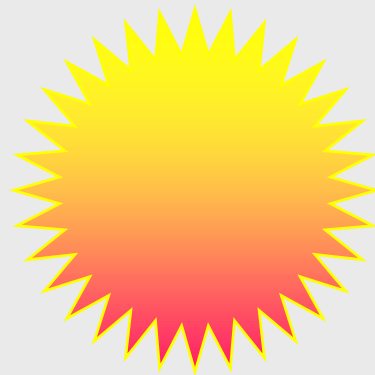


Insolação em placas planas



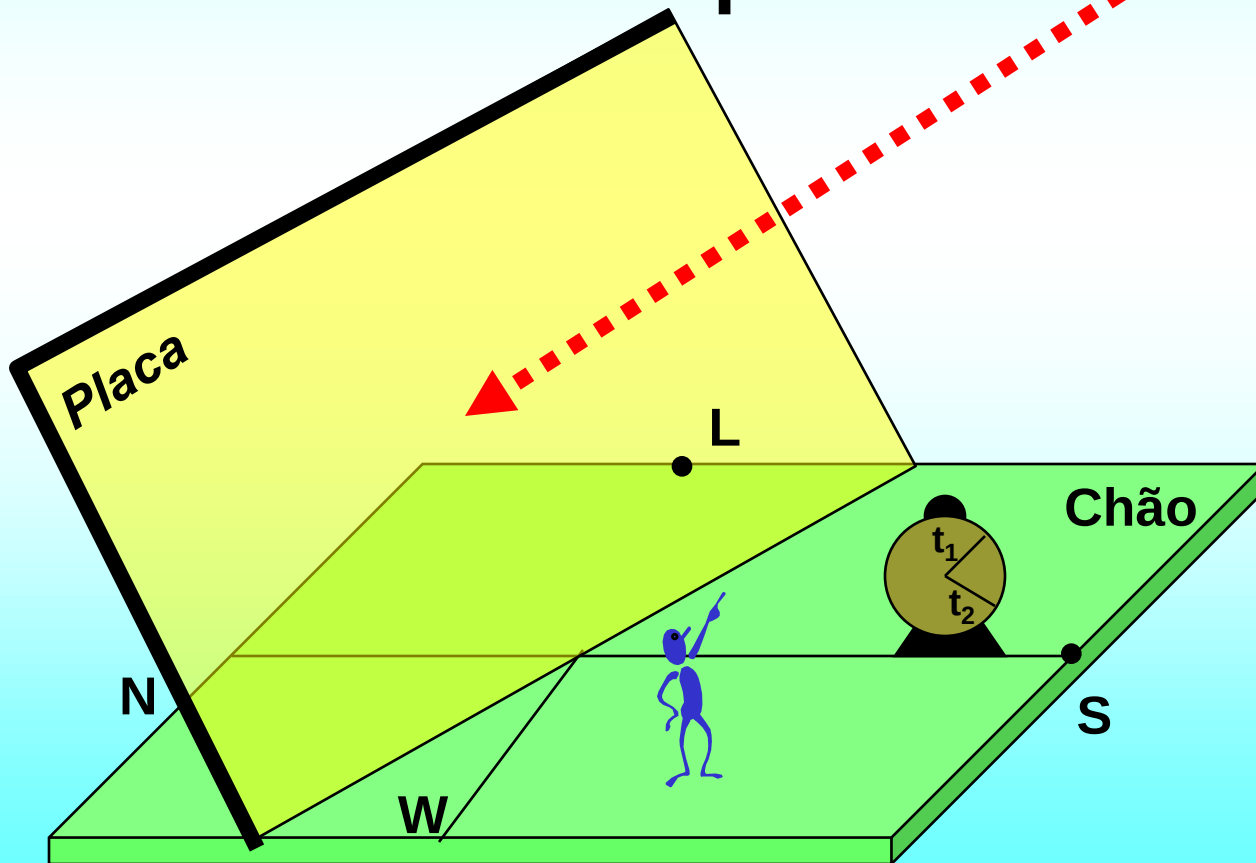
**Jorge Meléndez,
baseado na
aula do**

Prof. R. Boczko

IAG-USP

Quanta insolação (energia solar)

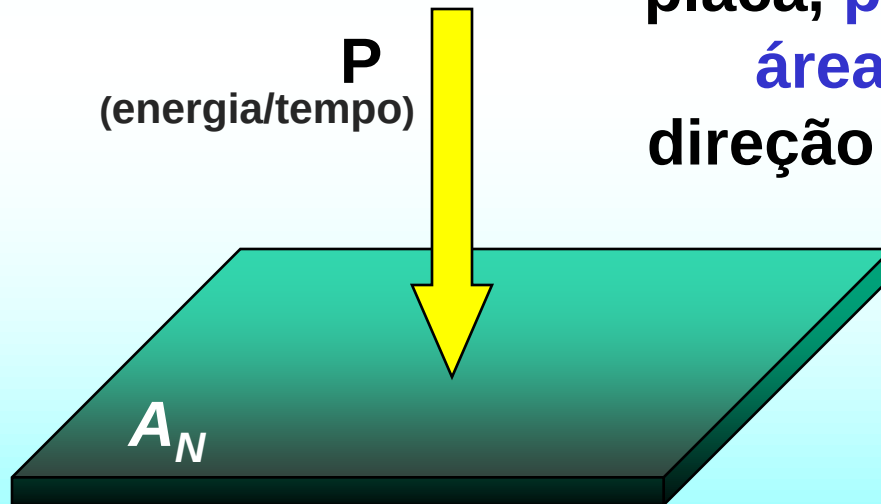
incide numa Placa Plana
Qualquer num certo intervalo
de tempo?



Irradiância

Irradiância Normal I_N

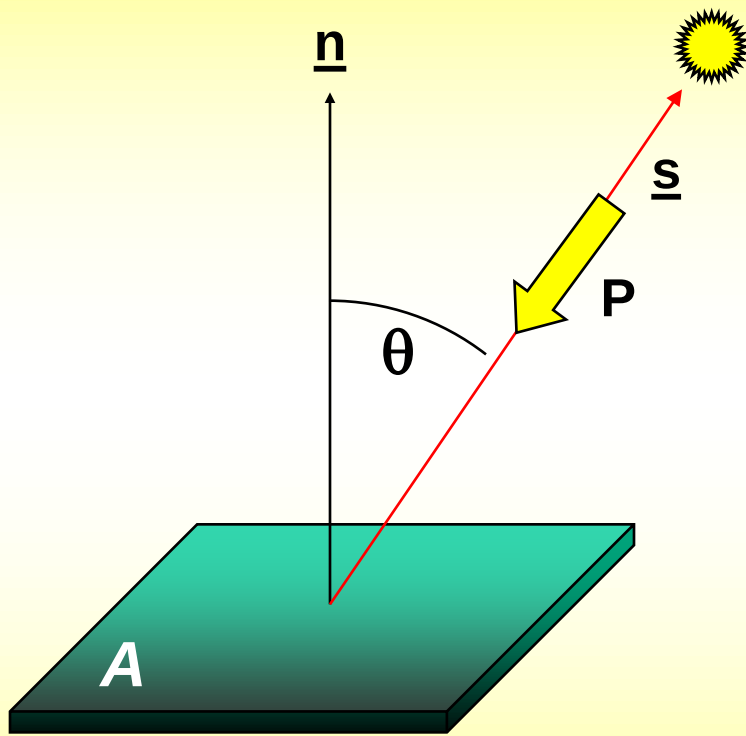
Irradiância normal I_N
é a potência P
incidente numa
placa, **por unidade de
área**, normal à
direção de incidência



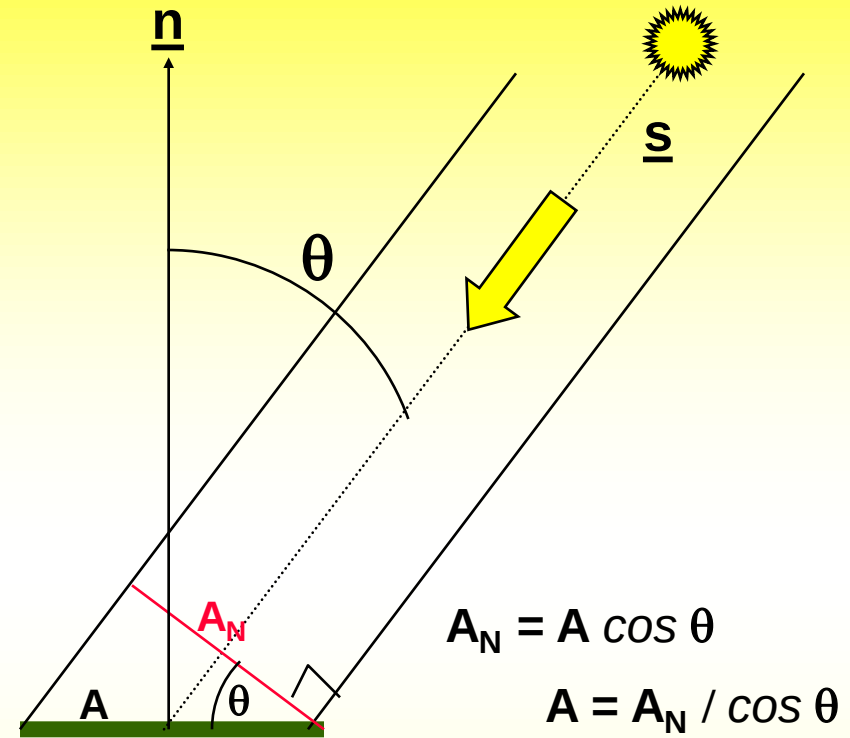
$$I_N \equiv P / A_N$$

$$I' = [W/m^2]$$

Irradiância $I \equiv P / A$



$$I \equiv P / A$$



$$I = P / (A_N / \cos \theta)$$

$$I = (P / A_N) \cos \theta$$

Como: $I_N \equiv P / A_N$

$$I = I_N \cos \theta$$

**Energia Incidente
numa placa**

Energia Incidente **E**

Lembrar que: $I \equiv P / A$
E que $P = E/t$

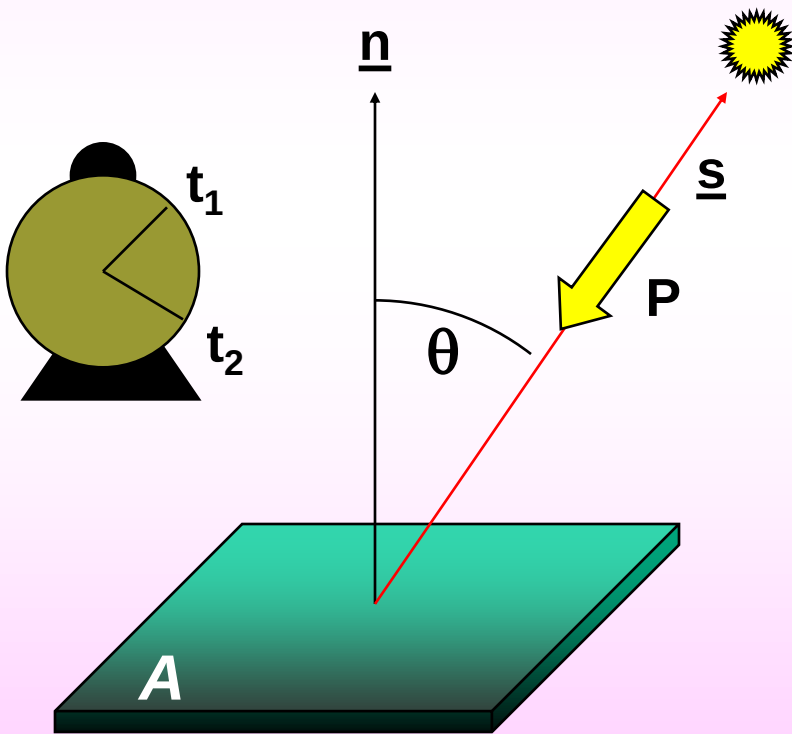
A energia **E** incidente na placa de área A no intervalo de tempo entre t_1 e t_2 pode ser dada por:

$$E = \int (I A) dt \quad [J, cal]$$

Como $I = I_N \cos \theta$

$$E = \int (I_N \cos \theta) A dt$$

$$E = \int I_N A \cos \theta dt$$

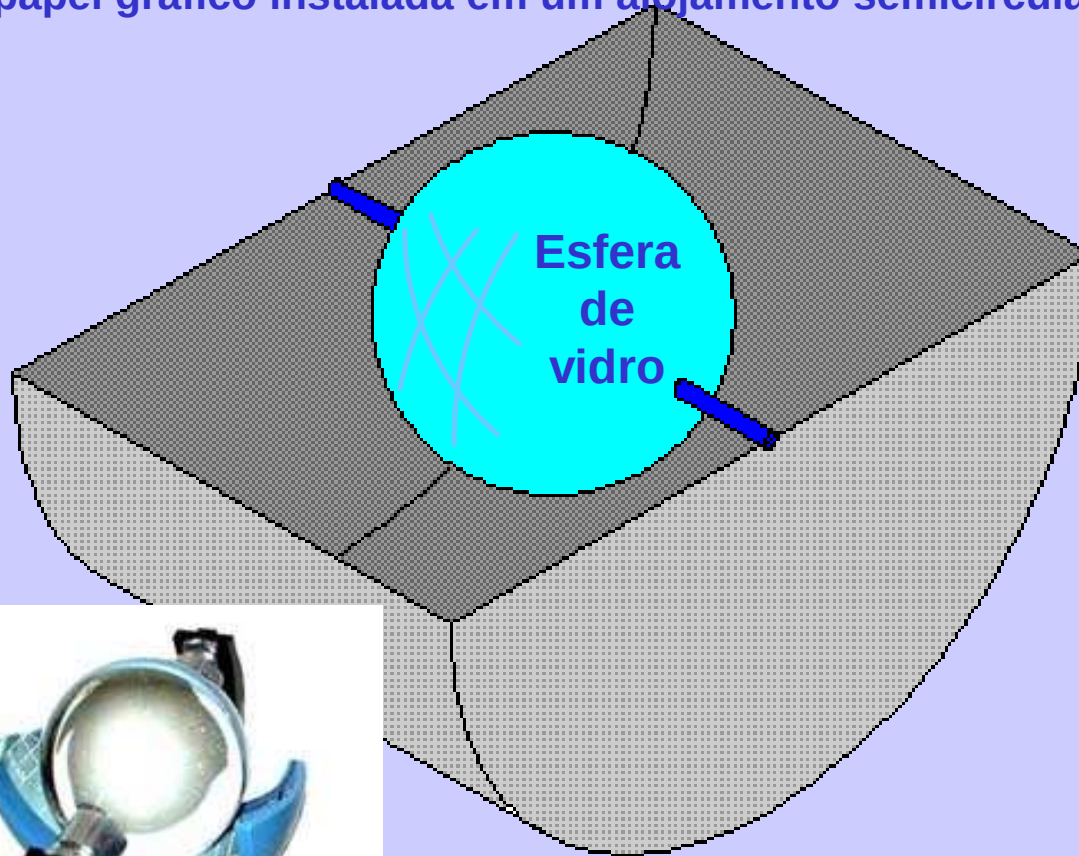


**Medindo a
insolação :
numero de horas de
brilho solar com um
heliógrafo**

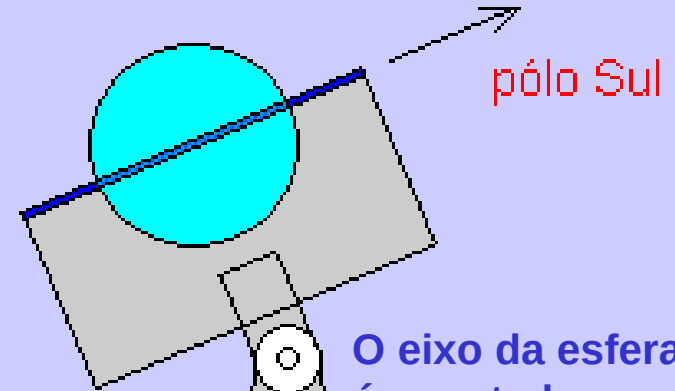
Esquema de um heliógrafo

Heliógrafo

esfera de vidro concentra os raios solares sobre uma fita de papel gráfico instalada em um alojamento semicircular



Esfera
de
vidro



pólo Sul

O eixo da esfera é apontado para o pólo, permitindo que o foco formado pelo Sol, no seu percurso anual atinja sempre toda a extensão da fita.



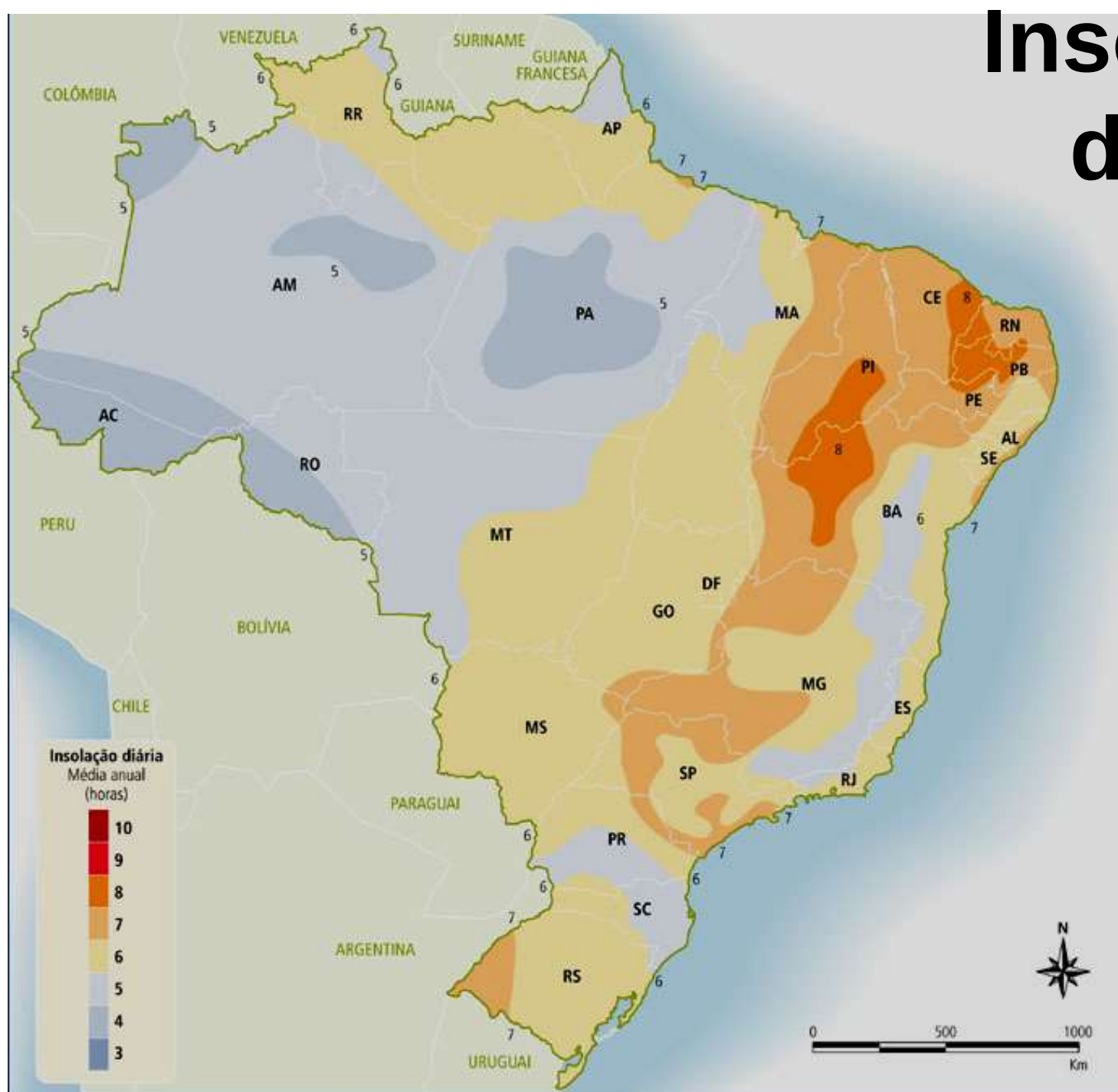


Heliógrafo



**Qual a região do
Brasil com o
maior número de
horas de Sol?**

Insoleção diária



Sol

Luminosidade L é a potência
(energia / unid. tempo)
emitida pelo Sol

Luminosidade

$3,845 \times 10^{26} \text{ W}$

~4 septilhões de lâmpadas de 100 W

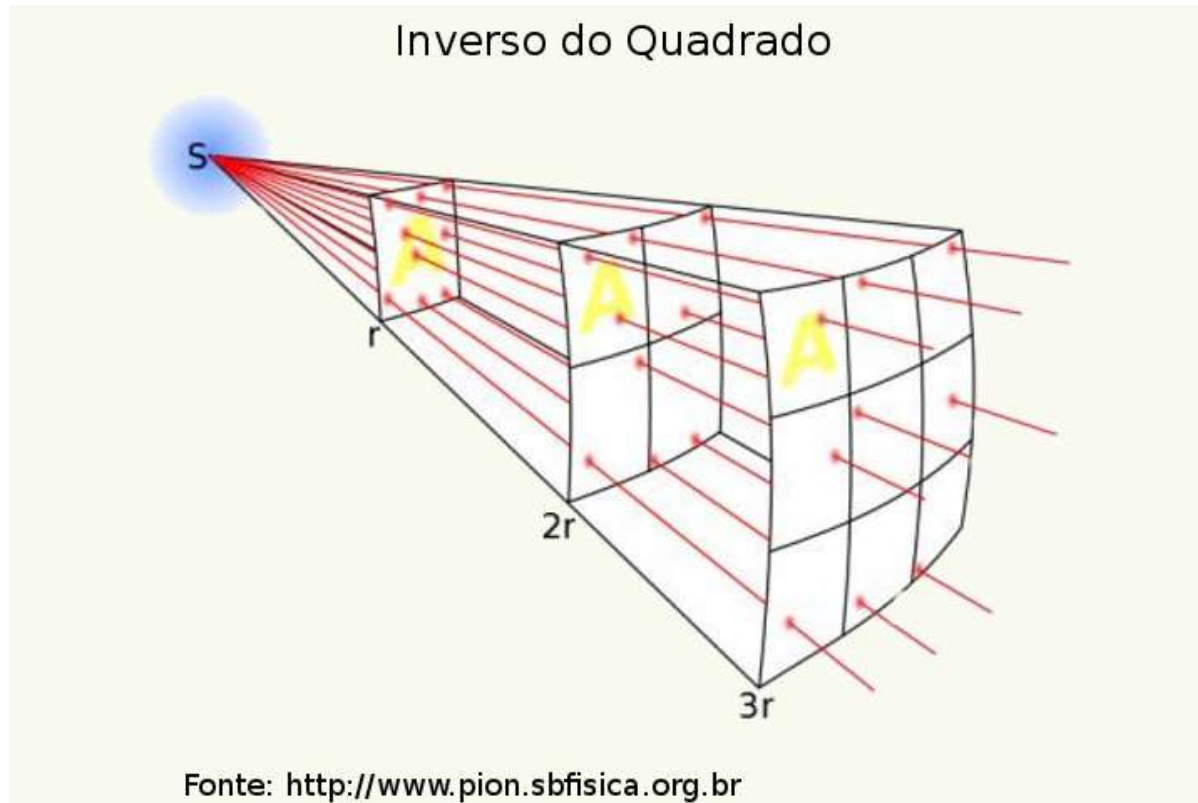


Raio
696 mil km

Luminosidade é constante
(em curtas escalas de tempo)

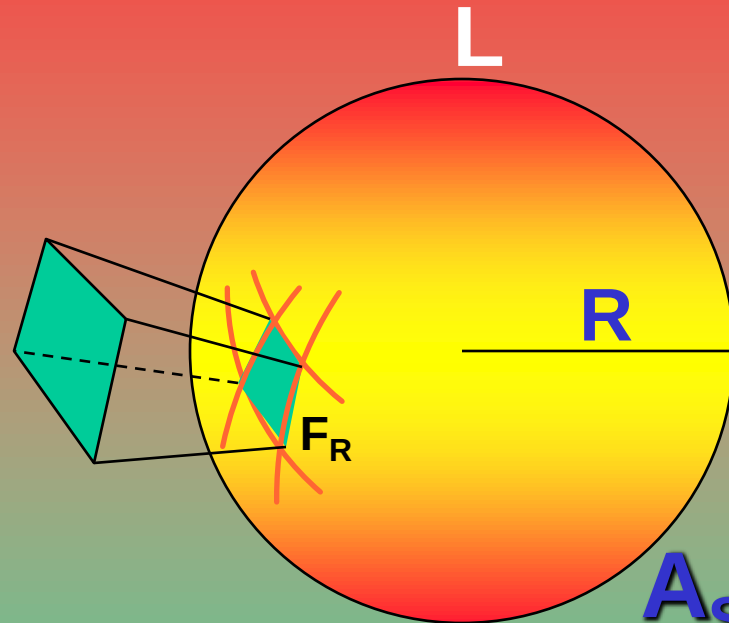
**O Fluxo ((energia/tempo)/area) não é
constante:**

**inversamente proporcional
ao quadrado da distância**



Fluxo Solar Superficial

É a potência emitida
por unidade de área do Sol.

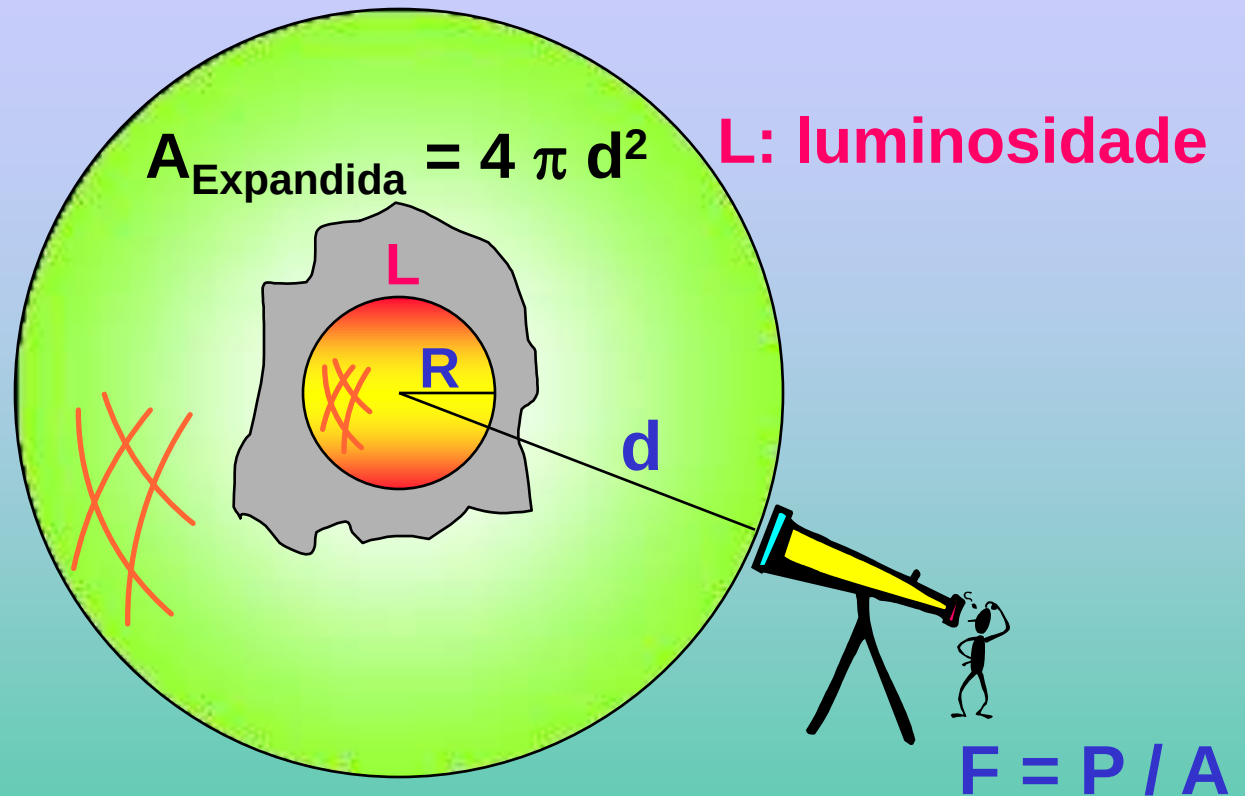


$$A_{\text{Superficial}} = 4 \pi R^2$$

$$F_R = L / (4 \pi R^2)$$

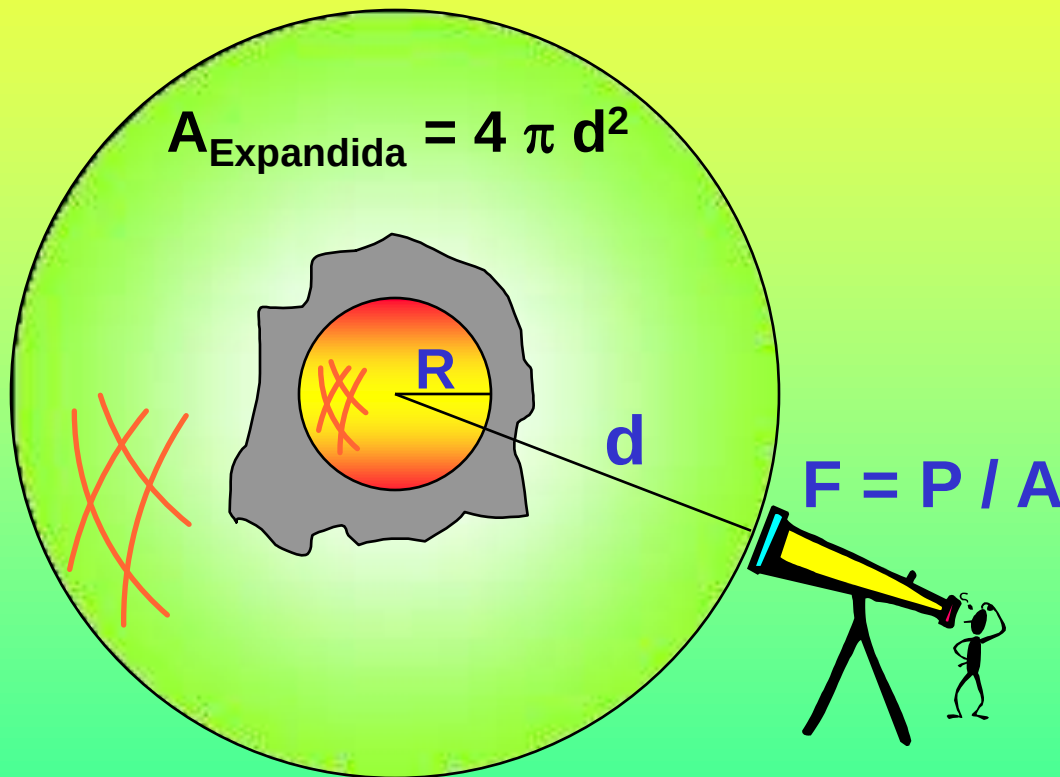
Fluxo Solar à distância d

É a potência medida por unidade de área a uma distância d do centro do Sol.



$$F = F_d = L / (4\pi d^2)$$

Variação do Fluxo em função da distância d



$$F_d = L / (4\pi d^2)$$

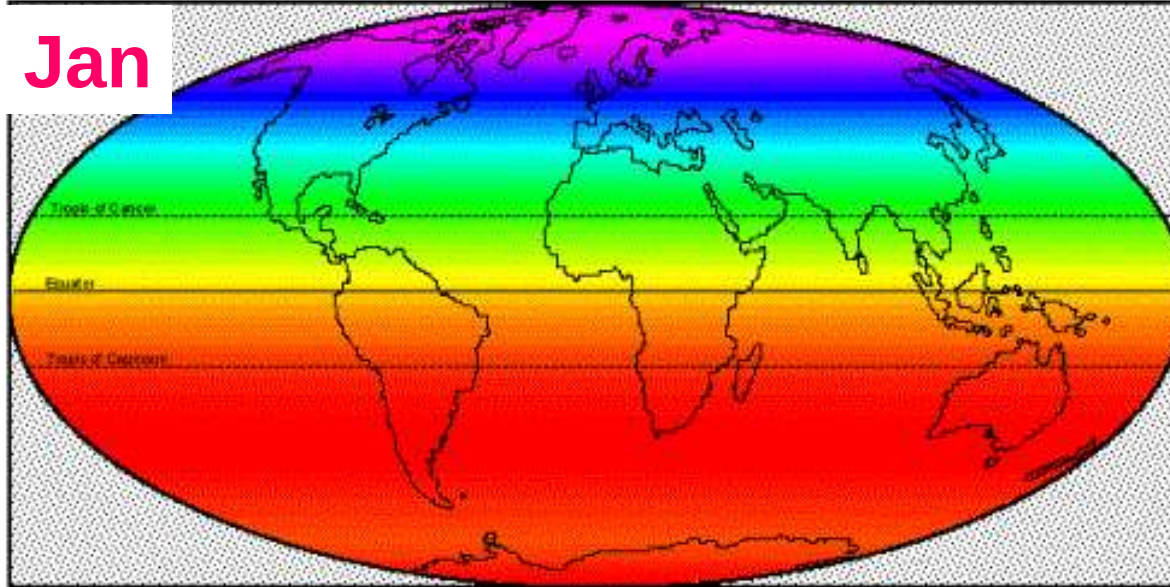
$$F_R = L / (4\pi R^2)$$

$$F_d / F_R = R^2 / d^2$$

$$F_d \cdot d^2 = F_R \cdot R^2$$

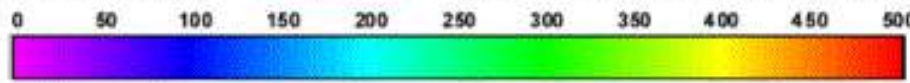
Insolação

Jan

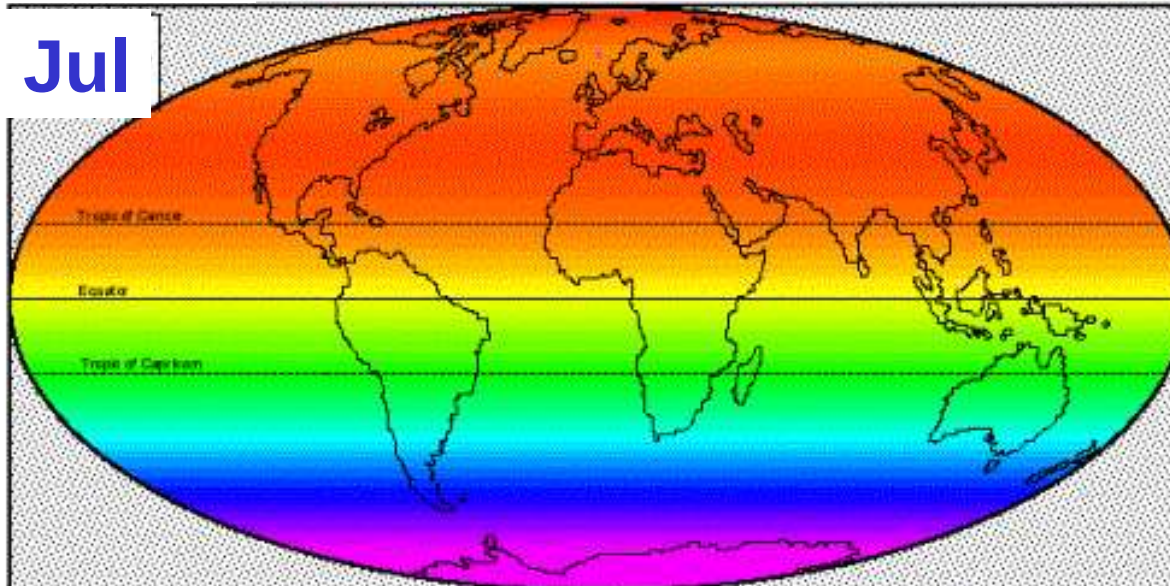


Insolação
W / m²

0 50 100 150 200 250 300 350 400 450 500



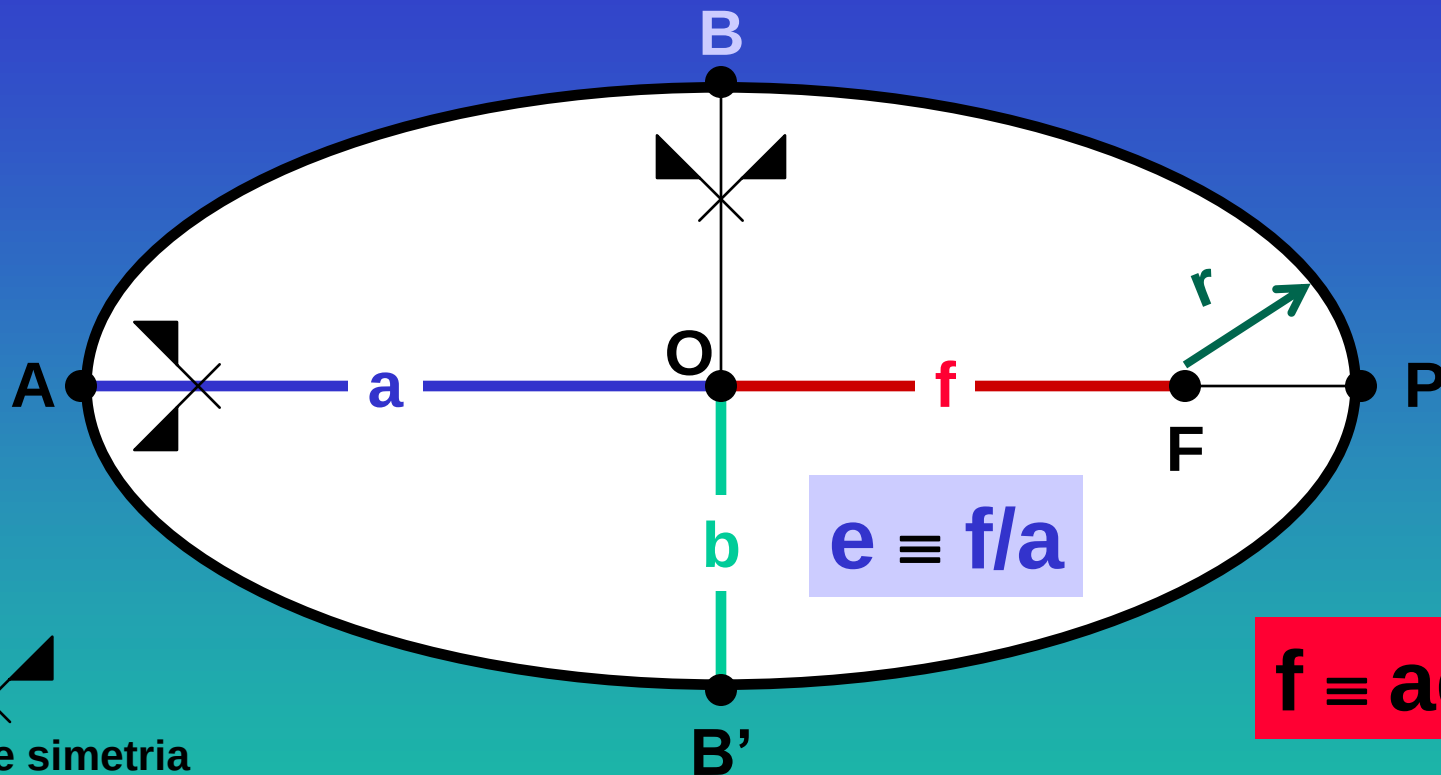
Jul



Radiação
solar no
verão do
H.Sul é
maior que
no verão
do H.Norte.

Why ?

Elementos de uma elipse



$$e \equiv f/a$$

$$f \equiv ae$$

a = semieixo maior

b = semieixo menor

f = distância focal

e = excentricidade

$$r_p = a(1 - e)$$

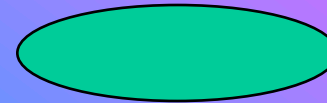
$$r_a = a(1 + e)$$

Forma da elipse em função da excentricidade

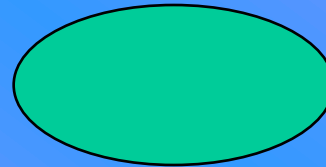
$e = 1$
Parábola



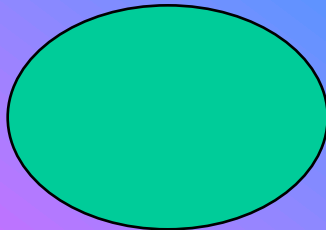
$e = 0,9$



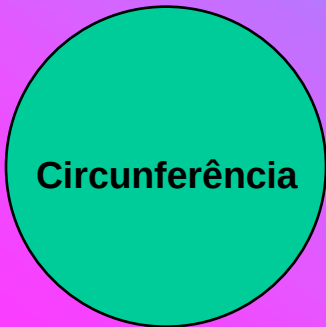
$e = 0,7$



$e = 0,5$



$e = 0,2$



Circunferência

$e = 0$

Variação da insolação devido à variação da distância Terra-Sol

Terra: $e = 0,0167$

$$F_{\text{afélio}} = L / (4\pi r_a^2)$$

$$F_{\text{periélio}} = L / (4\pi r_p^2)$$

$$F_{\text{afélio}} / F_{\text{periélio}} = r_p^2 / r_a^2$$

$$r_a = a (1 + e)$$

$$r_p = a (1 - e)$$

$$F_a / F_p = (1 - e)^2 / (1 + e)^2$$

Constante Solar

Sol: nossa estrela emissora de energia



Constante Solar S_o

1 UA

Seja uma placa plana de área unitária, fora da atmosfera terrestre, a 1 unidade astronômica do Sol, posicionada perpendicularmente à direção de incidência dos raios solares. Define-se **Constante Solar S_o** como a irradiância solar normal I_N nessa placa

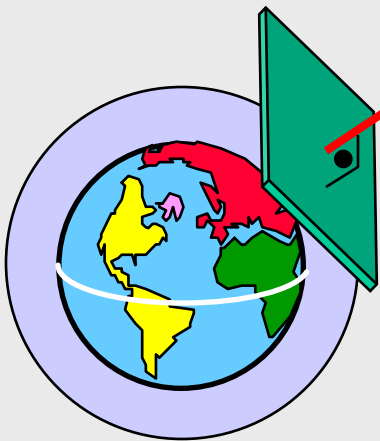
$$S_o = I_N \{\text{Solar}\}$$

$$S_o = 1367 \text{ W/m}^2$$

$$= 0,1367 \text{ W/cm}^2$$

$$= 1,367 \times 10^6 \text{ erg/s.cm}^2$$

$$= 1,95 \text{ cal/min.cm}^2$$



Insolação

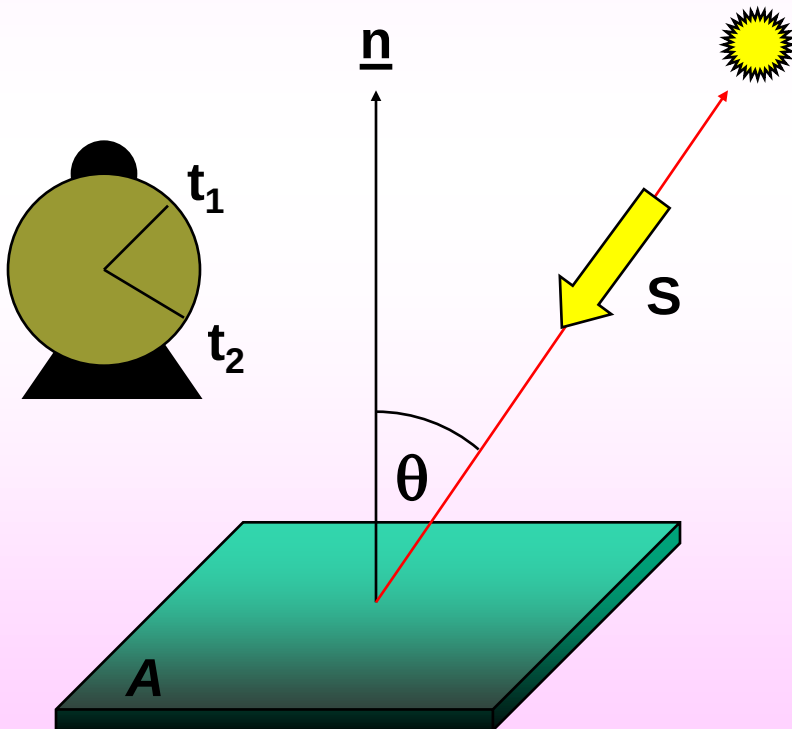
Insolação ou Energia Incidente **E**

A energia **E** incidente na placa de área A no intervalo de tempo entre t_1 e t_2 pode ser dada por:

Nomenclatura no caso solar:
 $S = I_N$

$$E = \int S A \cos \theta dt$$

cal cm² min
cal/(cm².min)



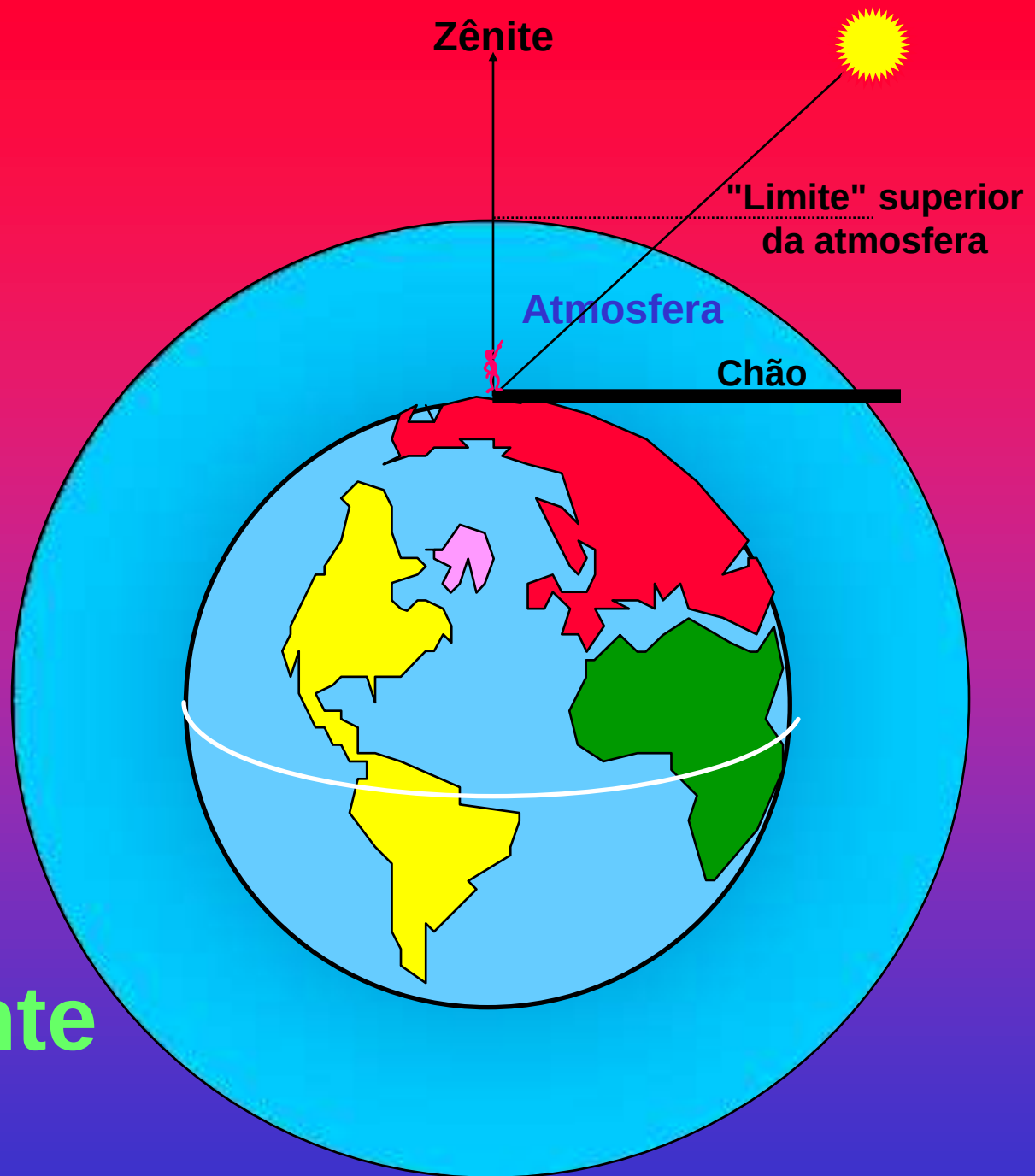
Problema!

$$E = \int S A \cos \theta dt$$

S
Irradiância
solar no solo

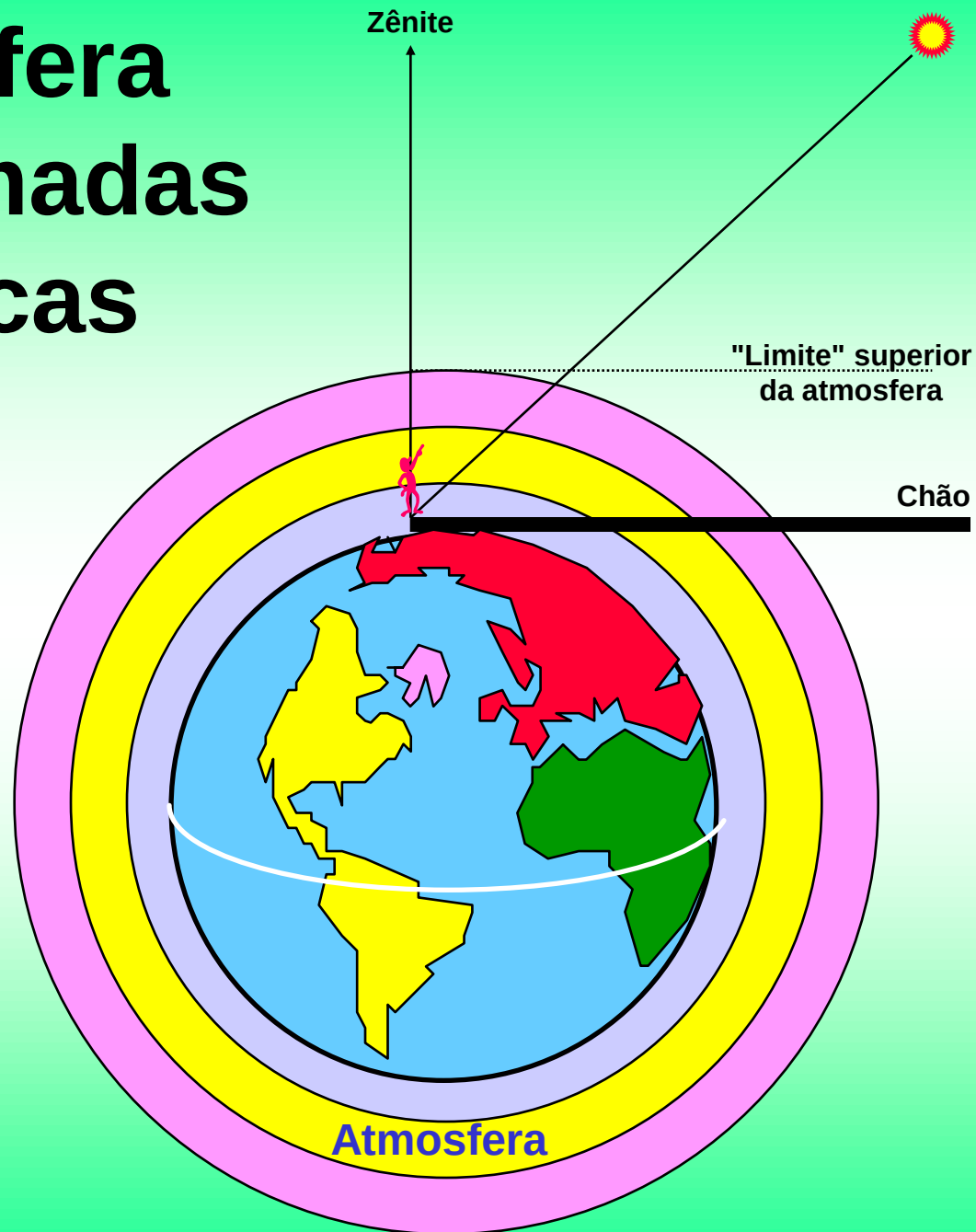
A atmosfera "rouba"
parte do calor solar
que deveria chegar
ao solo!

$S \neq \text{constante}$

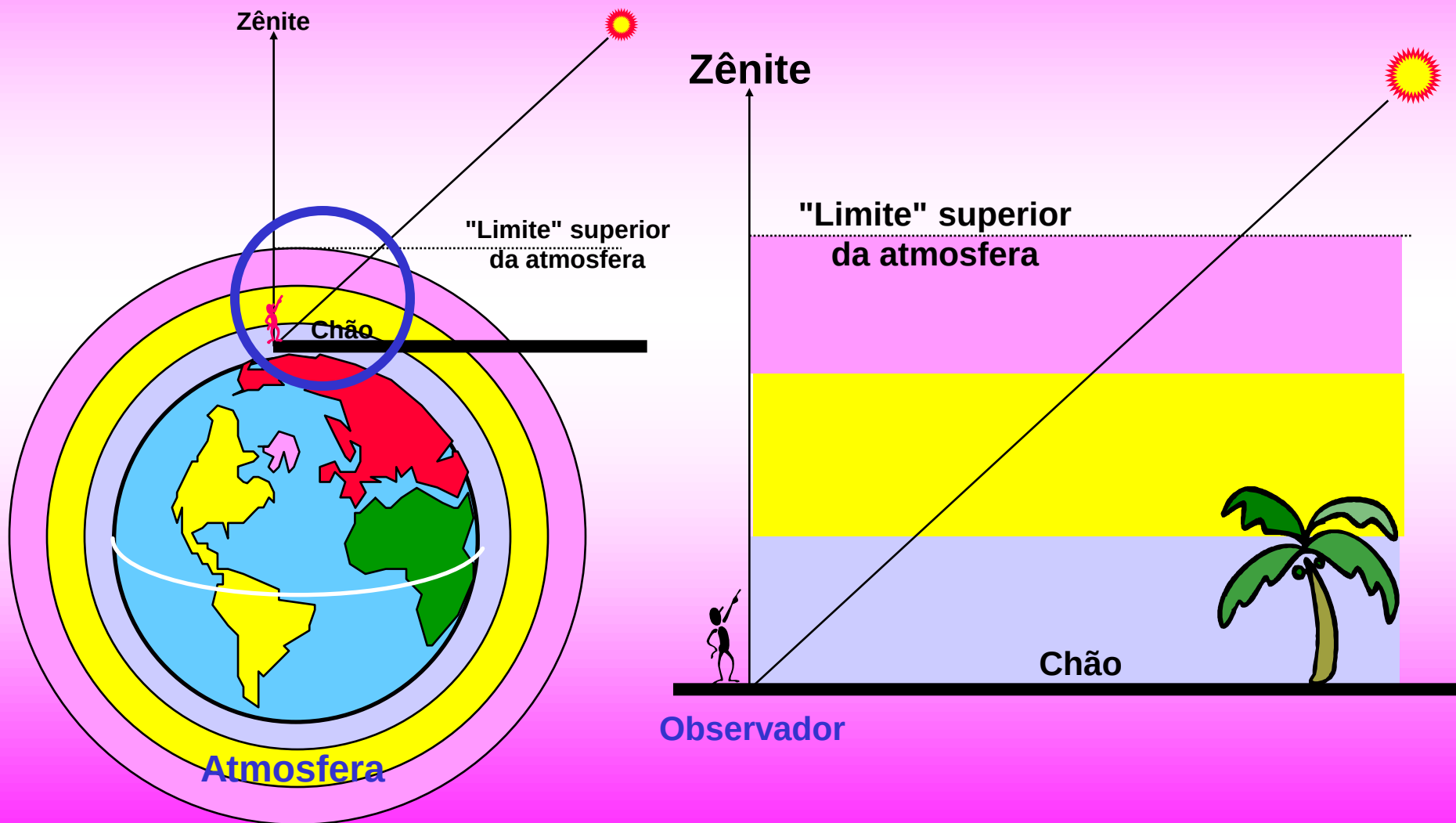


Geometria da atmosfera

Atmosfera com camadas esféricas

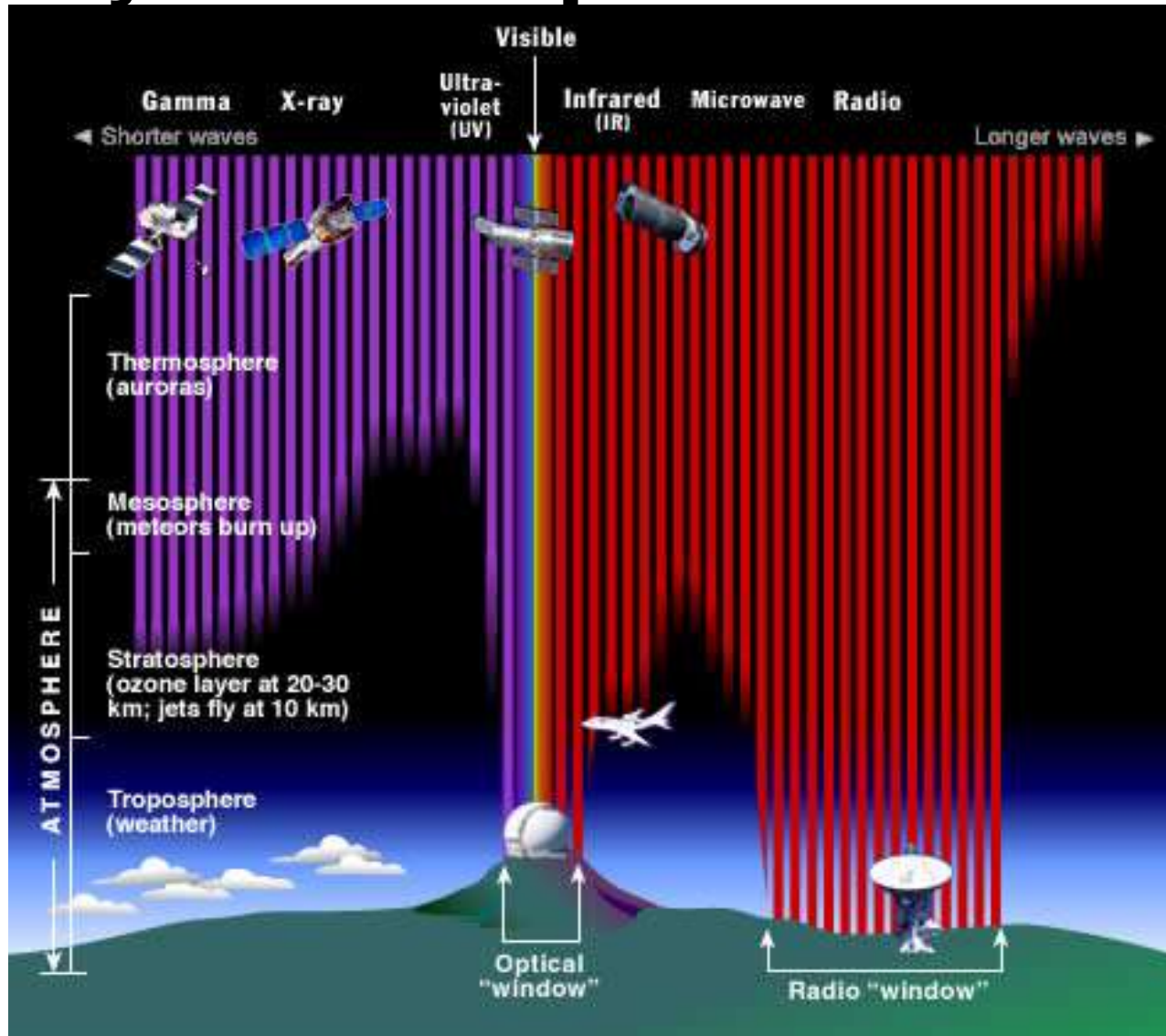


Aproximação da Atmosfera a camadas planas



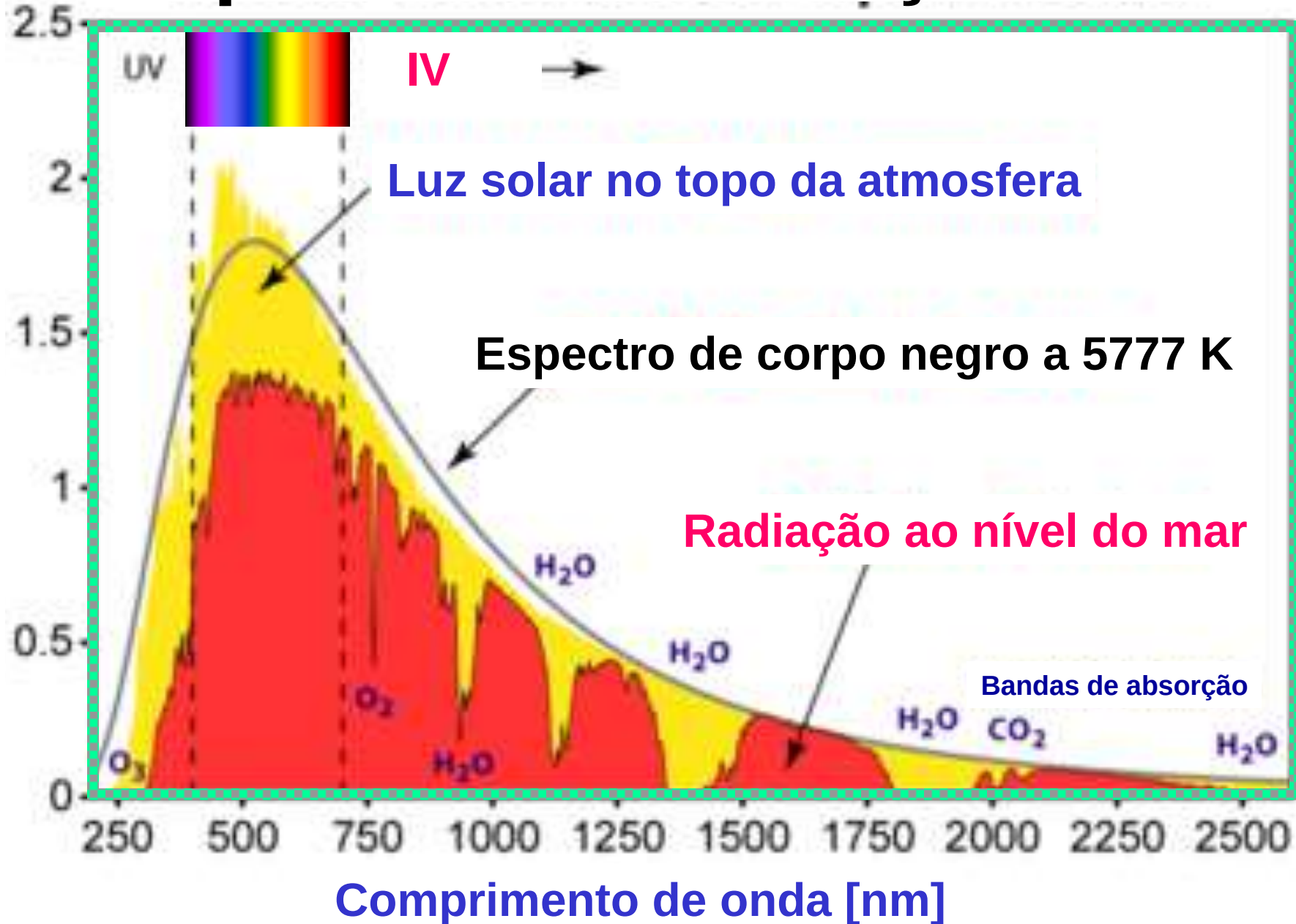
Extinção atmosférica

Atmosfera so deixa passar radiação UV-óptico-IR e rádio

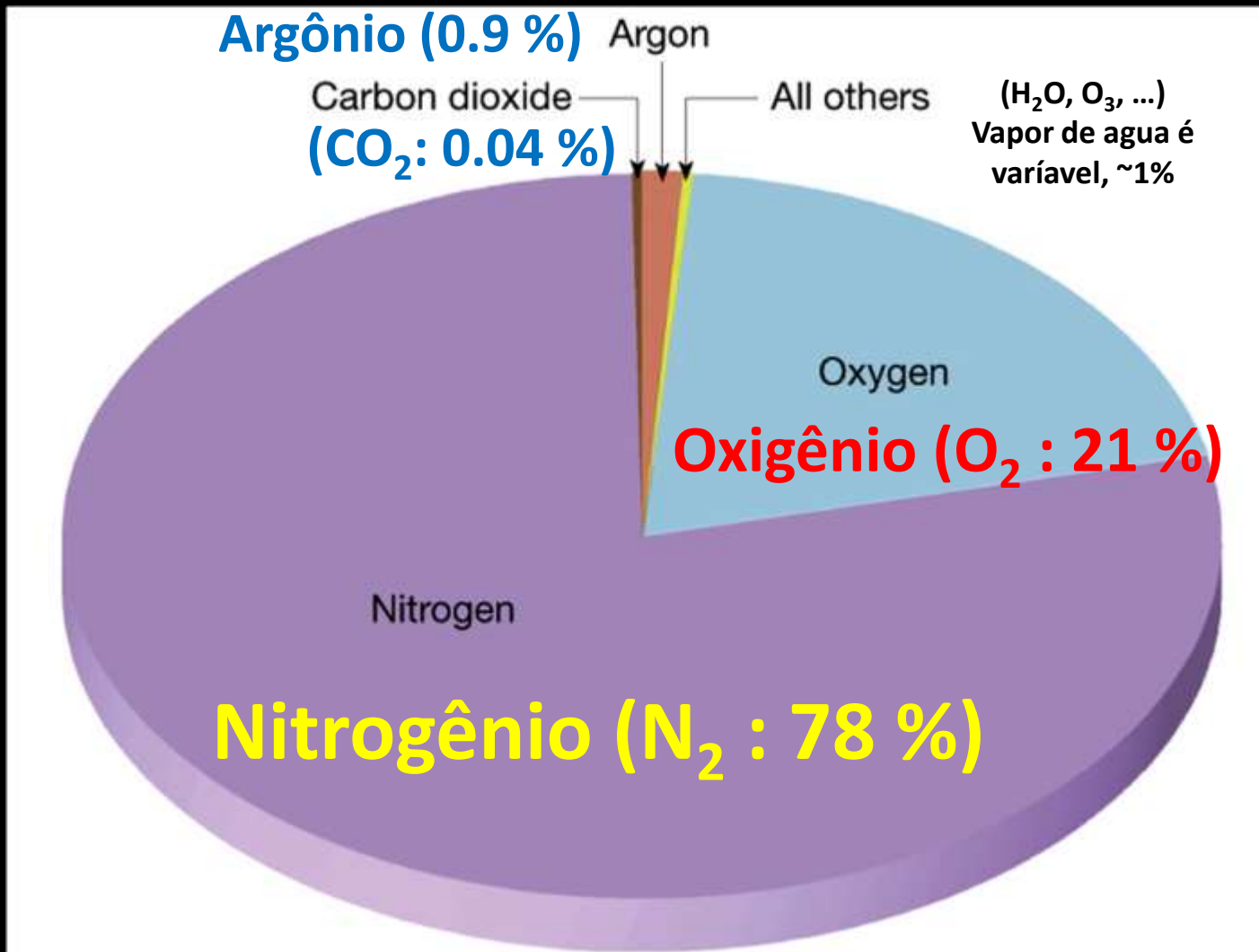


Espectro da irradiação solar

Irradiância espectral [W/m²/nm]



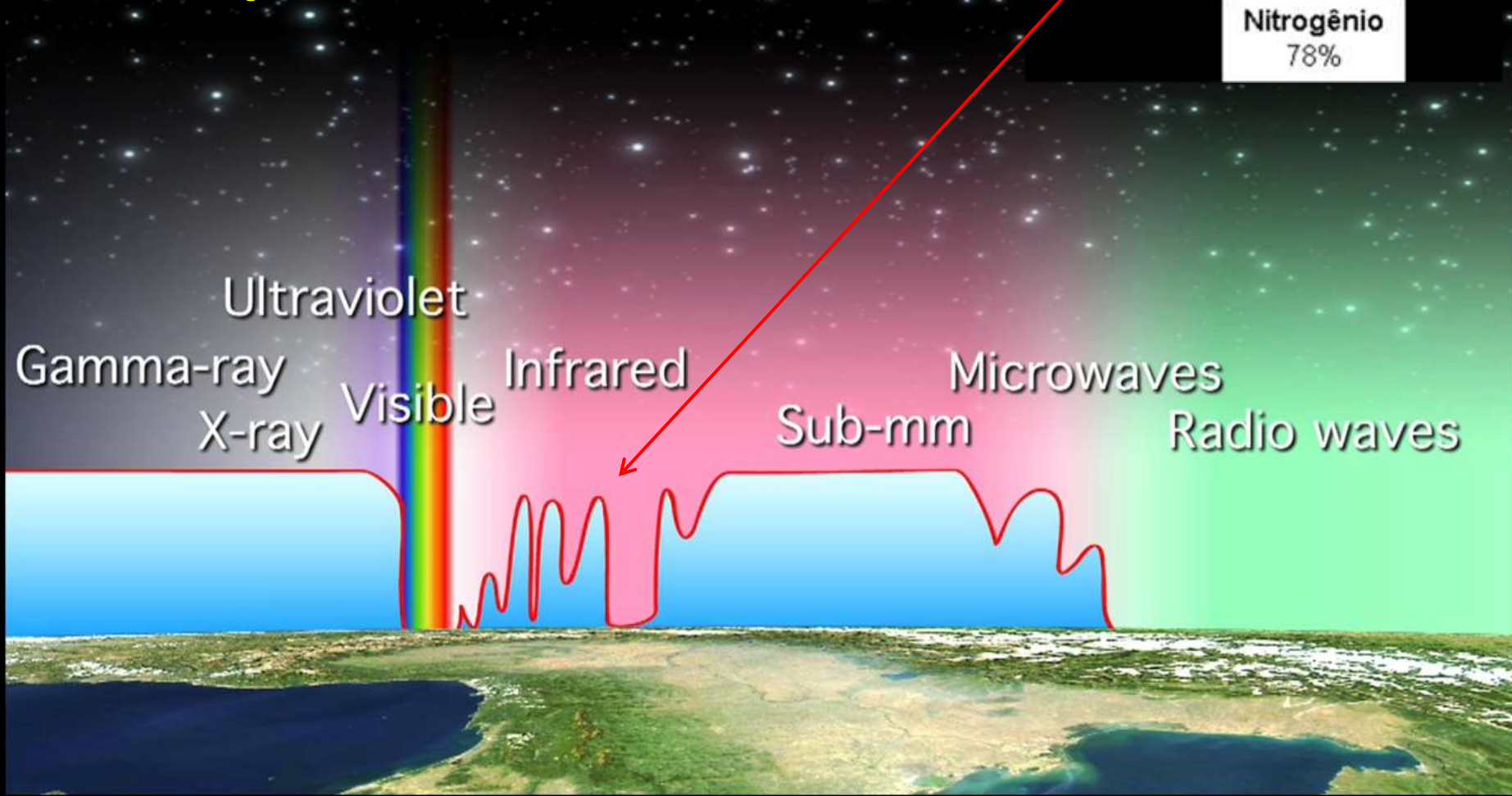
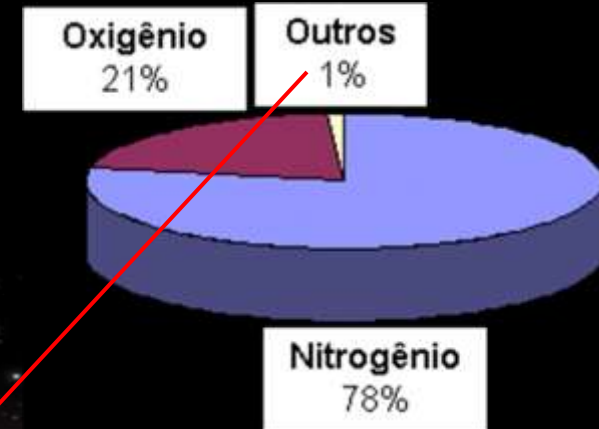
Constituintes da atmosfera



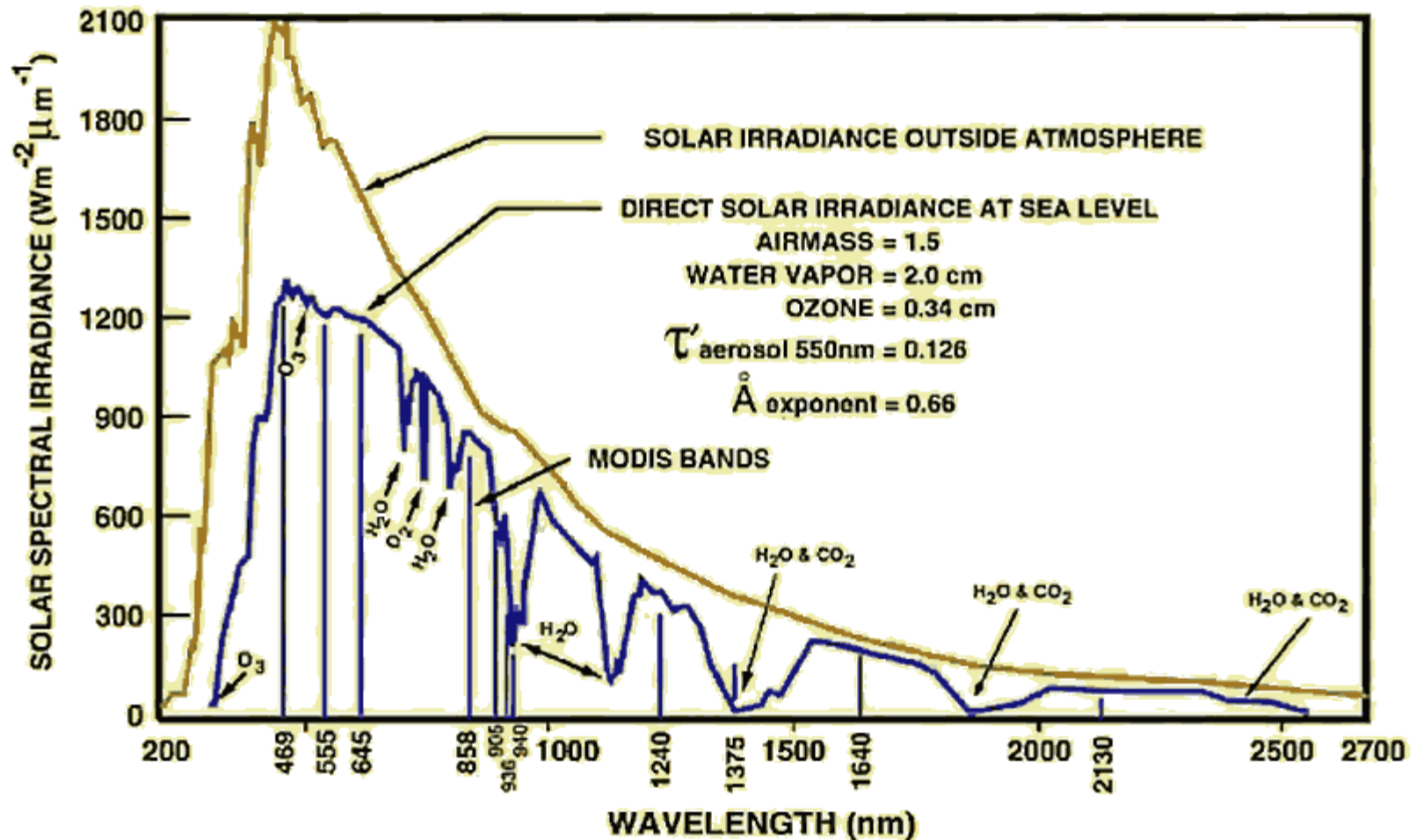
N e O são os principais constituintes. Sua proporção relativa é constante entre 0-90 km

Constituintes da atmosfera (CO_2 , H_2O , O_3 ,...)

Os constituintes menores (e variáveis) são importantes fontes de opacidade na atmosfera

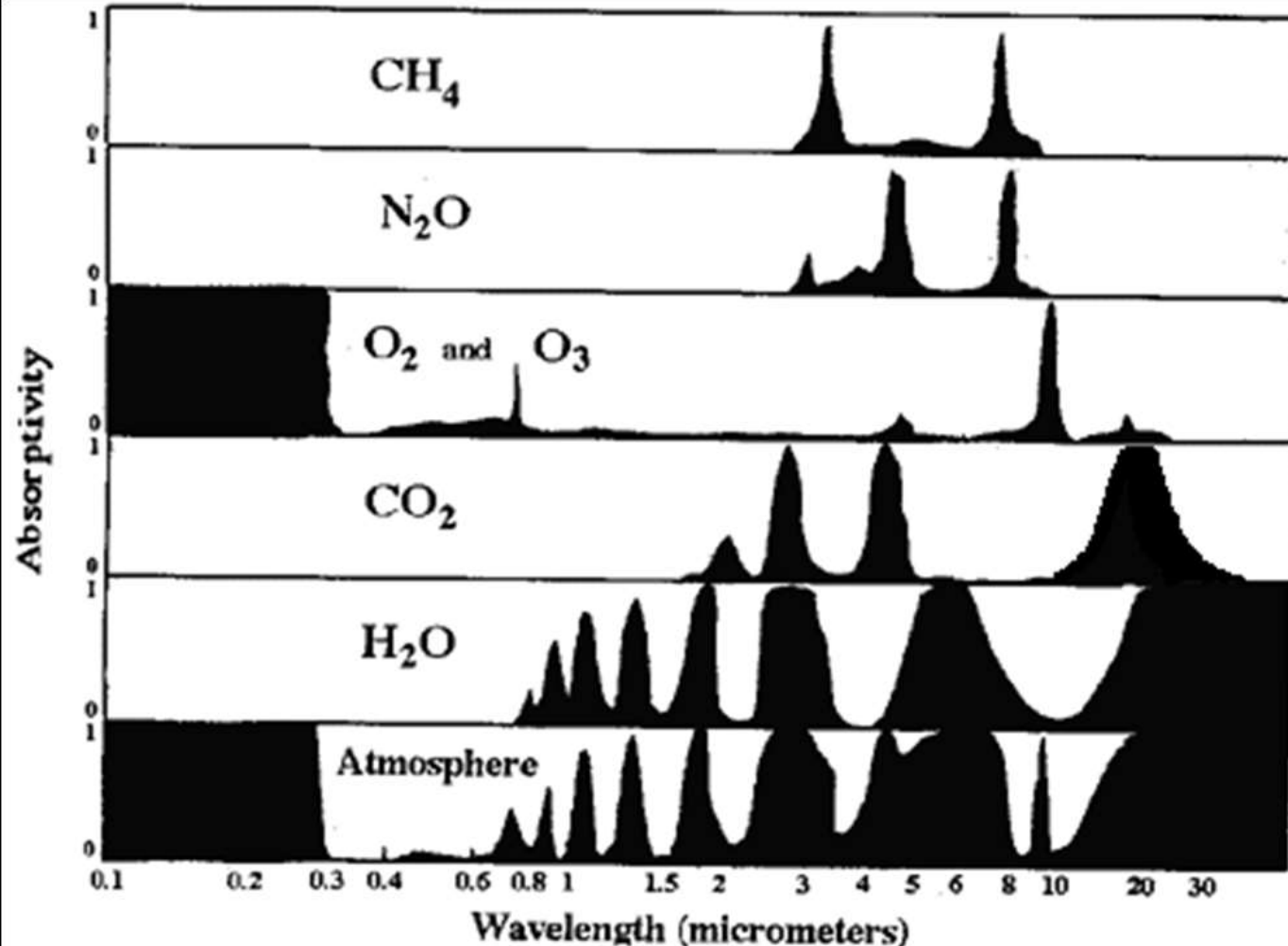


Vapor de água (H₂O) atrapalha no infravermelho e no ótico



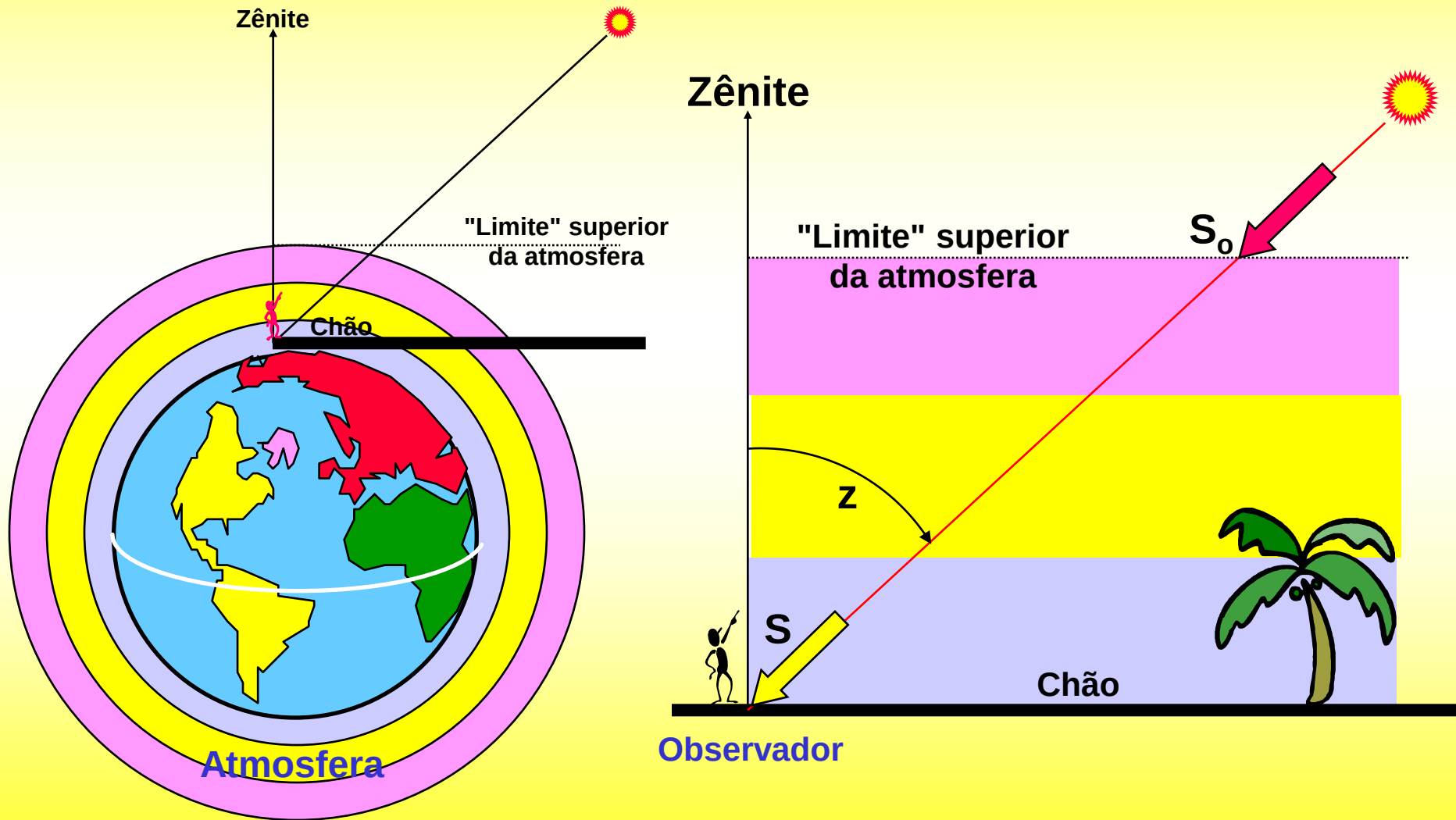
Bandas de absorção na atmosfera

bandas de absorção atmosférica
(bandas telúricas)



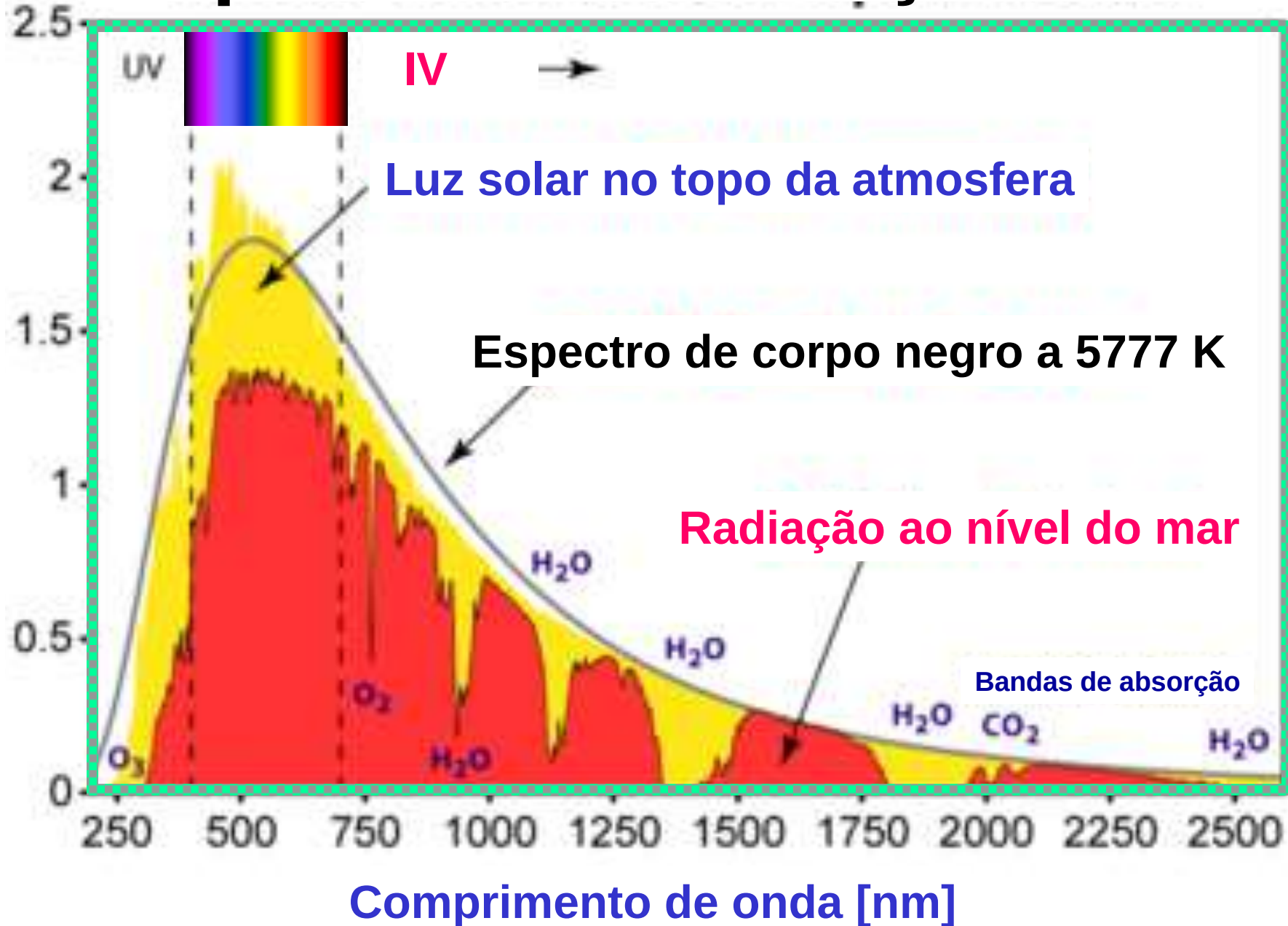
Absorptivity of various gases of the atmosphere and the atmosphere as a whole as a function of the wavelength of radiation. An absorptivity of zero means no absorption while a value of one means complete absorption. The dominant absorbers of infrared radiation are water vapor (H₂O) and carbon dioxide (CO₂). Oxygen (O₂) and ozone (O₃) absorb much of the sun's ultraviolet radiation.

Atmosfera plano-estratificada



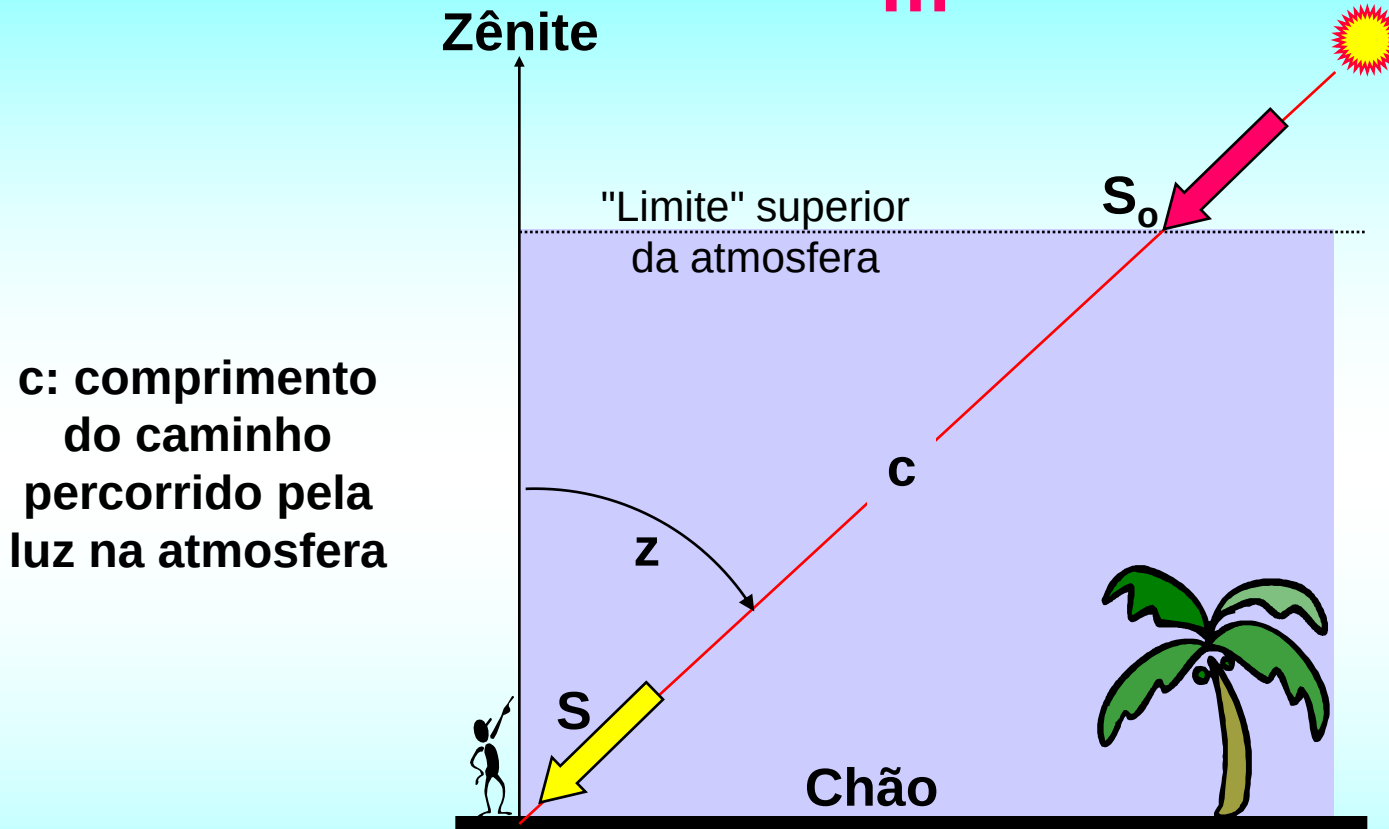
Espectro da irradiação solar

Irradiância espectral [$\text{W}/\text{m}^2/\text{nm}$]



Coeficiente de Extinção Médio

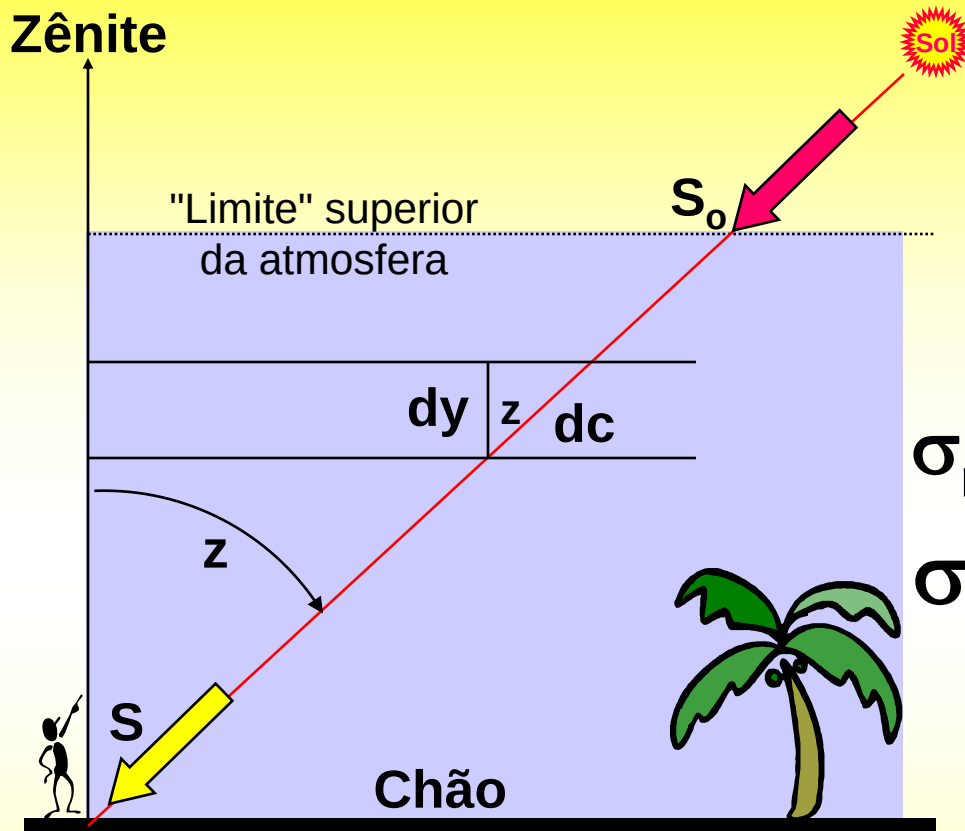
σ_m



$$\sigma_m \equiv - [(S - S_0) / S_0] / c$$

$$\sigma_m = [1/m \text{ ou } 1/cm]$$

Coeficiente de Extinção Local σ



$$\sigma_m = - [(S - S_0) / S_0] / c$$

$$\sigma \equiv - [dS / S] / dc$$

Espessura óptica (ou profundidade óptica)

τ

Coeficiente de extinção σ :

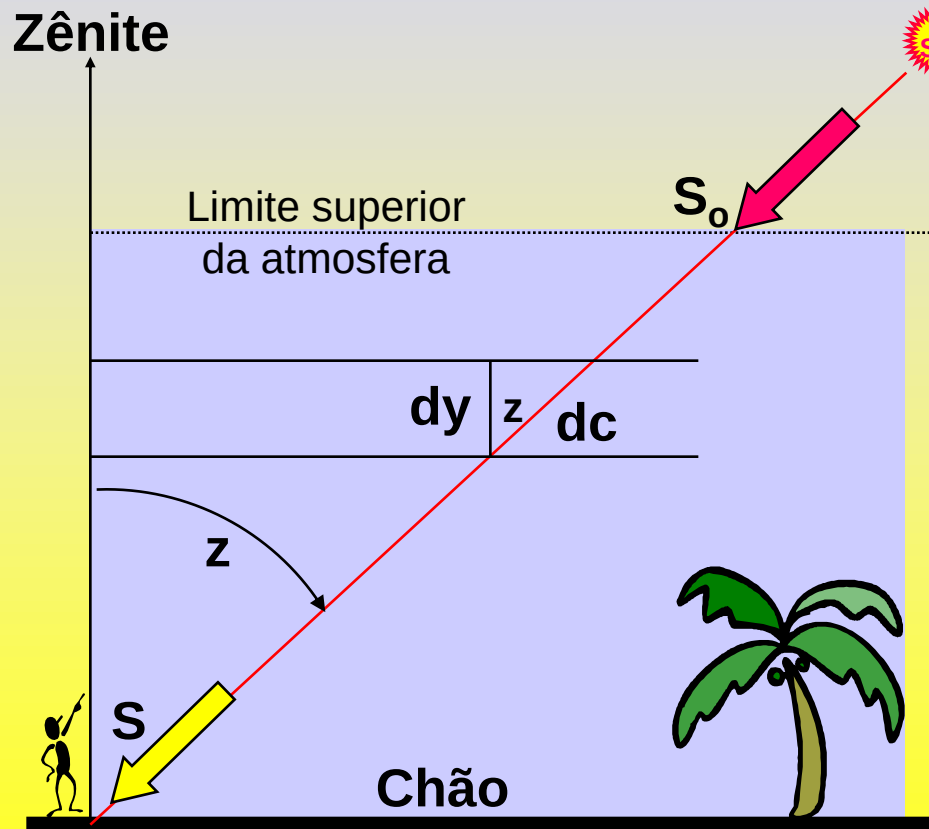
$$\sigma = - [dS / S] / dc$$

$$dS / S = - \sigma dc$$

$$\int (dS / S) = - \int \sigma dc$$

$$\tau \equiv \int \sigma dc$$

$$\int (dS / S) = - \tau$$





"Constante solar" no solo

$$\int (dS / S) = - \tau$$

$$[\ln S] = - \tau$$

S_0 = constante solar (fora da atmosfera)
 S = irradiância solar no local da medida

$$\ln S - \ln S_0 = - \tau$$

$$\ln (S/S_0) = - \tau$$

$$S = S_0 e^{-\tau}$$

$$e = \lim_{x \rightarrow \infty} (1 + 1/x)^x$$

$$e \cong 2,718\ 282 \dots$$

Base Neperiana

Base do logaritmo natural



"Espessura Óptica" zenital $\tau_z \equiv \zeta(\text{zeta})$

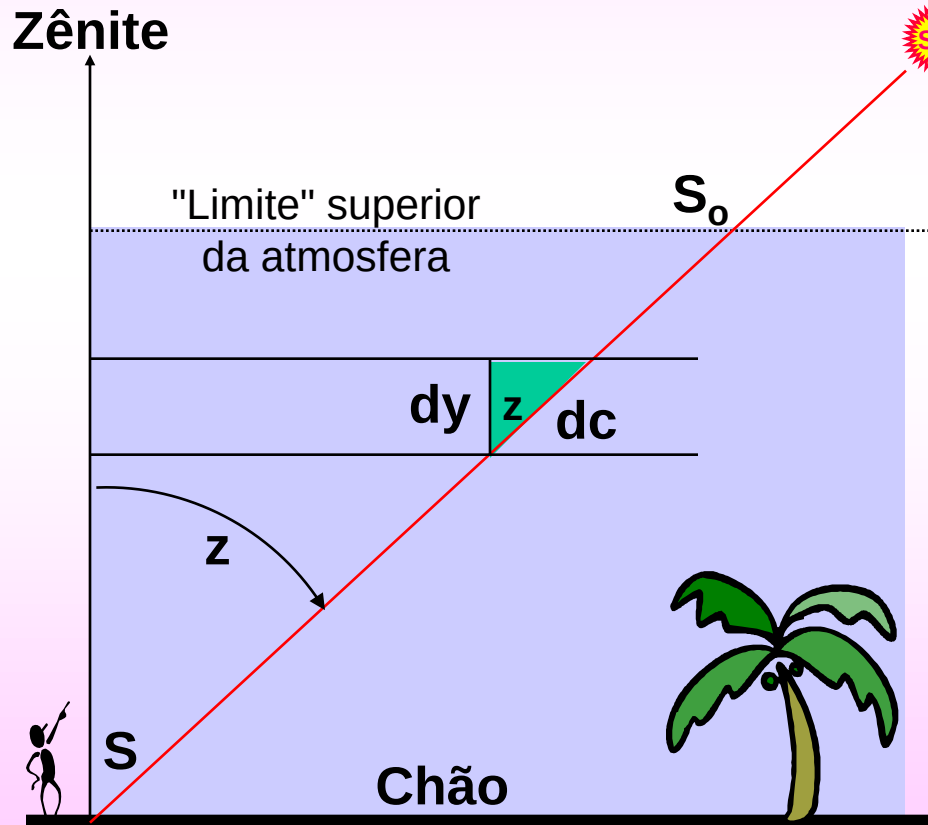
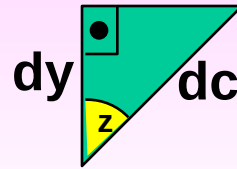
$$\tau \equiv \int \sigma \, dc$$

σ : Coeficiente de extinção

τ_z

$$dy = dc \cdot \cos z$$

$$dc = dy \cdot \sec z$$



$$\tau = \int \sigma \, dy \sec z$$

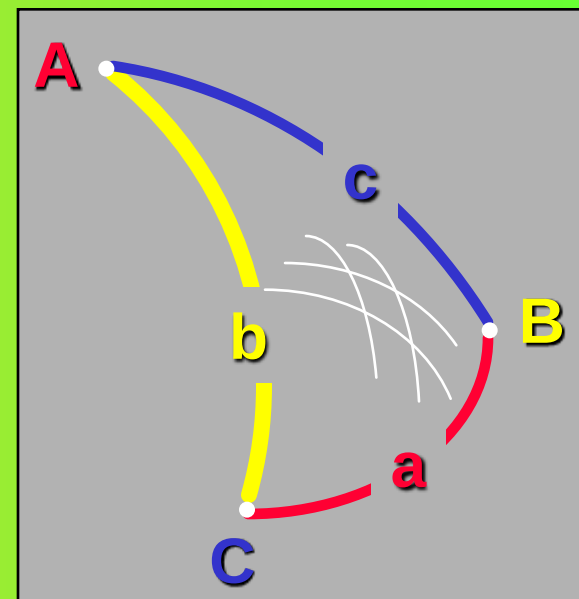
$$\tau = \sec z \int \sigma \, dy$$

$$\tau_z \equiv \int \sigma \, dy$$

$$\tau = \tau_z \sec z$$

$$\text{Sol: } \tau = \tau_z \sec z_s$$

Resumo das Fórmulas de Trigonometria Esférica



Co-seno

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

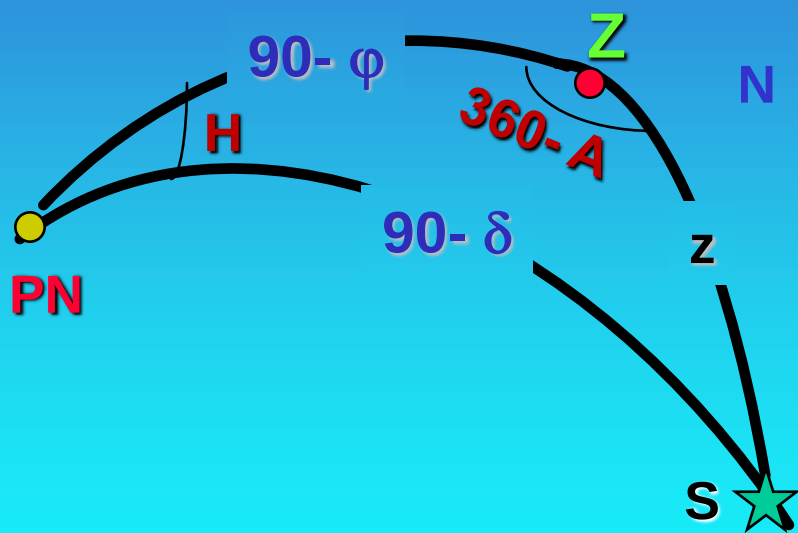
Seno

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Seno & Co-seno

$$\sin a \cdot \cos B = \cos b \cdot \sin c - \sin b \cdot \cos c \cdot \cos A$$

Distância zenital do Sol

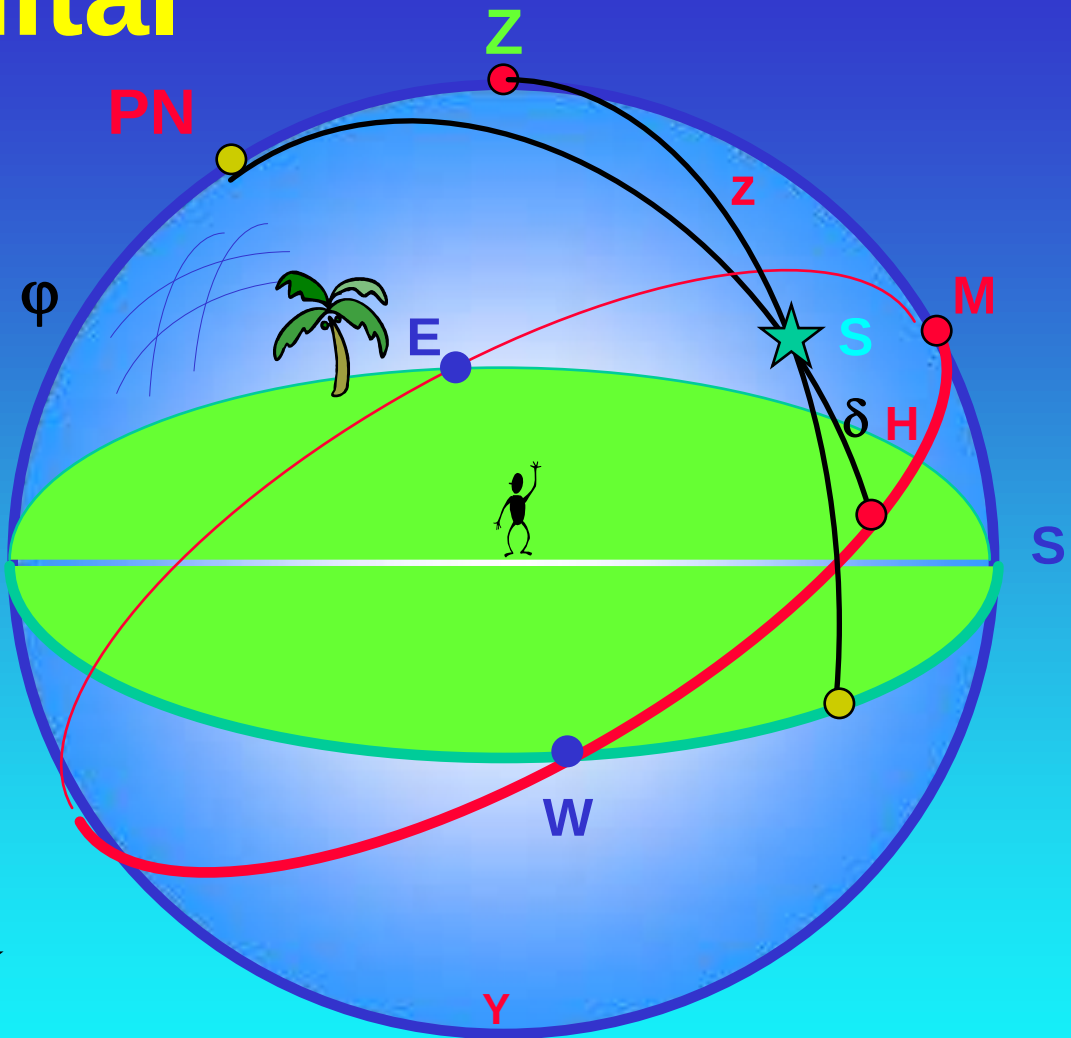


Lei de cosenos no triângulo esférico:

$$\cos z_S = \cos (90-\varphi) \cdot \cos (90-\delta_S) + \sin (90-\varphi) \cdot \sin (90-\delta_S) \cdot \cos H_S$$

$$\cos z_S = \sin \varphi \cdot \sin \delta_S + \cos \varphi \cdot \cos \delta_S \cdot \cos H_S$$

$$\sec z_S = 1 / \cos z_S$$



Irradiância Solar local **S**

$$\int (dS / S) = - \tau$$

$$\tau \equiv \tau_z \sec z$$

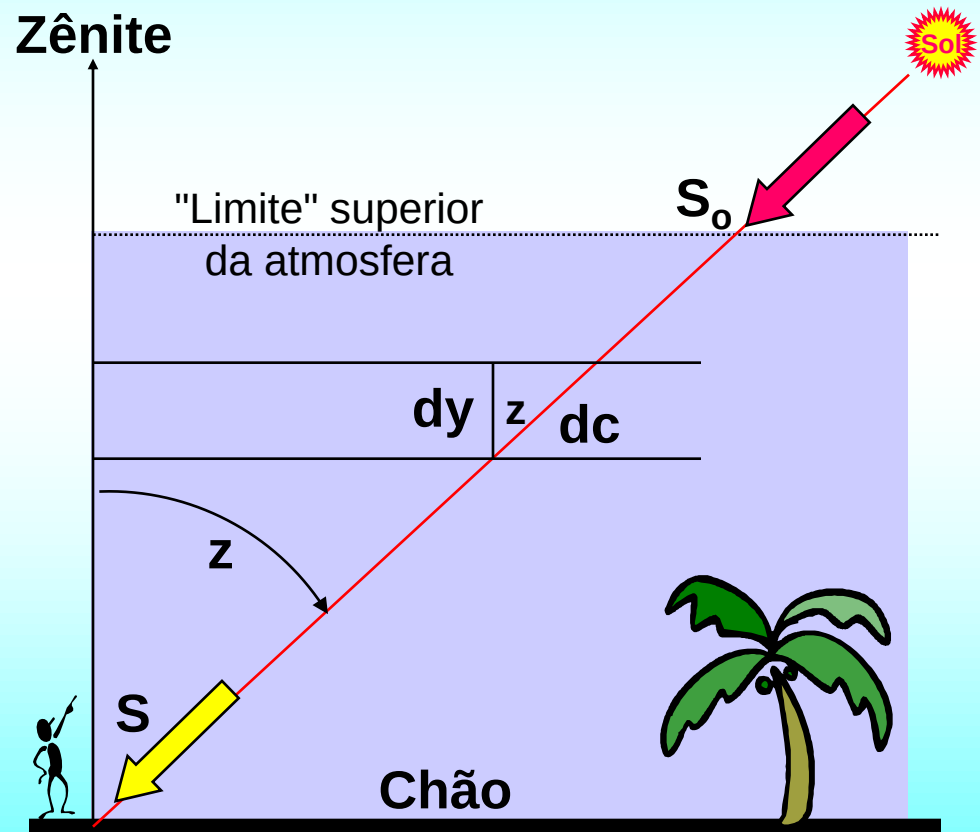
$$\ln (S / S_o) = - \tau_z \sec z$$

$$\ln (S / S_o) = - \zeta \sec z$$

$$\zeta \equiv \tau_z$$

$$S / S_o = e^{-\zeta \sec z}$$

$$S = S_o \cdot e^{-\zeta \sec z}$$



Cálculo da Constante Solar

Determinar S_o a partir do solo

$$\int (dS / S) = - \tau$$

$$\tau \equiv \tau_z \sec z$$

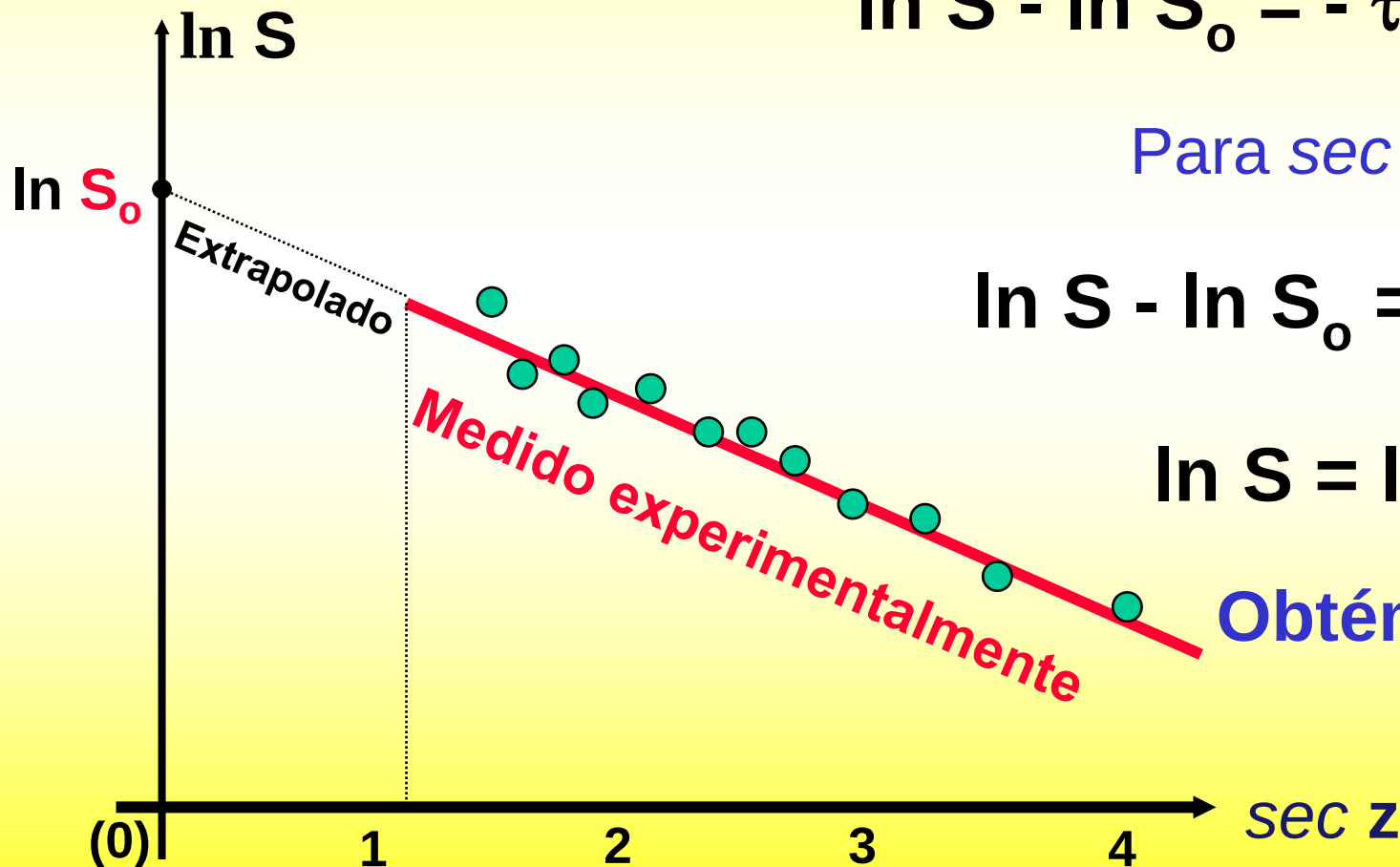
$$\ln S - \ln S_o = - \tau_z \sec z$$

Para $\sec z = 0$

$$\ln S - \ln S_o = - \tau_z \cdot 0$$

$$\ln S = \ln S_o$$

Obtém-se S_o



Determinação empírica da Irradiância Solar no solo

Irradiância Solar Empírica

$$S = K \cdot S_o \cdot e^{-\tau}$$

S_o = constante solar (fora da atmosfera) a 1 UA

K = correção que leva em conta a órbita elipsoidal

τ = espessura óptica da atmosfera

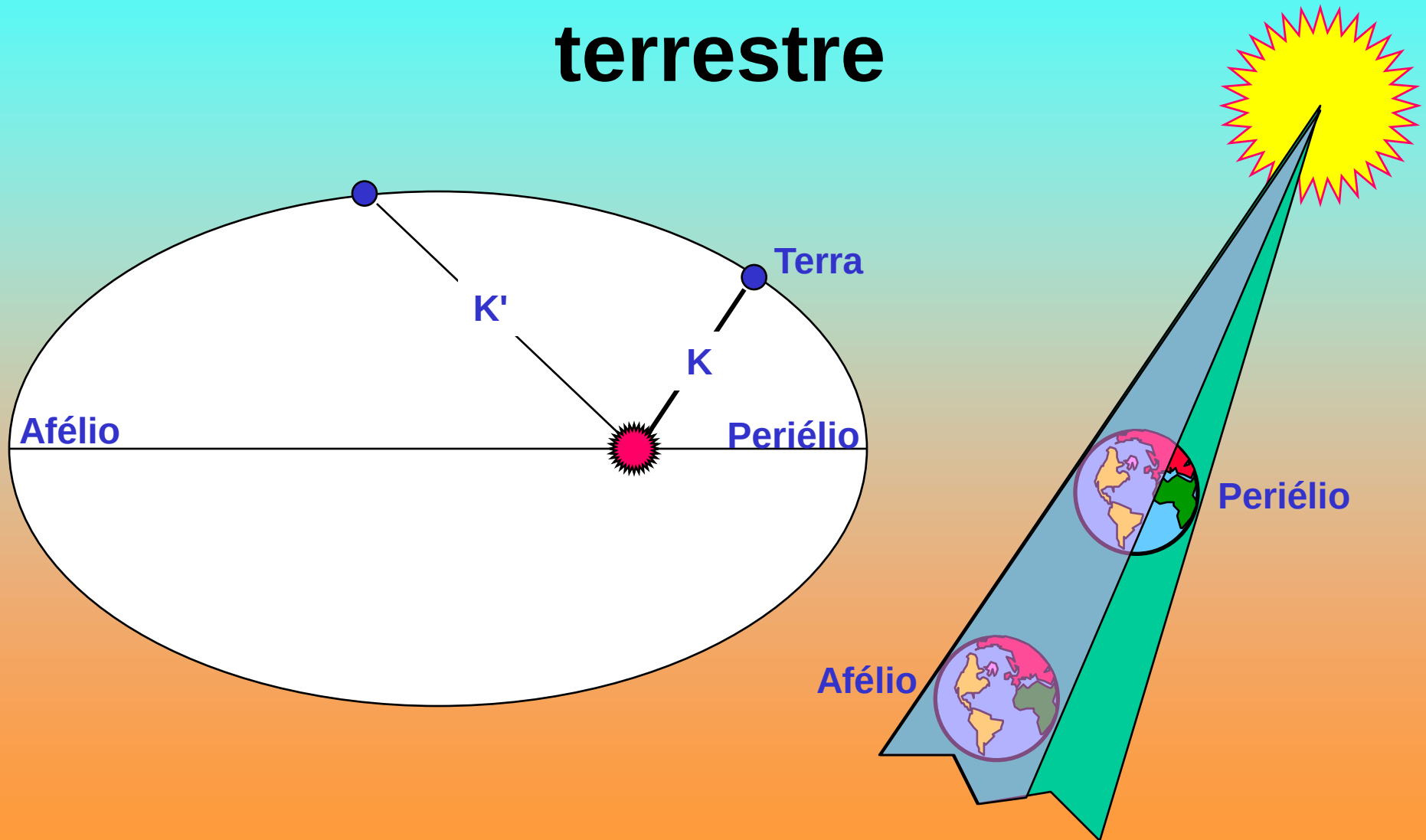
S = irradiância solar no local da medida.

$e \cong 2,718$

Neper



Variação da Irradiância Solar devido à excentricidade da órbita terrestre

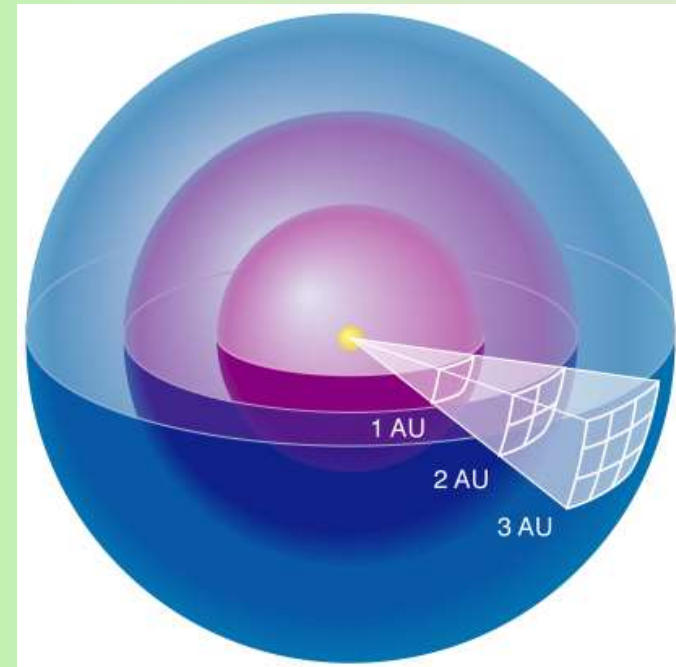
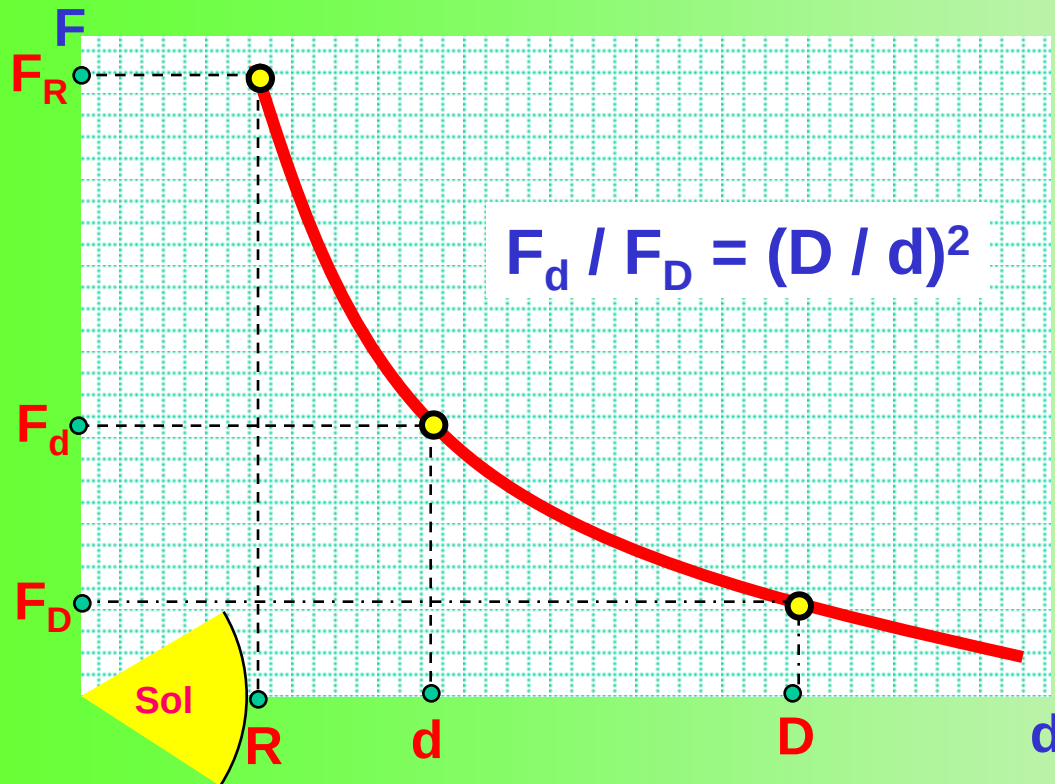


Fluxo em função da distância

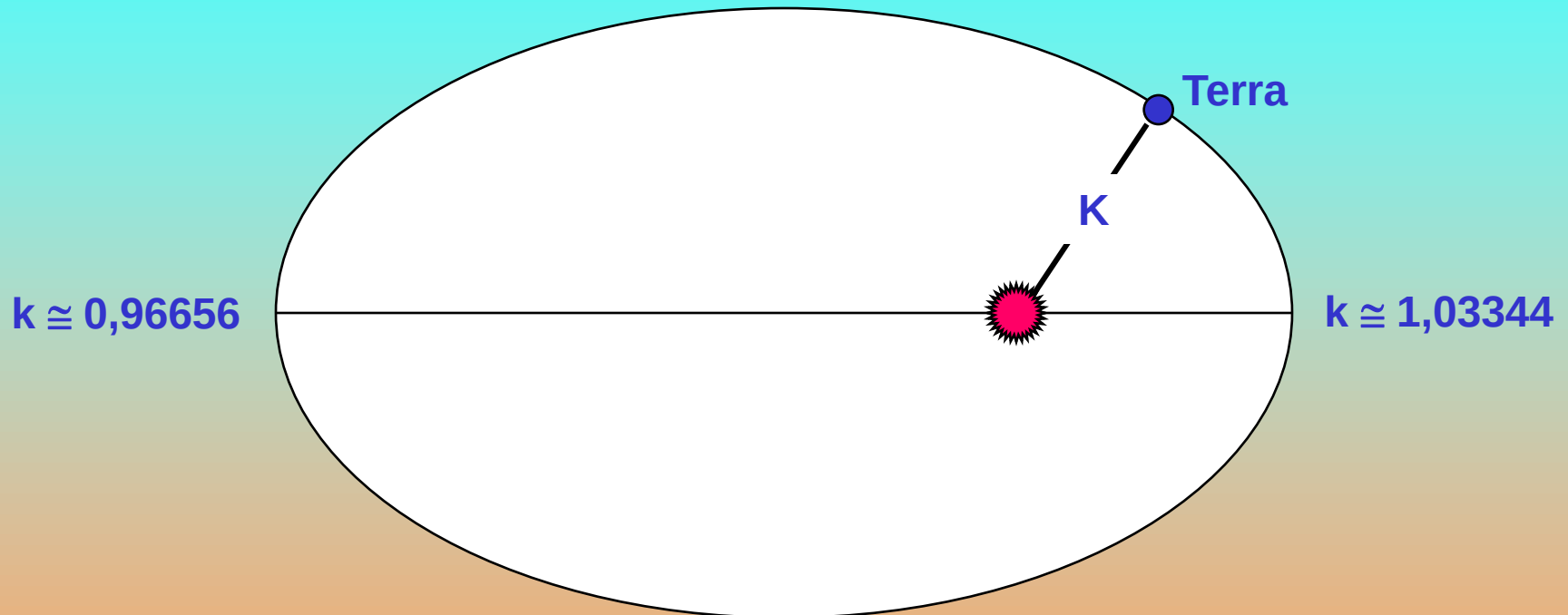
$$F_d = L / (4\pi d^2)$$

$$F_D = L / (4\pi D^2)$$

$$F_d / F_D = D^2 / d^2$$



Correção devida à excentricidade da órbita terrestre



$$K = 1 + 0,03344 \cos \left((360 d / 365,25) - 2,8^\circ \right)$$

d = número do dia do ano

$d \equiv 01$ para 01/jan

$d = 32$ para 01/fev

Espessura óptica

$$\tau$$

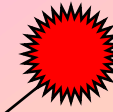
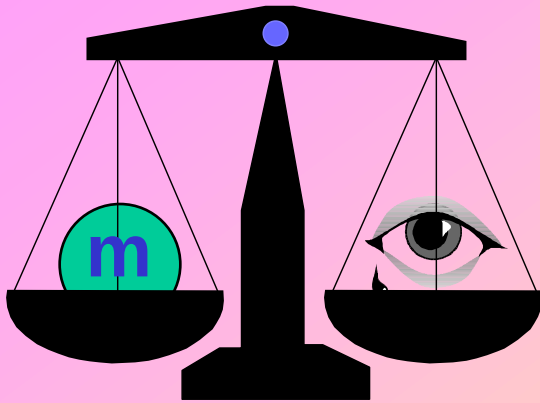
$$\tau = \tau_z \sec z$$

$$\tau \equiv \delta_R m$$

δ_R = espessura óptica de Rayleigh

m = massa óptica dependente da altura h do Sol

Massa óptica m



Para $h > 10^\circ$

$$m = (p/p_{\text{mar}}) \{ 1 / \text{sen } h \}$$



Para $h \leq 10^\circ$

$$m = (p/p_{\text{mar}}) \{ 1 / [\text{sen } h + 0,15 (h + 3,885)^{-1,235}] \}$$

Linha de divisão

h

h

10°



Solo

Razão entre pressões

Para L qualquer:

$$(p/p_{\text{mar}}) = e^{-(L/8.000)}$$

Linha divisória

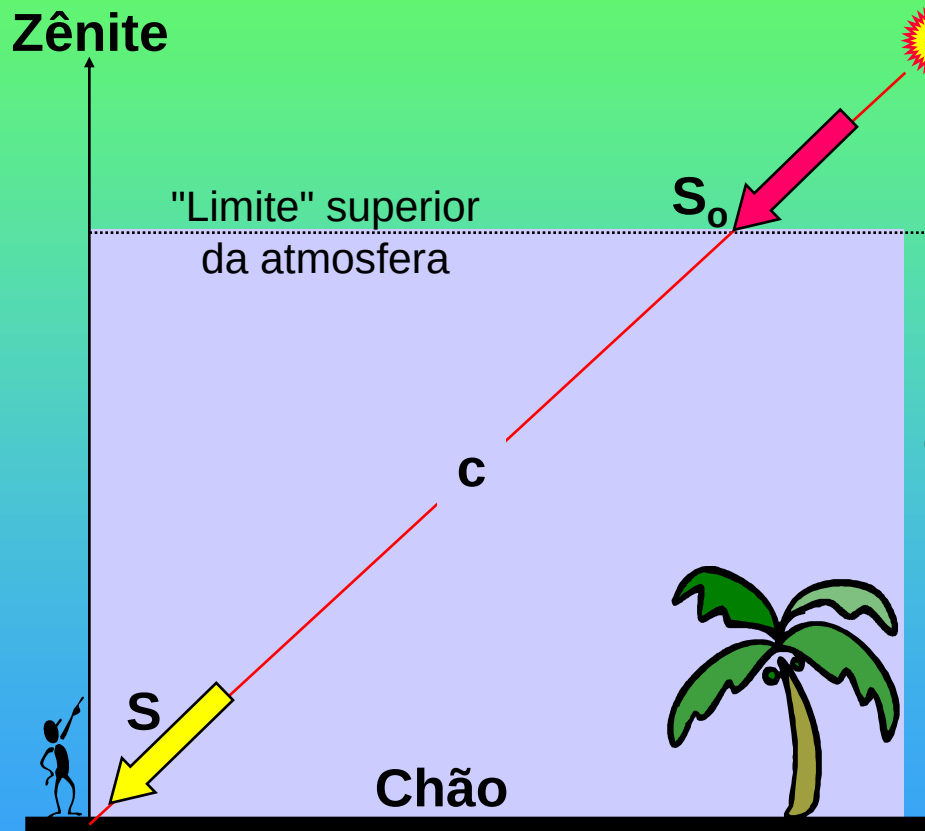
$L = 4.000 \text{ m}$

Para $L < 4000 \text{ m}$:

$$(p/p_{\text{mar}}) = 1 - (L/10.000)$$



Espessura óptica de Rayleigh



$$\delta_R = 1 / (0,9 \text{ m} + 9,4)$$

Obtenção da Irradiância Solar Empírica

$$K = 1 + 0,033.44 \cos ((360 d / 365,25) - 2,8^\circ)$$

$$\text{Para } L \text{ qualquer: } (p/p_{\text{mar}}) = e^{-(L/8.000)}$$

$$\text{Para } L < 4.000 \text{ m : } (p/p_{\text{mar}}) = 1 - (L/10.000)$$

$$\text{Para } h > 10^\circ \quad m = (p/p_{\text{mar}}) \{ 1 / \sin h \}$$

$$\text{Para } h \leq 10^\circ \quad m = (p/p_{\text{mar}}) \{ 1 / [\sin h + 0,15 (h + 3,885)^{-1,235}] \}$$

$$\delta_R = 1 / (0,9 m + 9,4)$$

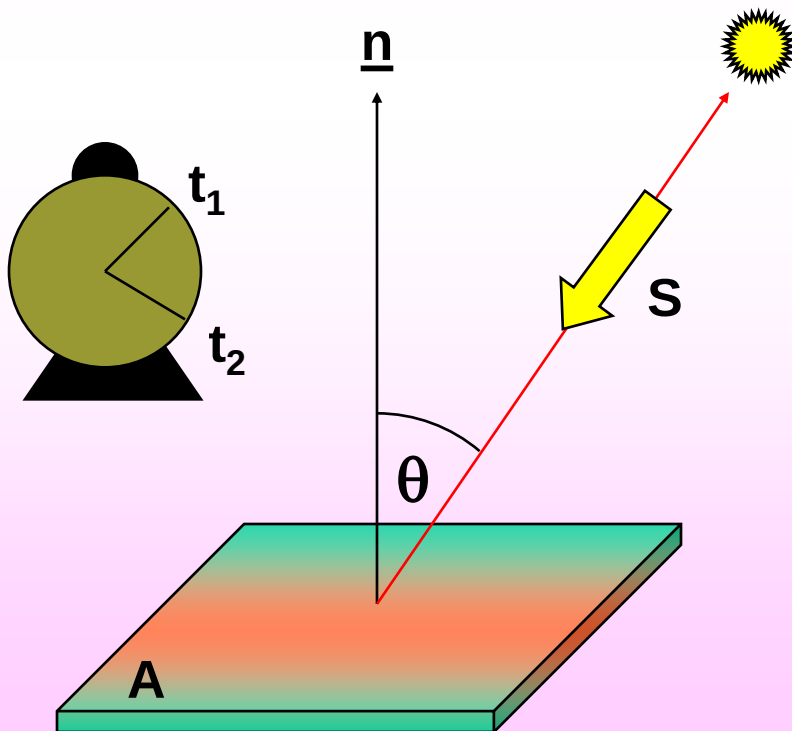
$$\tau \equiv \delta_R m$$

$$S = K \cdot S_o \cdot e^{-\tau}$$

Cálculo da insolação

Insolação ou Energia Incidente **E**

A energia **E** incidente na placa de área A no intervalo de tempo entre t_1 e t_2 pode ser dada por:



Nomenclatura:
 $S = I'$

$$E = \int S A \cos \theta dt$$

cal

cm²

min

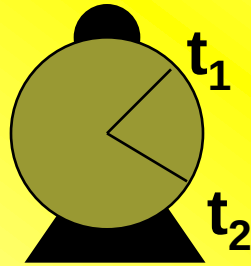
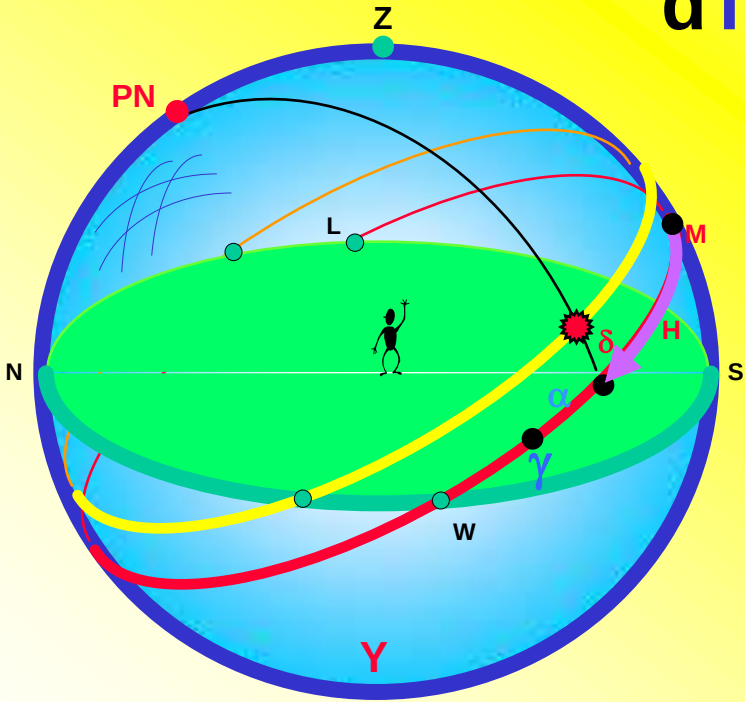
cal/cm².min

Diferencial do tempo

Relacionar diferenciais de tempo e de ângulo horário

Tempo sideral: $TS = \alpha + H$

$$dTS = d\alpha + dH$$



Hipótese: durante o intervalo de tempo considerado, a ascensão reta pode ser considerada constante. Nesse caso:

$$d\alpha = 0$$

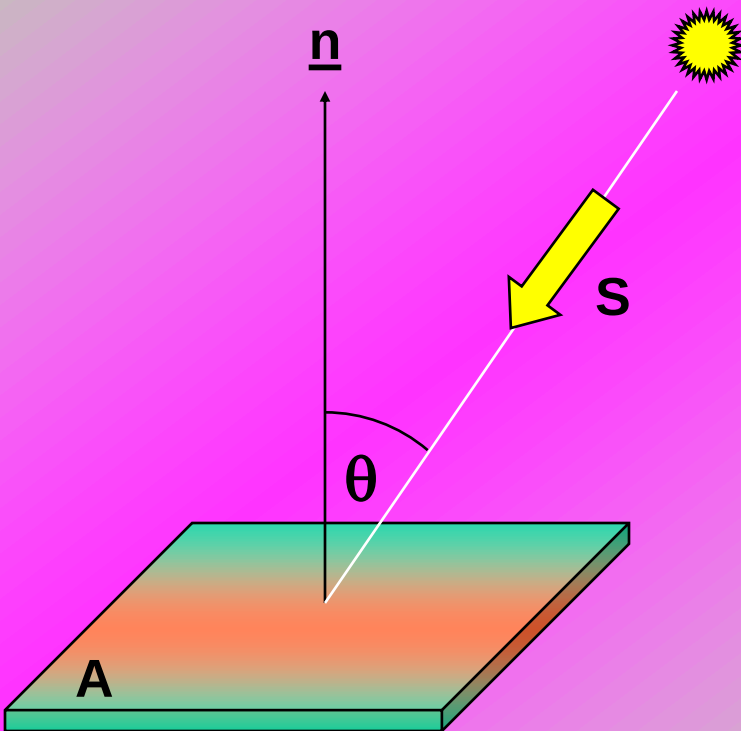
$$dTS = 0 + dH$$

$$dT_S = dH$$



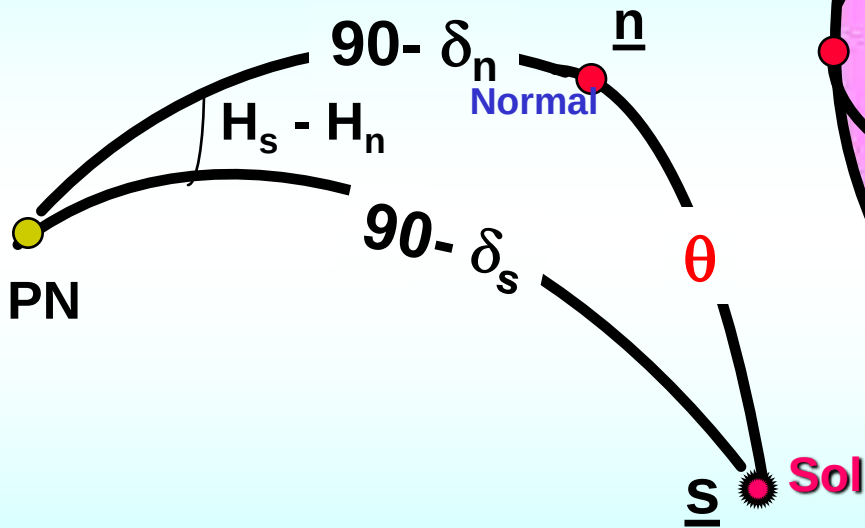
$$dt = dH$$

Obtenção do $\cos \theta$



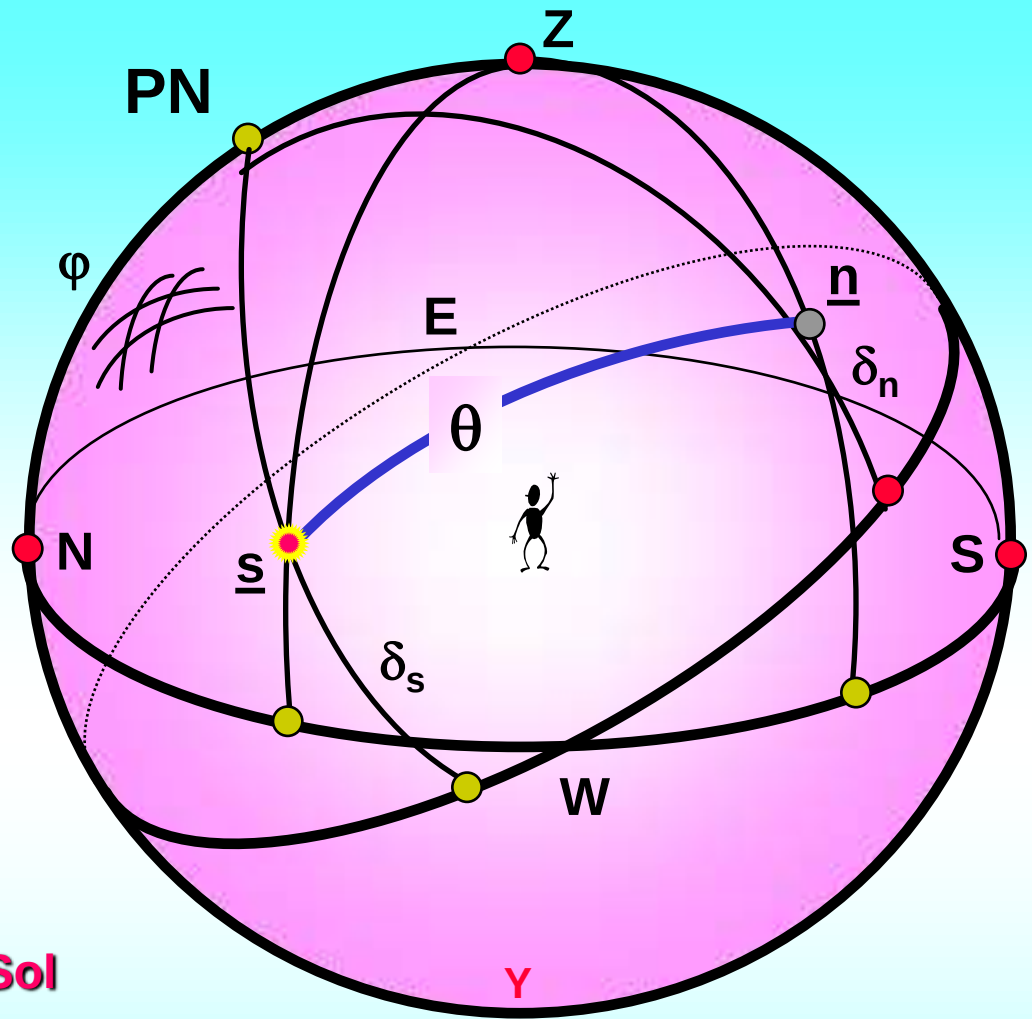
$$E = \int S A \cos \theta \, dt$$

$\cos \theta$ numa placa Plana Qualquer



$$\cos \theta = + \cos (90 - \delta_n) \cdot \cos (90 - \delta_s) + \sin (90 - \delta_n) \cdot \sin (90 - \delta_s) \cdot \cos (H_s - H_n)$$

$$\cos \theta = \sin \delta_n \cdot \sin \delta_s + \cos \delta_n \cdot \cos \delta_s \cdot \cos (H_s - H_n)$$



Hipóteses adicionais

Hipóteses adicionais

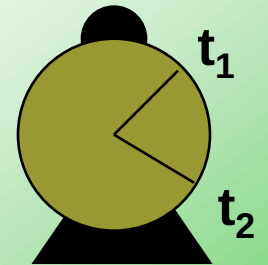
$$E = \int S A \cos \theta \, dt$$

Hipótese:

Durante o intervalo de tempo considerado, vamos impor que **S** e **A** possam ser consideradas constantes.

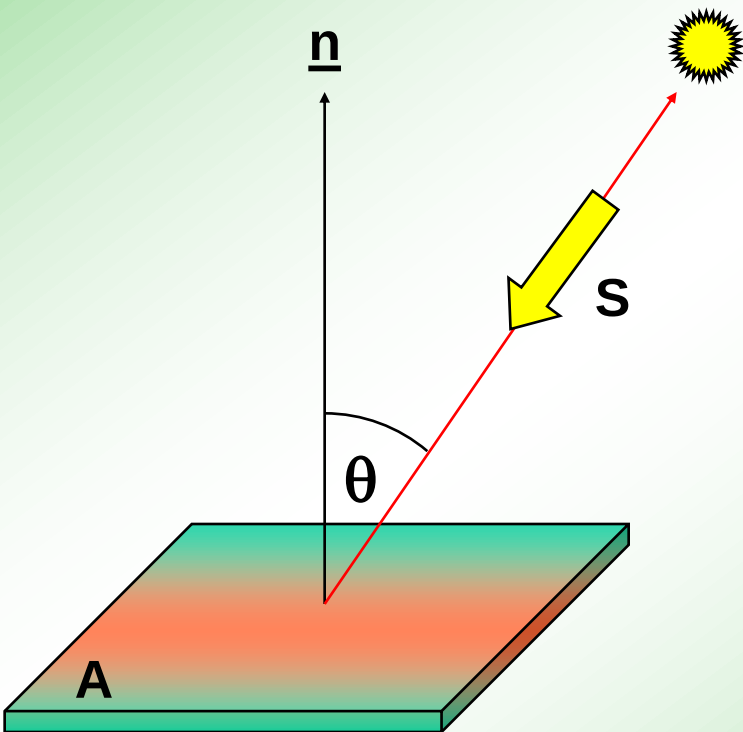
Nesse caso:

$$E = S A \int \cos \theta \, dt$$



Como: **$dt = dH$**

$$E = S A \int \cos \theta \, dH$$



Insolação em placas planas com diferentes posicionamentos

**Horizontal
Vertical Meridiana
Vertical Leste-Oeste
Vertical Qualquer
Qualquer**

Placa Plana Qualquer

Horizontal

Vertical Meridiana

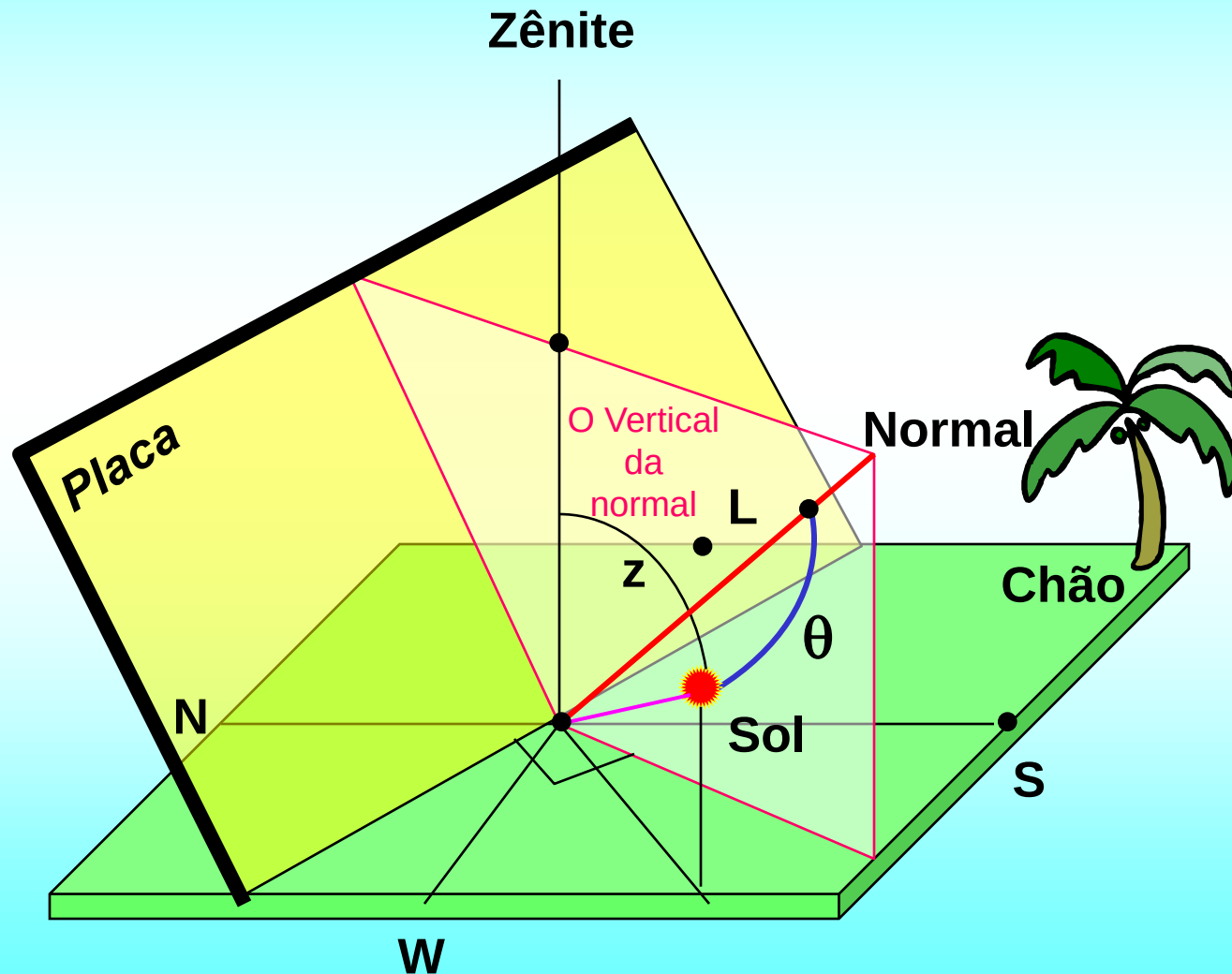
Vertical Leste-Oeste

Vertical Qualquer

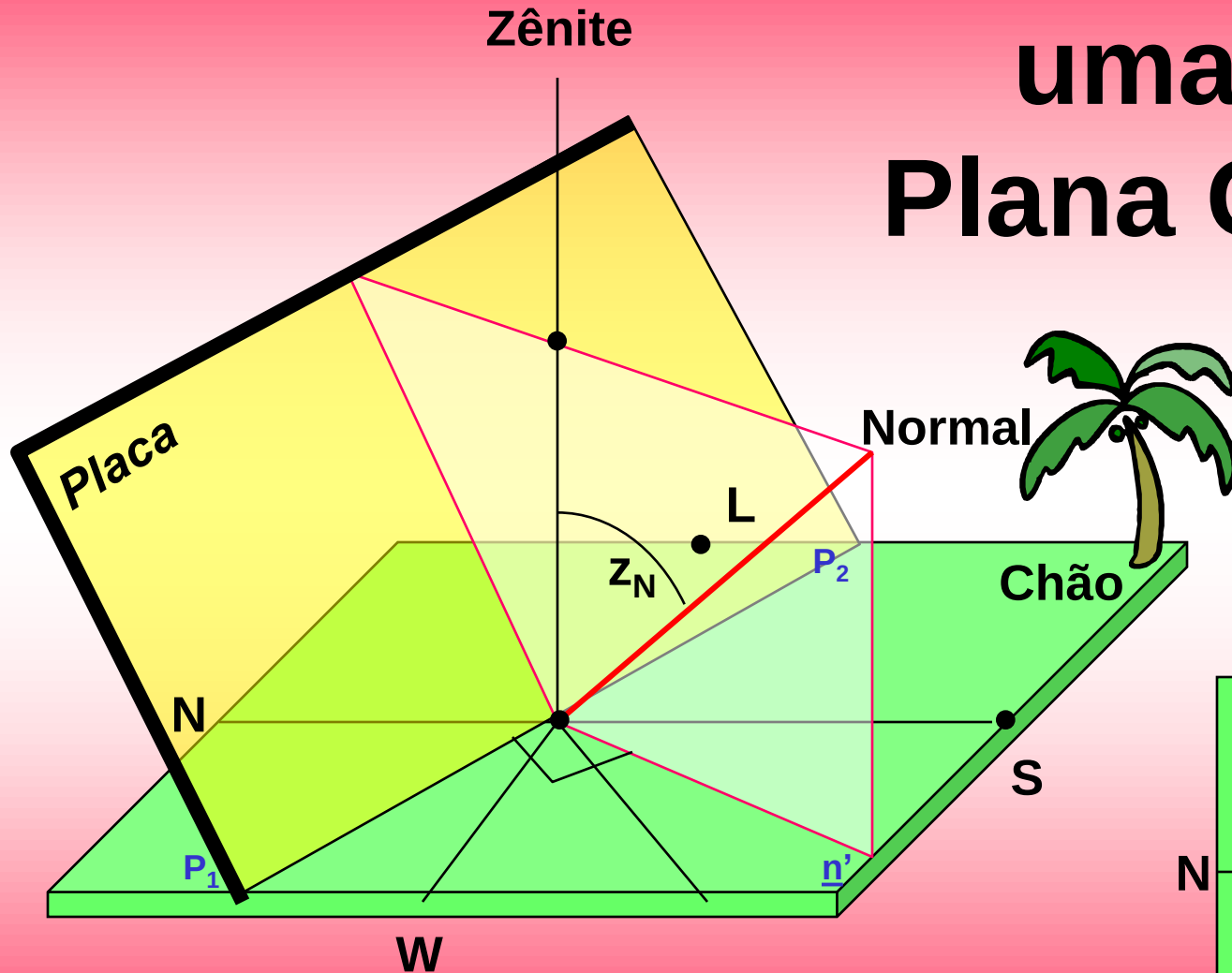


Qualquer

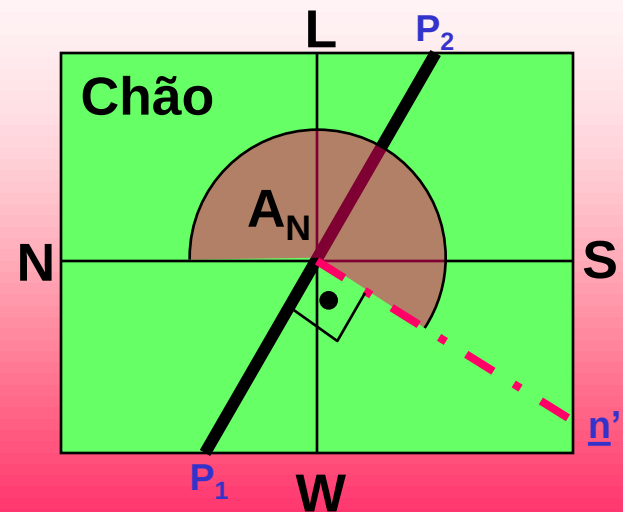
Placa Plana Qualquer



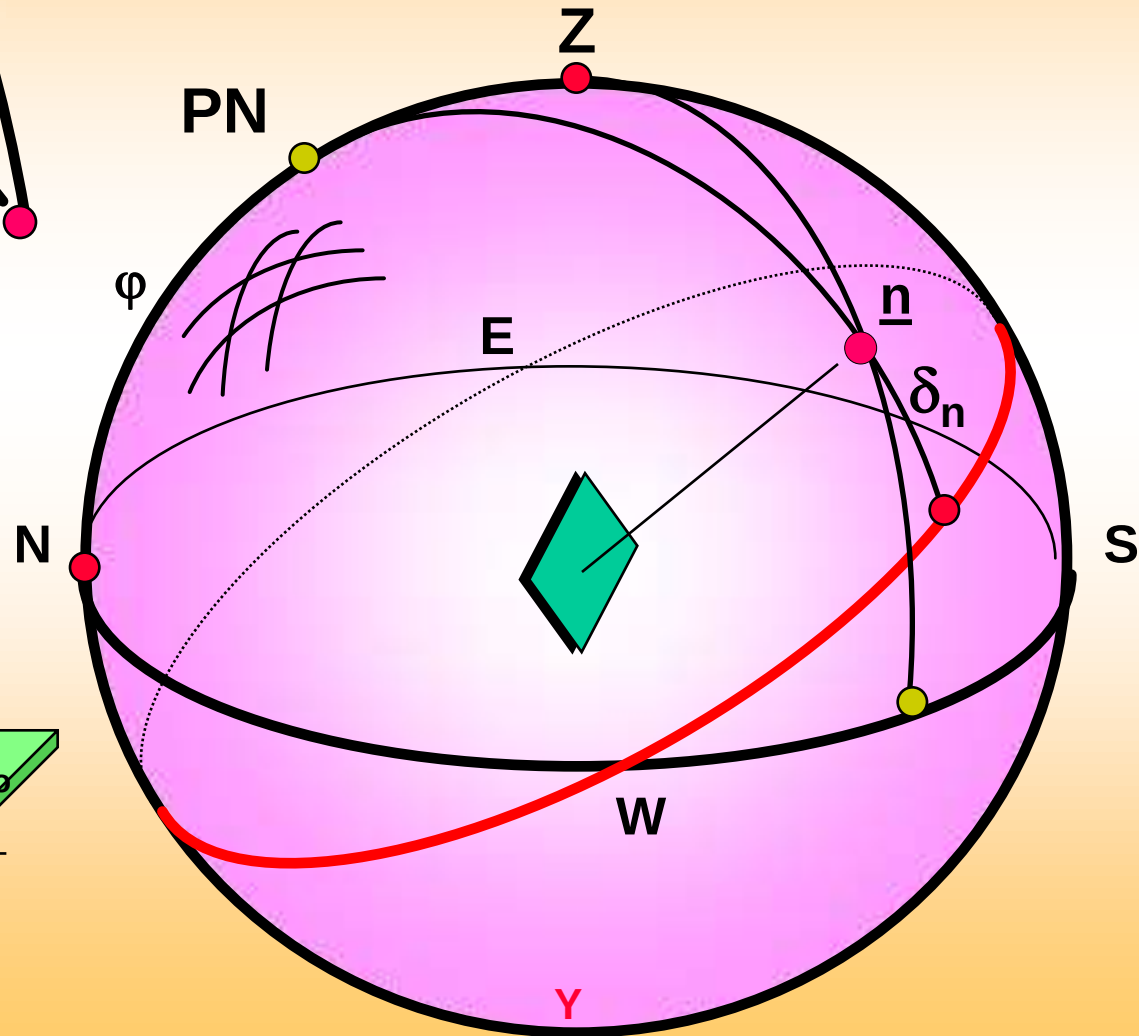
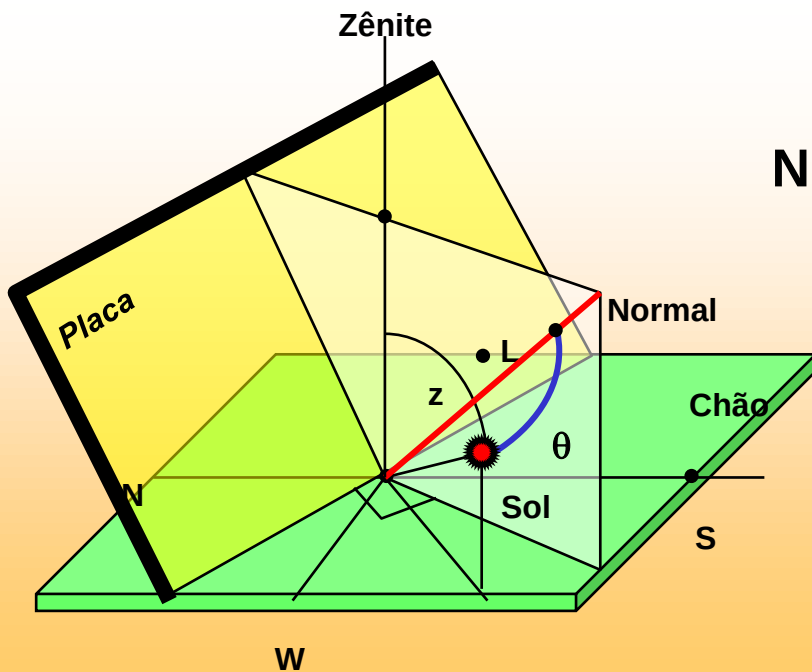
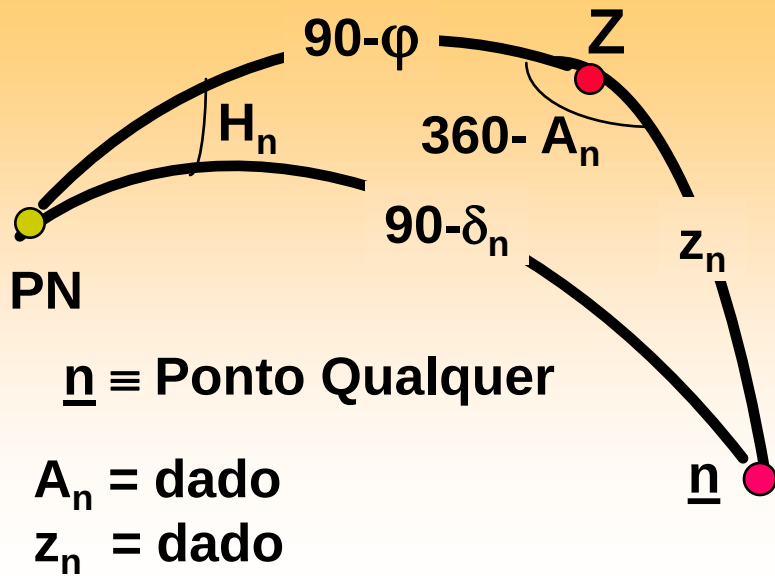
Posição de uma Placa Plana Qualquer



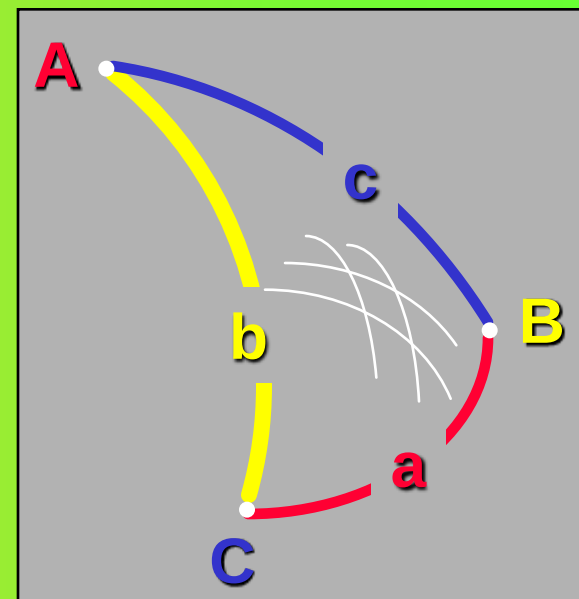
$$A_N = A_{P_2} + 90^\circ$$



Placa Plana Qualquer



Resumo das Fórmulas de Trigonometria Esférica



Co-seno

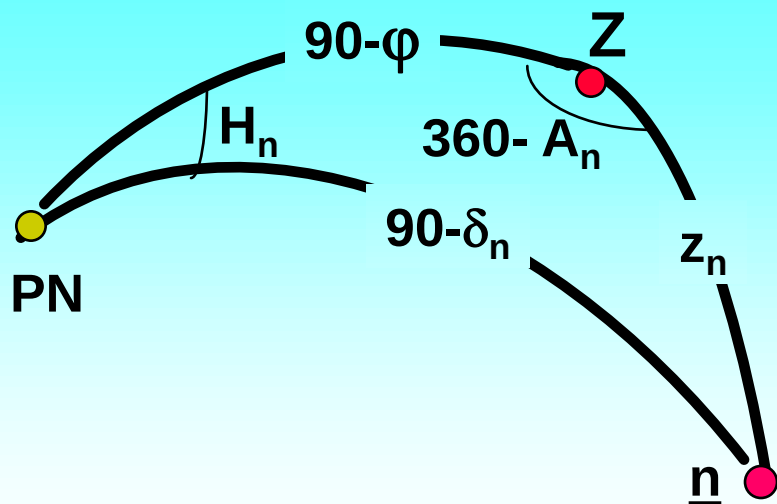
$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

Seno

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Seno & Co-seno

$$\sin a \cdot \cos B = \cos b \cdot \sin c - \sin b \cdot \cos c \cdot \cos A$$



Obter o H_n e o δ_n
de Placa Plana
Qualquer

$$\text{sen } \delta_n = \text{sen } \varphi \cdot \cos z_n + \cos \varphi \cdot \text{sen } z_n \cdot \cos A_n$$



δ_n

$$\cos H_n = [\cos z_n \cdot \cos \varphi - \text{sen } z_n \cdot \text{sen } \varphi \cdot \cos A_n] / \cos \delta_n$$

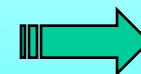


$$\Rightarrow 0 \leq \underline{H}_n \leq 180^\circ$$

$$\text{sen } H_n = - \text{sen } z_n \cdot \text{sen } A_n / \cos \delta_n$$

Se $\text{sen } H_n \geq 0$ então $H_n = \underline{H}_n$

Se $\text{sen } H_n < 0$ então $H_n = 360^\circ - \underline{H}_n$



H_n

Insolação numa placa plana

qualquer...

$$E = S A \int \cos \theta \, dH$$

$$\cos \theta = \sin \delta_n \cdot \sin \delta_s + \cos \delta_n \cdot \cos \delta_s \cdot \cos (H_s - H_n)$$

$$E = S A \int [\sin \delta_n \cdot \sin \delta_s + \cos \delta_n \cdot \cos \delta_s \cdot \cos (H_s - H_n)] \, dH_s$$

Integral de uma soma = soma das integrais

$$E = S A \int [\sin \delta_n \cdot \sin \delta_s] \, dH_s + S A \int [\cos \delta_n \cdot \cos \delta_s \cdot \cos (H_s - H_n)] \, dH_s$$

Extrair as constantes do interior das integrais

$$E = S A \sin \delta_n \cdot \sin \delta_s \int dH_s + S A \cos \delta_n \cdot \cos \delta_s \int [\cos (H_s - H_n)] \, dH_s$$

...Insolação numa placa plana qualquer...



$$E = S A \sin \delta_n \cdot \sin \delta_s \int dH_s + S A \cos \delta_n \cdot \cos \delta_s \int [\cos (H_s - H_n)] dH_s$$

Integrando dH

$$E = S A \sin \delta_n \cdot \sin \delta_s \{H_s\} + S A \cos \delta_n \cdot \cos \delta_s \int [\cos (H_s - H_n)] dH_s$$

Co-seno de uma diferença

$$\cos (H_s - H_n) = \cos H_s \cdot \cos H_n + \sin H_s \cdot \sin H_n$$

$$E = S A \sin \delta_n \cdot \sin \delta_s \{H_s\} + S A \cos \delta_n \cdot \cos \delta_s \int [\cos H_s \cdot \cos H_n + \sin H_s \cdot \sin H_n] dH_s$$

Integral indefinida em definida

Integral de uma soma = soma das integrais

$$E = S A \sin \delta_n \cdot \sin \delta_s (H_F - H_I) + S A \cos \delta_n \cdot \cos \delta_s \int [\cos H_s \cdot \cos H_n] dH_s + S A \cos \delta_n \cdot \cos \delta_s \int [\sin H_s \cdot \sin H_n] dH_s$$

...Insolação numa placa plana qualquer...



$$\begin{aligned} E = & S A \operatorname{sen} \delta_n \cdot \operatorname{sen} \delta_s (H_F - H_l) + \\ & + S A \cos \delta_n \cdot \cos \delta_s \int [\cos H_s \cdot \cos H_n] dH_s + \\ & + S A \cos \delta_n \cdot \cos \delta_s \int [\operatorname{sen} H_s \cdot \operatorname{sen} H_n] dH_s \end{aligned}$$

Extrair as constantes do interior das integrais

$$\begin{aligned} E = & S A \operatorname{sen} \delta_n \cdot \operatorname{sen} \delta_s (H_F - H_l) + \\ & + S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n \int [\cos H_s] dH_s + \\ & + S A \cos \delta_n \cdot \cos \delta_s \cdot \operatorname{sen} H_n \int [\operatorname{sen} H_s] dH_s \end{aligned}$$

Integrando $\cos H$ e $\operatorname{sen} H$

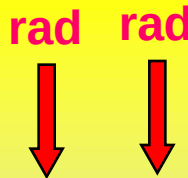
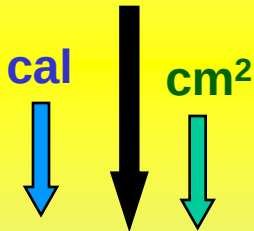
$$\begin{aligned} E = & S A \operatorname{sen} \delta_n \cdot \operatorname{sen} \delta_s (H_F - H_l) + \\ & + S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n \{ \operatorname{sen} H_s \} + \\ & + S A \cos \delta_n \cdot \cos \delta_s \cdot \operatorname{sen} H_n \{ -\cos H_s \} \end{aligned}$$

...Insolação numa placa plana qualquer...



$$\begin{aligned} E = & S A \operatorname{sen} \delta_n \cdot \operatorname{sen} \delta_s (H_F - H_I) + \\ & + S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n \{ \operatorname{sen} H_s \} + \\ & + S A \cos \delta_n \cdot \cos \delta_s \cdot \operatorname{sen} H_n \{ -\cos H_s \} \end{aligned}$$

cal / (rad.cm²)



Onde H_F e H_I são
argumentos do *sen*
ou do *cos* eles
devem entrar em
graus.

$$\begin{aligned} E = & S A \operatorname{sen} \delta_n \cdot \operatorname{sen} \delta_s \cdot (H_F - H_I) + \\ & + S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n (\operatorname{sen} H_F - \operatorname{sen} H_I) + \\ & - S A \cos \delta_n \cdot \cos \delta_s \cdot \operatorname{sen} H_n (\cos H_F - \cos H_I) \end{aligned}$$

...Insolação numa placa plana qualquer...



Onde H_F e H_I são argumentos do *sen* ou do *cos* eles devem entrar em graus.

$$\begin{aligned}
 E = & S A \sin \delta_n \cdot \sin \delta_s \cdot (H_F - H_I) + \\
 & + \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n (\sin H_F - \sin H_I) + \\
 & - \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \sin H_n (\cos H_F - \cos H_I)
 \end{aligned}$$

$$\left[\left[\left[x \right]^{\text{rad}} \times 180 / \pi \right]^0 / 15 \right]^{\text{horas}} \times 60 \right]^{\text{min}}$$

ψ : *psi*

$$\left[x \right]^{\text{rad}} \times 720 / \pi$$

$$\psi \equiv 720 / \pi$$

...Insolação numa placa plana qualquer



cal / (min.cm²)

cal
↓

cm²
↓

graus
↓

graus
↓

Onde H_F e H_I são argumentos do $\sin$ ou do $\cos$. Eles devem entrar em graus.

$$E = 4 S A \sin \delta_n \cdot \sin \delta_s \cdot (H_F - H_I) +$$

$$+ \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n (\sin H_F - \sin H_I) +$$

$$- \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \sin H_n (\cos H_F - \cos H_I)$$

[[[x]⁰ / 15]^{horas} x 60]^{min}

[x]⁰ x 4

Expressão simplificada da insolação

cal / (min.cm²)

cal
↓

cm²
↓

graus
↓

graus
↓

Onde H_F e H_I são argumentos do *sen* ou do *cos*. Eles devem entrar em graus.

Expressão geral

$$E = 4 S A \operatorname{sen} \delta_n \cdot \operatorname{sen} \delta_s \cdot (H_F - H_I) +$$

$$+ \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n (\operatorname{sen} H_F - \operatorname{sen} H_I) +$$

$$- \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \operatorname{sen} H_n (\cos H_F - \cos H_I)$$

$$\psi \equiv 720 / \pi$$

Expressão simplificada

graus
↓

graus
↓

Batizados:

$$W \equiv 4 S A \operatorname{sen} \delta_n \cdot \operatorname{sen} \delta_s$$

$$Q \equiv + \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n$$

$$C \equiv - \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \operatorname{sen} H_n$$

$$E = W \cdot (H_F - H_I) + Q \cdot (\operatorname{sen} H_F - \operatorname{sen} H_I) + C \cdot (\cos H_F - \cos H_I)$$

Finalmente: Expressão simplificada da insolação

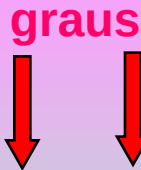
$$\psi \equiv 720 / \pi$$

Batizados:

$$W \equiv 4 S A \operatorname{sen} \delta_n \cdot \operatorname{sen} \delta_s$$

$$Q \equiv + \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n$$

$$C \equiv - \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \operatorname{sen} H_n$$

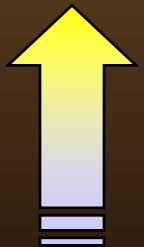
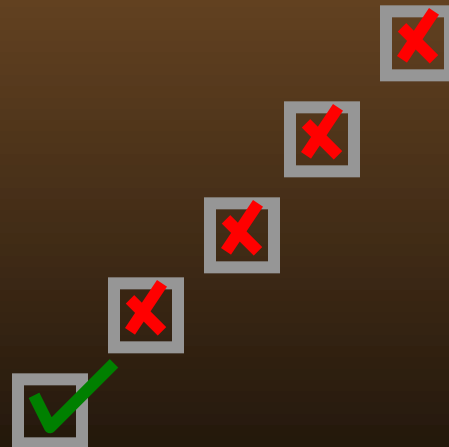


$$E = W \cdot (H_F - H_I) + Q \cdot (\operatorname{sen} H_F - \operatorname{sen} H_I) + C \cdot (\cos H_F - \cos H_I)$$

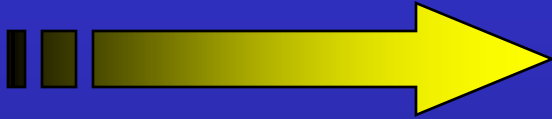
Próximo passo

**Adaptação da fórmula geral para
calcular a insolação em placas
planas com posicionamentos
particulares:**

Horizontal
Vertical Meridiana
Vertical Leste-Oeste
Vertical Qualquer
Qualquer

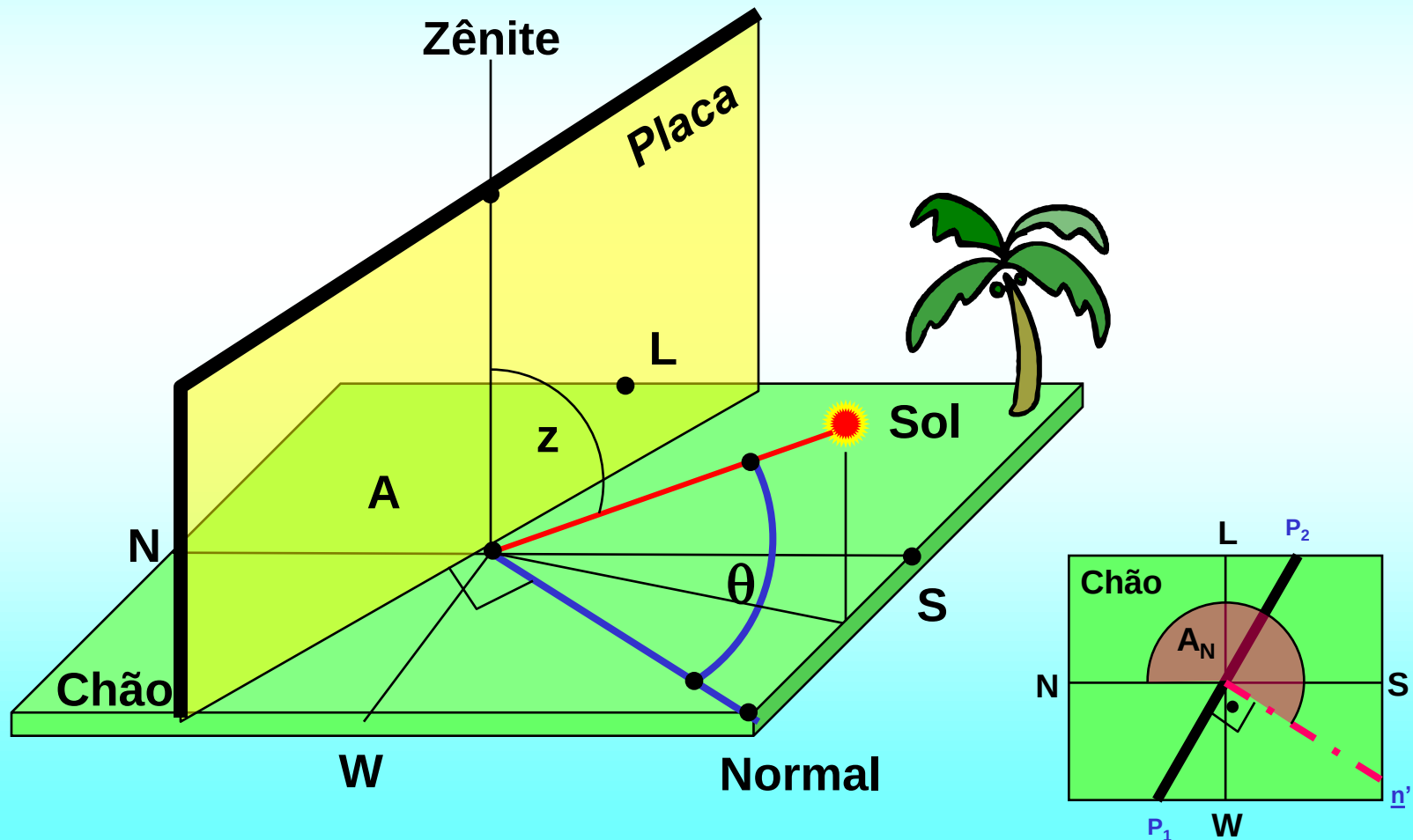


Horizontal
Vertical Meridiana
Vertical Leste-Oeste
Vertical Qualquer
Qualquer

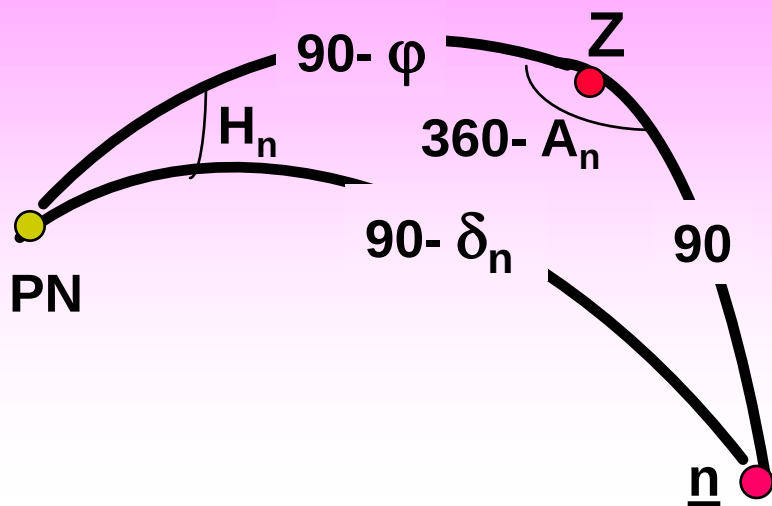


**Placa Vertical
Qualquer**

Placa Vertical Qualquer



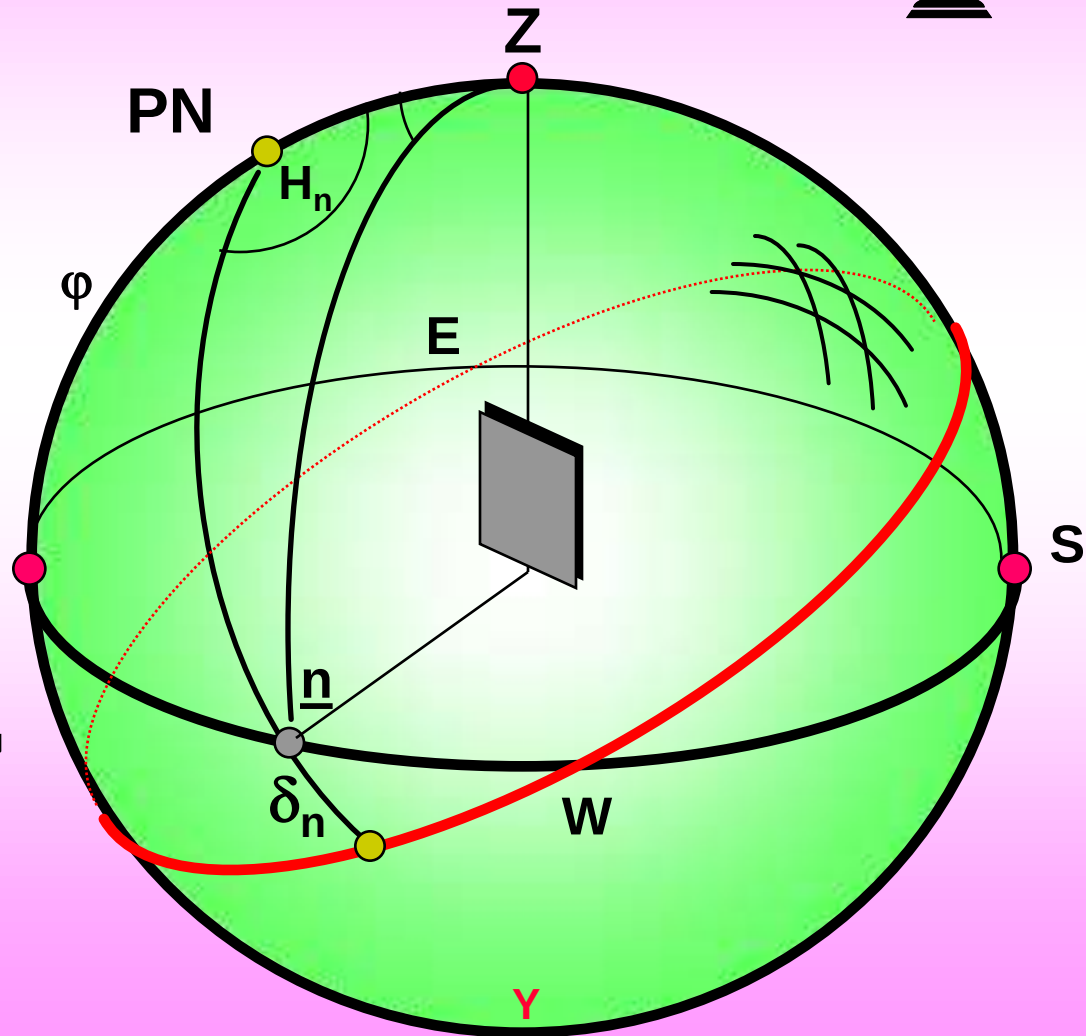
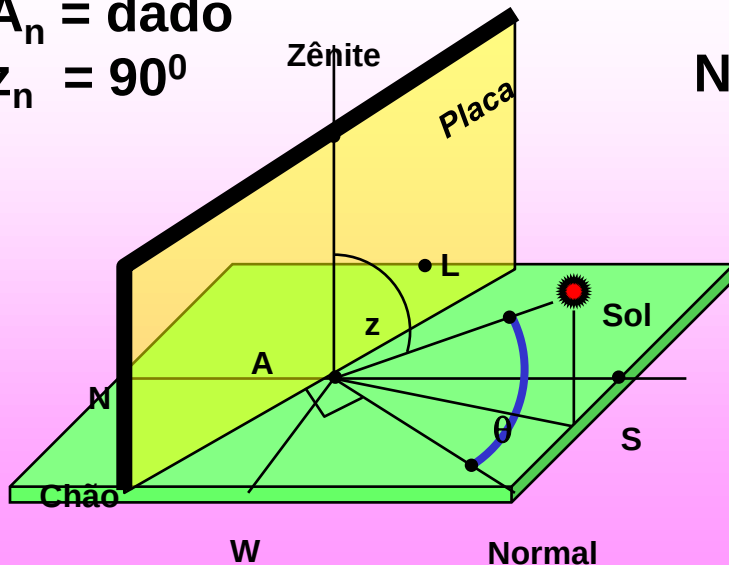
H e δ de Placa Vertical Qualquer no HN



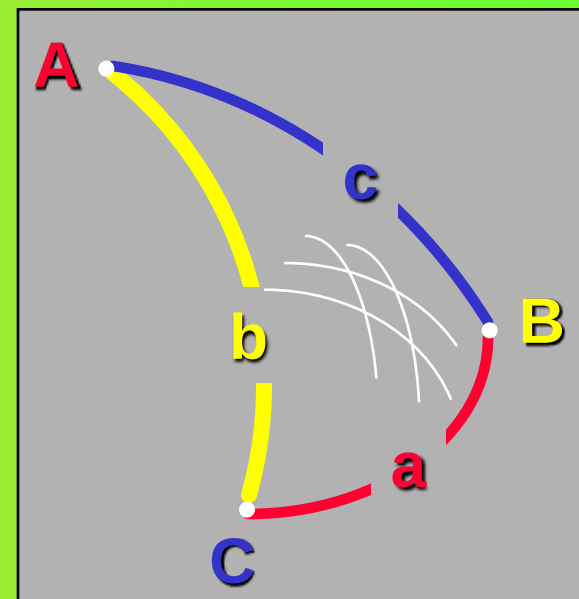
$\underline{n} \equiv$ Ponto do Horizonte

$A_n = \text{dado}$

$z_n = 90^\circ$



Resumo das Fórmulas de Trigonometria Esférica



Co-seno

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

Seno

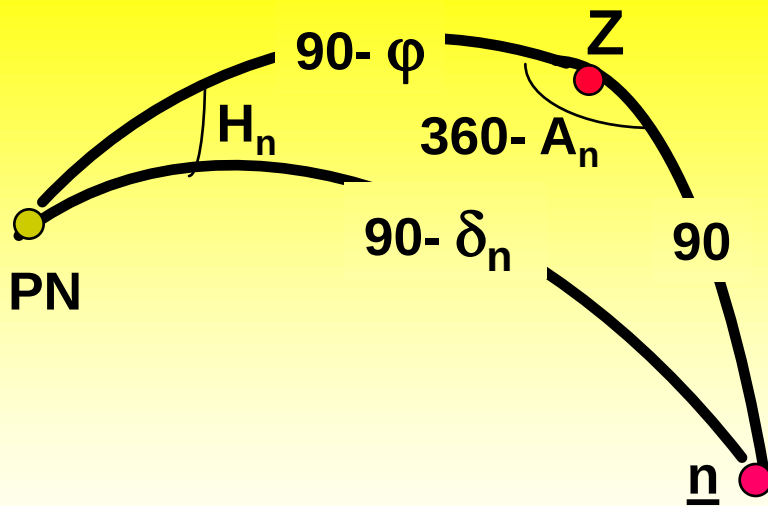
$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Seno & Co-seno

$$\sin a \cdot \cos B = \cos b \cdot \sin c - \sin b \cdot \cos c \cdot \cos A$$

H e δ de Placa Vertical Qualquer no HN

$A_n = \text{dado}$



C $\cos (90 - \delta_n) = \cos (90) \cdot \cos (90 - \varphi) + \sin (90) \cdot \sin (90 - \varphi) \cdot \cos (360 - A_n)$

$$\sin \delta_n = \cos \varphi \cdot \cos A_n$$



$$\delta_n$$

S&C

$$\sin (90 - \delta_n) \cdot \cos H_n = \cos (90) \cdot \sin (90 - \varphi) - \sin (90) \cdot \cos (90 - \varphi) \cdot \cos (360 - A_n)$$

$$\cos H_n = - \sin \varphi \cdot \cos A_n / \cos \delta_n$$

S

$$\sin (90) / \sin H_n = \sin (90 - \delta_n) / \sin (360 - A_n)$$



$$\Rightarrow 0 \leq \underline{H}_n \leq 180^\circ$$

$$\sin H_n = - \sin A_n / \cos \delta_n$$

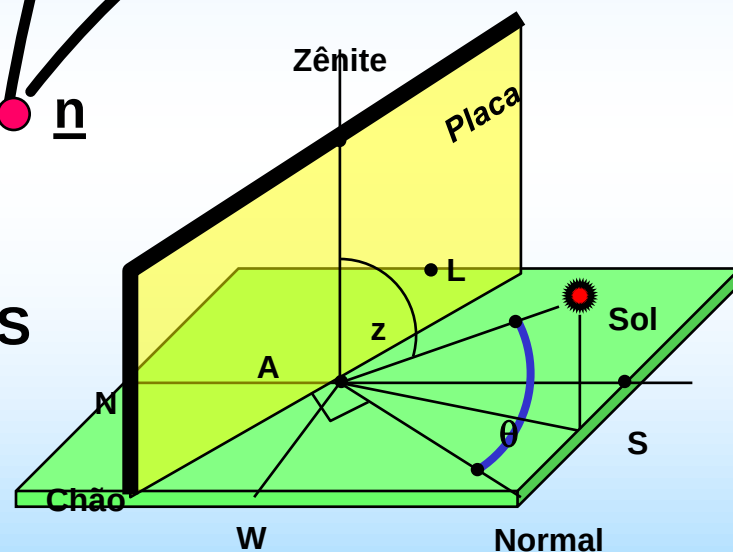
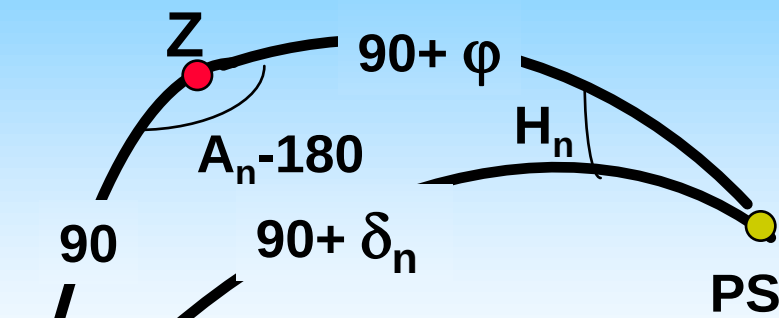
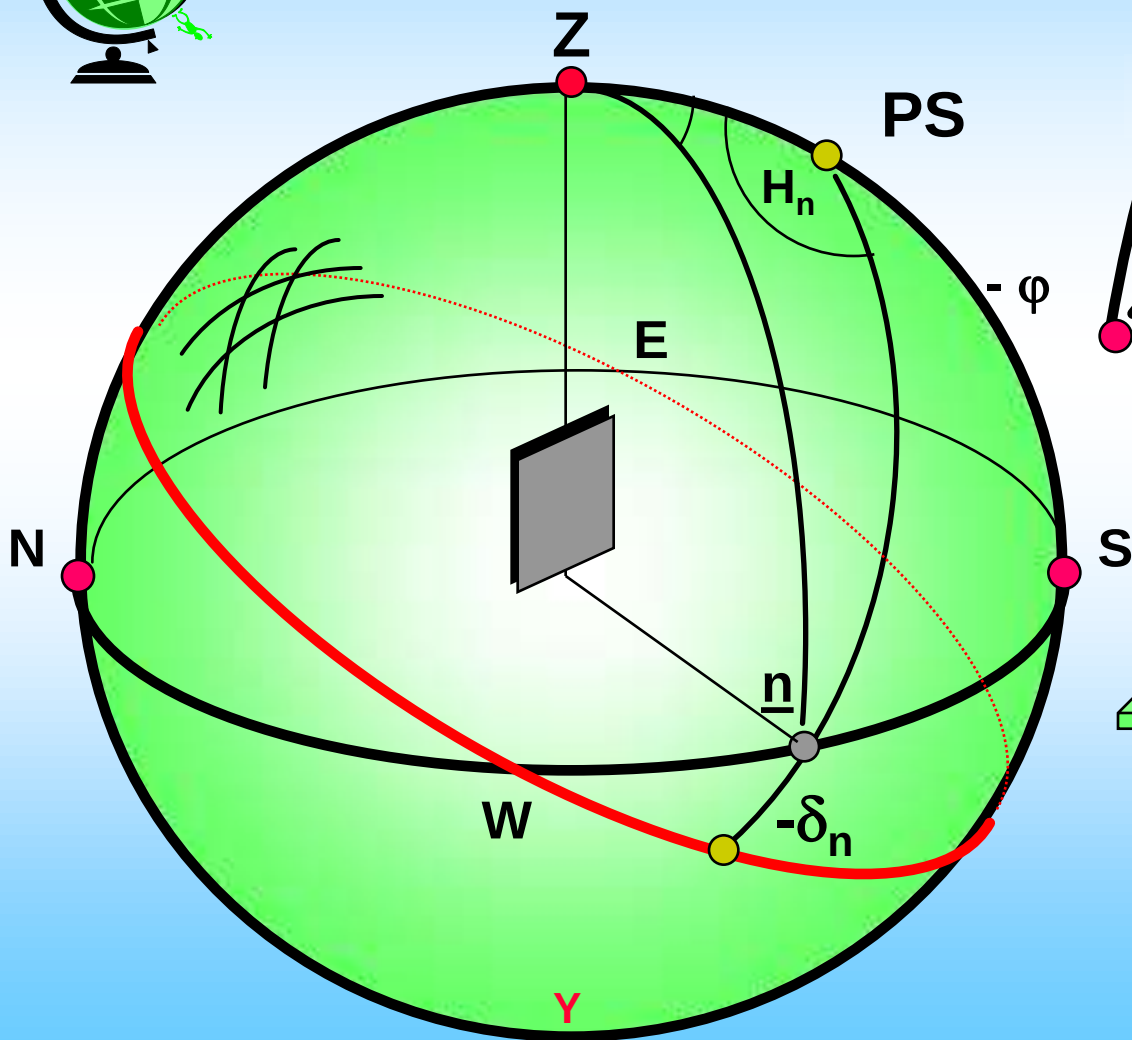
Se $\sin H_n \geq 0$ então $H_n = \underline{H}_n$

Se $\sin H_n < 0$ então $H_n = 360 - \underline{H}_n$



$$H_n$$

H e δ de Placa Vertical Qualquer no HS



$\underline{n} \equiv$ Ponto do Horizonte

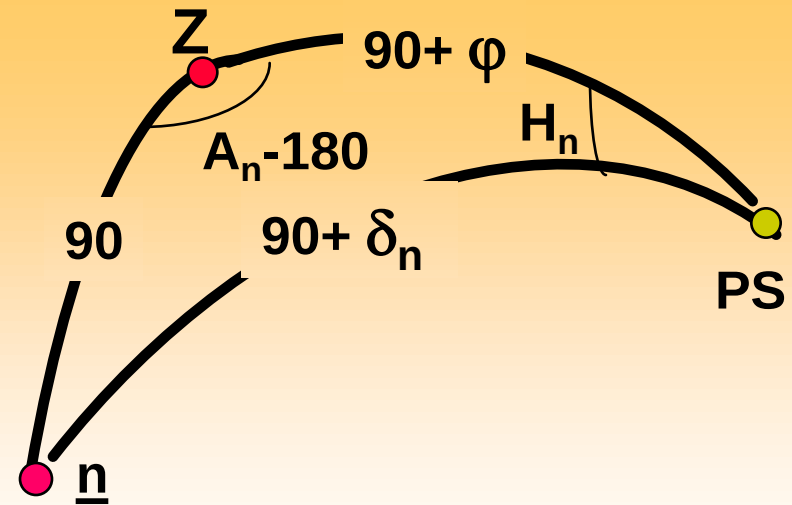
$A_n =$ dado

$z_n = 90^\circ$

H e δ de Placa Vertical Qualquer no HS



$A_n = \text{dado}$



$$\cos (90+\delta_n) = \cos (90) \cdot \cos (90+\varphi) + \sin (90) \cdot \sin (90+\varphi) \cdot \cos (A_n-180) \\ - \sin \delta_n = \cos \varphi \cdot (-\cos A_n)$$

$$\sin \delta_n = \cos \varphi \cdot \cos A_n$$



δ_n



$$\sin (90+\delta_n) \cos H_n = \cos (90) \cdot \sin (90+\varphi) - \sin (90) \cdot \cos (90+\varphi) \cdot \cos (A_n-180)$$

$$\cos H_n = -\sin \varphi \cdot \cos A_n / \cos \delta_n$$



$$\sin (90) / \sin H_n = \sin (90+\delta_n) / \sin (A_n-180)$$



$$\Rightarrow 0 \leq H_n \leq 180^\circ$$

$$\sin H_n = -\sin A_n / \cos \delta_n$$

Se $\sin H_n \geq 0$ então $H_n = H_n$

Se $\sin H_n < 0$ então $H_n = 360 - H_n$



H_n

Insolação numa Placa Vertical Qualquer

Batizados:

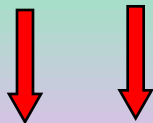


$$W \equiv \text{4 S A } \sin \delta_n \cdot \sin \delta_s$$

$$Q \equiv + \text{\textcolor{red}{\psi} S A } \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n$$

$$C \equiv - \text{\textcolor{red}{\psi} S A } \cos \delta_n \cdot \cos \delta_s \cdot \sin H_n$$

graus



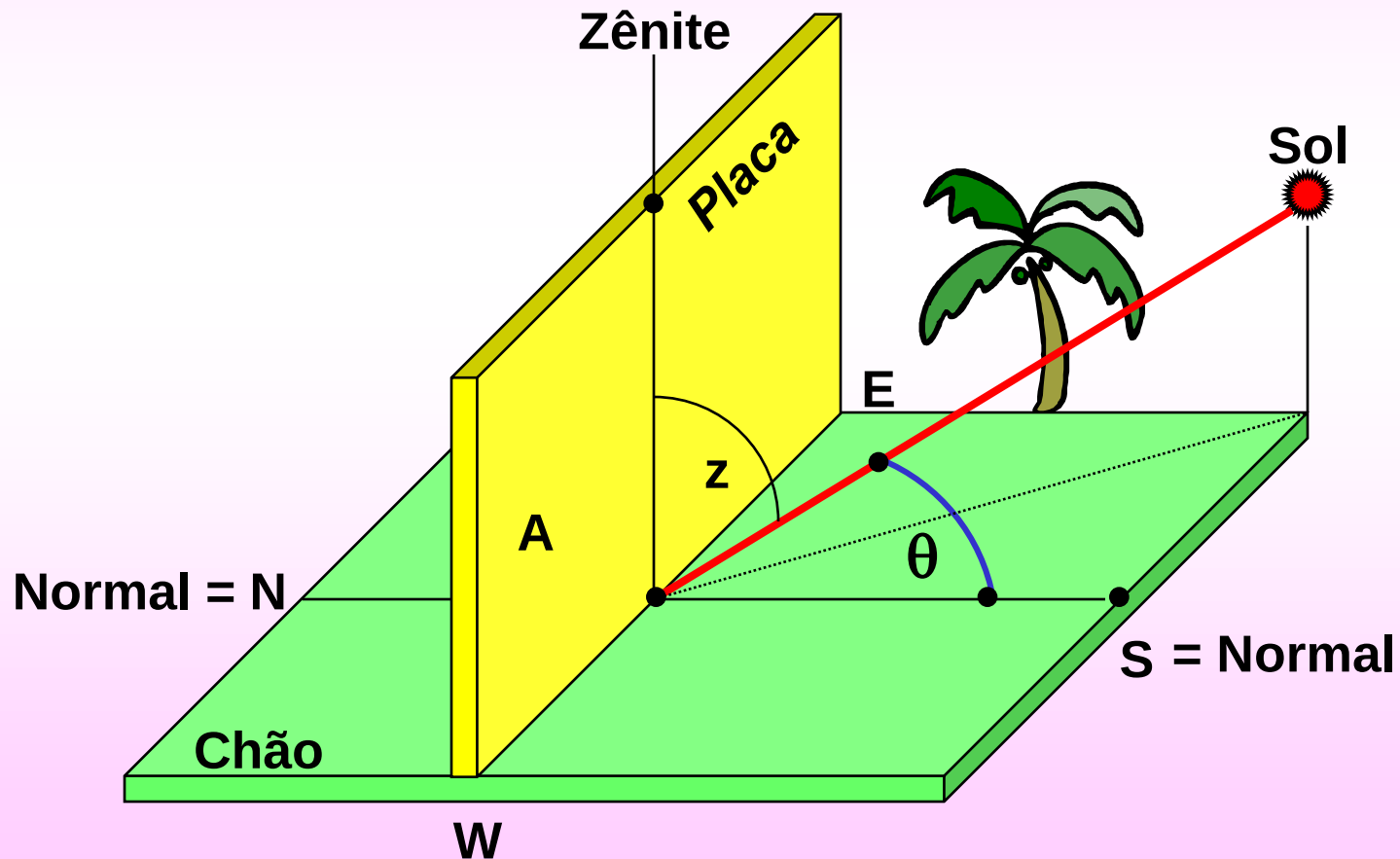
$$E = W \cdot (\text{\textcolor{red}{H}_F} - \text{\textcolor{red}{H}_I}) + Q \cdot (\sin H_F - \sin H_I) + C \cdot (\cos H_F - \cos H_I)$$



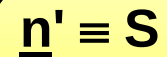
Horizontal
Vertical Meridiana
Vertical Leste-Oeste
Vertical Qualquer
Qualquer

**Placa vertical
Leste-Oeste**

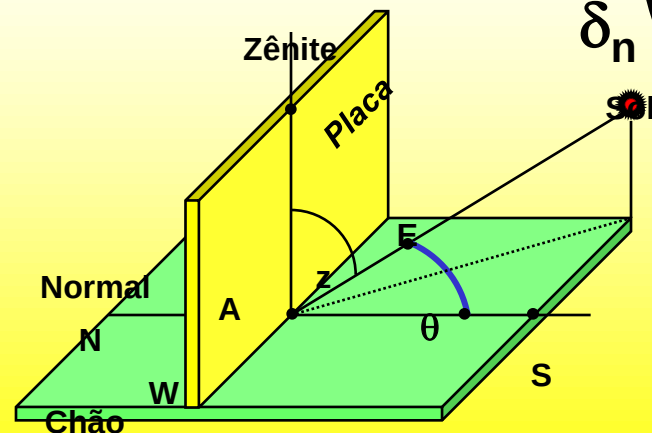
Placa vertical Leste-Oeste



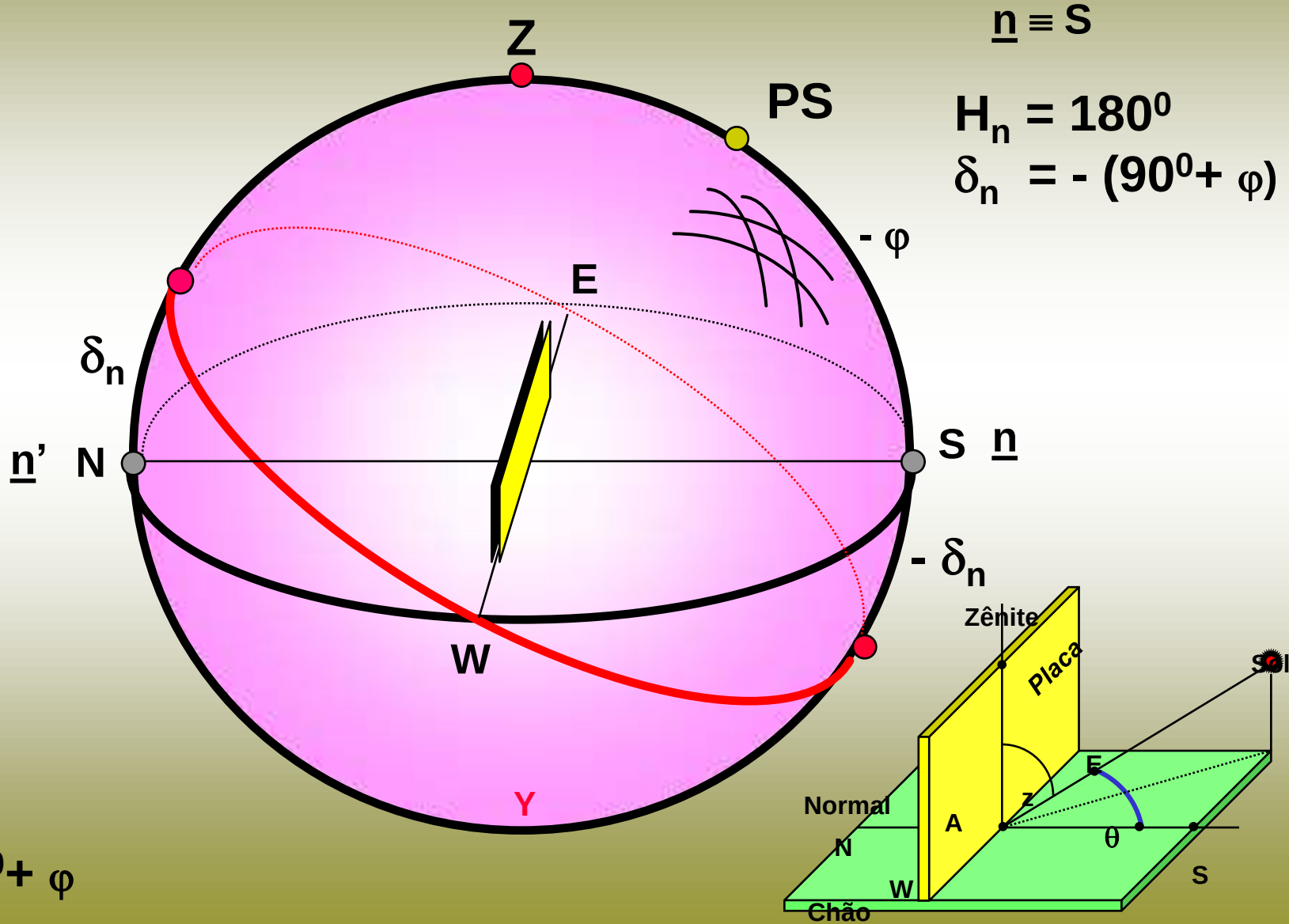
$$\delta_n = 90^\circ - \varphi$$



$$\begin{aligned} \mathbf{H}_n &= \mathbf{0}^0 \\ \delta_n &= - (90^\circ - \varphi) \end{aligned}$$



H e δ de Placa Vertical Leste-Oeste no HS



Insolação numa Placa Vertical Leste-Oeste

Como

$$H_n = 0$$

ou

$$H_n = 180^\circ$$

$$\rightarrow \sin H_n = 0$$

$$\rightarrow \cos H_n = \pm 1$$

Batizados:

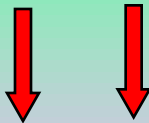


$$W \equiv 4 S A \sin \delta_n \cdot \sin \delta_s$$

$$Q \equiv + \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n$$

$$C \equiv - \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \sin H_n$$

graus



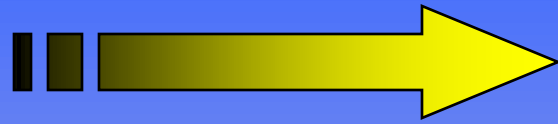
$$W = 4 S A \sin \delta_n \cdot \sin \delta_s$$

$$Q = + \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n$$

$$C = - \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \sin H_n = 0$$

$$E = W \cdot (H_F - H_I) + Q \cdot (\sin H_F - \sin H_I) + C \cdot (\cos H_F - \cos H_I)$$

$$E = W \cdot (H_F - H_I) + Q \cdot (\sin H_F - \sin H_I)$$



Horizontal

Vertical Meridiana

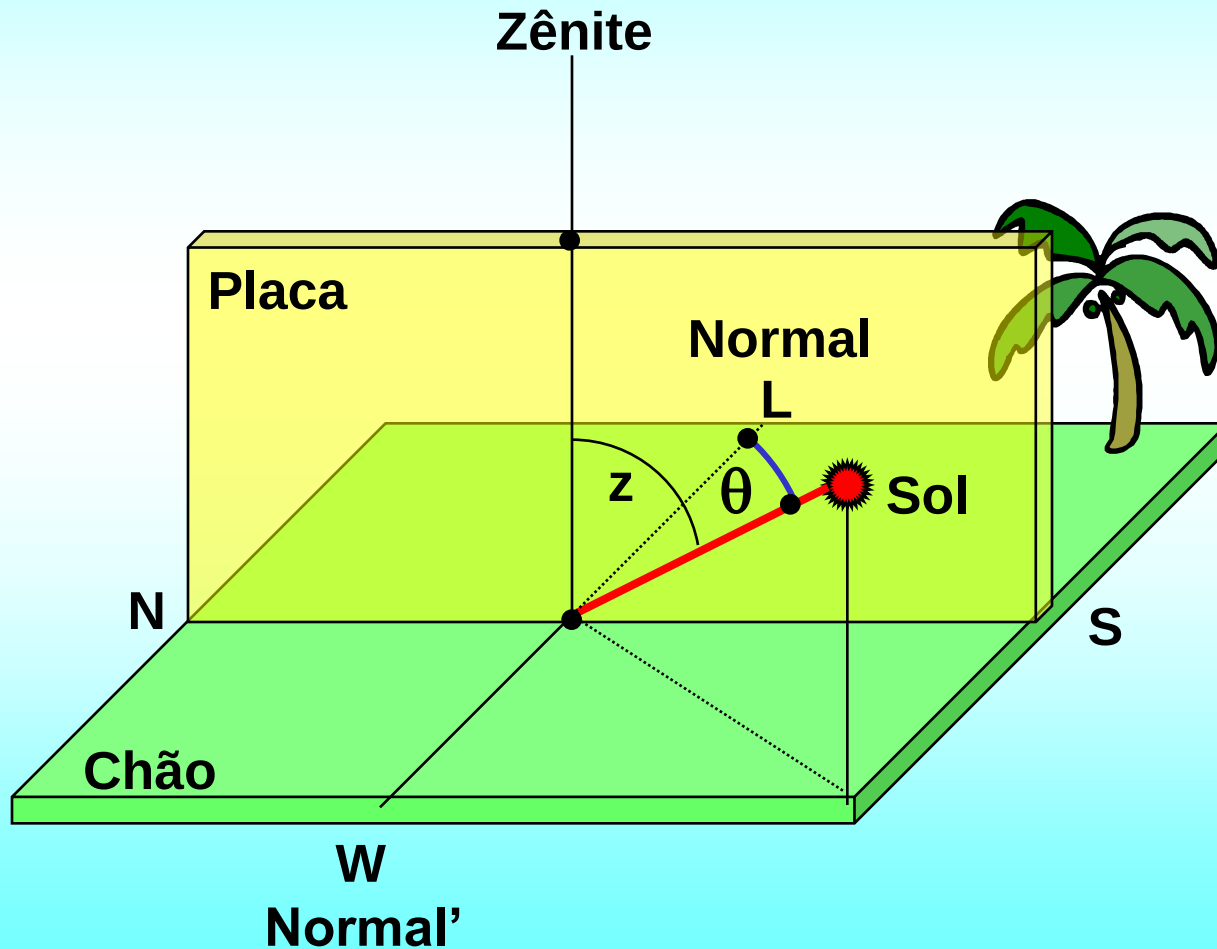
Vertical Leste-Oeste

Vertical Qualquer

Qualquer

**Plana vertical
meridiana**

Placa Vertical Meridiana



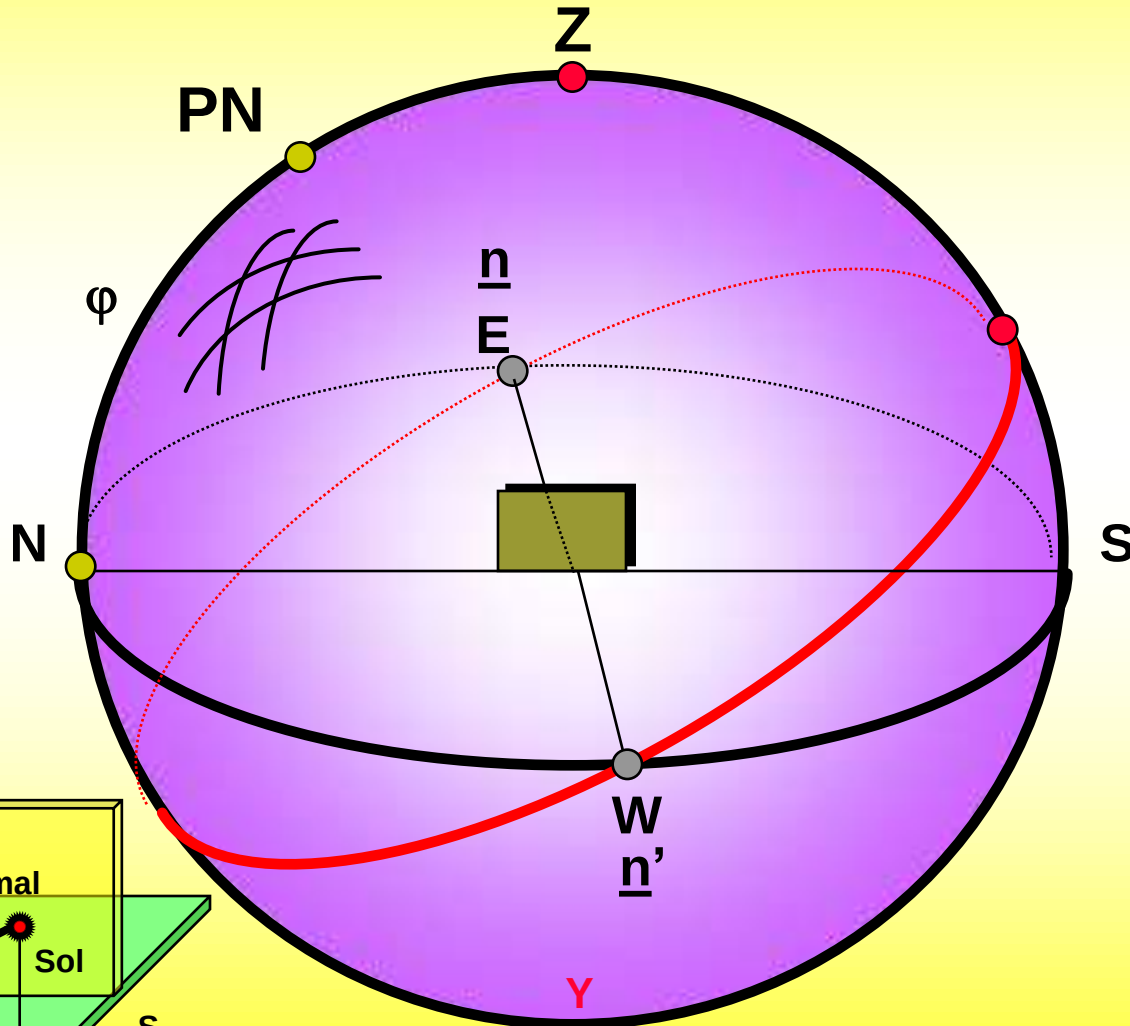
H e δ de Placa Vertical Meridiana no HN



$$\underline{n} \equiv E$$

$$H_n = -90^\circ$$

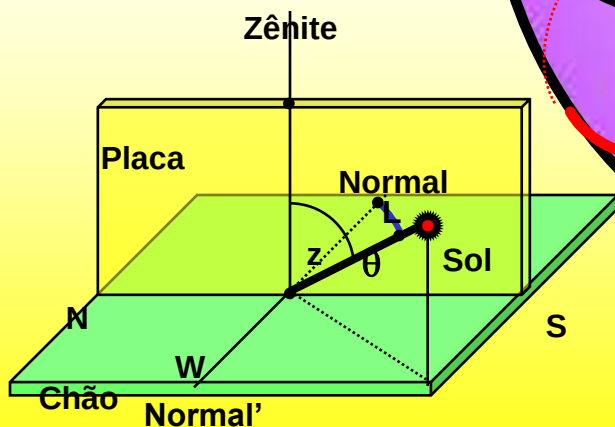
$$\delta_n = +0$$



$$\underline{n}' \equiv W$$

$$H_n = +90^\circ$$

$$\delta_n = +0$$



Insolação numa Placa Vertical Meridiana

Como $\delta_n = 0 \rightarrow \sin \delta_n = 0$
 $\cos \delta_n = 1$

Como $H_n = \pm 90 \rightarrow \cos H_n = 0 \rightarrow \sin H_n = \pm 1$



Batizados:

$$W = 4 S A \sin \delta_n \cdot \sin \delta_s$$

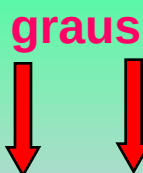
$$Q = + \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n$$

$$C = - \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \sin H_n$$

$$W = 4 S A \cancel{\sin \delta_n} \cdot \sin \delta_s = 0$$

$$Q = + \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \cancel{\cos H_n} = 0$$

$$C = - \psi S A \cdot \cos \delta_s \cdot \sin H_n$$



$$E = W \cdot (H_F - H_I) + Q \cdot (\sin H_F - \sin H_I) + C \cdot (\cos H_F - \cos H_I)$$

$$E = C \cdot (\cos H_F - \cos H_I)$$

Placa plana horizontal



Horizontal

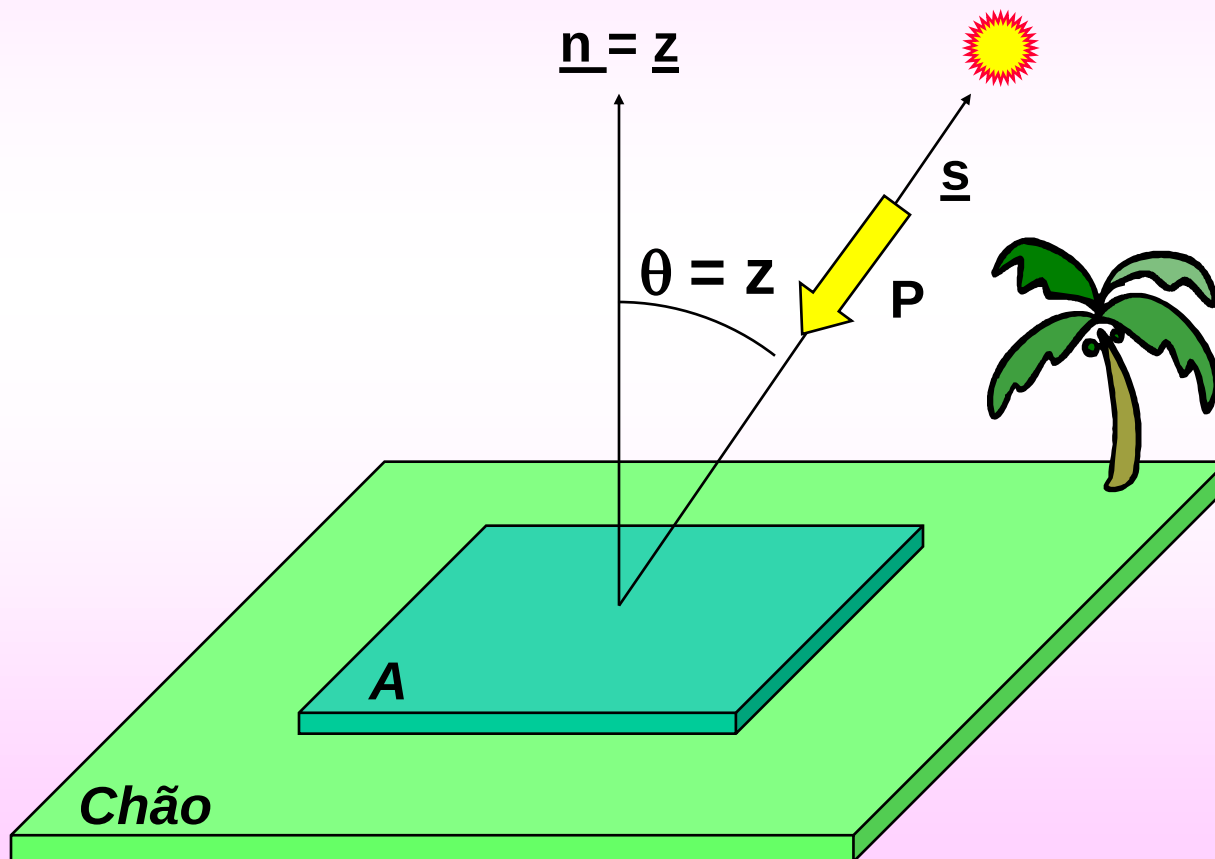
Vertical Meridiana

Vertical Leste-Oeste

Vertical Qualquer

Qualquer

Placa horizontal

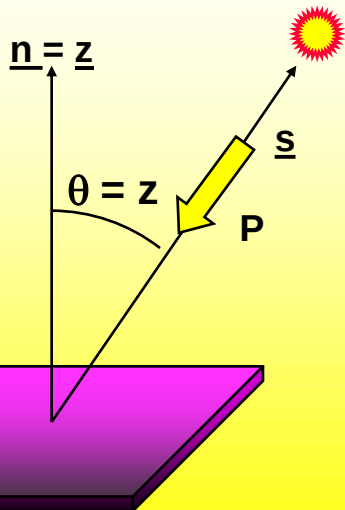
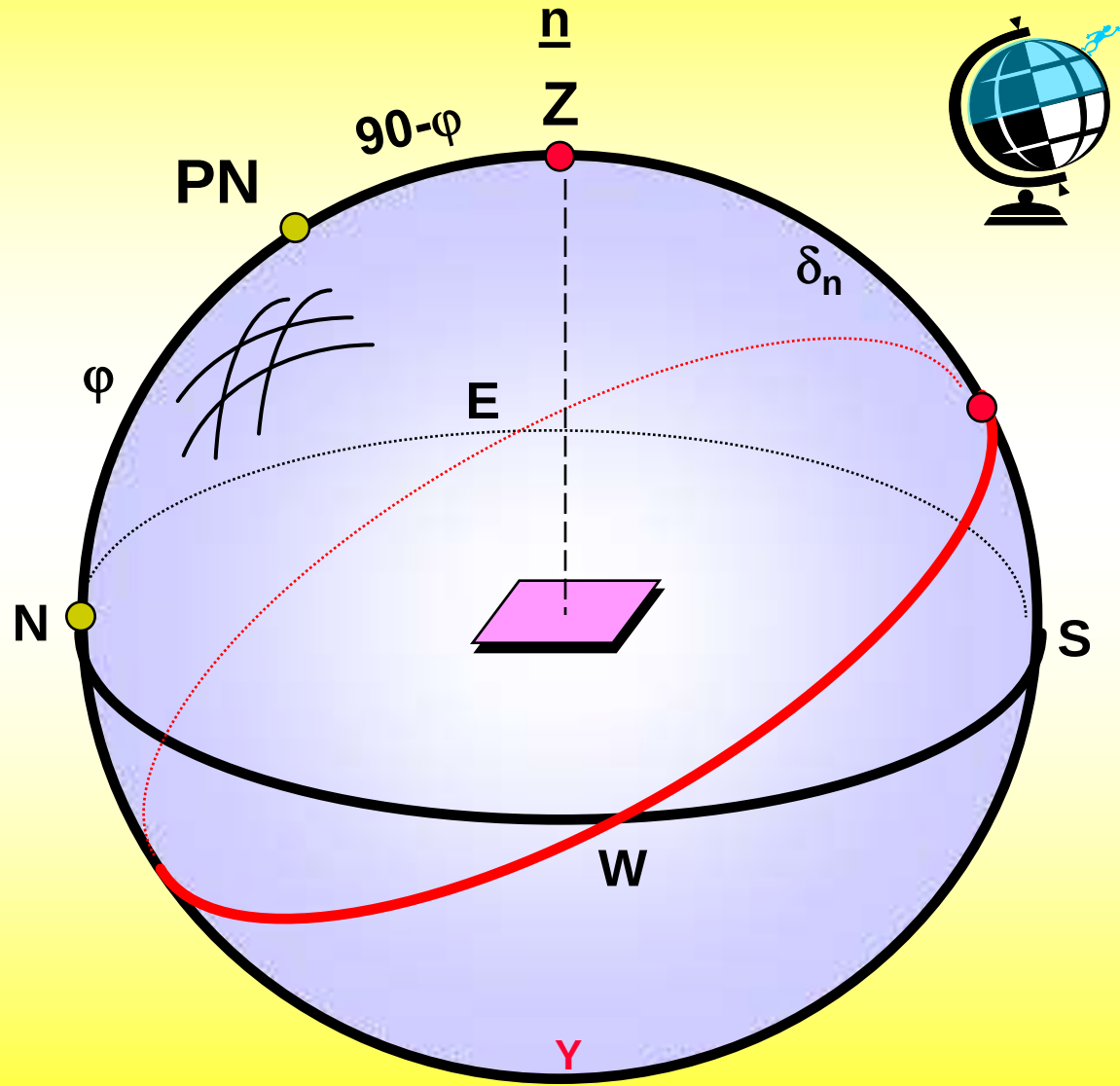


H e δ de Placa horizontal no HN

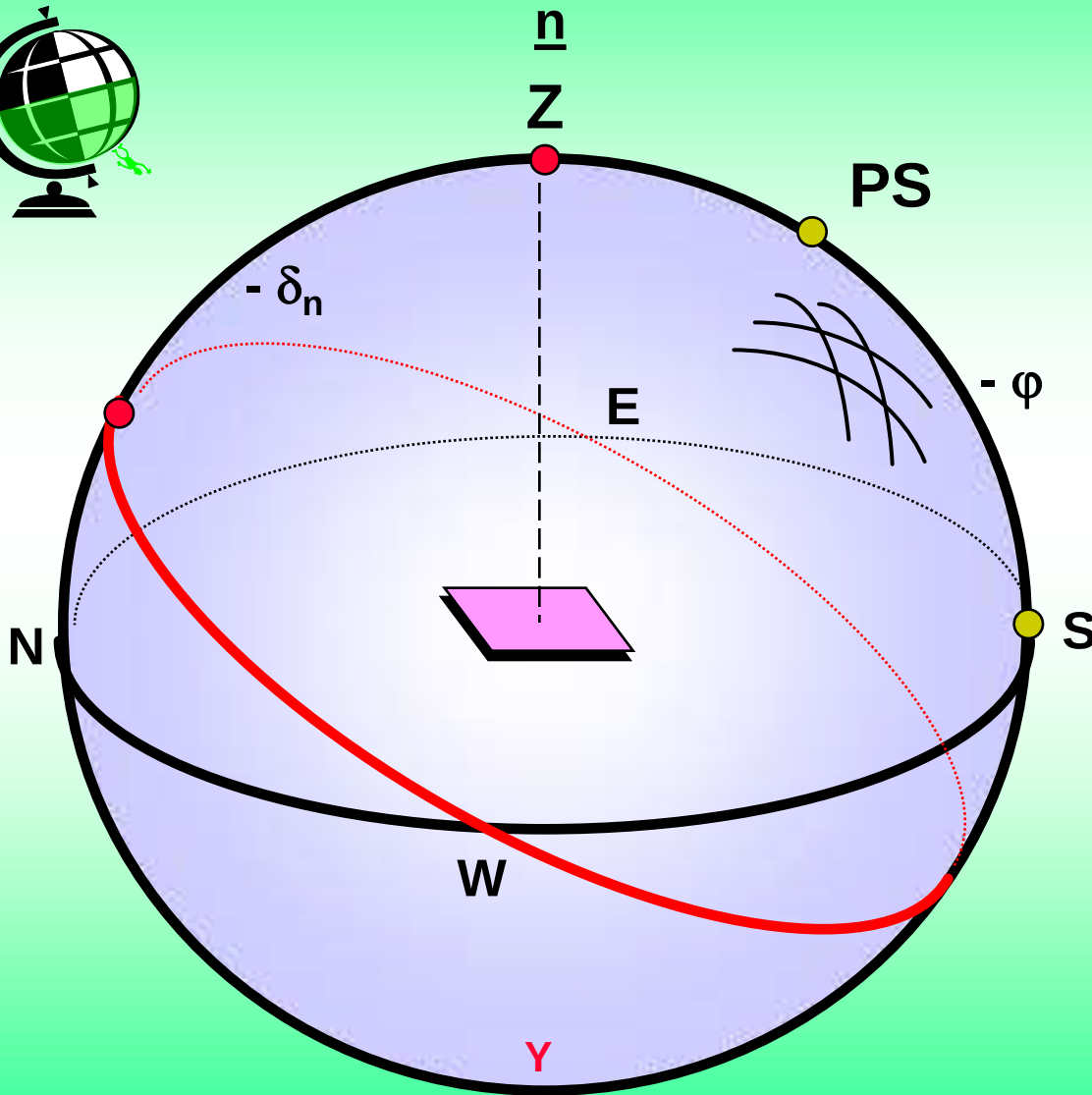
$$\underline{n} \equiv Z$$

$$H_n = 0$$

$$\delta_n = \varphi$$



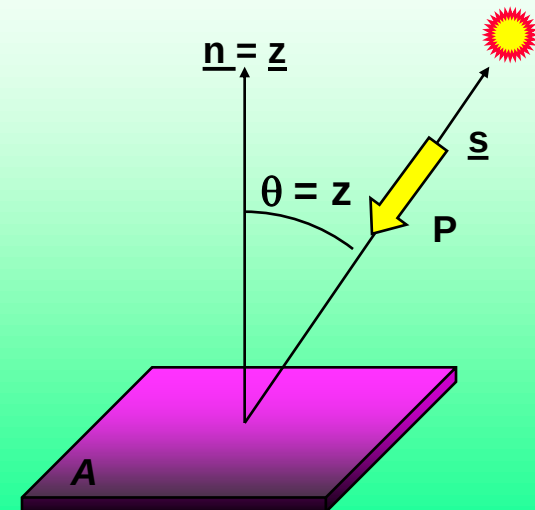
H e δ de Placa horizontal no HS



$$\underline{n} \equiv Z$$

$$H_n = 0$$

$$\delta_n = \phi$$



Insolação numa Placa Horizontal

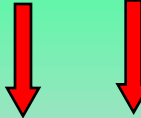
Como $\delta_n = \varphi \rightarrow \sin \delta_n = \sin \varphi$
 $\cos \delta_n = \cos \varphi$

Como $H_n = 0 \rightarrow \sin H_n = 0$
 $\cos H_n = 1$



$$\begin{aligned} W &= 4 S A \sin \delta_n \cdot \sin \delta_s \\ Q &= + \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n \\ C &= - \psi S A \cos \delta_n \cdot \cos \delta_s \cdot \sin H_n \end{aligned}$$

$$\begin{aligned} W &= 4 S A \sin \varphi \cdot \sin \delta_s \\ Q &= + \psi S A \cos \varphi \cdot \cos \delta_s \cdot \underline{1} \\ C &= - \psi S A \cos \varphi \cdot \cos \delta_s \cdot \underline{\sin H_n} = 0 \end{aligned}$$

graus


$$E = W \cdot (H_F - H_I) + Q \cdot (\sin H_F - \sin H_I) + C \cdot (\cos H_F - \cos H_I)$$

$$E = W \cdot (H_F - H_I) + Q \cdot (\sin H_F - \sin H_I)$$

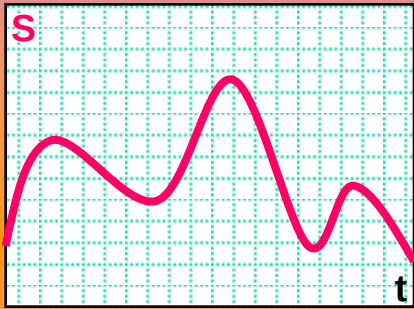
Quando S não
puder ser

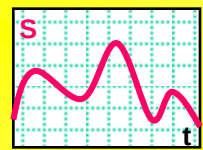
considerado

constante no

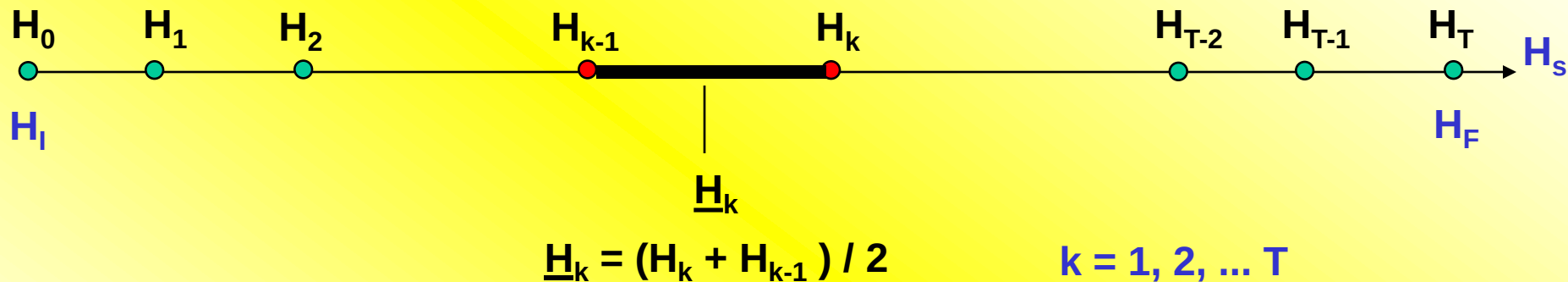
intervalo de tempo

$$t_F - t_I$$



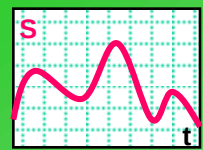


Partição do intervalo ($t_F - t_I$)



1. Dividir o intervalo H_I a H_F em T subintervalos
2. Calcular o ângulo horário médio $\underline{H}_k$ de cada subintervalo
3. Calcular a distância zenital z_s do Sol em cada subintervalo no instante $\underline{H}_k$

$$\cos \underline{z}_k = \sin \varphi \cdot \sin \delta_s + \cos \varphi \cdot \cos \delta_s \cdot \cos \underline{H}_k$$



Correção da distância heliocêntrica

$$K = 1 + 0,03344 \cos \left((360 \text{ d} / 365,25) - 2,8^\circ \right)$$

Cálculo da influência da pressão atmosférica

Para L qualquer: $(p/p_{\text{mar}}) = e^{-(L/8.000)}$

Para $L < 4000 \text{ m}$: $(p/p_{\text{mar}}) = 1 - (L/10.000)$

Cálculo da massa óptica

$$h = 90^\circ - z_k$$

Para $h > 10^\circ$ $m_k = (p/p_{\text{mar}}) \{ 1 / \text{sen } h \}$

Para $h \leq 10^\circ$ $m_k = (p/p_{\text{mar}}) \{ 1 / [\text{sen } h + 0,15 (h + 3,885)^{-1,235}] \}$

Cálculo da espessura óptica de Rayleigh

$$\delta_R = 1 / (0,9 m_k + 9,4)$$

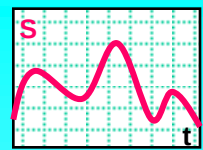
Cálculo da espessura óptica

$$\tau_k \equiv \delta_R m_k$$

Calcular a
"Constante
Solar" para
cada
subintervalo k
 $k = 1, 2, \dots T$

Cálculo da "Constante Solar" no local

$$S_k = K \cdot S_o \cdot \left(e^{-\tau} \right)_k$$



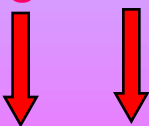
Calcular a insolação em cada subintervalo e no total

Para cada $k = 1, 2, \dots T$

$$\begin{aligned}
 W &\equiv \text{⚡ } 4 S_k A \sin \delta_n \cdot \sin \delta_s \\
 Q &\equiv + \psi S_k A \cos \delta_n \cdot \cos \delta_s \cdot \cos H_n \\
 C &\equiv - \psi S_k A \cos \delta_n \cdot \cos \delta_s \cdot \sin H_n
 \end{aligned}$$

Insolação
em cada
intervalo

graus



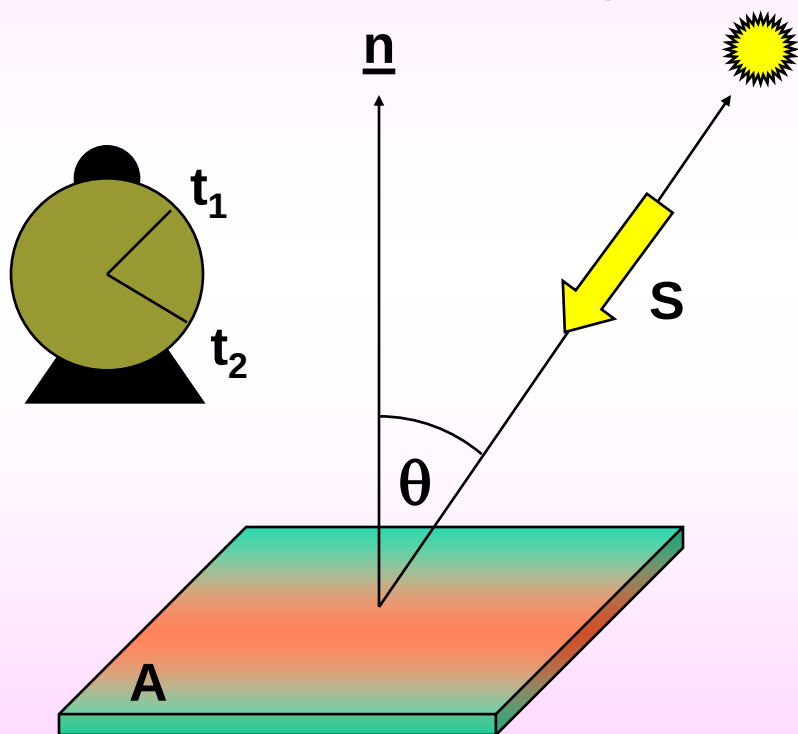
$$E_k = W_k \cdot (H_k - H_{k-1}) + Q_k \cdot (\sin H_k - \sin H_{k-1}) + C_k \cdot (\cos H_k - \cos H_{k-1})$$

Insolação
total

$$E = E_0 + E_1 + E_2 + \dots + E_T$$

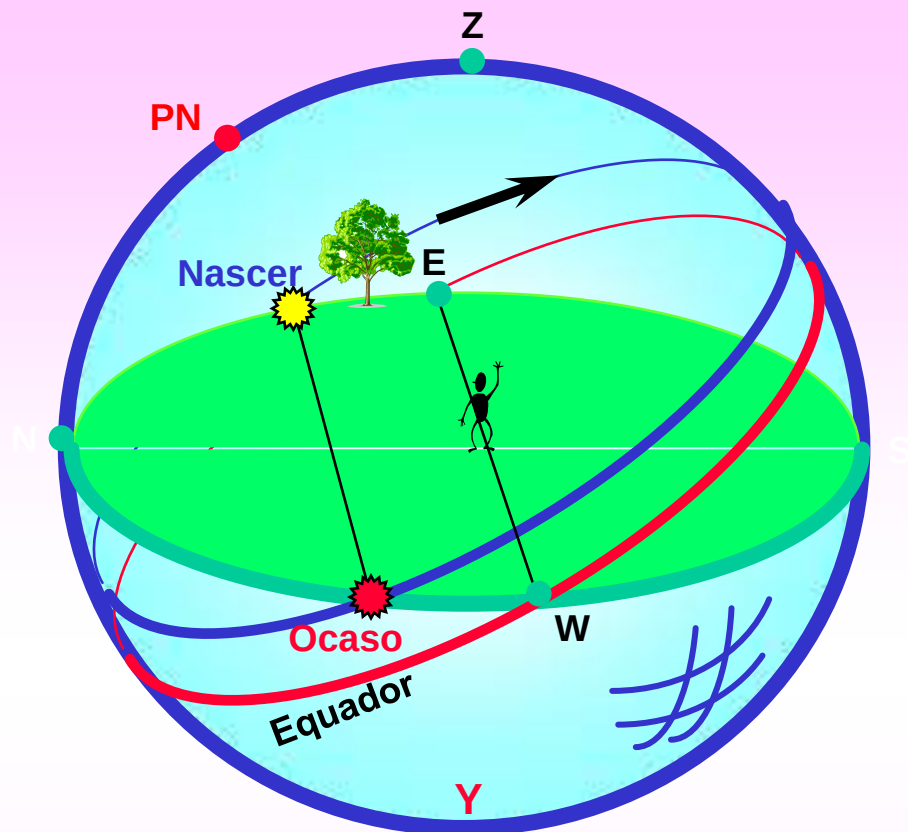
Limites temporais da Integração

Limite temporal da integração da Insolação



Limites de integração:

- ❖ Sol entre seu nascer e seu ocaso
- ❖ Sol deve estar na frente da placa.



$$E = \int S A \cos \theta dt$$

cal

cm²

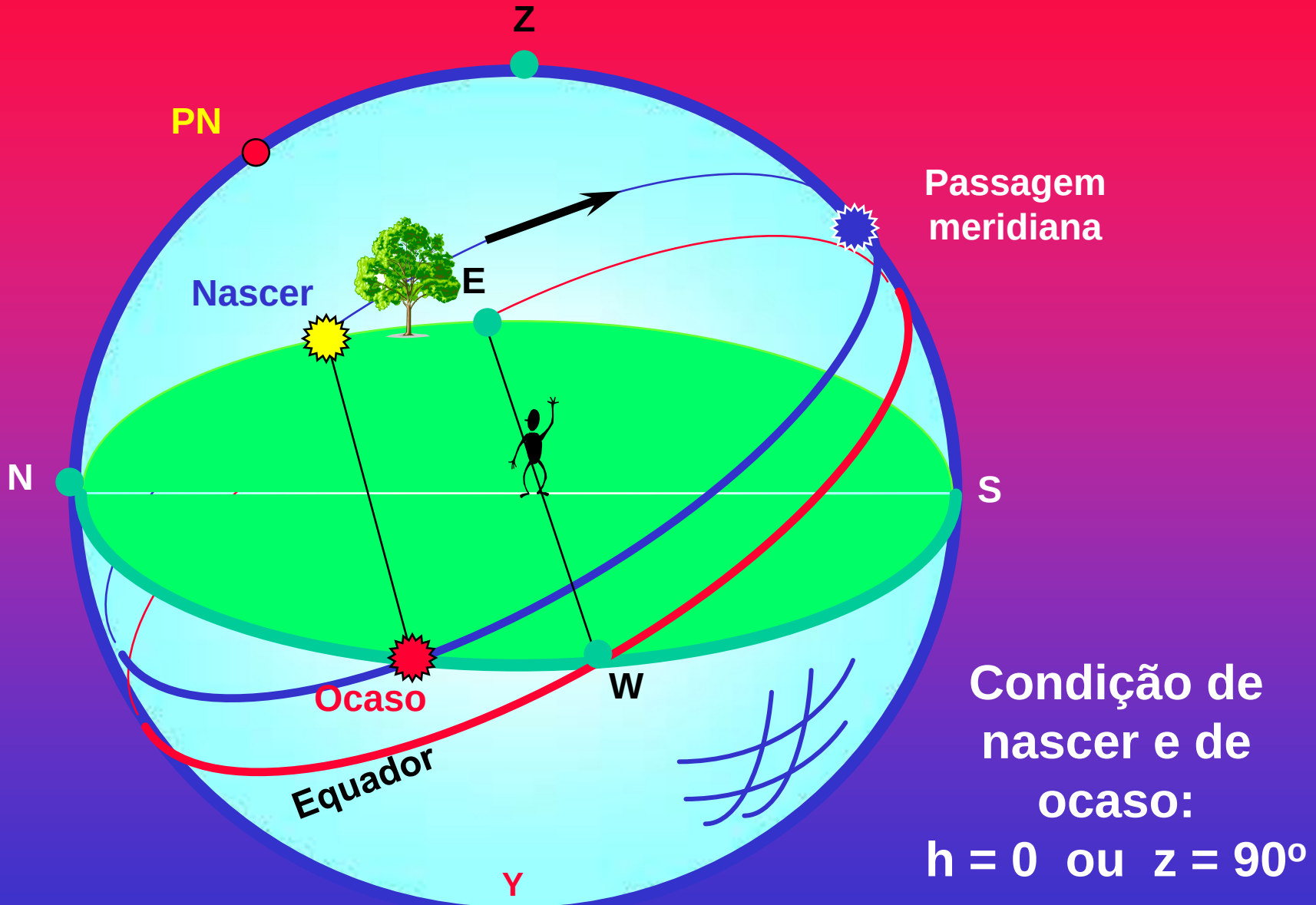
min

cal/cm².min

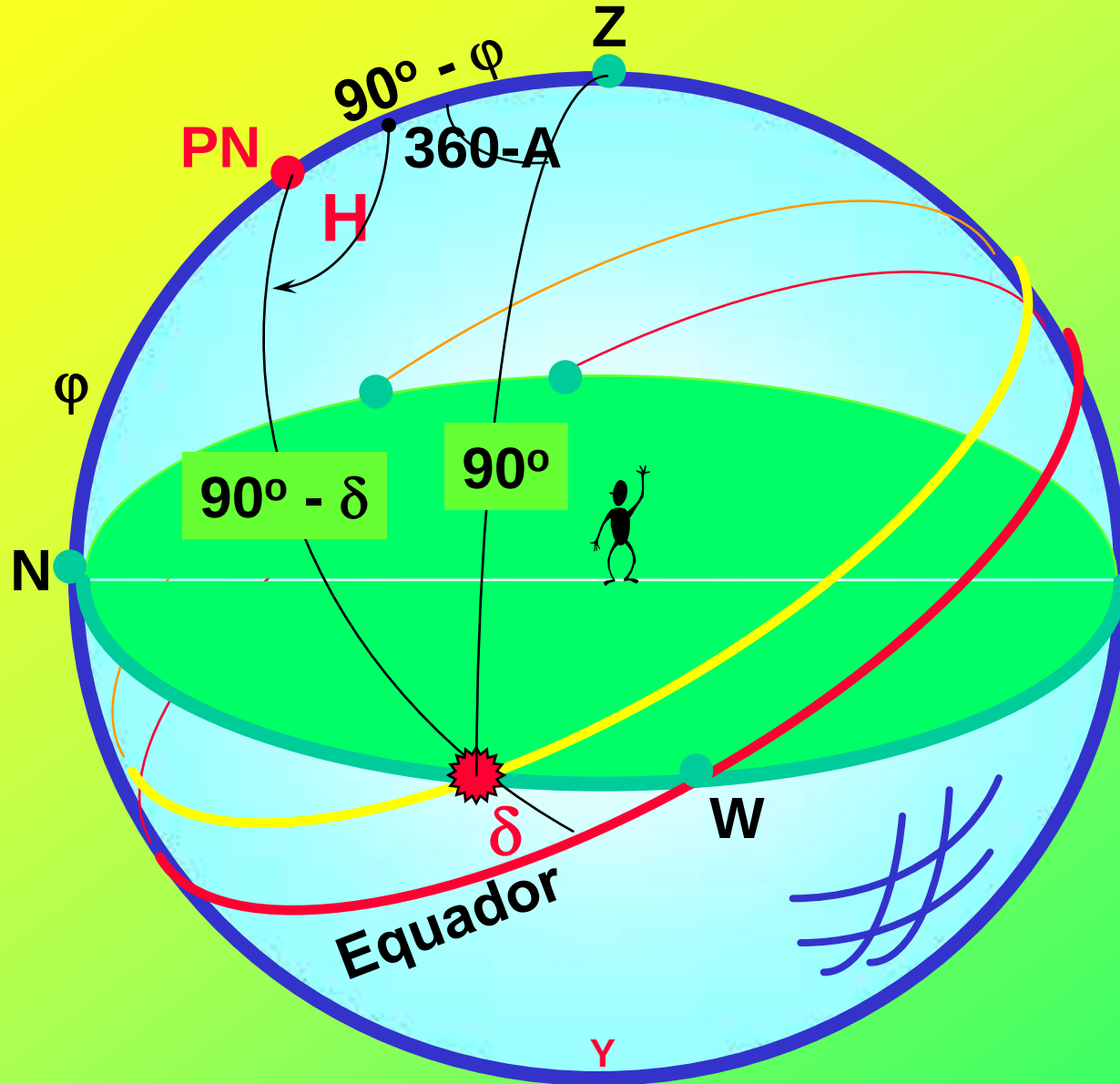


**Ângulo horário e Azimute do
nascer e do ocaso, sem
considerar a refração**

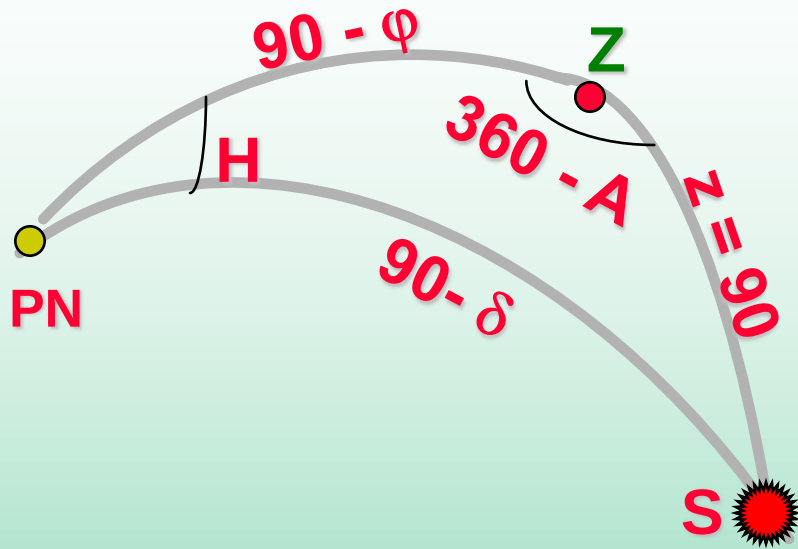
Nascer e Ocaso de um astro



Triângulo no ocaso de um astro



Azimute do Nascer e do Ocaso



$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$



$$\cos (90-\delta) = \cancel{\cos z} \cdot \cos(90-\varphi) + \sin (90-\varphi) \cdot \sin z \cdot \cos (360-A)$$

$$\sin \delta = \cos \varphi \cdot \cos A$$

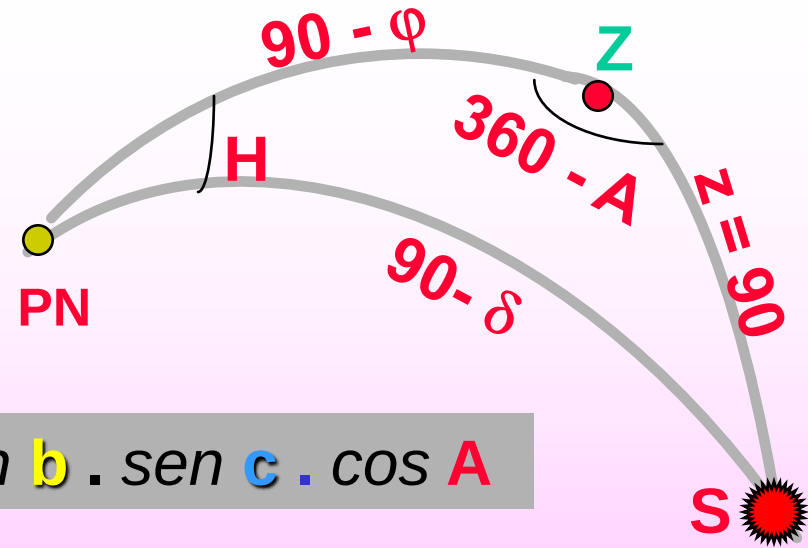
$$\cos A = \sin \delta / \cos \varphi$$

$$\Rightarrow 0 \leq \underline{A} \leq 180^\circ$$

No Nascer: $A = \underline{A}$

No Ocaso: $A = 360^\circ - \underline{A}$

Ângulo horário no nascer e no ocaso



$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$



$$\cos z = \cos (90-\varphi) \cdot \cos (90-\delta) + \sin (90-\varphi) \cdot \sin (90-\delta) \cdot \cos H$$

$$0 = \sin \varphi \cdot \sin \delta + \cos \varphi \cdot \cos \delta \cdot \cos H$$

$$\cos H = - \sin \varphi \cdot \sin \delta / \cos \varphi \cdot \cos \delta$$

$$\cos H = - \tan \varphi \cdot \tan \delta$$

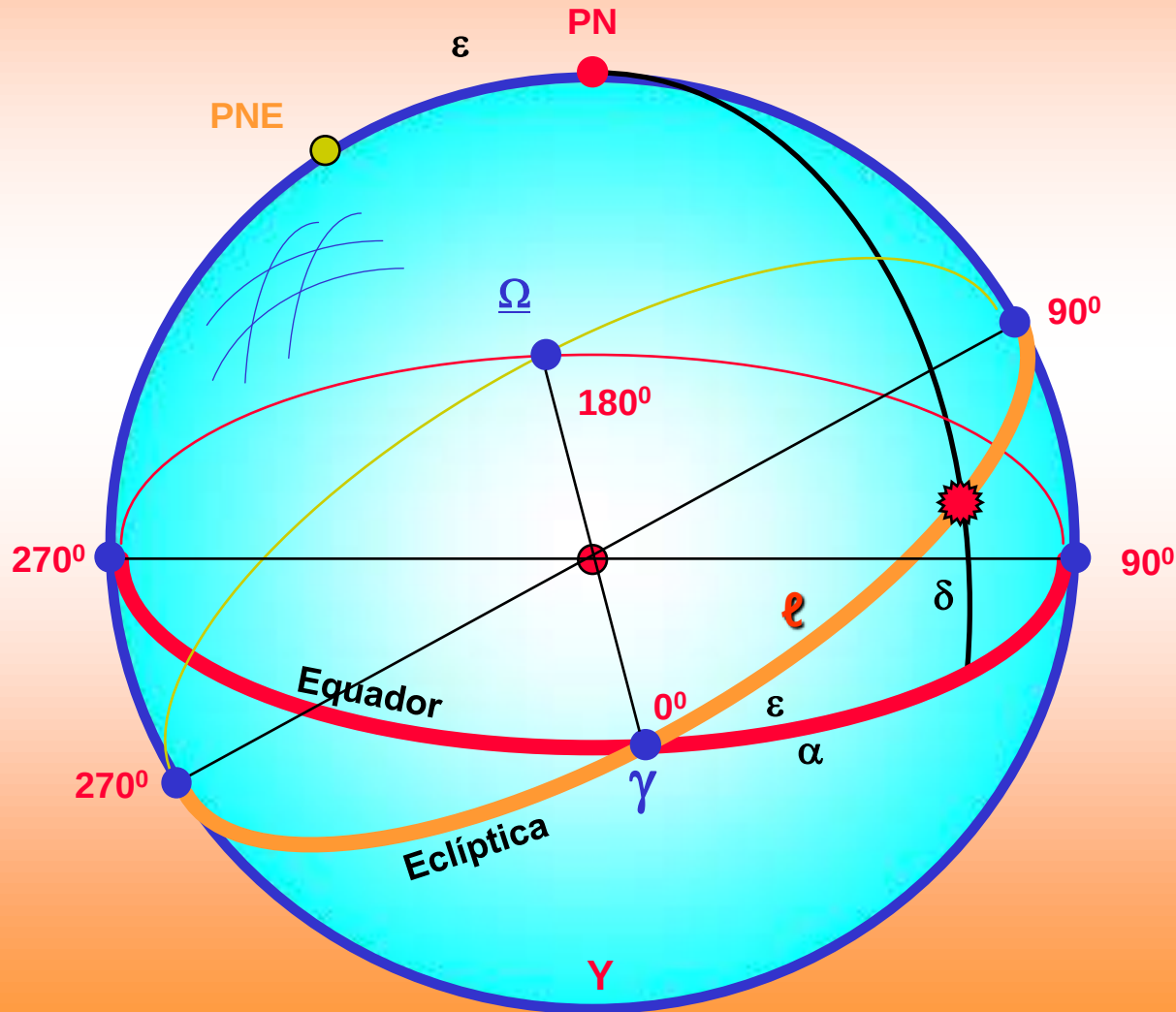
$$\Rightarrow 0 \leq \underline{H} \leq 180^\circ$$

$$\text{No Ocaso: } H_o = \underline{H}$$

$$\text{No Nascer: } H_n = - \underline{H}$$

Coordenadas aproximadas do Sol

do Sol



Coordenadas aproximadas do Sol

(precisão de $0,01^0$ entre 1950 e 2050)

$$DJ2000 = 2\,451\,545,0$$

$$n = DJ - DJ2000 \quad (\text{Número de dias desde o meio-dia de 01/jan/2000})$$

$$\varepsilon \cong 23,439^0 - 0,000\,000\,4\,n \quad (\text{Obliquidade da eclíptica})$$

$$L \cong 280,461^0 + 0,985\,6474\,n \quad (\text{Longitude média})$$

$$0 \leq L < 360^0 \quad (\text{Imposição})$$

$$g \cong 357,528^0 + 0,985\,6003\,n \quad (\text{Anomalia média})$$

$$0 \leq g < 360^0 \quad (\text{Imposição})$$

$$R \cong 1,000\,14^{UA} - 0,016\,71 \cos g - 0,000\,14 \cos 2g \quad (\text{Raio vetor do Sol})$$

$$\ell_{\bullet} = L^0 + 1,915^0 \sin g + 0,020 \sin 2g \quad (\text{Longitude eclíptica do Sol})$$

$$f = 180^0 / \pi$$

$$t = \tan^2 (\varepsilon / 2)$$

$$\alpha \cong \ell_{\bullet}^0 - f \cdot t \cdot \sin 2 \ell_{\bullet} + (f/2) \cdot t^2 \cdot \sin 4 \ell_{\bullet} \quad (\text{Ascensão reta do Sol})$$

$$Eq.T^{\text{minutos}} \cong -4 \cdot (L^0 - \alpha^0) \quad (\text{Equação do tempo: precisão de } 0,1^{\text{min}})$$

$$\delta \cong \arcsen (\sin \varepsilon \cdot \sin \ell_{\bullet}) \quad (\text{Declinação do Sol})$$

Bibliografia

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Editor: J.K. Page

Department of Building Sciences of University of Sheffield,
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Muhammad Iqbal, Toronto, Academic Press, 1983

Film