptions, especially the youngest groups in both samples, but even in this case the probability \( P(r; n) \) is still small. In other words, the probability of obtaining a large correlation coefficient as shown in Table 3, on the basis of samples of the given sizes from an uncorrelated population is generally very small.

### 3.2 Average uncertainties

It is interesting to compare the uncertainties associated with the linear fits with those associated with a simple dispersion around some average value, that is, assuming no variations of the abundances with the galactocentric distance. Since the observational abundance uncertainties are generally taken to be in the range 0.1 – 0.2 dex, we find that the uncertainties of the linear correlations are usually much lower than the uncertainties obtained assuming no systematic variations with \( R \).

### 3.3 \( \chi^2 \) analysis

Assuming that the abundances are distributed around an average value, characterized by the mean \( \mu \) and standard deviation \( \sigma \), we can estimate the reduced \( \chi^2 \) and the probability \( P(\chi^2; v) \) that a random sample of data points would yield a value of \( \chi^2 \) as large as or larger than the observed if the parent distribution, assumed gaussian, were equal to the assumed distribution. If the probability is approximately \( P(\chi^2; v) \approx 1 \), the assumed distribution describes the spread of data points well. If the probability is small, either the assumed distribution is not a good estimate of the parent distribution or the data are not a representative sample.

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### Table 3. Probability of uncorrelated parent population \( P(r; n) \)

<table>
<thead>
<tr>
<th>Group</th>
<th>( r )</th>
<th>( n )</th>
<th>( P(r; n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>O/H – PN</td>
<td>I</td>
<td>0.64</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.94</td>
<td>99</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>0.75</td>
<td>69</td>
</tr>
<tr>
<td>S/H – PN</td>
<td>I</td>
<td>0.66</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.76</td>
<td>72</td>
</tr>
<tr>
<td>OC – Friel</td>
<td>I</td>
<td>0.33</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.77</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>0.73</td>
<td>10</td>
</tr>
<tr>
<td>OC – Chen</td>
<td>I</td>
<td>0.22</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.69</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>0.71</td>
<td>20</td>
</tr>
<tr>
<td>Cepheids</td>
<td></td>
<td>0.82</td>
<td>127</td>
</tr>
</tbody>
</table>

### Table 4. Average uncertainties

<table>
<thead>
<tr>
<th></th>
<th>gradient</th>
<th>no gradient</th>
</tr>
</thead>
<tbody>
<tr>
<td>O/H PN</td>
<td>I</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>0.12</td>
</tr>
<tr>
<td>S/H</td>
<td>I</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.14</td>
</tr>
<tr>
<td>OC Friel</td>
<td>I</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>0.46</td>
</tr>
<tr>
<td>OC Chen</td>
<td>I</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>0.29</td>
</tr>
</tbody>
</table>
that the probability is very low in all cases, so that to a gaussian distribution around some average value. Studied elements are probably correct within 20%, which

\[ P \]

and \[ \sigma \]

value.

In Tables 1 and 2. Here we have \( n = 2 \) degrees of freedom, so that we can estimate \( \chi^2_n \) and \( P(\chi^2, v) \) provided we have a reliable estimate of the uncertainty of the data. For planetary nebulae, recent discussions by Pottasch et al. (2005) of objects with ISO data suggest that the abundances of the best-studied elements are probably correct within 20%, which corresponds to 0.10 dex for oxygen. This is probably a lower limit for other nebulae for which no infrared data is available, so that their abundances depend more heavily on ionization correction factors. We may then adopt \( \sigma \simeq 0.15 - 0.20 \) dex for O/H and \( \sigma \simeq 0.20 - 0.25 \) dex for S/H as realistic estimates for planetary nebulae. The latter can also be attributed to the open clusters, in view of the heterogeneity of the data and the use of photometric abundances. For cepheid variables, which have the best determinations, an average uncertainty \( \sigma \simeq 0.10 - 0.15 \) seems appropriate. The results are shown in column 3 of Table 4, under the heading “linear”. Again the probability is given within brackets. We can see that in all cases the \( \chi^2 \) values are lower than the corresponding values for the averages, so that the probability \( P(\chi^2, v) \) is higher for the linear correlation than for the simple averages. In fact, these probabilities are very close to unity in most cases, especially if we consider the more realistic, higher uncertainties. It can also be seen that for cepheid variables the probability given in column 3 is essentially unity, reinforcing our conclusion about systematic abundance variations with the galactocentric distance.

<table>
<thead>
<tr>
<th>Group</th>
<th>( \chi^2 ) [( P(\chi^2, v) )]</th>
<th>( \chi^2 ) [( P(\chi^2, v) )]</th>
</tr>
</thead>
</table>
| PN    | \( \begin{array}{l}
\text{O/H} \\
\text{I} \\
\text{II} \\
\text{III} \\
\text{S/H} \\
\text{I} \\
\text{II} \\
\text{OC} \\
\text{Friel} \\
\text{I} \\
\text{II} \\
\text{III} \\
\text{Chen} \\
\text{I} \\
\text{II} \\
\text{III} \\
\text{Cepheids} \\
\end{array} \) | \( \begin{array}{l}
\text{I} \\
\text{II} \\
\text{III} \\
\text{I} \\
\text{II} \\
\text{I} \\
\text{II} \\
\text{I} \\
\text{II} \\
\text{I} \\
\text{II} \\
\text{I} \\
\text{II} \\
\text{I} \\
\text{II} \\
\text{I} \\
\text{II} \\
\end{array} \) |

Column 2 of Table 5 shows the estimated values of \( \chi^2 \) and \( P(\chi^2, v) \) [within brackets] assuming average values, that is, no linear variations. The results for PNe show that the probability is very low in all cases, so that the data points are probably not distributed according to a gaussian distribution around some average value. However, it is interesting to note that, if we restrain the galactocentric distances to a smaller range, such as from \( R = 6 \) kpc to 8 kpc, or \( R = 8 \) kpc to 10 kpc, the probability \( P(\chi^2, v) \) increases, showing that, for a given galactocentric bin, the abundances show a better agreement with the gaussian distribution around some average value.

For the open clusters, the table shows a generally better agreement with the gaussian distribution around a mean value, both for the Friel and Chen samples, in agreement with our conclusions in sect. 3.2. However, for cepheid variables we have the same results as for the PNe, that is, the cepheid data are apparently not consistent with a gaussian distribution around a mean value.

We can also estimate \( P(\chi^2, v) \) in each case taking into account the derived linear correlations which are displayed in Tables 1 and 2. Here we have \( v = n - 2 \) for the number of degrees of freedom, so that we can estimate \( \chi^2 \) and \( P(\chi^2, v) \) provided we have a reliable estimate of the uncertainty of the data. For planetary nebulae, recent discussions by Pottasch et al. (2005) of objects with ISO data suggest that the abundances of the best-studied elements are probably correct within 20%, which corresponds to 0.10 dex for oxygen. This is probably a lower limit for other nebulae for which no infrared data is available, so that their abundances depend more heavily on ionization correction factors. We may then adopt \( \sigma \simeq 0.15 - 0.20 \) dex for O/H and \( \sigma \simeq 0.20 - 0.25 \) dex for S/H as realistic estimates for planetary nebulae. The latter can also be attributed to the open clusters, in view of the heterogeneity of the data and the use of photometric abundances. For cepheid variables, which have the best determinations, an average uncertainty \( \sigma \simeq 0.10 - 0.15 \) seems appropriate. The results are shown in column 3 of Table 4, under the heading “linear”. Again the probability is given within brackets. We can see that in all cases the \( \chi^2 \) values are lower than the corresponding values for the averages, so that the probability \( P(\chi^2, v) \) is higher for the linear correlation than for the simple averages. In fact, these probabilities are very close to unity in most cases, especially if we consider the more realistic, higher uncertainties. It can also be seen that for cepheid variables the probability given in column 3 is essentially unity, reinforcing our conclusion about systematic abundance variations with the galactocentric distance.

### ACKNOWLEDGMENTS

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