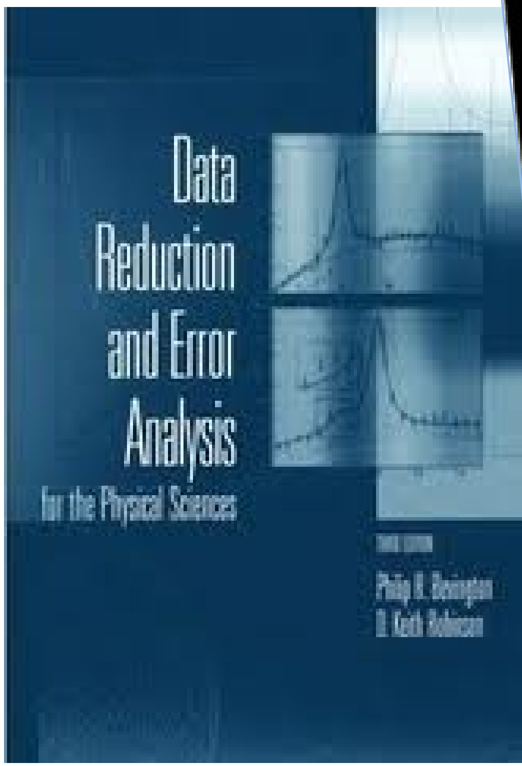
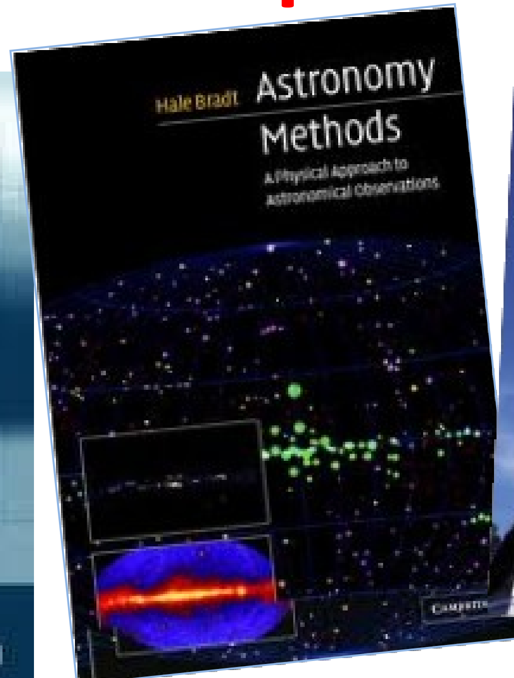
Data, errors & visualization

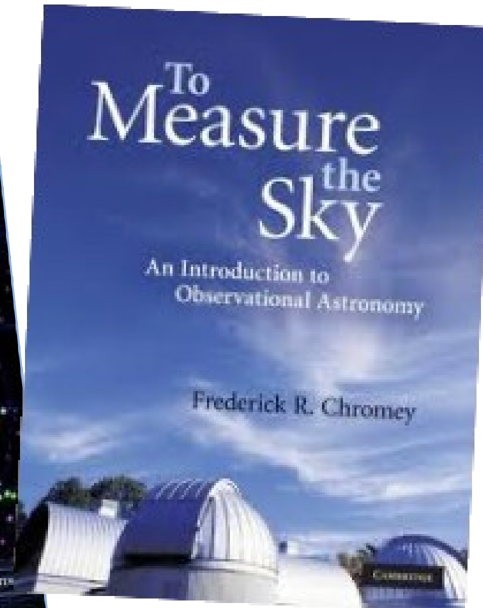
Simple statistics



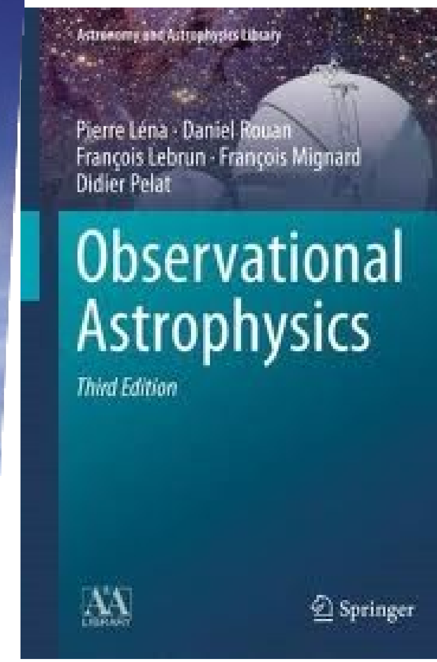
Philip R. Bevington & D. Keith Robinson 2002, 3rd edition



Astronomy Methods. H. Bradt, 2004. Section 6.5, pp. 151 - 172

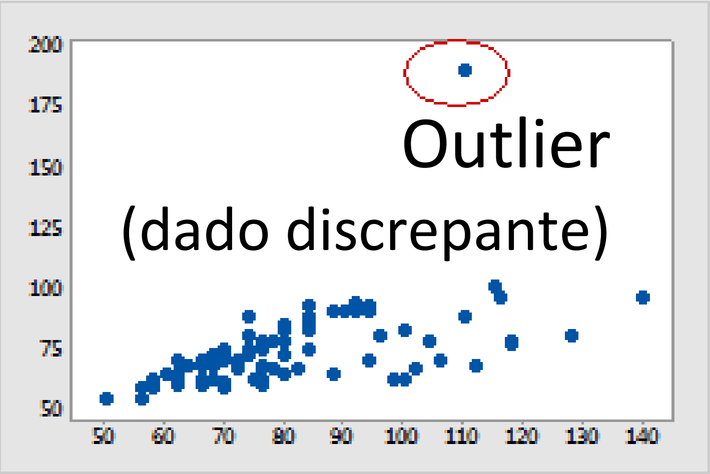


To Measure the Sky. An Introduction to Observational Astronomy. F. R. Chromey, 2010. Chapter 2.



Observational Astrophysics, 3rd Ed., P. Lena et al., 2012. Appendix B

Robust statistics



M.Sc. in Applied Statistics MT2004

Robust Statistics

http://www.stats.ox.ac.uk/pub/StatMeth/Robust.pdf

©1992–2004 B. D. Ripley[1]

Economic Statistics

http://www.scribd.com/doc/75349300/Economic-Statistics

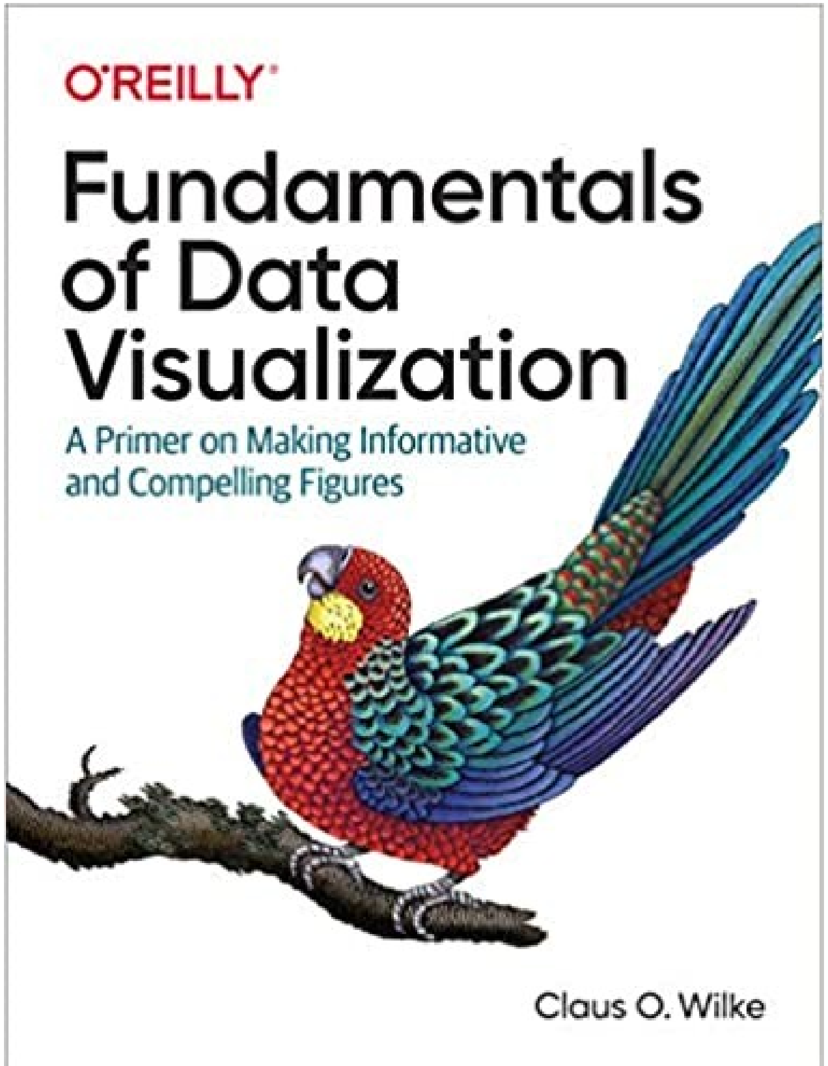
Estatística Robusta Aplicada aos Títulos do Tesouro Direto

Rhayssa Maia Costa Pinto

https://bdm.unb.br/bitstream/10483/13245/1/2015_RhayssaMaiaCostaPinto.pdf

Visualization

Free version at:
https://clauswilke.com/dataviz/



Data Visualization

Measurements (or estimates)

Distance to the Galactic Center

$$R_0 = 8.0 \pm 0.25 \text{ kpc}$$

Estimate of the real value

Estimate of the uncertainty

Unit

Malkin, Zinovy.
Statistical analysis of the determinations of the Sun's Galactocentric distance

2013, IAU Symp 289, 406

Significant figures

Distance to the Galactic Center

$$R_0 = 8.0 \pm 0.25 \text{ kpc}$$

8,0 ± 0,3 kpc   or

8,00 ± 0,25 kpc

3 significant figures

2 significant figures

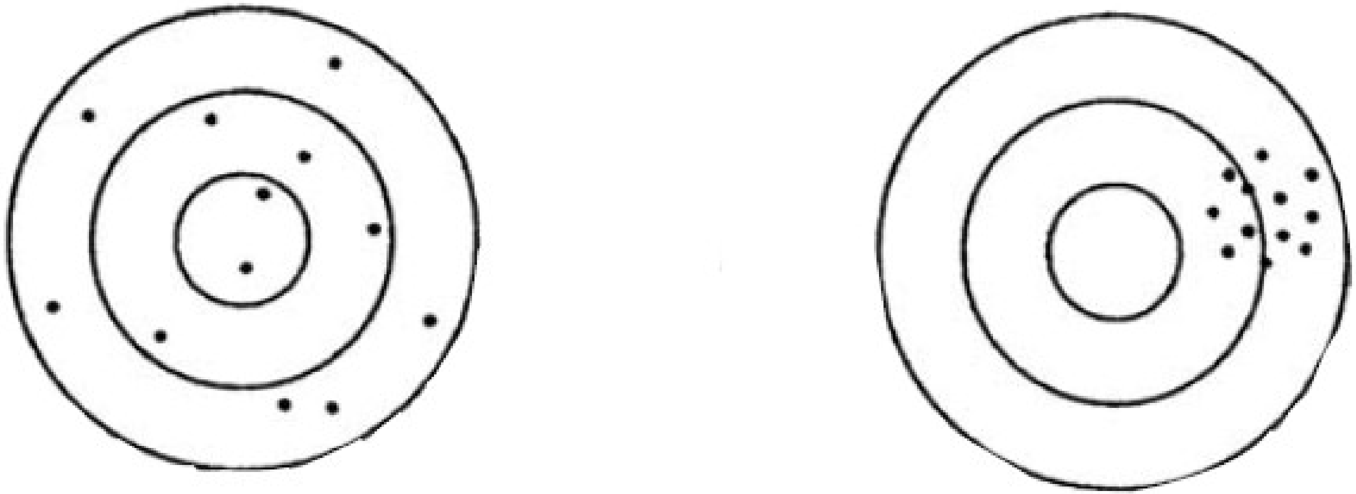
Malkin, Zinovy. Statistical analysis of the determinations of the Sun's Galactocentric distance 2013, IAU Symp 289, 406

Significant figures

Table 1: Previous average estimates of R_0

Paper	Period covered	R_0, kpc
Kerr & Lynden-Bell (1986)	1974–1986	8.5 ± 1.1
Reid (1989)	1974–1987	7.7 ± 0.7
Reid (1993)	1974–1992	8.0 ± 0.5
Nikiforov (2004)	1974–2003	7.9 ± 0.2
Avedisova (2005)	1992–2005	7.8 ± 0.32

Accuracy and precision



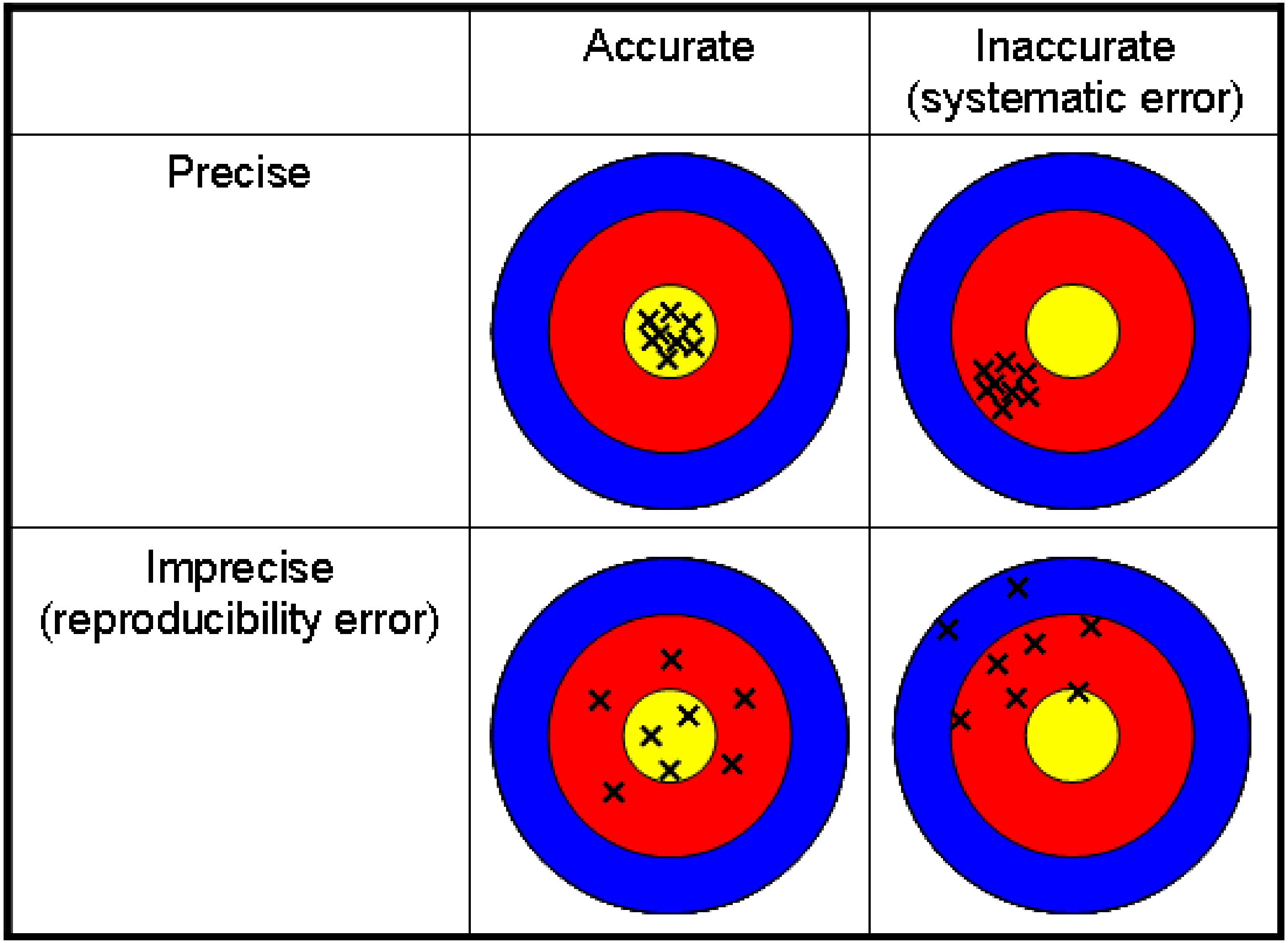
(a) Precision: low
Accuracy: fair
Precisão: baixa
Exactidão: razoável

(b) Precision: high
Accuracy: low
Precisão: elevada
Exactidão: baixa

figura 3 *Visualização dos conceitos de precisão e exactidão num alvo. Em (a) o conjunto de tiros (resultados) apresenta uma baixa precisão pois apresentam uma dispersão apreciável e uma exactidão razoável visto que não apresentam um desvio sistemático do centro do alvo. Em (b) a precisão é mais elevada (os tiros estão menos dispersos) e a exactidão é mais baixa pois os tiros encontram-se "sistematicamente" afastados para a direita do alvo.*

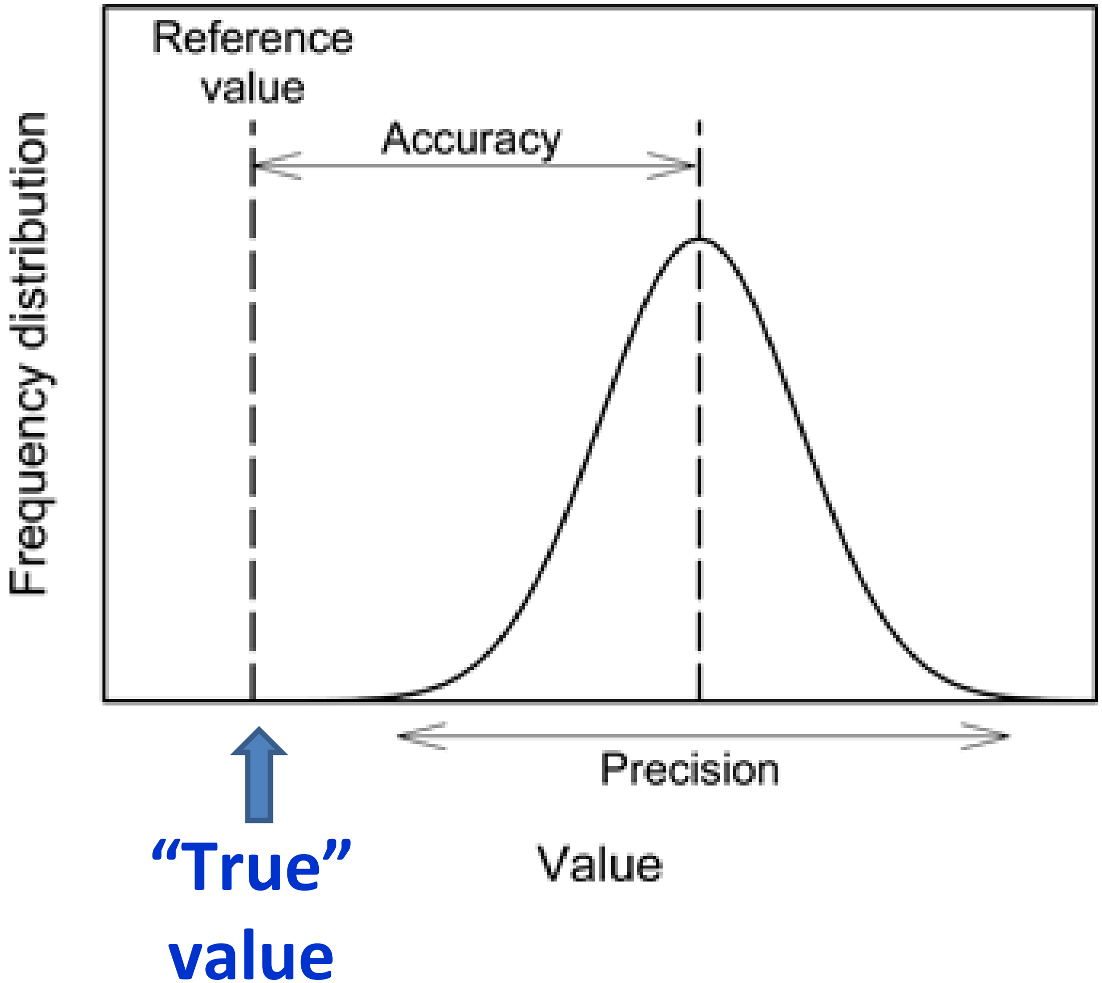
Erros experimentais – uma abordagem pedagógica.
I S A B E L M . A . F O N S E C A

Accuracy and precision



http://www.wellesley.edu/Chemistry/Chem105manual/Appendices/uncertainty_analysis.html

Accuracy and precision



http://dels-old.nas.edu/ilar_n/ilarjournal/49_2/html/v4902Simmons.shtml

How to define the uncertainty (or error) ?

- Error = “True Value” – Measurement

- If we know the true value, why bothering with the measurement?

Definitions of error

- Error of a measurement x_i (precision) : $\boldsymbol{\delta x_i = x_i - \langle x \rangle}$
- Relative error: $\boldsymbol{\delta x_i / \langle x \rangle}$
- Discrepancy (related to accuracy) = $\boldsymbol{x_i - x_{true}}$
- Relative discrepancy = $\boldsymbol{(x_i - x_{true})/x_{true}}$
- **Statistical "error"**: random fluctuations of the measurements that limit the **precision** of the result
- **Systematic error**: tends to deviate the measurement from the real value, limiting the **accuracy** of the result
- **spread = largest result - smallest result**

Example

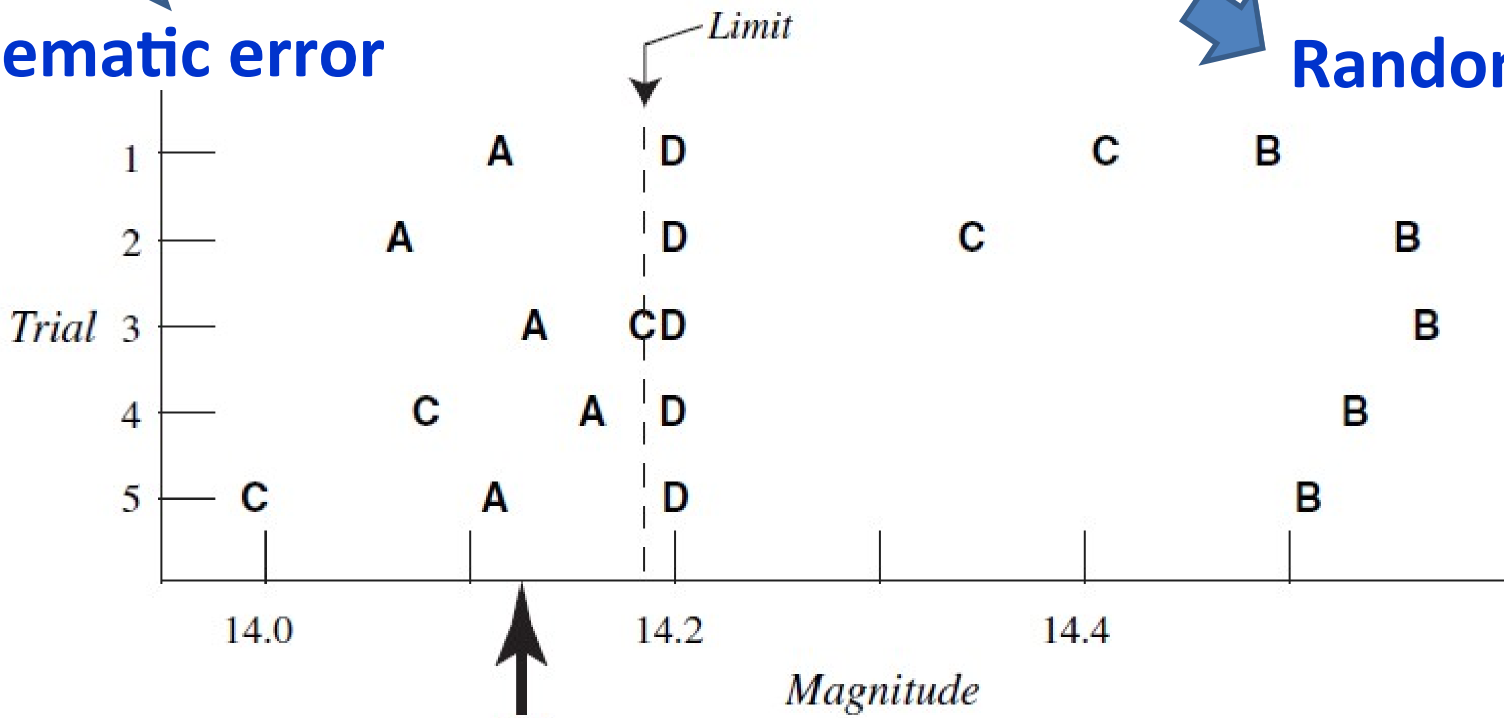
- “True” value = 100,0 cm
- Measurements: 99,4 99,2 99,5 99,3 99,1 cm
- <x> = 99,3 cm (mean)
- **δx_i** = 0,1   -0,1 +0,2 0,0 -0,2 cm
- **δx_i/<x>** = +0,001 -0,001 +0,002 0,000 -0,002

  +0,1 %   -0,1%   +0,2% 0.0%   -0,2%
- Discrepancy = -0,6 -0,8 -0,5 -0,7 -0,9 cm
- Relative discrepancy = -0,6, -0,8 -0,5 -0,7 -0,9 %
- Systematic error = ???

Accuracy and precision

Systematic error

Random error



Astronomer	A	B	C	D
Trial 1	14.115	14.495	14.386	14.2
Trial 2	14.073	14.559	14.322	14.2
Trial 3	14.137	14.566	14.187	14.2
Trial 4	14.161	14.537	14.085	14.2
Trial 5	14.109	14.503	13.970	14.2
Mean	14.119	14.532	14.190	14.2
Deviation from truth	−0.004	+0.409	+0.067	+0.077
Spread	0.088	0.071	0.418	0
σ	0.033	0.032	0.174	0
s	0.029	0.029	0.156	0
Uncertainty of the mean	0.013	0.013	0.070	(0.05)

Distance to the GC
Random and systematic errors

R_0 determinations with estimation of both statistical and systematic errors

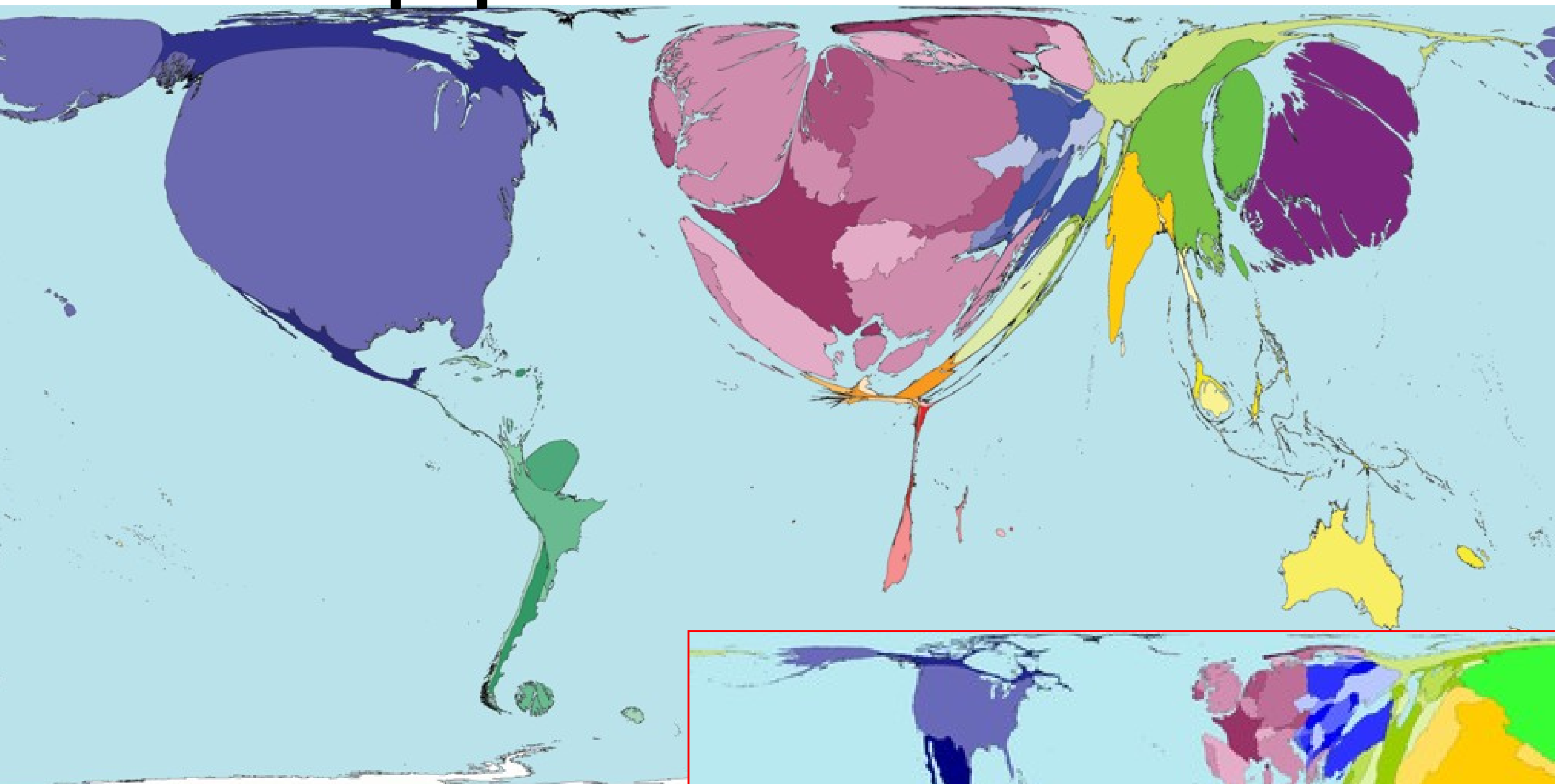
Paper	R_0, kpc
Nishiyama et al. (2006)	$R_0 = 7.52 \pm 0.10\,\|_{stat} \pm 0.35\,\|_{syst}$
Groenewegen et al. (2008)	$R_0 = 7.94 \pm 0.37\,\|_{stat} \pm 0.26\,\|_{syst}$
Trippe et al. (2008)	$R_0 = 8.07 \pm 0.32\,\|_{stat} \pm 0.13\,\|_{syst}$
Gillessen et al. (2009b)	$R_0 = 8.33 \pm 0.17\,\|_{stat} \pm 0.31\,\|_{syst}$
Gillessen et al. (2009a)	$R_0 = 8.28 \pm 0.15\,\|_{stat} \pm 0.29\,\|_{syst}$
Matsunaga et al. (2009)	$R_0 = 8.24 \pm 0.08\,\|_{stat} \pm 0.42\,\|_{syst}$
Sato et al. (2010)	$R_0 = 8.3 \pm 0.46\,\|_{stat} \pm 1.0\,\|_{syst}$

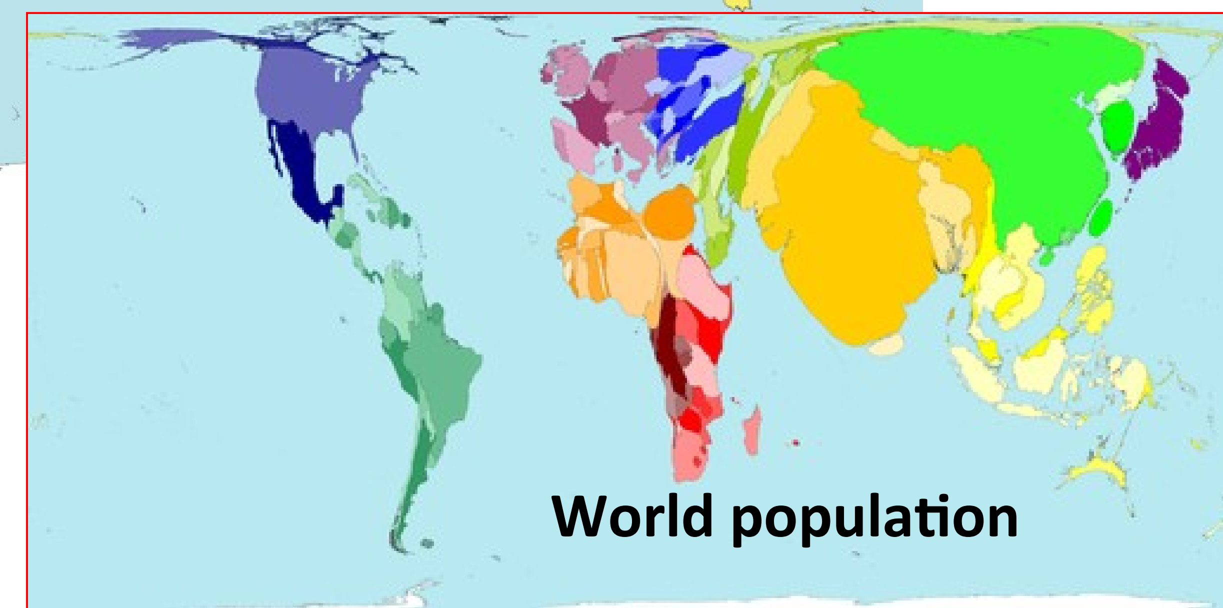
Population and Sample of a population

- **Population** is the **whole** set of measurements
- **Sample** is a **part** (representative or not) of the population
- Small populations could be fully studied. *Ex.: age of students of AGA5802*
- Big populations can be studied through samples. *Ex.: weight of each person on Earth (7 billions !!!)*

Population of papers/person

Published papers





Be careful about bias in your sample !!!

http://www.worldmapper.org/

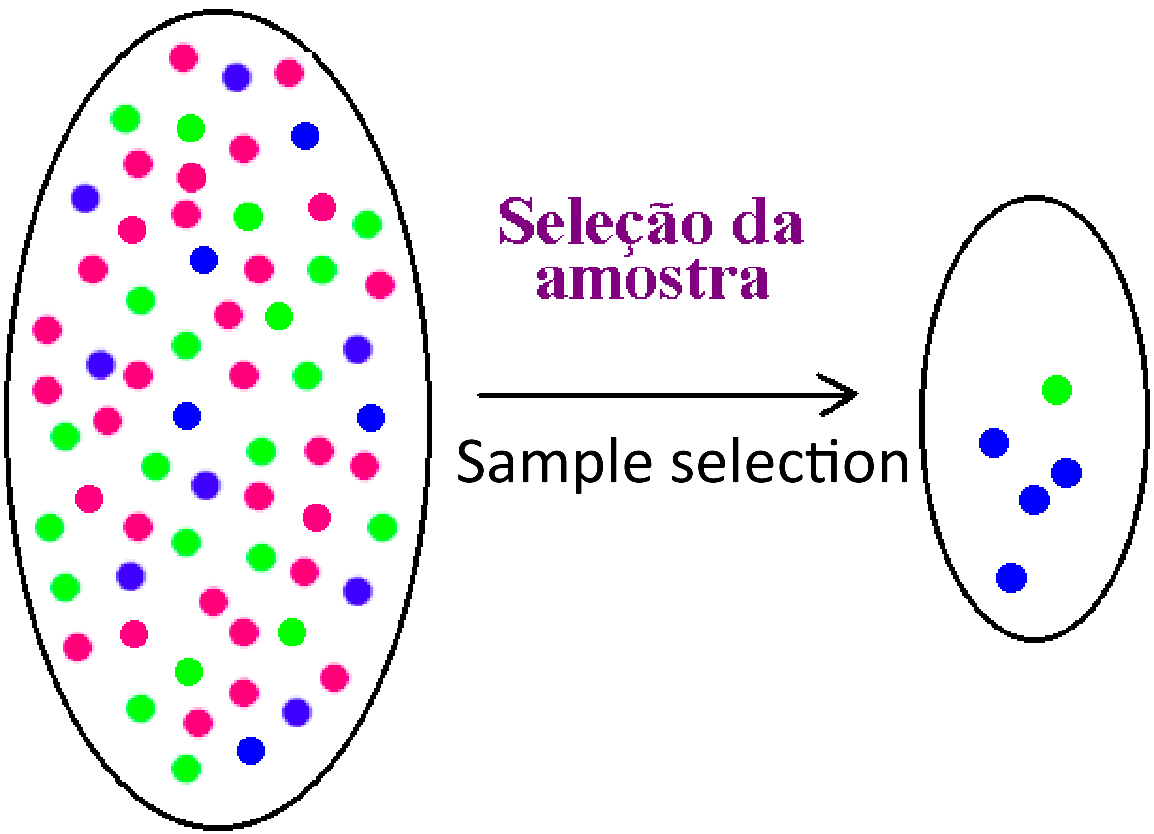
Population of height (adults) on Earth

How to select a representative sample (without *bias*) ?

Country	Average male height	Average female height	Ratio (male to female)	Sample population / age range	Share of pop. over 15 covered[3]	Methodology	Year
Argentina	1.73 m	1.60 m	1.08	17 (healthy)	N/A	Measured	2000
Bolivia / Aimara	1.600 m	1.422 m	1.13	20–29	N/A	Measured	1970
Brazil	1.71 m	1.59 m	1.07	18+	93.2%	Measured	2008–2009
China	1.66 m	1.57 m	1.06	Rural, 17	N/A	Measured	2002
China	1.70 m	1.59 m	1.07	Urban, 17	N/A	Measured	2002
Germany	1.81 m	1.68 m	1.08	18–25	N/A	Self-reported	2009
Germany	1.78 m	1.65 m	1.08	18+	96.5%	Self-reported	2009
Netherlands	1.83 m	1.70 m	1.08	20–30	N/A	Self-reported	2010
Perú	1.64 m	1.510 m	1.09	20+	85.4%	Measured	2005

Selection bias

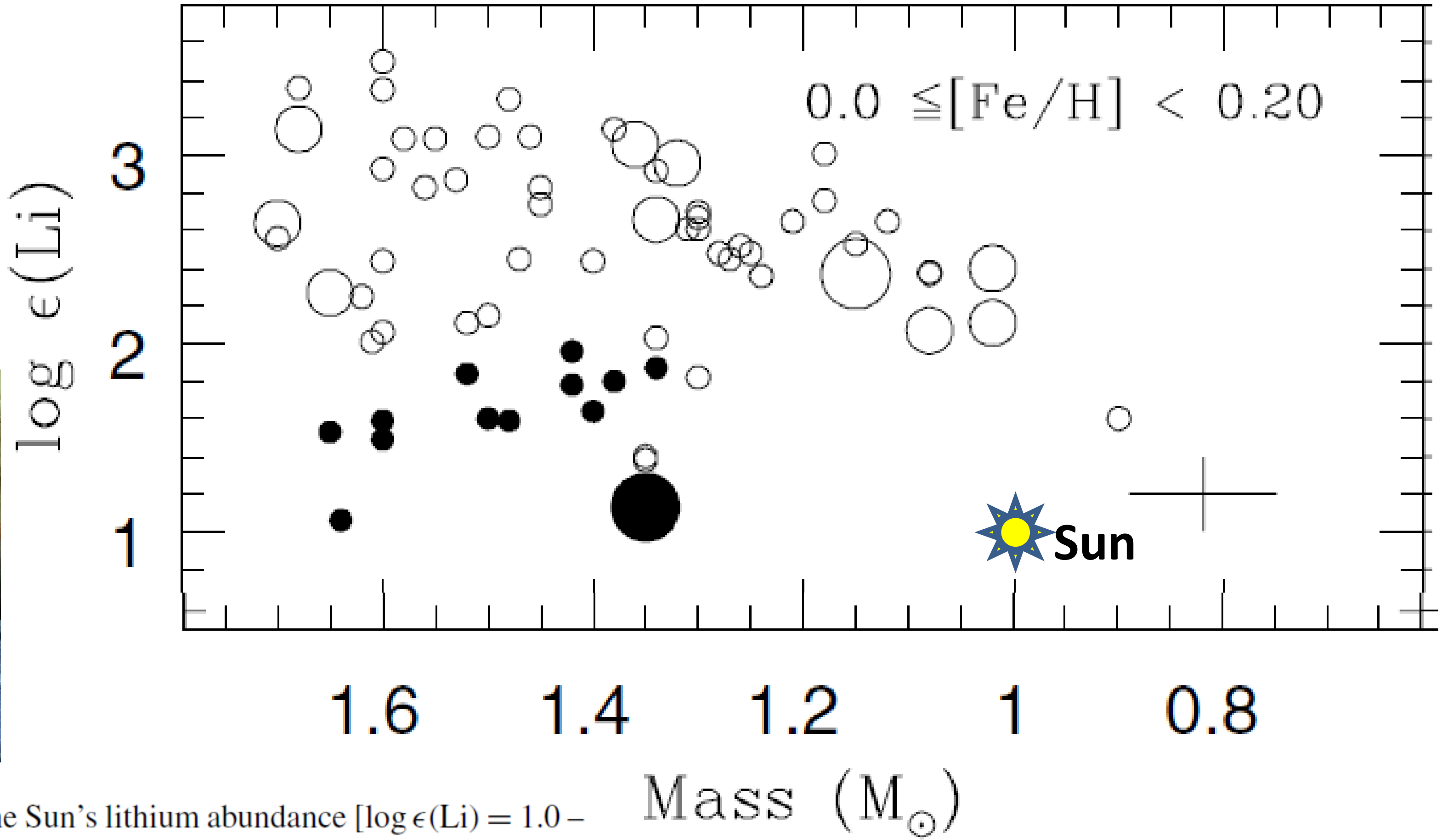
Is your sample representative of the population? If there is a bias → wrong conclusions



Mon. Not. R. Astron. Soc. **349,** 757–767 (2004)

Lithium abundances of the local thin disc stars

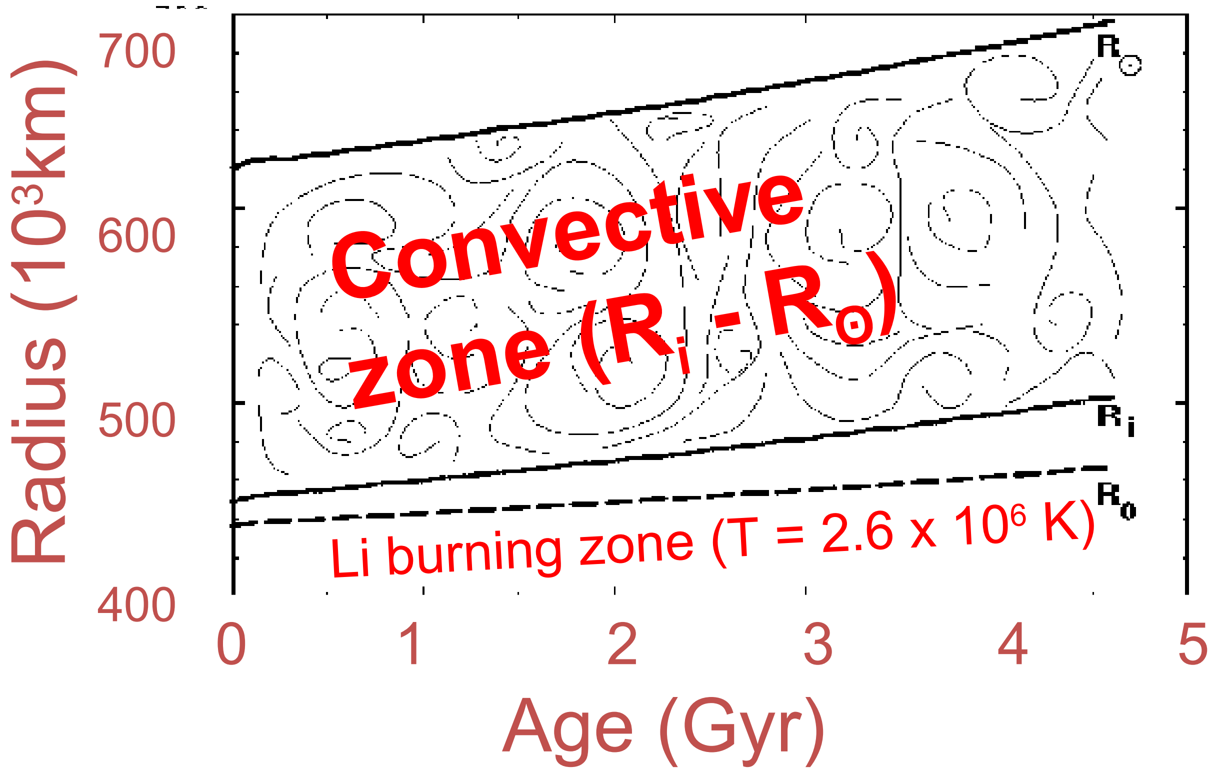
David L. Lambert[1] and Bacham E. Reddy[1,2★]



A curiosity is that the Sun's lithium abundance [log ϵ(Li) = 1.0 – Müller, Peytremann & de la Reza 1975] appears to fall by more than 1 dex below the trend defined by the field stars (see Fig. 3). If placed among NGC 188's stars, the Sun would be deemed very Li-poor. Among M67's stars, the Sun would be one of the most Li-poor stars. This hint that the Sun may be 'peculiar' as regards the depletion of lithium weakens its value as a calibrator for prescriptions of non-standard modes of lithium astration.

Studied 450 dwarf stars of type F and G in different mass and metallicity regimes

Is the solar Li abundace peculiar ?



Astrophys Space Sci (2010) 328: 193–200

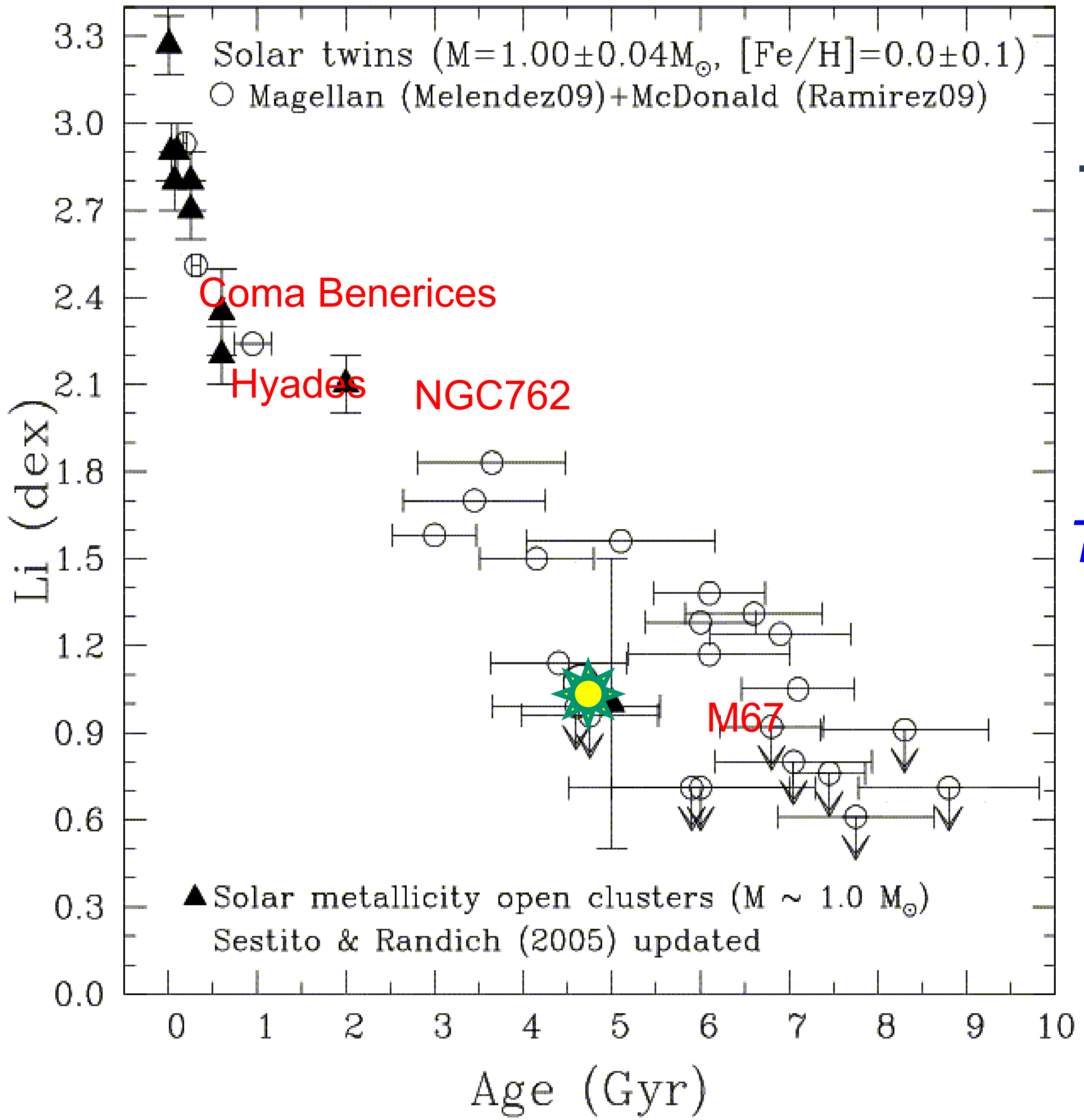
The solar, exoplanet and cosmological lithium problems

J. Meléndez · I. Ramírez · L. Casagrande · M. Asplund · B. Gustafsson · D. Yong · J.D. do Nascimento Jr. · M. Castro · M. Bazot

A&A 519, A87 (2010)

Lithium depletion in solar-like stars: no planet connection

P. Baumann[1], I. Ramírez[1], J. Meléndez[2], M. Asplund[1], and K. Lind[3]



Solar twins in open cluster and field stars

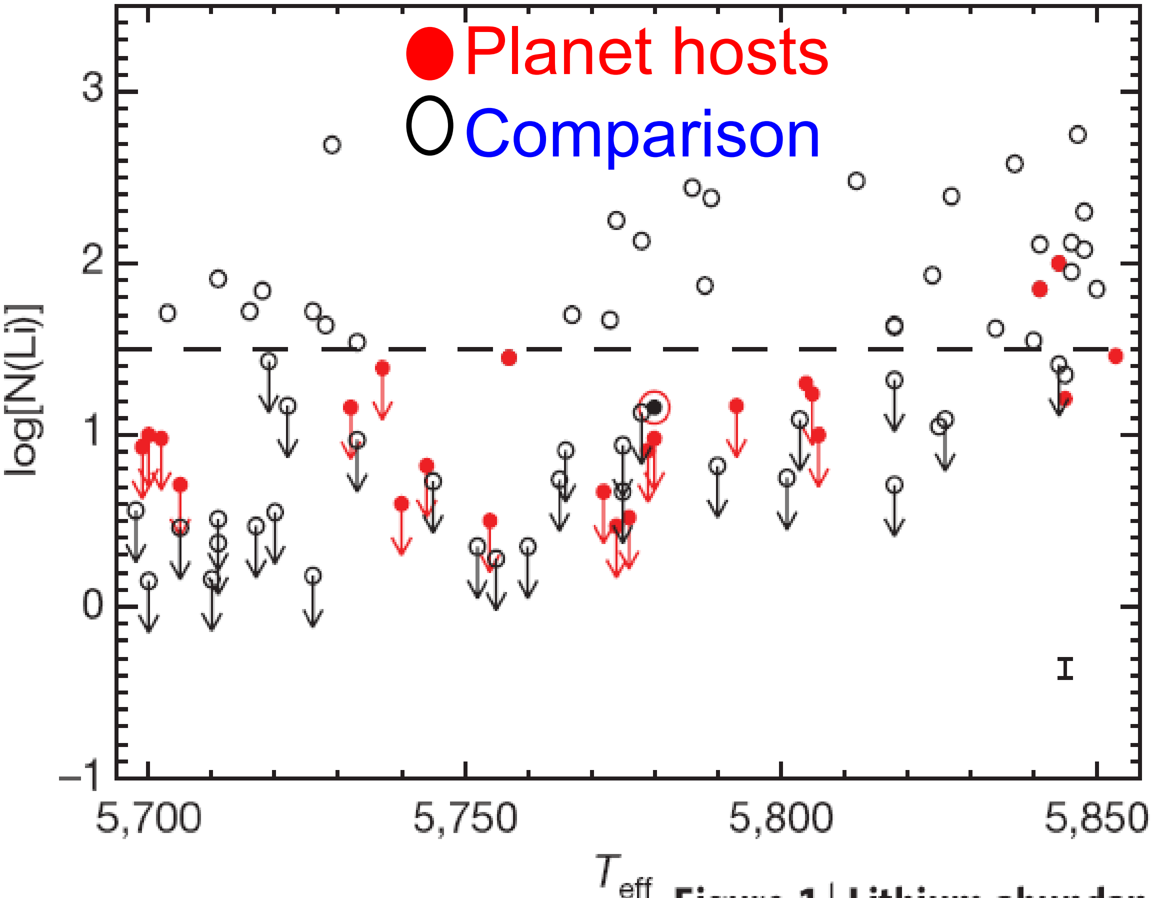
The Sun is normal in lithium compared to others 1-solar-mass stars at 4.6 Gyr

(Melendez et al. 2010; Baumann et al. 2010)

nature Vol 462|12 November 2009 LETTERS

Enhanced lithium depletion in Sun-like stars with orbiting planets

Garik Israelian[1,2], Elisa Delgado Mena[1,2], Nuno C. Santos[3,4], Sergio G. Sousa[1,3], Michel Mayor[4], Stephane Udry[4], Carolina Domínguez Cerdeña[1,2], Rafael Rebolo[1,2,5] & Sofia Randich[6]

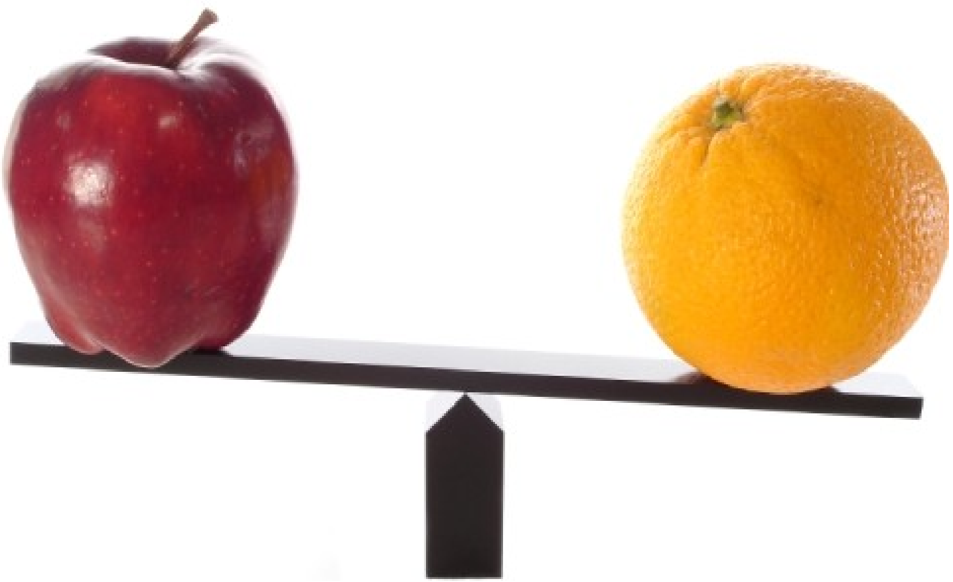


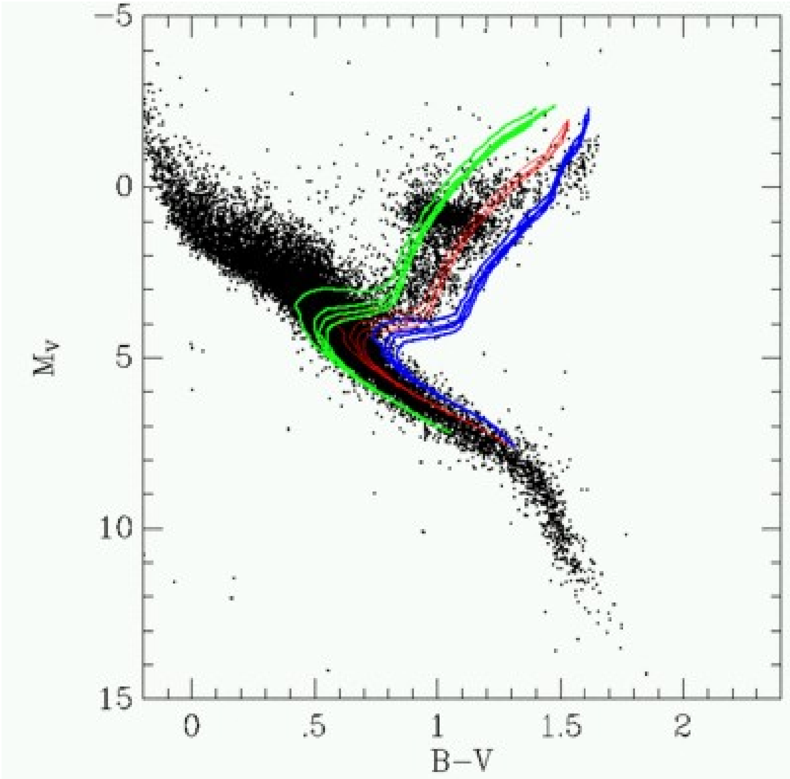
Planet-host stars around solar T_{eff} seem depleted in Li

Figure 1 | Lithium abundance plotted against effective temperature in solar-analogue stars with and without detected planets. The planet-

You cannot compare apples and oranges ...

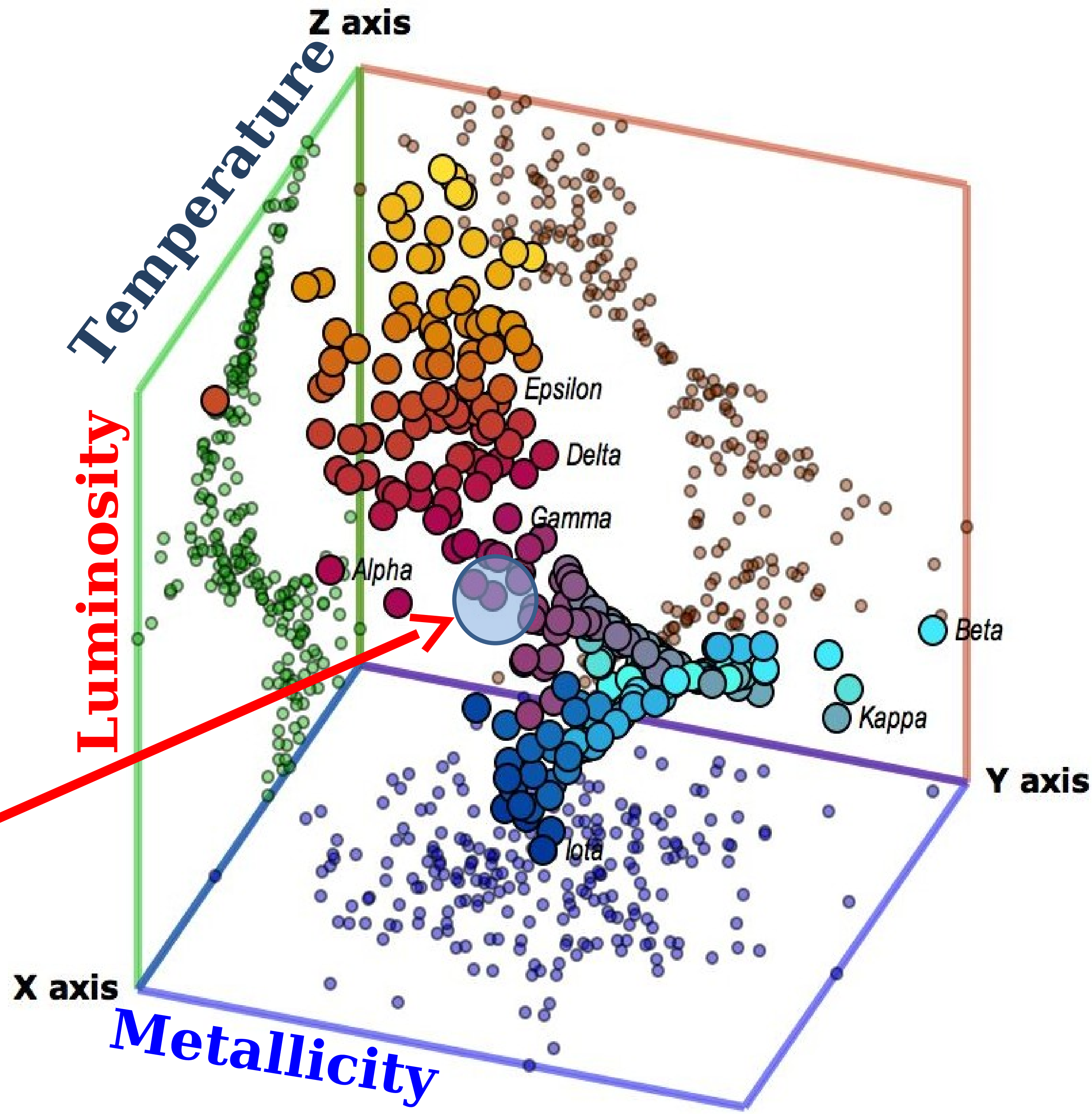
comparer des pommes avec des oranges
comparer des pommes et des poires





You cannot add pears and apples ...
No puedes sumar peras con manzanas

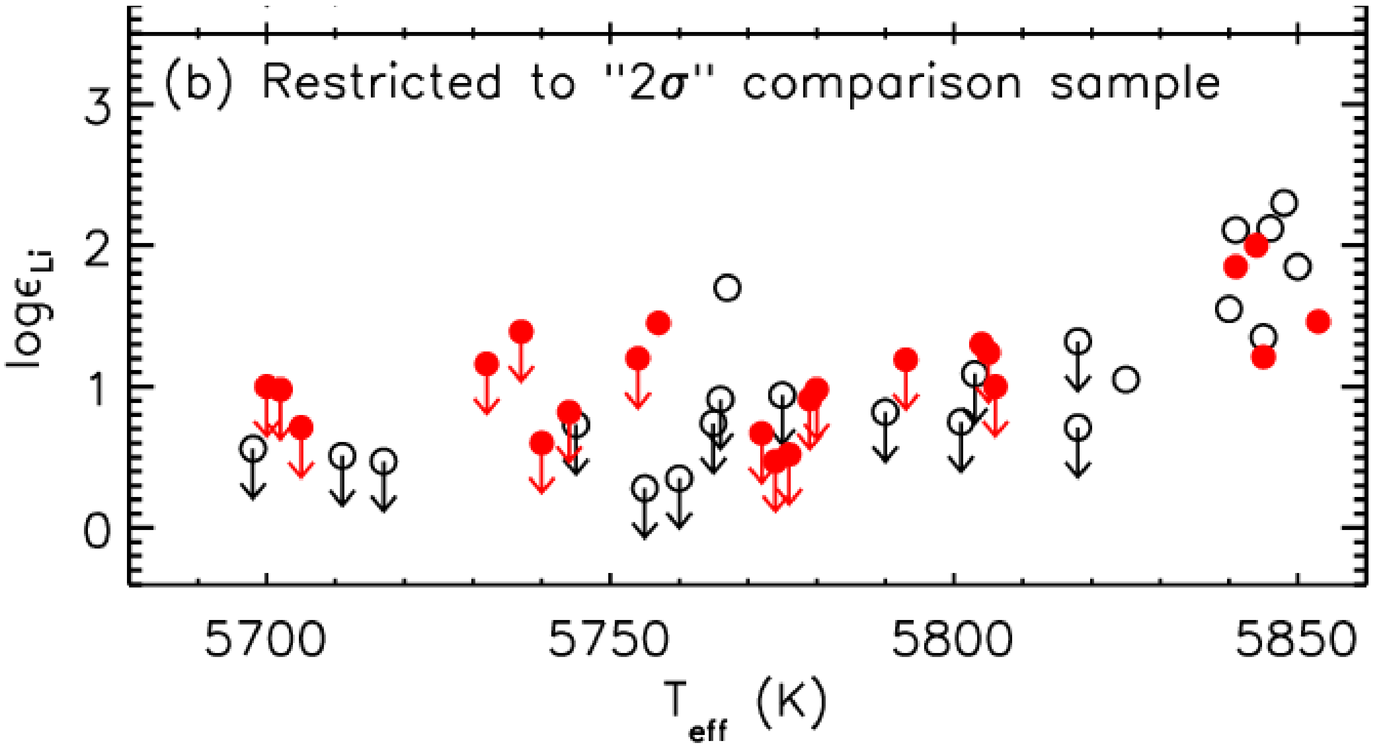
Comparing apples & apples



Li depletion is not enhanced in planet hosts !

Comparing apples & apples

(only stars with similar stellar parameters within 2-sigma)

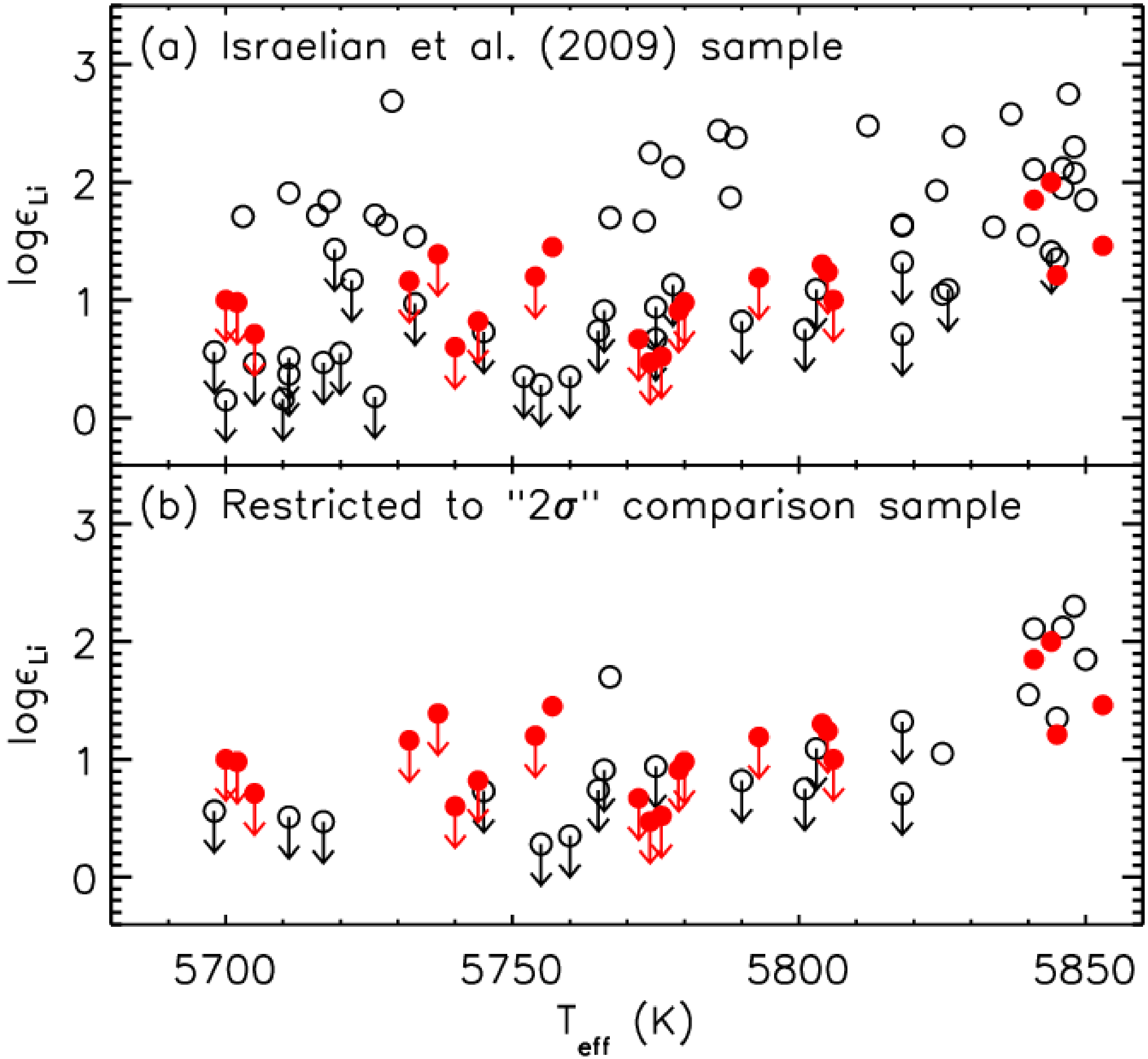


● Planet hosts

○ Comparison

Baumann, Ramírez, Meléndez, & Asplund 2010, A&A, 519, A87

Li depletion is not enhanced in planet hosts !



Apples & oranges

Israelian et al. 2009, Nature

Planet hosts

Comparison

Apples & apples

Baumann, Ramírez, Meléndez, & Asplund 2010, A&A, 519, A87

Conclusion (year 2022) on lithium in stars with and without planets (sem viés na comparação):

there is no difference in Li abundance between stars with and without planets

But, in 2023:

Actually, stars with planets may be somewhat less abundant in lithium (~0,25 dex) relative to stars without planets (based on 194 stars from Carlos et al. 2019, Giulia Martos et al. 2023, Anne Rathsam et al. 2023)

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MNRAS **522,** 3217–3226 (2023)
Advance Access publication 2023 April 21
https://doi.org/10.10

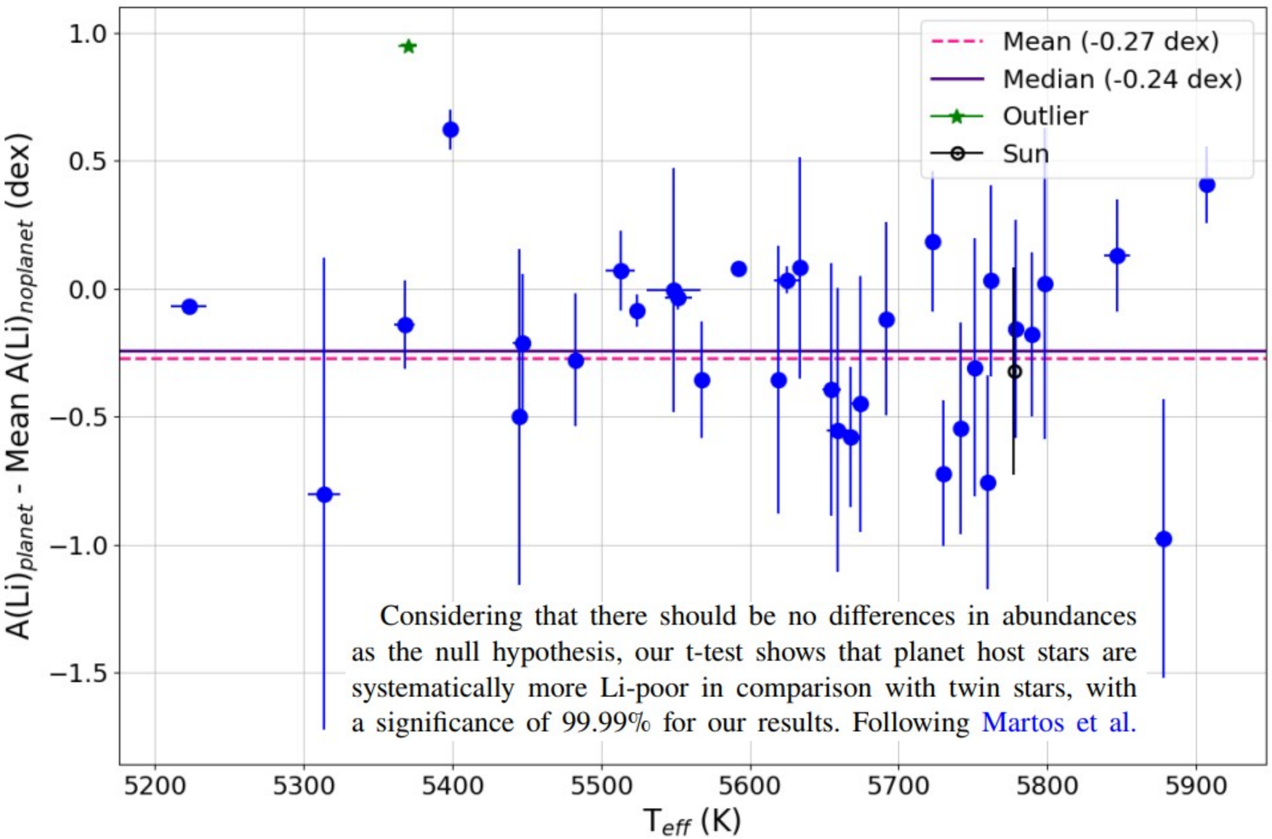
Metallicity and age effects on lithium depletion in solar analogues

Preprint 27 May 2023 Compiled

Giulia Martos,★ Jorge Meléndez, Anne Rathsam and Gabriela Carvalho Silva
Departamento de Astronomia, IAG, Universidade de São Paulo, Rua do Matão 1226, São Paulo 05508-090, Brazil

Lithium depletion in solar analogs: age and mass effects

Anne Rathsam,[1★] Jorge Meléndez[1] and Gabriela Carvalho Silva[1]
a e Ciências Atmosféricas, Rua do Matão 1226, São Paulo



Considering that there should be no differences in abundances as the null hypothesis, our t-test shows that planet host stars are systematically more Li-poor in comparison with twin stars, with a significance of 99.99% for our results. Following Martos et al.

Comparison between the Li abundance of planet host stars and the average Li of stars without planets of similar parameters (age ± 0.5 Gyr; mass ± 0.05 M⊙; [Fe/H] ± 0.15 dex), based on 194 stars (35 planet hosts) in Rathsam et al. 2023, Martos et al. 2023 and Carlos et al. 2019.

Example of populations & samples

Population	Sample	Better sample
1000 colored marbles mixed in a container: 500 red, 499 blue, 1 purple	5 marbles drawn at random from the container	50 marbles drawn at random
The luminosities of each star in the Milky Way galaxy (about 10^{11} values)	The luminosities of each of the nearest 100 stars (100 values)	The luminosities of 100 stars at random locations in the galaxy (100 values)
The weights of every person on Earth	The weights of each person in this room	The weights of 100 people drawn from random locations on Earth

Age of each star in the Galaxy?

- **Open cluster?**
- **Globular cluster?**
- **Spiral arms?**
- **Halo?**
- **Bulge?**

20 stars in 50 "random" places (actually, representative) in the Galaxy (1000 values)

Central Value & Standard Deviation of a **Population x_i of M elements** (in total)

Central value:

Mean (average): $$\mu = \frac{1}{M}\sum_{i=1}^{M} x_i$$

Median: value that divides the population exactly in half

Mode: is the value that occurs most often.

Standard deviation:

$$\sigma = \sqrt{\frac{1}{M}\sum_{i=1}^{M} (x_i - \mu)^2}$$

Central Value & Standard deviation **estimated** of a population using a **sample of N elements**

Average <x> : $$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Estimated standard deviation: $$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Use *N – 1* instead of *N* because <x> is only an estimate of μ.

Example, population of heights in 5 girls:

Population: 149, 151, 153, 152, 169 cm. μ= 154,8cm. σ = 7,22cm

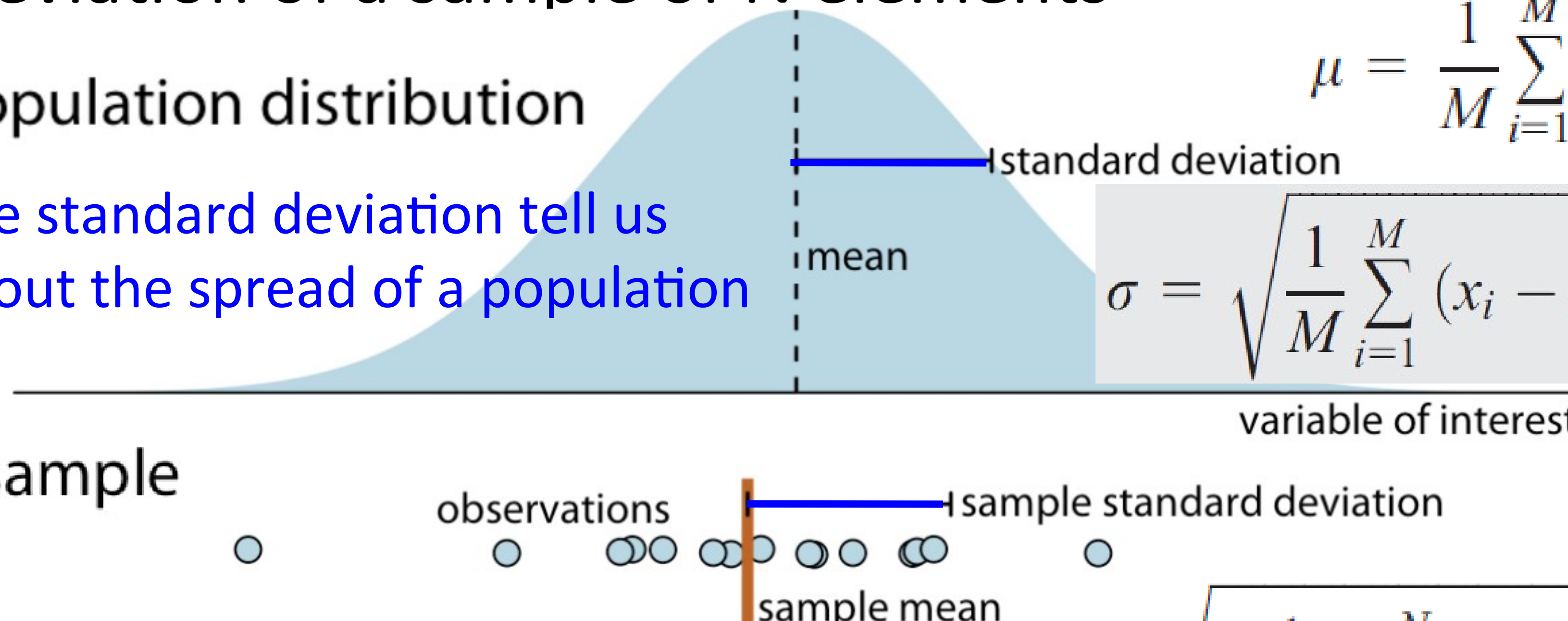
Sample: 151, 152 cm. <x> = 151,5 cm, σ = 0,5cm , s = 0,70cm

Sample: 149, 169 cm. <x> = 159 cm, σ = 10cm , s = 14cm

Sample: 149,151,169 cm. <x> = 156,3 cm, σ = 9cm , s = 11cm

Mean μ & standard deviation σ of a population of M elements and sample mean & sample standard deviation of a sample of N elements

population distribution



$$\mu = \frac{1}{M}\sum_{i=1}^{M} x_i$$

$$\sigma = \sqrt{\frac{1}{M}\sum_{i=1}^{M}(x_i - \mu)^2}$$

The standard deviation tell us about the spread of a population

sample

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

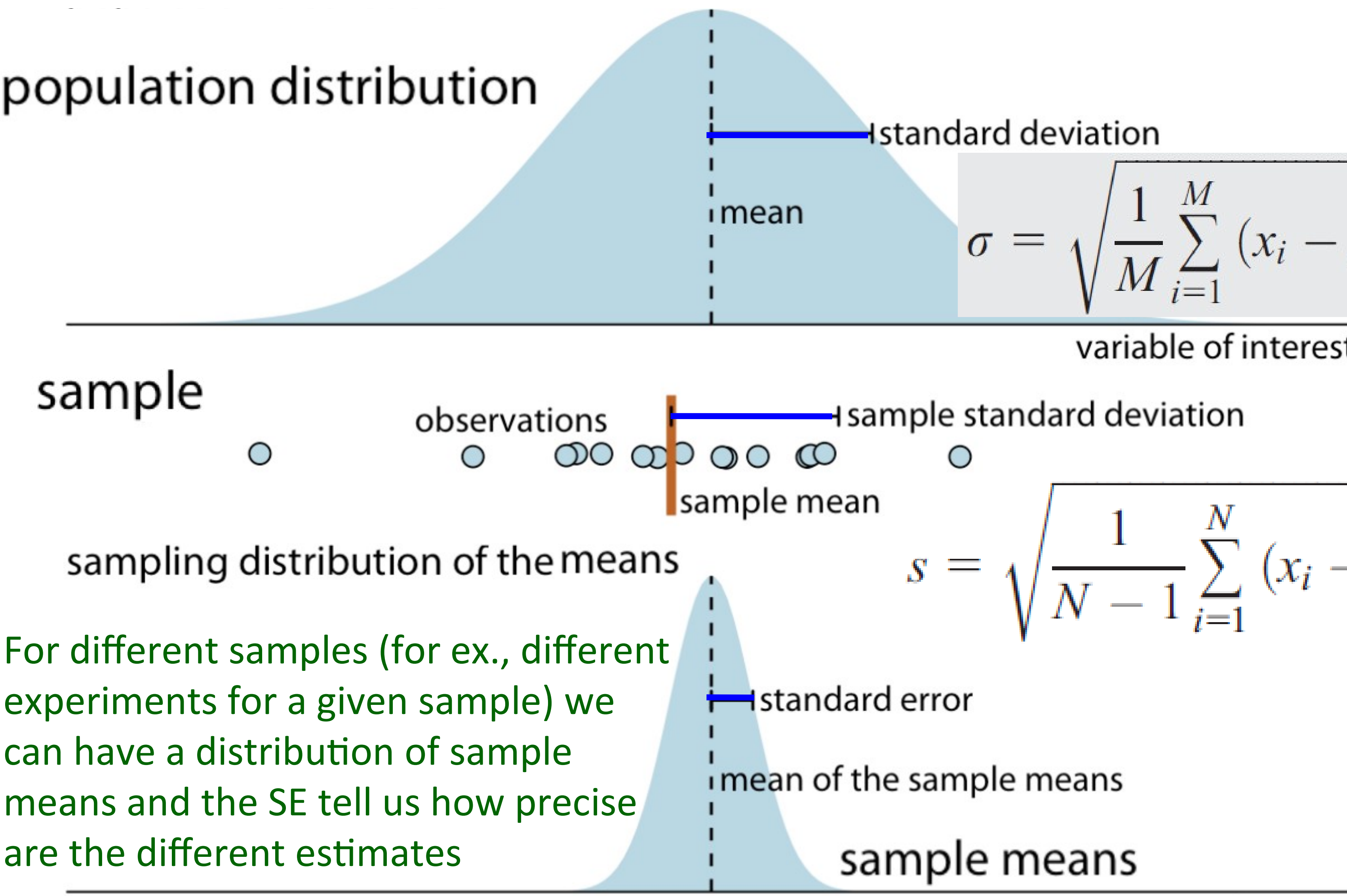
$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

The sample mean & sample standard deviation are estimates of the mean μ and σ of the population

Standard deviation of the mean of n samples, σ_n

$$\sigma_\mu(n) = \frac{s}{\sqrt{n}}$$

Also known as standard error (SE)



$$\sigma = \sqrt{\frac{1}{M}\sum_{i=1}^{M}(x_i - \mu)^2}$$

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

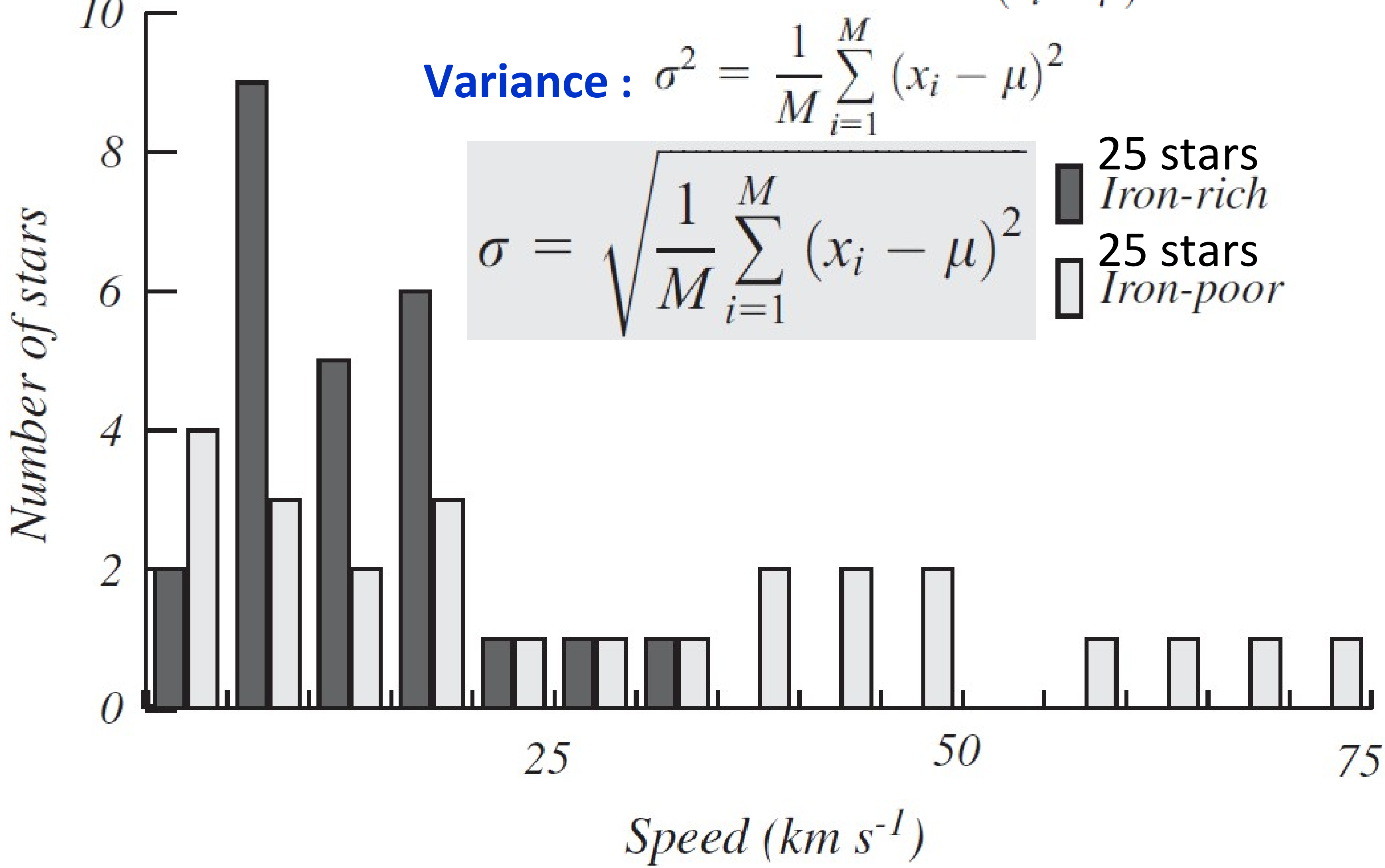
For different samples (for ex., different experiments for a given sample) we can have a distribution of sample means and the SE tell us how precise are the different estimates

Stellar populations in our galaxy

deviation from the mean $= (x_i - \mu)$

Variance : $\sigma^2 = \frac{1}{M}\sum_{i=1}^{M}(x_i - \mu)^2$

$$\sigma = \sqrt{\frac{1}{M}\sum_{i=1}^{M}(x_i - \mu)^2}$$



GROUP A: Variance = 57.25 km^2 / s^2, σ = 7.57 km/s. Mean =12.85 km/s

Stellar populations in our galaxy

$$\mathrm{TD/D} = \frac{X_{\mathrm{TD}}}{X_{\mathrm{D}}} \cdot \frac{f_{\mathrm{TD}}}{f_{\mathrm{D}}}$$

$$\mathrm{TD/H} = \frac{X_{\mathrm{TD}}}{X_{\mathrm{H}}} \cdot \frac{f_{\mathrm{TD}}}{f_{\mathrm{H}}}$$

Bensby, T. et al. 2003, A&A, 410, 527

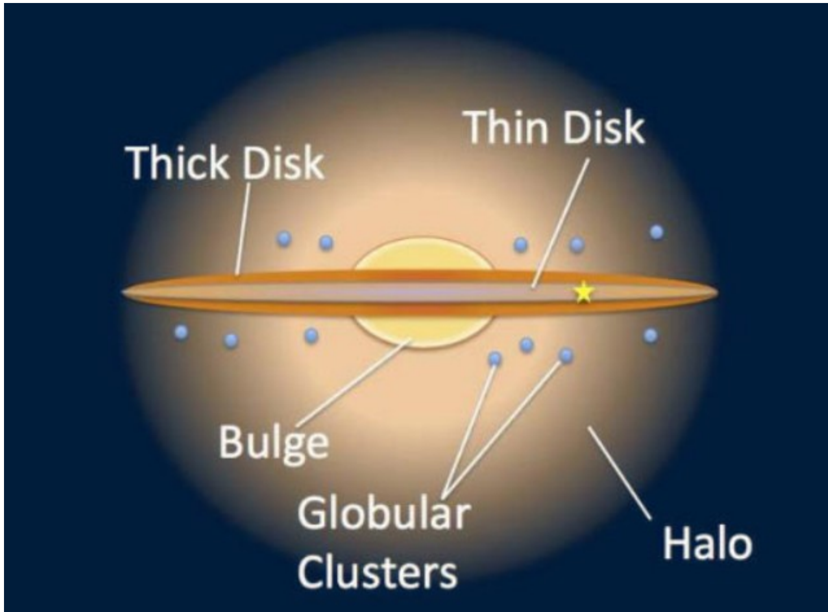
The selection of thick and thin disk stars is done by assuming that the Galactic space velocities (U_{LSR}, V_{LSR}, and W_{LSR}, see Appendix A) of the stellar populations in the thin disk, the thick disk, and the halo have Gaussian distributions,

$$f(U,\,V,\,W) = k \cdot \exp\left(-\frac{U_{\mathrm{LSR}}^2}{2\,\sigma_U^2} - \frac{(V_{\mathrm{LSR}} - V_{\mathrm{asym}})^2}{2\,\sigma_V^2} - \frac{W_{\mathrm{LSR}}^2}{2\,\sigma_W^2}\right), \quad (1)$$

where

$$k = \frac{1}{(2\pi)^{3/2}\,\sigma_U \sigma_V \sigma_W}$$

	X	σ_U	σ_V	σ_W	V_{asym}
			[km s^{-1}]		
Thin disk (D)	0.94	35	20	16	−15
Thick disk (TD)	0.06	67	38	35	−46
Halo (H)	0.0015	160	90	90	−220



Estimated central value (sample) & "True" central value (μ) of a population

$$\mu = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} x_i = \lim_{N \to \infty} \bar{x}$$

$$\mu \approx \bar{x}$$

Variance & estimated standard deviation are similar to the "true values of a population" for N >> 1

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

$$\sigma^2 \approx s^2$$

$$\sigma^2 = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2 = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 = \lim_{N\to\infty}\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

$$= \lim_{N\to\infty} s^2$$

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} \approx \sigma$$

Weighted mean

$$y_c = \sigma_c^2 \sum_{i=1}^{n} (y_i/\sigma_i^2)$$

$$1/\sigma_c^2 = \sum_{i=1}^{n} (1/\sigma_i^2)$$

- Example:

y_1 = 18 ± 3 cm, y_2= 16 ± 4 cm

y_c = (3)*144/25 = 17,3 cm ± 2,4 cm

$1/\sigma^2$ = (1/16) + (1/9) = 25/144, σ^2 = 144/25

Distribuição das medidas

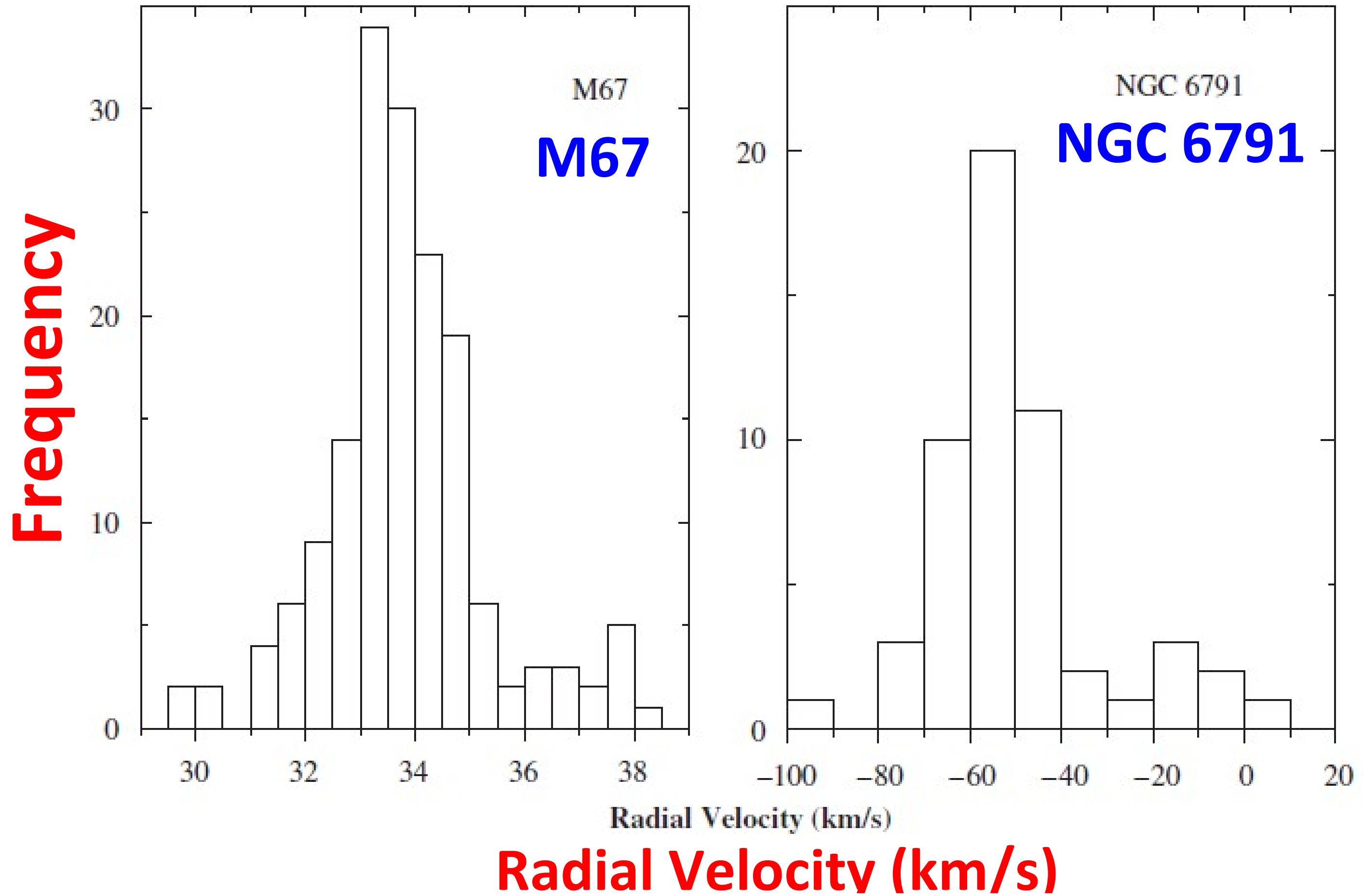


Fig. 2. Radial velocity histogram of four stellar clusters after removing stars according to the second selection method, that is, star velocity difference to the central peak of the distribution larger than 3σ.

Physica A 384 (2007) 507–515

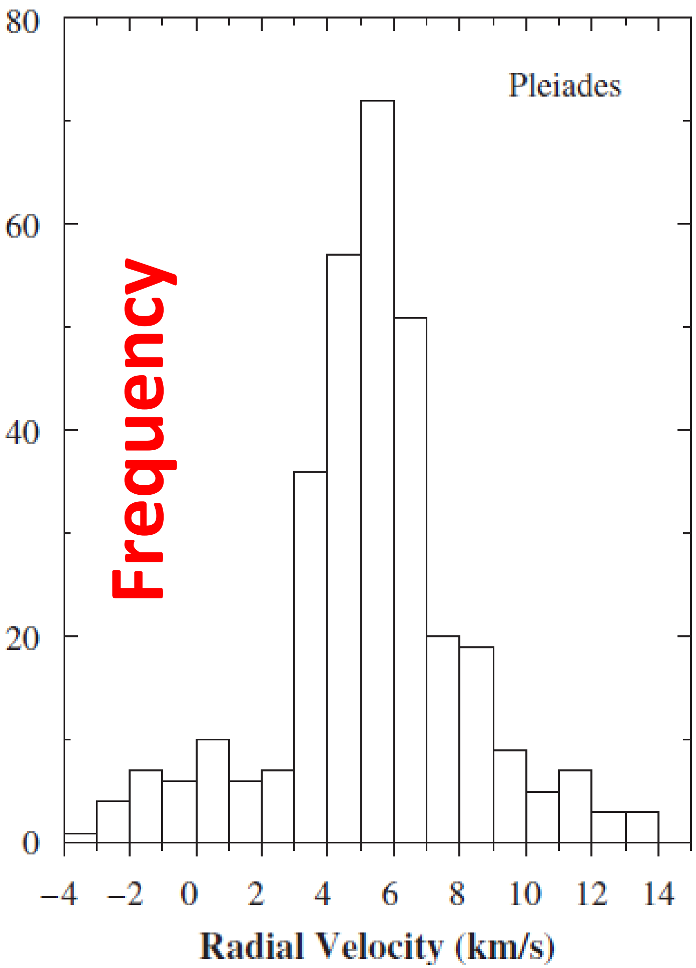
Radial velocities of open stellar clusters: A new solid constraint favouring Tsallis maximum entropy theory

J.C. Carvalho[a], B.B. Soares[a], B.L. Canto Martins[a], J.D. do Nascimento Jr.[a], A. Recio-Blanco[b], J.R. De Medeiros[a,*]

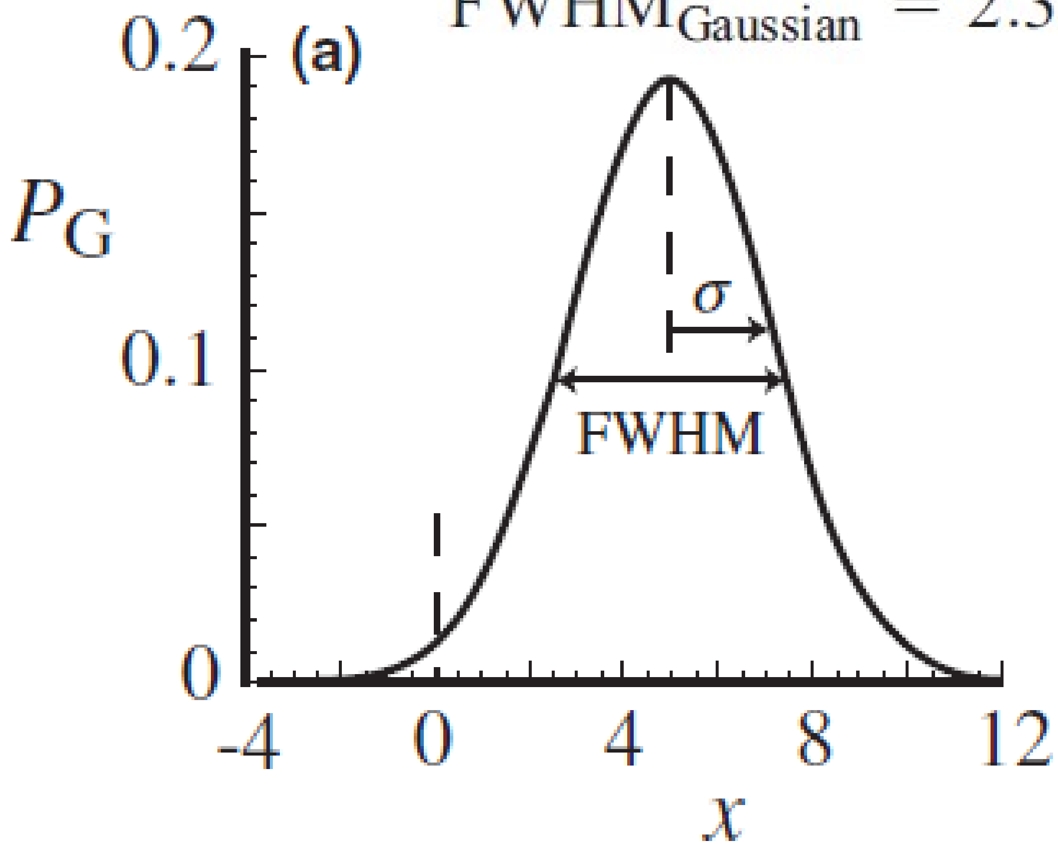
The Gaussian, or normal, distribution

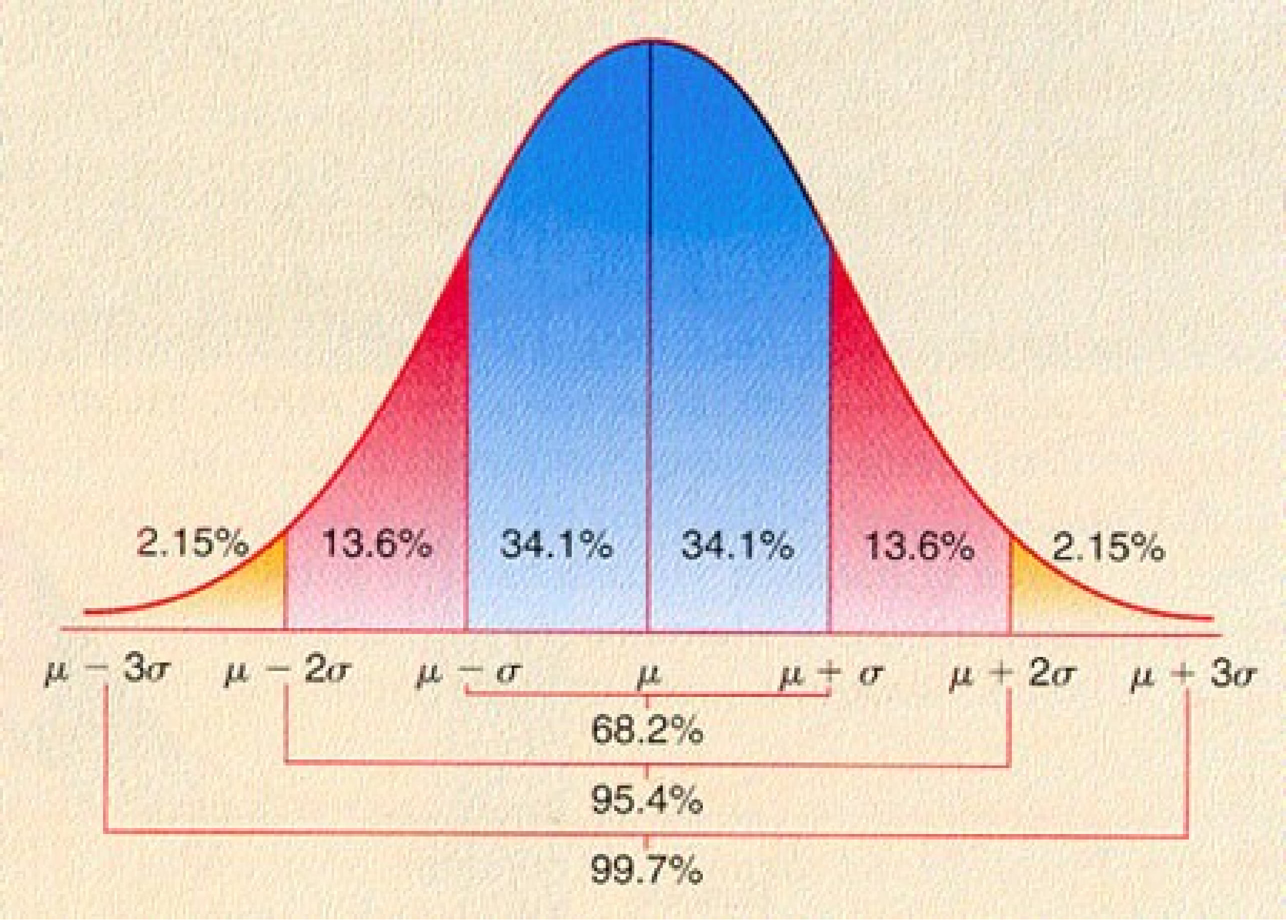
$$P_G(x,\mu,\sigma)\mathrm{d}x = \frac{\mathrm{d}x}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

When the distribution arises just due to random uncertainties in different measurements, it could be described by a Gaussian probability distribution



$$\mathrm{FWHM}_{\mathrm{Gaussian}} = 2.354\sigma$$




2.15%
13.6%
34.1%
34.1%
13.6%
2.15%
$\mu - 3\sigma$
$\mu - 2\sigma$
$\mu - \sigma$
μ
$\mu + \sigma$
$\mu + 2\sigma$
$\mu + 3\sigma$
68.2%
95.4%
99.7%

Sum of the variance

$$\sigma^2 = \sigma_1^2 + \sigma_2^2$$

- Example: σ_1 = 3 cm, σ_2 = 4 cm
- Total error? σ = 5cm

Robust statistics (ordem)

- Trimean = (Q1 + 2 Median + Q3)/4
- interquartile deviation: IQ = Q3 – Q1
- quartile deviation : QD = IQ/2
- MAD = median { $|x_i - \text{median}|$ }

pseudo-σ :

- σ_{MAD} = 1,4826 MAD
- σ_{QD} = 1,4826 QD
- σ_{QD} = IQ/1,349

Median = Q2

Example

- 2 5 **5** 6 6 **6** 9 9 **9** 9 150 (sorted)
- 11 elements
- <x> = 19,6 σ = 41,3
- Mode = 9
- Q2 = Median (50% of population) = 6
- Q1 (25% of population) = 5
- Q3 (75% of population) = 9
- Trimean = 6,5 IQ = 4 QD = 2 σ(QD) = 3,0
- NOTA: if we eliminate the last point (150) we obtain <x> = 6,6 e σ = 2,2

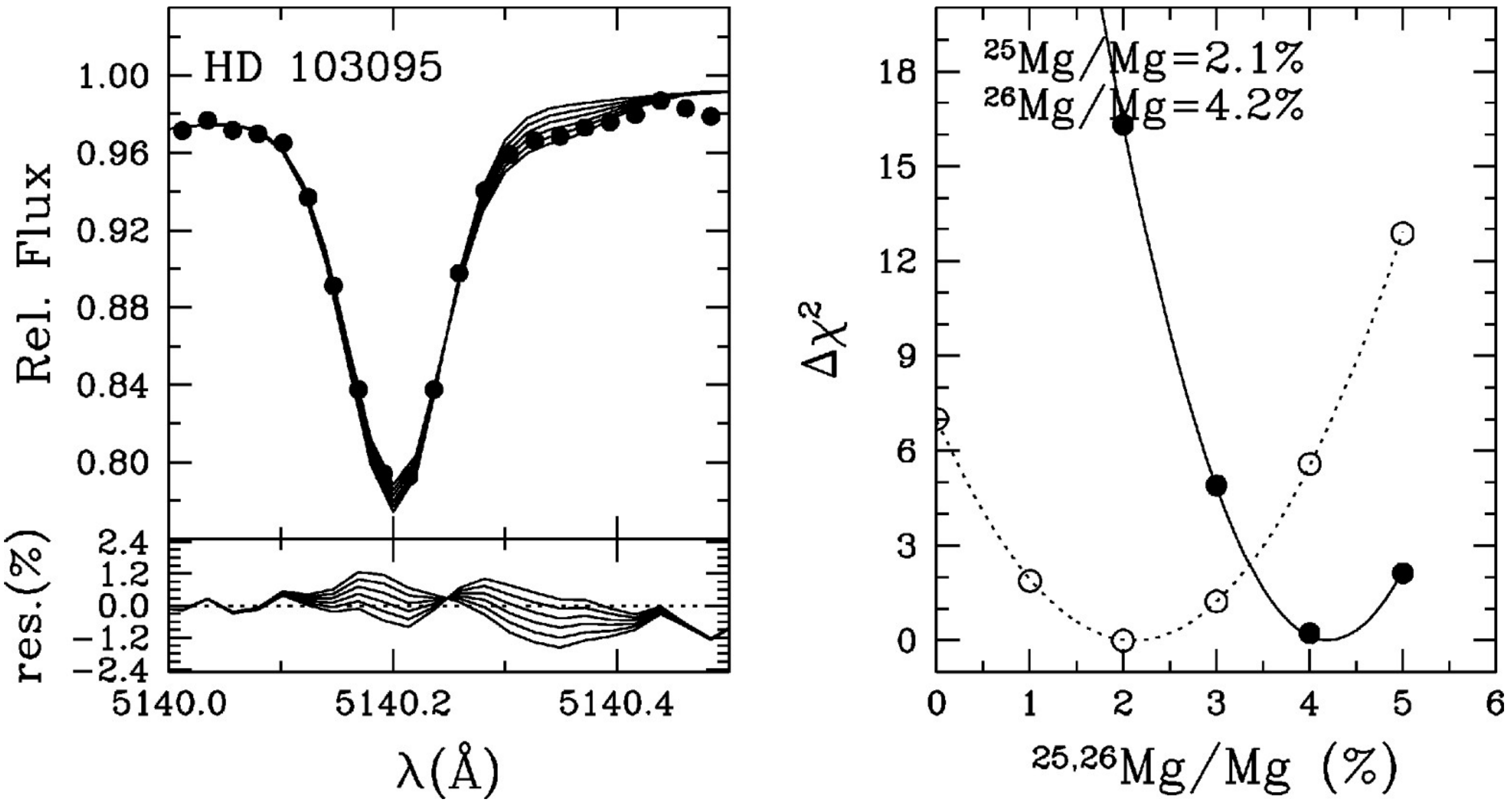
chi squared, χ^2

$$\chi^2 \equiv \sum_i \left[\frac{y_{\mathrm{ob},i} - y_{\mathrm{th},i}}{\sigma_i} \right]^2$$

THE ASTROPHYSICAL JOURNAL, 659:L25–L28, 2007 April 10

MAGNESIUM ISOTOPES IN METAL-POOR DWARFS: THE RISE OF AGB STARS AND THE FORMATION TIMESCALE OF THE GALACTIC HALO[1]

JORGE MELÉNDEZ AND JUDITH G. COHEN



How to prepare observing proposals

Tip #13: Justify your sample size

- Important to justify any sample size (1, 10, 1000)

- Is half the sample enough for your aims? Or actually you need twice as many objects?

In some cases assume binomial distribution

The Astrophysical Journal, 757:164 (13pp), 2012 October 1 doi:10.1088/0004-637X/757/2/164

OXYGEN ABUNDANCES IN LOW- AND HIGH-α FIELD HALO STARS AND THE DISCOVERY OF TWO FIELD STARS BORN IN GLOBULAR CLUSTERS

I. Ramírez[1], J. Meléndez[2], and J. Chanamé[3]
[1] McDonald Observatory and Department of Astronomy, University of Texas a
[2] Departamento de Astronomia do IAG/USP, Universidade de São Pa
[3] Departamento de Astronomía y Astrofísica, Pontificia Universidad Católica de
Received 2012 May 23; accepted 2012 Augus

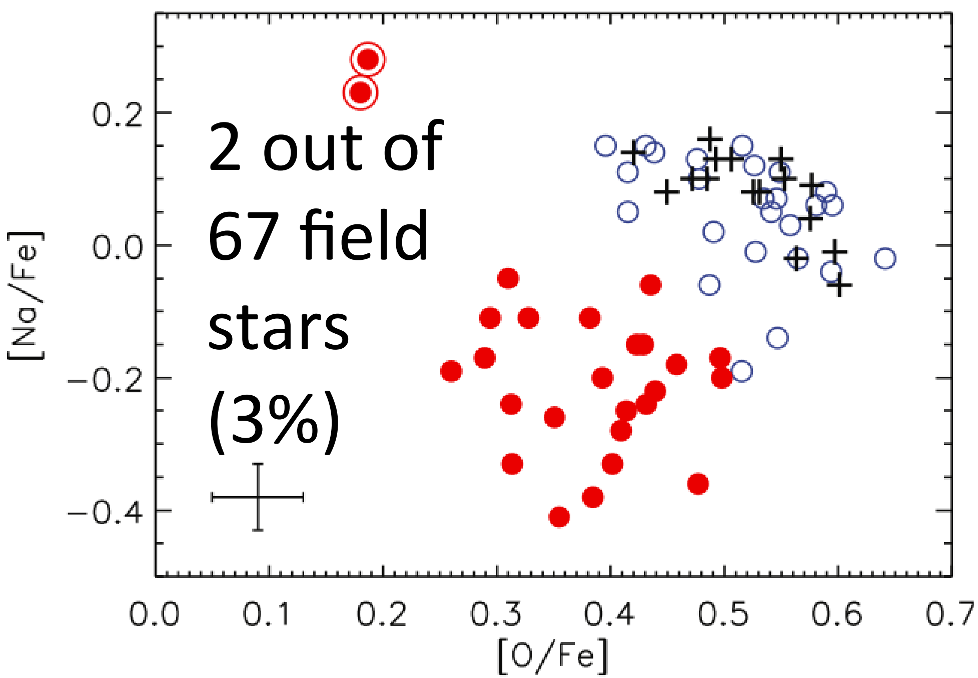


Figure 7. [Na/Fe] vs. [O/Fe] relation for the stars in Figure 1. Sodiun abundances are from Nissen & Schuster (2010). Typical error bars are shown a the bottom left corner.

Since we have analyzed 67 stars, the fraction of metal-poor field stars originating from second-generation globular cluster (GC) stars is ~ 3% (2/67). Adopting a binomial distribution (i.e., field and GC), an error bar can be estimated from the variance of the probability distribution (e.g., Bevington 1969, Chapter 3):

$\sigma^2 = np(1 - p)$, where $n = 67$ is the number of stars and p is the probability of "success" ($p = 2/67 = 0.03$). We find $\sigma = 1.4$, which implies a probability error of $1.4/67 = 2\%$.

Example adopting a binomial distribution

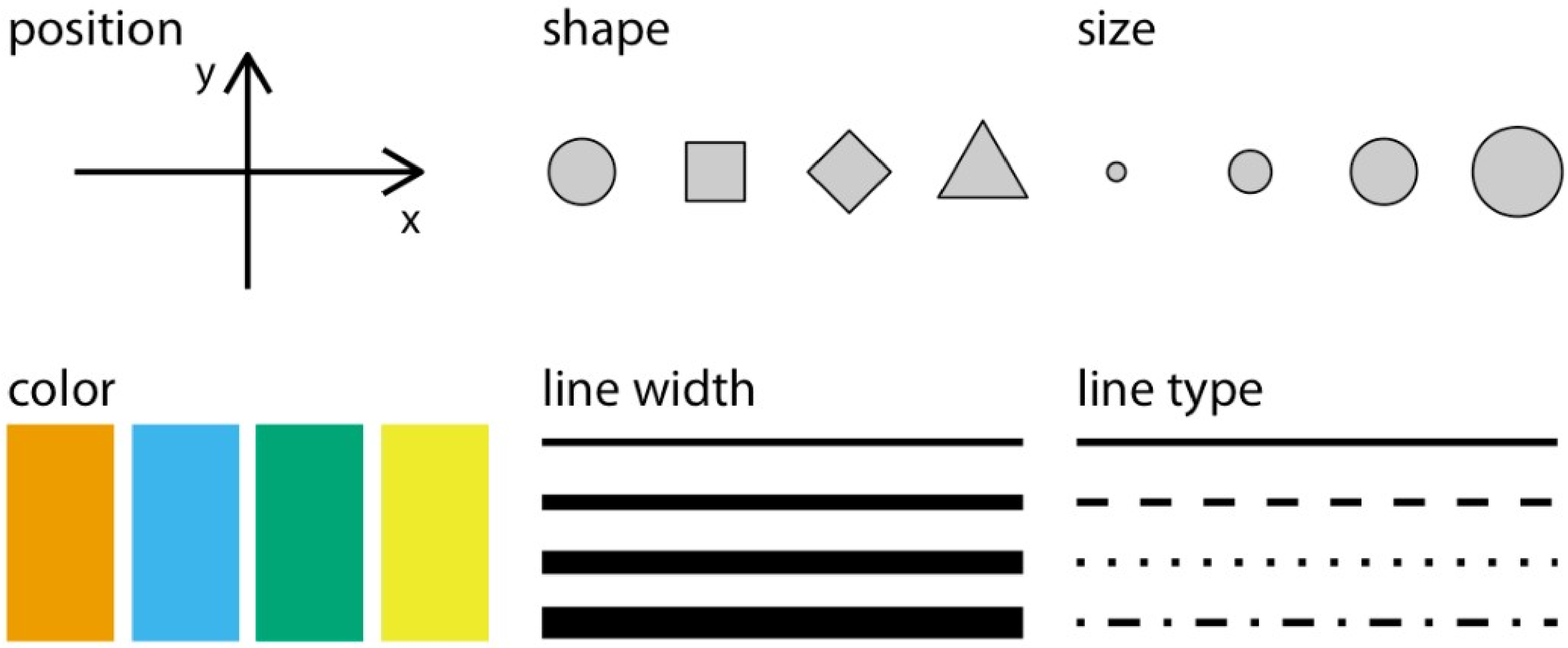
- You know that roughly 2% of objects are of a given class in a random sample of stars
- If you want to discover 1 such object, you will need to observe at least 50 stars. What is the error?
- $\sigma^2 = np(1 - p) = 50 \times 0.02\ (1 - 0.02) = 0.98$

→ $\sigma = 0.99$ star; *in percent: 100% x (0.99/50) = 2%*

- What about observing 200 stars?
- $\sigma^2 = 200 \times 0.02\ (1 - 0.02) = 3.92 \rightarrow \sigma = 1.98$ *stars*

→Fraction 2% ± 100% x (1.98/200) = 2.0 ± 1.0 %

Visualization



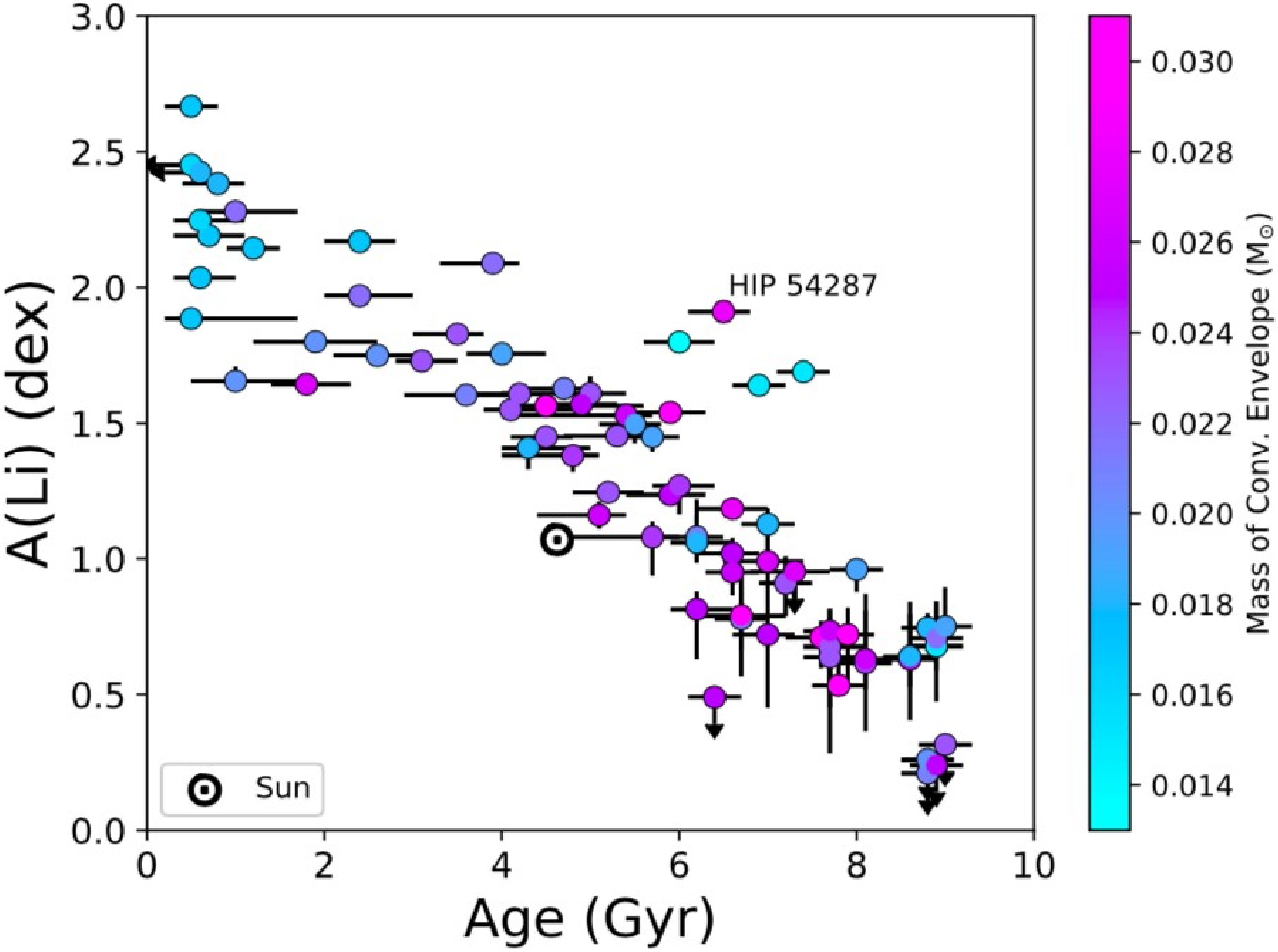


Figure 4. Lithium abundances versus stellar age colour coded by [Fe/H] (top panel), mass (middle panel), and the mass of the convective envelope (bottom panel). HIP 54287 is labelled in the lower panel because, as discussed in the text, it could have engulfed a planet.

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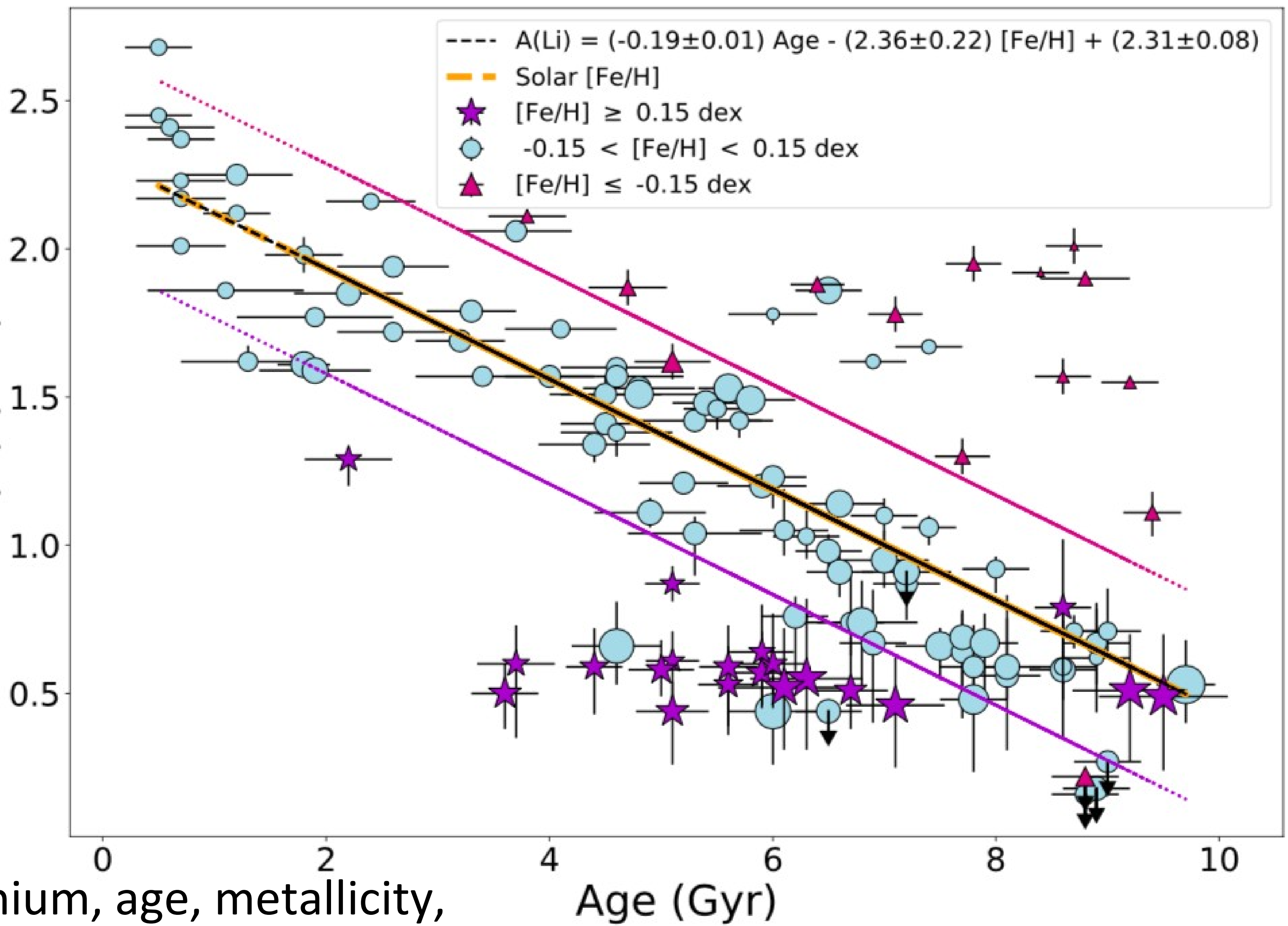
The Li–age correlation: the Sun is unusually Li deficient for its age

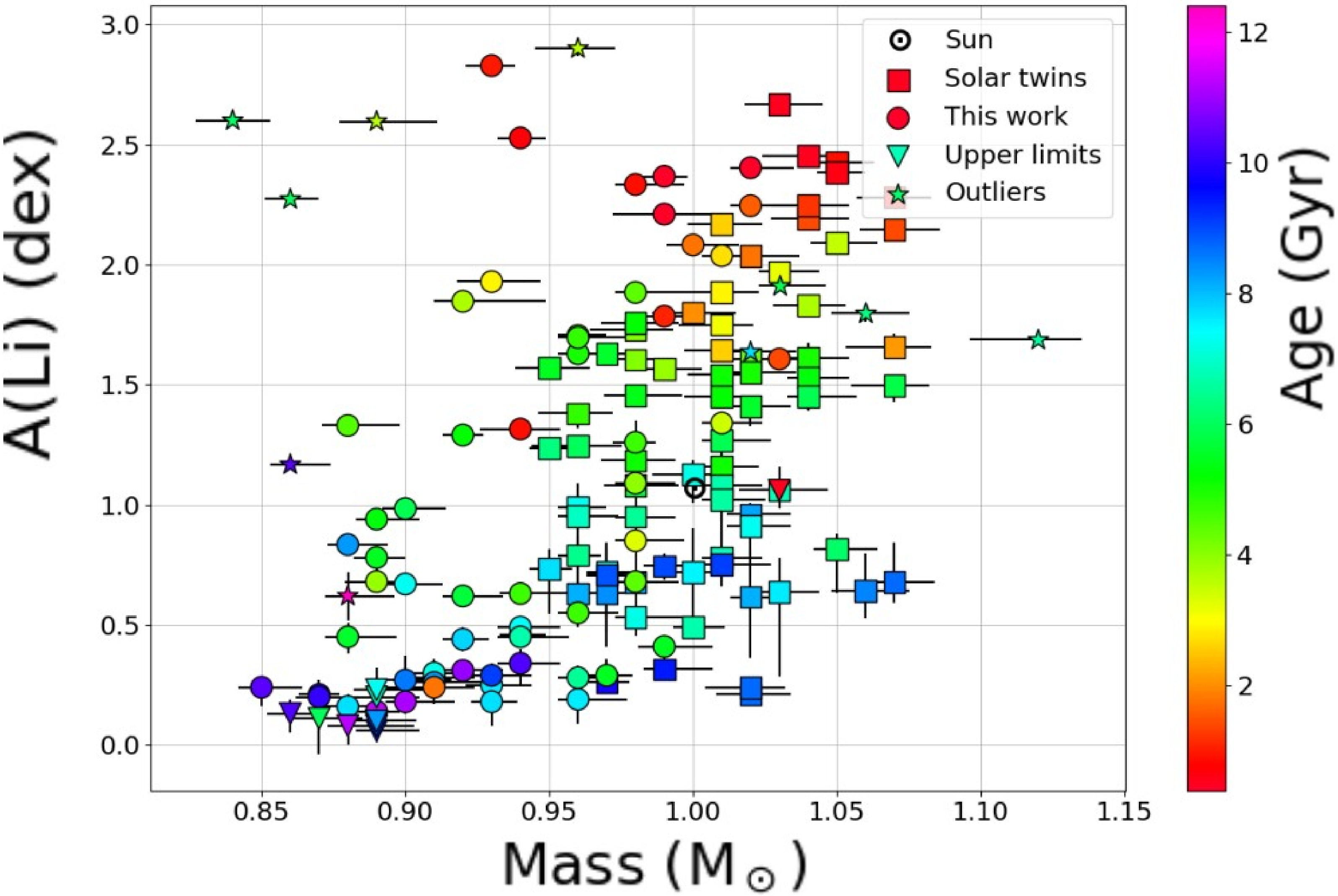
M. Carlos,[1★] J. Meléndez,[1] L. Spina,[2] L. A. dos Santos,[3] M. Bedell,[4] I. Ramirez,[5] M. Asplund,[6] J. L. Bean,[7] D. Yong,[6] J. Yana Galarza[1] and A. Alves-Brito[8]



Lithium, age, metallicity, convection zone

Giulia Martos, TG, 2022

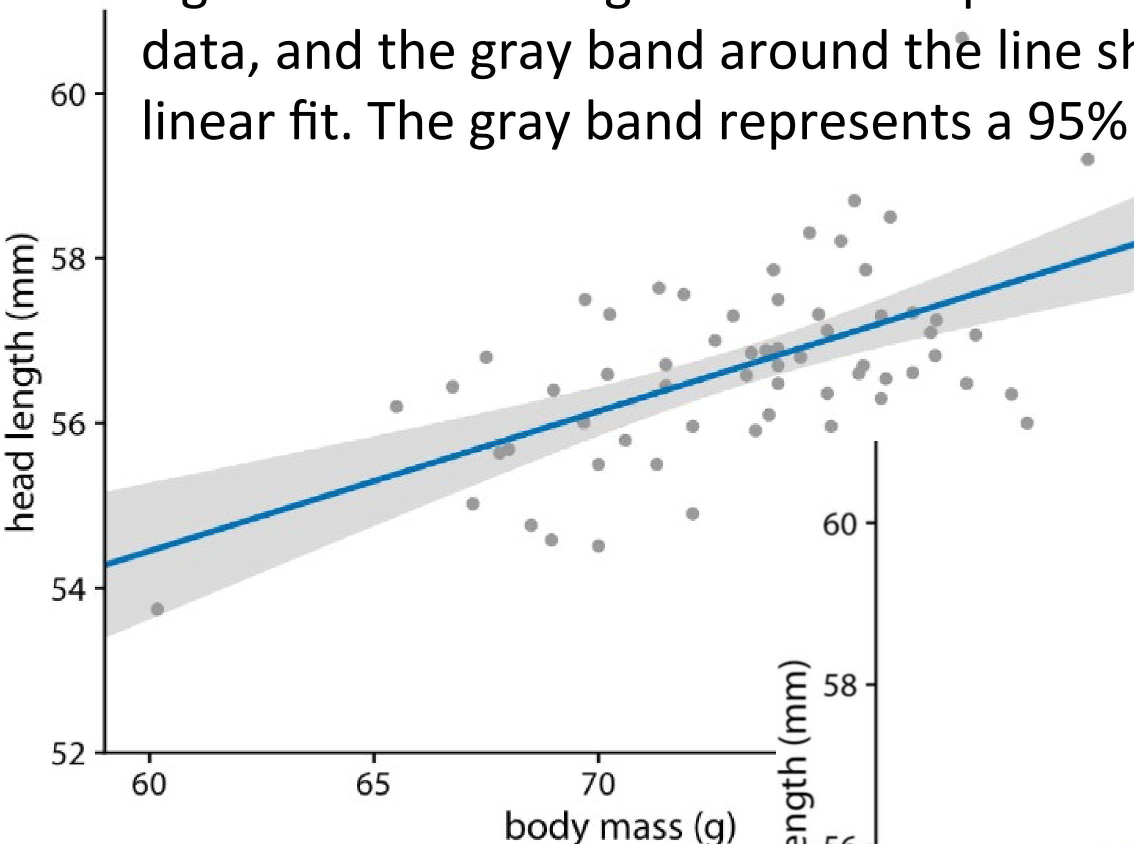
Size of the symbol proportional to mass of the convection zone



Lithium, mass and age

(Anne Rathsam et al. submitted to MNRAS, 2023)

Fig. 16.15: The straight blue line represents the best linear fit to the data, and the gray band around the line shows the uncertainty in the linear fit. The gray band represents a 95% confidence level.



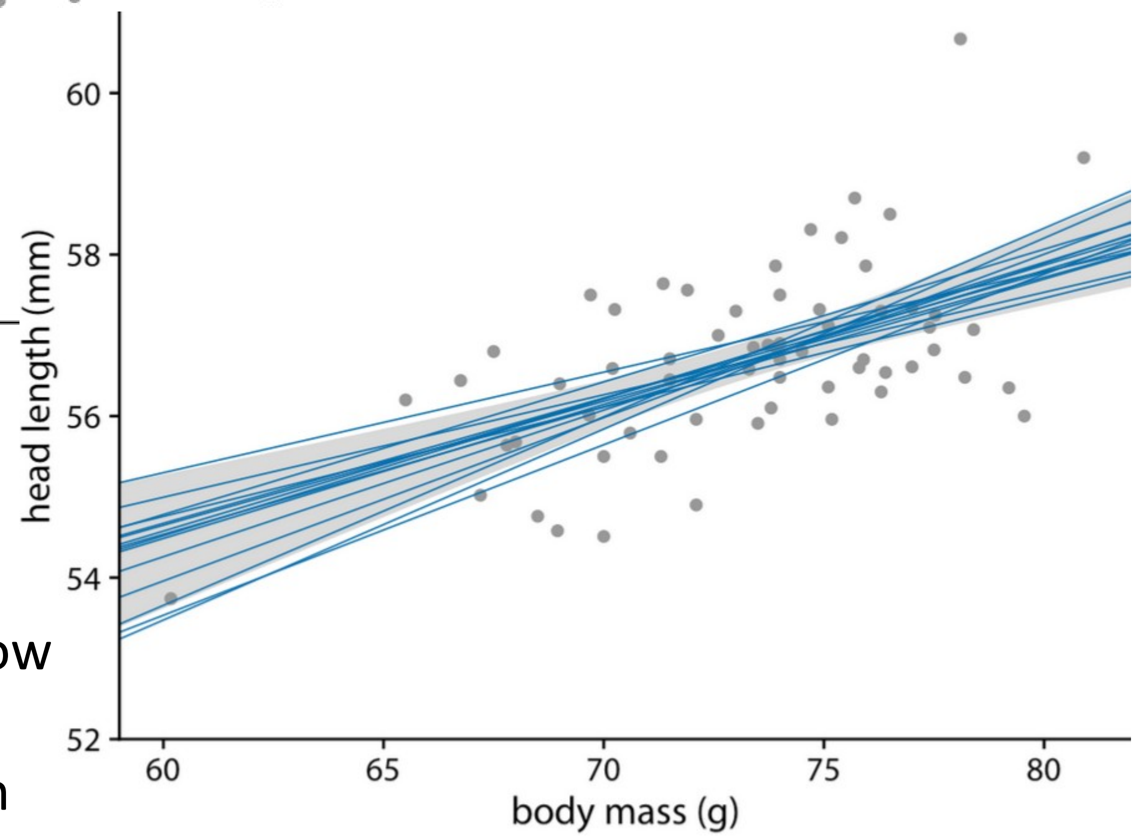


Fig. 16.16: In contrast to Fig. 16.15, the straight blue lines now represent equally likely alternative fits randomly drawn from the posterior distribution.

https://clauswilke.com/dataviz/visualizing-uncertainty.html