Selected Radiation Processes in High Energy Astrophysics

Roberto Aloisio

Gran Sasso Science Institute – L'Aquila (Italy)

GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

Plan of the lectures

1. Synchrotron emission
   1.a Total power emitted
   1.b Spectrum
   1.c Distribution of emitting particles
   1.d Coherent emission

2. Inverse Compton scattering
   2.a Power emitted in a single scattering
   2.b Spectrum
   2.c Distribution of electrons
   2.d Synchrotron self Compton

3. Proton-proton interactions
4. Gamma rays emission in SNR and molecular clouds
5. UHE Protons and nuclei interaction with photon backgrounds
   5.a Basic processes: pair production, photo-pion production, photo-disintegration
   5.b Secondary gamma rays and neutrinos
   5.c Diffuse extragalactic gamma ray background
   5.d Gamma rays from distant AGN

Basic bibliography

Books:

G.B. Ribiky and A.B. Lightman "Radiative Processes in Astrophysics"
Wiley and Sons, **ISBN-13:** 978-0471827597 **ISBN-10:** 0471827592

G. Ghisellini "Radiative Processes in High Energy Astrophysics" (Lecture Notes in Physics) Springer, **ISBN-13:** 978-3319006116 **ISBN-10:** 3319006118

V.S. Berezinskii, S.V. Bulanov, V.A. Dogiel, V.L. Ginzburg, V.S. Ptuskin "Astrophysics of Cosmic Rays" North Holland, **ISBN-13:** 978-0444886415 **ISBN-10:** 0444886419

Several relevant papers are cited in the text

Synchrotron Emission

A particle with charge q, mass m and velocity **v** in an external E.M. field is subject to the Lorentz force and has the Eq. of motion

$$\frac{d(\gamma m c^2)}{dt} = q\vec{v}\cdot\vec{E} \qquad \frac{d(\gamma m\vec{v})}{dt} = q\left[\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right]$$

The E.M. radiation emitted by a particle moving in a static magnetic field is called synchrotron radiation, in this case **E**=0 and $\mathbf{B}=\mathbf{B_0}$ from the first equation above follows

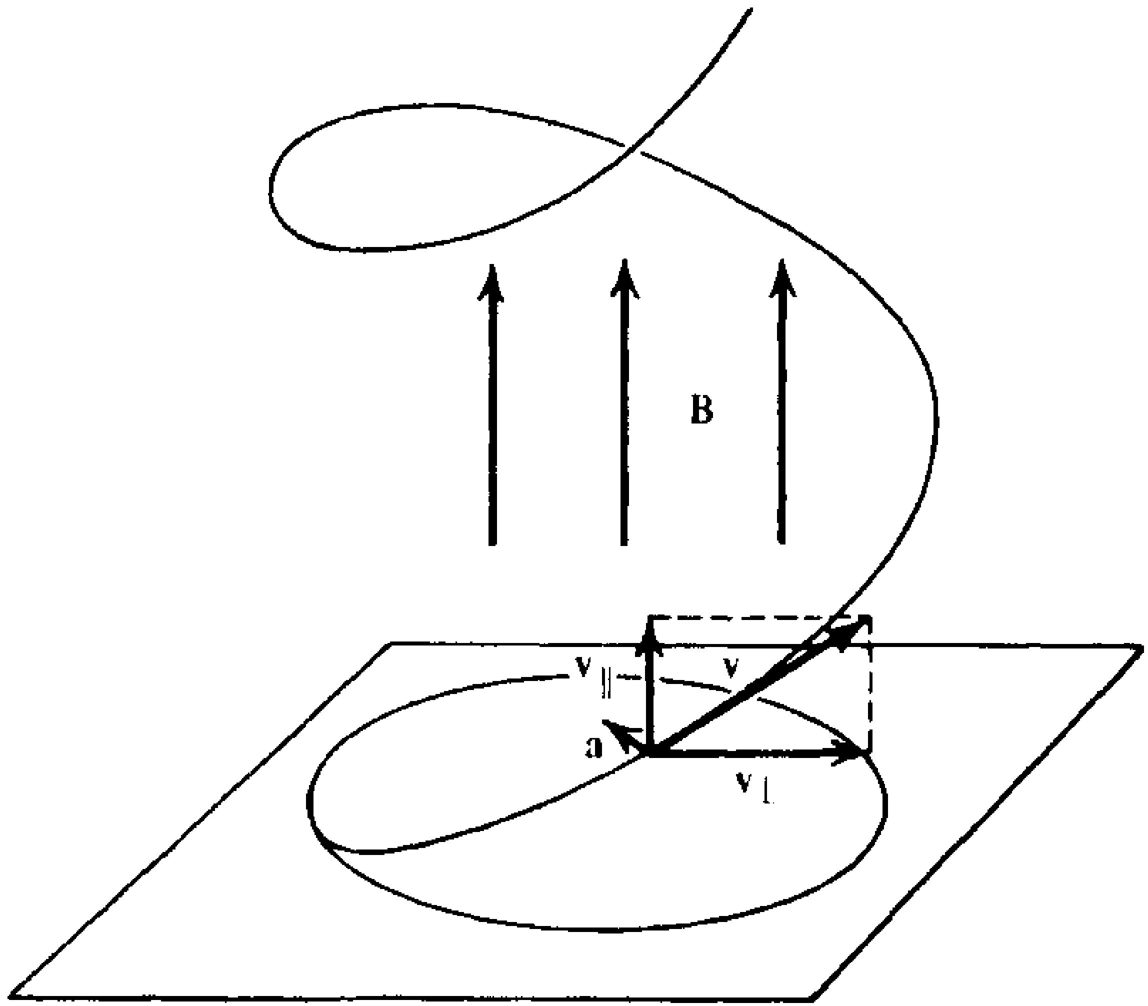
$$\gamma = \gamma_0 \quad |\vec{v}| = const \qquad \gamma m\frac{d\vec{v}}{dt} = \frac{q}{c}\vec{v}\times\vec{B}_0$$

Let us divide the motion in the two components parallel and perpendicular to the magnetic field the previous Eq. becomes:

$$\frac{d\vec{v}_{\|}}{dt} = 0 \qquad m\gamma\frac{d\vec{v}_{\perp}}{dt} = \frac{q}{c}\vec{v}_{\perp}\times\vec{B}$$

It follows that the parallel component of the velocity remains constant and, since the modulus of the velocity is constant, also the perpendicular component of the velocity will be constant in modulus. The solution of the Eq. of motion above will be a uniform circular motion on the plane normal to the magnetic field direction with a translational motion along the magnetic field direction. The so-called helical motion. The frequency of the rotation (gyration) is

$$\omega_B = \frac{qB_0}{\gamma mc}$$



Total power emitted by synchrotron

Let us consider now the properties of the power emitted by the charged particle moving in a constant magnetic field B_0. The motion is accelerated, hence we cannot define a steady inertial reference frame. We define an "instantaneous" rest frame of the particle in which (time by time) the particle can be considered non-relativistic (it would be never at rest being the actual motion accelerated). Under this hypothesis in the instantaneous rest frame we can use the Larmor formula, the emission will be isotropic with no variation of the particle's momentum.

Using "prime" to indicate quantities in the instantaneous rest frame one has (for energy dW and time dt in the LAB reference frame)

$$dW = \gamma dW' \qquad dt = \gamma dt'$$

Hence the power emitted is an invariant:

$$\frac{dW}{dt} = \frac{\gamma dW'}{\gamma dt'} = \frac{dW'}{dt'}$$

NOTE: the isotropy of the emission in the instantaneous rest frame is a fundamental hypothesis in order to get the invariance. As it implies no change of the particle's momentum dp'=0.

The Larmor formula gives us the power emitted in the instantaneous rest frame (so in the LAB frame)

$$P' = \frac{dW'}{dt'} = \frac{2q^2}{3c^3}\left|\vec{a}'\right|^2 = \frac{dW}{dt} = P$$

NOTE: Using the properties of the 4-acceleration and the 4-velocity in the instantaneous rest frame we can rewrite the power emitted in an explicit covariant form

$$P = \frac{dW}{dt} = \frac{2q^2}{3c^3} a_\mu a^\mu$$

It is useful to express P in terms of the components parallel and perpendicular to the magnetic field. The relation that links acceleration in the LAB and instantaneous rest frames are

$$a'_{\|} = \gamma^3 a_{\|} \qquad a'_{\perp} = \gamma^2 a_{\perp}$$

Using these relations we can rewrite the power emitted as

$$P = \frac{2q^2}{3c^3}|\vec{a}'|^2 = \frac{2q^2}{3c^3}(a_{\|}^{\prime 2} + a_{\perp}^{\prime 2}) = \frac{2q^2}{3c^3}\gamma^4(a_{\perp}^2 + \gamma^2 a_{\|}^2)$$

NOTE: this is a general result that holds in the case of the emission due to accelerated charged particles.

In the particular case of synchrotron emission as we saw before

$$a_{\|} = 0 \qquad a_{\perp} = \omega_B v_{\perp}$$

So we can rewrite the total power emitted by synchrotron as

$$r_0 = \frac{q^2}{mc^2}$$

Classical radius of the electron

$$P = \frac{2q^2}{3c^3}\gamma^4\omega_B^2 v_\perp^2 = \frac{2q^2}{3c^3}\gamma^4\frac{q^2B^2}{m^2c^2\gamma^2}v_\perp^2 = \frac{2}{3}r_0^2 c\beta_\perp^2\gamma^2 B^2$$

$$\beta_\perp = \beta\sin\alpha = \frac{v}{c}\sin\alpha$$

α is the pitch angle, formed by particle's velocity and the direction of the magnetic field.

Integrating over all possible pitch angles one gets the total power emitted by synchrotron

$$\langle\beta_\perp^2\rangle = \frac{\beta^2}{4\pi}\int \sin^2(\alpha)\ d\Omega = \frac{2}{3}\beta^2$$

$$P = \left(\frac{2}{3}\right)^2 r_0^2 c\beta^2\gamma^2 B^2$$

That is commonly written as the energy lost by the particle per unit time over the whole frequency range of the synchrotron emission

$$\left(\frac{dE}{dt}\right)_{Syn} = \frac{4}{3}\sigma_T c\beta^2\gamma^2 U_B$$

$$U_B = \frac{B^2}{8\pi}$$

Magnetic field energy density

$$\sigma_T = \frac{8\pi}{3}r_0^2$$

Thompson cross-section

Relativistic beaming

Consider a light ray that arrives at the origin of a fixed reference frame with an angle θ. The effect of Lorentz transformations is to change this angle θ → θ' if one moves to a reference frame moving with some velocity respect to the first reference frame. This angle variation is called relativistic aberration. To be more quantitative let's consider the laws of transformation of velocities in special relativity (note: we are interested in normal velocities and not 4-velocities).

$$\vec{u} = \frac{d\vec{r}}{dt} \qquad \vec{u}' = \frac{d\vec{r}'}{dt'}$$

Using the Lorentz transformations for space and time

$$dx' = \gamma(dx - \beta dt) \quad dy' = dy \quad dz' = dz \quad dt' = \gamma(dt - \beta dx)$$

$$u'_x = \frac{dx'}{dt'} = \frac{dx - \beta dt}{dt - \beta dx} = \frac{u_x - \beta}{1 - \beta u_x}$$

$$u'_y = \frac{dy'}{dt'} = \frac{dy}{\gamma(dt - \beta dx)} = \frac{u_y}{\gamma(1 - \beta u_x)}$$

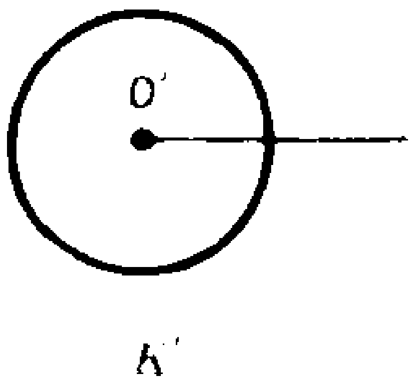
$$u'_z = \frac{dz'}{dt'} = \frac{dz}{\gamma(dt - \beta dx)} = \frac{u_z}{\gamma(1 - \beta u_x)}$$

$$\cos(\theta') = \frac{\cos(\theta) + \beta}{1 + \beta\cos(\theta)}$$

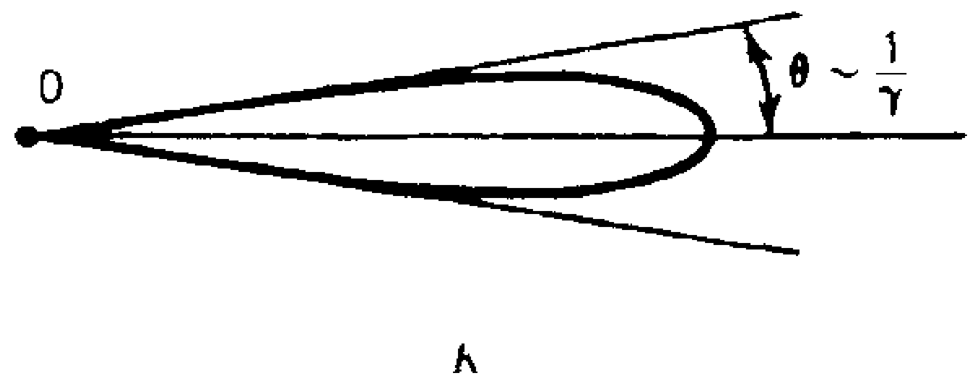
$$\sin(\theta') = \frac{\sin(\theta)}{\gamma(1 + \beta\cos(\theta))}$$

NOTE: it is important to stress the factor 1/γ that enters the transformations for angles and the fact that changing v →-v and u → -u the denominator of angle transformations remains unchanged. From these facts follows that light rays that are received or emitted isotropically in a given reference frame are emitted or received at smaller angles in a reference frame moving respect to the first one. This effect is called relativistic "beaming" and transforms angles as θ' →θ/γ .

Rest system of the emitting particle Isotropic emission



LAB system beamed emission with 1/γ aperture of the emission cone.



Relativistic Doppler effect

To discuss the relativistic Doppler effect let us consider the 4-velocity of the source u^μ, of the observer v^μ and the 4-momentum of photons k^μ.

$$u^\mu = \gamma(u)(1, \vec{u}) \qquad v^\mu = \gamma(v)(1, \vec{v}) \qquad k^\mu = (\omega, \vec{k}) \qquad (\vec{k} = \omega \hat{n})$$

In the rest system of the source $u^\mu=(1,0)$ and $k^\mu u_\mu=\omega_0$, being $k^\mu u_\mu$ a Lorentz invariant we can rewrite for both the source and the observer the following relations

$$\omega_0 = \gamma(u)\omega(1 - \hat{n} \cdot \vec{u}) \qquad \omega' = \gamma(v)\omega(1 - \hat{n} \cdot \vec{v})$$

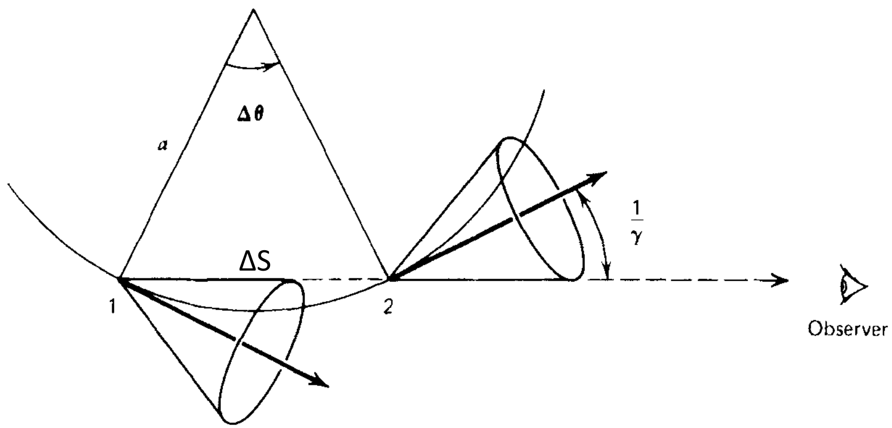
$$\frac{\omega'}{\omega_0} = \frac{\gamma(v)(1 - \hat{n} \cdot \vec{v})}{\gamma(u)(1 - \hat{n} \cdot \vec{u})} \qquad \frac{\lambda'}{\lambda_0} = \frac{\gamma(u)(1 - \hat{n} \cdot \vec{u})}{\gamma(v)(1 - \hat{n} \cdot \vec{v})}$$

In the case of light emitted by a source which is running away from an observer at rest $\vec{v} = 0$, $\hat{n} \cdot \vec{u} = -u_r$ (radial velocity of the source).

$$\lambda' = \lambda_0 \gamma(u)(1 + u_r) = \lambda_0 D(u) \qquad D = \gamma(u)(1 + u_r) \simeq 1 + \frac{u_r}{c}$$

NOTE: even in the case of radial velocity zero, with a non-zero tangential velocity (normal to the line of sight) the observed wavelength will be increased.

Because of the relativistic beaming the radiation emitted by synchrotron is concentrated in a cone and the observer will receive the emission only when the cone axis coincides with the line of sight of the observer



NOTE: the observer receives the emission in 1 and 2 when the cone of aperture 1/γ intersects the line of sight.

The distance ΔS between 1 and 2 can be determined using the gyration radius a, the geometry tells us that

$$\Delta S = a\Delta\theta = \frac{2a}{\gamma}$$

The gyration radius can be determined from the Eq. of motion

$$\gamma m \frac{\Delta\vec{v}}{\Delta t} = \frac{q}{c}\vec{v}\times\vec{B} \qquad \gamma m \frac{\Delta\vec{v}}{\Delta t} = \frac{q}{c} vB\sin(\alpha)$$

being $\Delta v = v\Delta\theta \quad \Delta S = v\Delta t$

$$a = \frac{\Delta S}{\Delta\theta} = \frac{\gamma mcv}{qBsin(\alpha)} = \frac{v}{\omega_B\sin(\alpha)} = \frac{r_L}{\sin(\alpha)}$$

NOTE: the gyration radius differs from the Larmor radius by a factor 1/sin(α) (pitch angle). The Larmor radius r_L is the gyration radius on the plane ortogonal to the magnetic field.

From the previous relations one gets

$$\Delta S = \frac{2a}{\gamma} = \frac{2v}{\gamma \omega_B \sin(\alpha)}$$

The time interval during which the particle passes between the points 1 and 2 is given by

$$\Delta S = v(t_2 - t_1) \qquad (t_2 - t_1) = \frac{2}{\gamma \omega_B \sin(\alpha)}$$

Because of the relativistic Doppler effect the time interval of arrival of the radiation to the observer is squeezed respect to the time interval of emission

$$(t_2^A - t_1^A) = (t_2 - t_1)(1 - \beta) = \frac{2}{\gamma \omega_B \sin(\alpha)}(1 - \beta)$$

$$\gamma \gg 1 \qquad 1 - \beta = 1 - \frac{v}{c} \simeq \frac{1}{2\gamma^2} \qquad \Delta t^A = \frac{1}{\gamma^3 \omega_B \sin(\alpha)} = \frac{T_{gyr}}{\gamma^3}$$

Δt^A is the time duration of the radiation pulse (at the observer) which is reduced by a factor $1/\gamma^3$ respect to the gyration period of the particle. The scale Δt^A fixes also the characteristics of the emitted radiation which will show all the frequencies until $1/\Delta t^A$. As we will analytically show below the critical synchrotron frequency is

$$\omega_c = \frac{3}{2}\gamma^3 \omega_B \sin(\alpha) \qquad \nu_c = \frac{3}{4\pi}\gamma^3 \omega_B \sin(\alpha)$$

Synchrotron spectrum

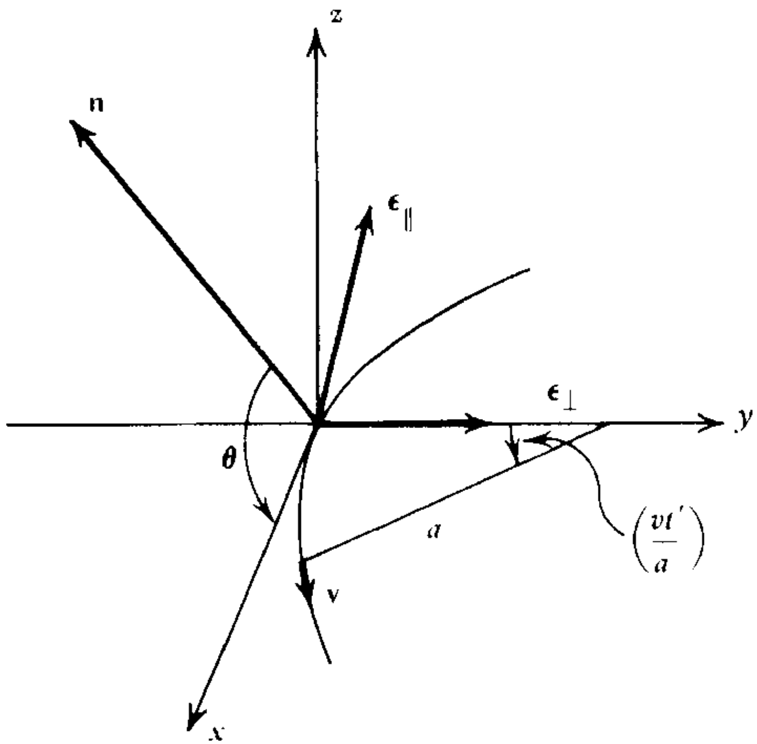
In general the emitted spectrum (energy per unit frequency and solid angle) of a moving charge can be expressed as (see Rybicki Lightman chapter 3 Eq. (3.13))

$$\frac{dW}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left| \int_{-\infty}^{\infty} \hat{n}\times(\hat{n}\times\vec{\beta})\exp\left[i\omega\left(t' - \frac{1}{c}\hat{n}\cdot\vec{r}(t')\right)\right] dt'\right|^2$$

$\hat{n}$ line of sight

$\vec{\beta}$ particle's velocity

and t' is the retarded time that takes into account the movement of the source. Let us assume that in t'=0 the particle is at the origin of the reference system (x=y=z=0) with the line of sight along x and the magnetic field along z



After a time t' the particle has covered an angle vt'/a and we can evaluate the vector in the Eq. above dividing it into the two components parallel and perpendicular to the magnetic field direction

$$\hat{n}\times(\hat{n}\times\vec{\beta}) = -\hat{\epsilon}_\perp \sin\left(\frac{vt'}{a}\right) + \hat{\epsilon}_\parallel \cos\left(\frac{vt'}{a}\right)\sin\theta$$

and

$$t' - \frac{1}{c}\hat{n}\cdot\vec{r}(t') = t' - \frac{a}{c}\cos\theta\sin\left(\frac{vt'}{a}\right)$$

hence we can rewrite the Eq. for the energy emitted as

$$\frac{dW}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt' \left[-\hat{\epsilon}_\perp \sin\left(\frac{vt'}{a}\right) + \hat{\epsilon}_\parallel \cos\left(\frac{vt'}{a}\right)\sin\theta\right]\exp\left[i\omega\left(t' - \frac{a}{c}\cos\theta\sin\left(\frac{vt'}{a}\right)\right)\right]\right|^2$$

We can separate the two components

$$\frac{dW}{d\omega d\Omega} = \left(\frac{dW}{d\omega d\Omega}\right)_\perp + \left(\frac{dW}{d\omega d\Omega}\right)_\parallel$$

Being relativistic particles β≈1 (γ>>1) the observer receives the signal only when vt'/a≈0 and θ≈0. By series expanding the Eq. above one has

$$\left(\frac{dW}{d\omega d\Omega}\right)_\perp = \frac{e^2\omega^2}{4\pi^2 c}\left(\frac{a\theta_\gamma^2}{\gamma^2 c}\right)^2\left|\int_{-\infty}^{\infty} y\exp\left[\frac{3}{2}i\eta\left(y + \frac{1}{3}y^3\right)\right]dy\right|^2$$

$$\left(\frac{dW}{d\omega d\Omega}\right)_\parallel = \frac{e^2\omega^2\theta^2}{4\pi^2 c}\left(\frac{a\theta_\gamma}{\gamma c}\right)^2\left|\int_{-\infty}^{\infty} \exp\left[\frac{3}{2}i\eta\left(y + \frac{1}{3}y^3\right)\right]dy\right|^2$$

being $\quad y = \gamma\frac{ct'}{a\theta_\gamma} \quad \theta_\gamma^2 = 1 + \gamma^2\theta^2$

$$\eta = \frac{\omega a\theta_\gamma^3}{3c\gamma^3} = \frac{\omega}{2\omega_c}\theta_\gamma^3 \simeq \frac{\omega}{2\omega_c} \quad \omega_c = \frac{3}{2}\gamma^3\omega_B\sin(\alpha)$$

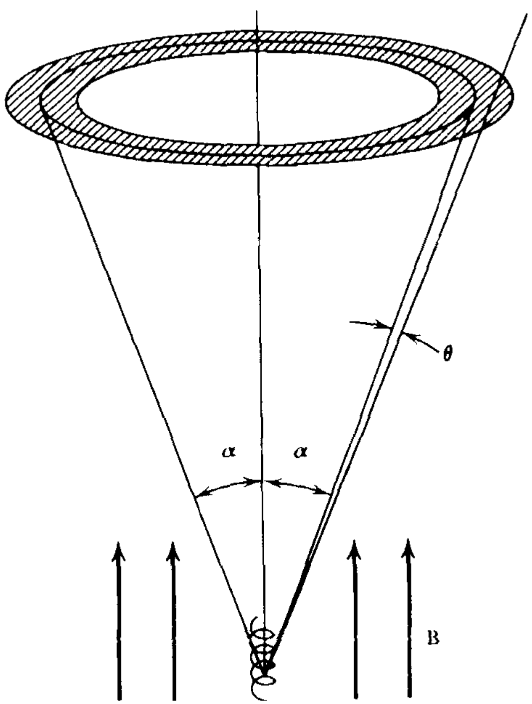
NOTE: the frequency dependence is contained in η and the angular dependence in γθ, as a consequence of the relativistic limit. Moreover, the frequency dependence is always through the ration ω/ω_c .

Functions present in the expressions above are the modified Bessel functions of order 2/3 and 1/3, so that we can rewrite

$$\left(\frac{dW}{d\omega d\Omega}\right)_{\perp} = \frac{e^2\omega^2}{4\pi^2 c}\left(\frac{a\theta_\gamma^2}{\gamma^2 c}\right)^2 K_{2/3}^2(\eta)$$

$$\left(\frac{dW}{d\omega d\Omega}\right)_{\parallel} = \frac{e^2\omega^2\theta^2}{4\pi^2 c}\left(\frac{a\theta_\gamma}{\gamma c}\right)^2 K_{1/3}^2(\eta)$$

We can now integrate these relations over the solid angle, obtaining in this way the energy emitted per unit frequency for a complete orbit on the plane normal to the magnetic field.



During one such orbit the emitted radiation is almost completely confined to the solid angle shown shaded in figure, which lies within an angle 1/γ of a cone of aperture twice the pitch angle α. Thus:

$$d\Omega = 2\pi\sin(\alpha)d\theta$$

$$\left(\frac{dW}{d\omega d\Omega}\right)_{\perp} = \frac{2e^2\omega^2 a^2\sin\alpha}{3\pi c^3\gamma^4}\int_{-\infty}^{\infty}\theta_\gamma^4 K_{2/3}^2(\eta)d\theta$$

$$\left(\frac{dW}{d\omega d\Omega}\right)_{\parallel} = \frac{2e^2\omega^2 a^2\sin\alpha}{3\pi c^3\gamma^2}\int_{-\infty}^{\infty}\theta_\gamma^2\theta^2 K_{1/3}^2(\eta)d\theta$$

The integral functions are non-zero only in an interval Δθ centered around α with a spread of 1/γ. Therefore the integration limits are not relevant and integrals can be put in a simpler form

$$\left(\frac{dW}{d\omega}\right)_{\perp} = \frac{\sqrt{3}e^2\gamma\sin\alpha}{2c}\left[F(x)+G(x)\right]$$

$$\left(\frac{dW}{d\omega}\right)_{\parallel} = \frac{\sqrt{3}e^2\gamma\sin\alpha}{2c}\left[F(x)-G(x)\right]$$

$$F(x) = x\int_x^\infty K_{5/3}(\xi)d\xi$$

$$G(x) = xK_{2/3}(x)$$

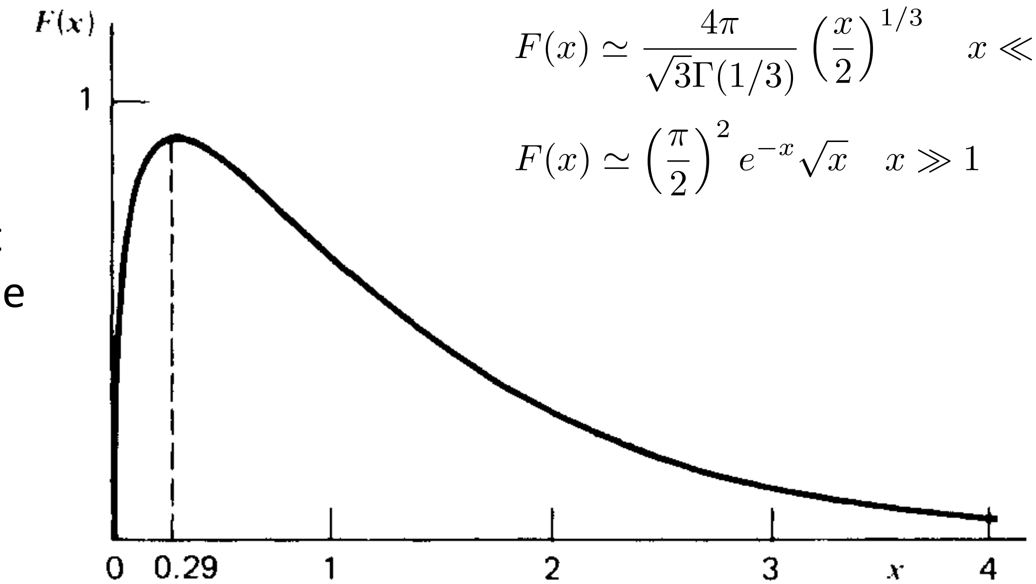
$$x = \frac{\omega}{\omega_c}$$

The total emitted energy per unit frequency is the sum of the two contributions above, it is given by

$$\left(\frac{dW}{d\omega}\right) = \frac{\sqrt{3}e^2\gamma\sin\alpha}{2c}F(x)$$

The total power emitted per unit frequency is obtained dividing the energy by the gyration period of the particle $2\pi/\omega_B$.

$$P(\omega) = \frac{\sqrt{3}e^3B\sin\alpha}{2\pi mc^2}F(x)$$



$$F(x) \simeq \frac{4\pi}{\sqrt{3}\Gamma(1/3)}\left(\frac{x}{2}\right)^{1/3} \quad x \ll 1$$

$$F(x) \simeq \left(\frac{\pi}{2}\right)^2 e^{-x}\sqrt{x} \quad x \gg 1$$

Distribution of emitting particles

Until now we have considered only the case of a single emitting particle. Obviously this is not what happens in astrophysics, where one deals always with distribution of particles. Therefore, let us consider many particles distributed over energy as a power law with spectral index p>0

$$\frac{dN}{dE} \propto E^{-p}$$

In this case, integrating the emitted power over the distribution of particles one gets the general result:

$$P_{tot}(\omega) = \int_m dE \frac{dN}{dE} P_E(\omega) \propto \omega^{-\frac{p-1}{2}}$$

NOTE: it can be easily proven just using the fact that P is non-zero only around ω_c

Summary

1. The angular distribution from a single radiating particle lies close (within $1/\gamma$) to the cone with half-angle equal to the pitch angle α.
2. The single particle spectrum extends up to something of the order of the critical frequency ω_c , being a function of ω/ω_c alone.
3. For a distribution of emitting (identical) particles with a power law (energy) distribution and power law index p, the emitted radiation will be a power law in frequency with a power law index (p-1)/2.

Coherent Synchrotron emission

The coherent emission can be realized if the synchrotron radiation emitted by different particles has some phase relation. In other words the coherent effect is realized if the electric field of the radiation emitted by different particles has some relation that links their phases. In this case a system of a Z particles all with the same Lorentz factor γ shows an emission which is Z^2 time the emission of a single particle. Hence, the coherent effect can represent a strong amplification of the emitted signal.

Let us consider the case of many particles each with the same Lorentz factor γ and different phases α_i in this case the relation for the wave vector will be

$$\hat{n} \times (\hat{n} \times \vec{\beta}_i) = -\hat{\epsilon}_\perp \sin\left(\frac{vt'}{a} - \alpha_i\right) + \hat{\epsilon}_\| \cos\left(\frac{vt'}{a} - \alpha_i\right)\sin(\theta)$$

repeating the analysis done before we can rewrite the total energy emitted per unit frequency and solid angle as

$$\left(\frac{dW}{d\omega d\Omega}\right) = \left(\frac{dW}{d\omega d\Omega}\right)^{(1)} \left|\sum_{i=1}^{Z} \exp\left(i\frac{\omega}{\omega_B}\alpha_i\right)\right|^2 = \left(\frac{dW}{d\omega d\Omega}\right)^{(1)} S(Z,\omega)$$

$$S(Z,\omega) = \left|\sum_{i=1}^{Z} \exp\left(i\frac{\omega}{\omega_B}\alpha_i\right)\right|^2$$

Being the first factor the emission produced by one particle. The term S(Z,ω) takes into account the interference among the electric fields of the radiation emitted by different particles.

The two simplest cases are:

(i) coherent emission – when the phases are all the same α_i=const;

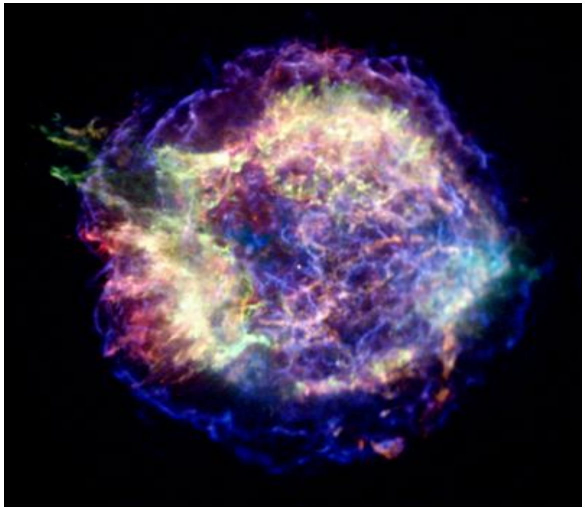
(ii) incoherent emission – in the case in which α_i are randomly distributed in the interval (0,2π) without any relation among them.

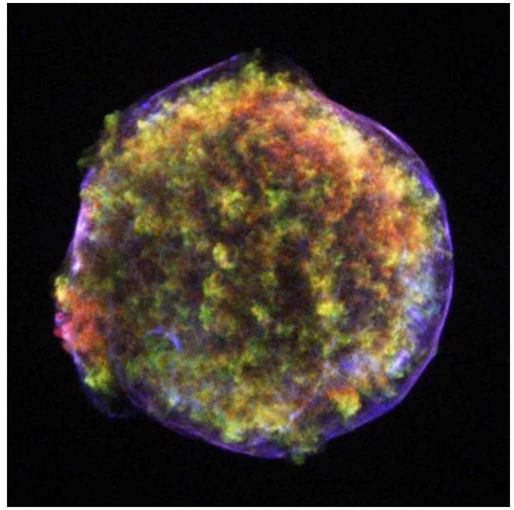
Coherent emission $$S(Z,\omega) = Z^2$$

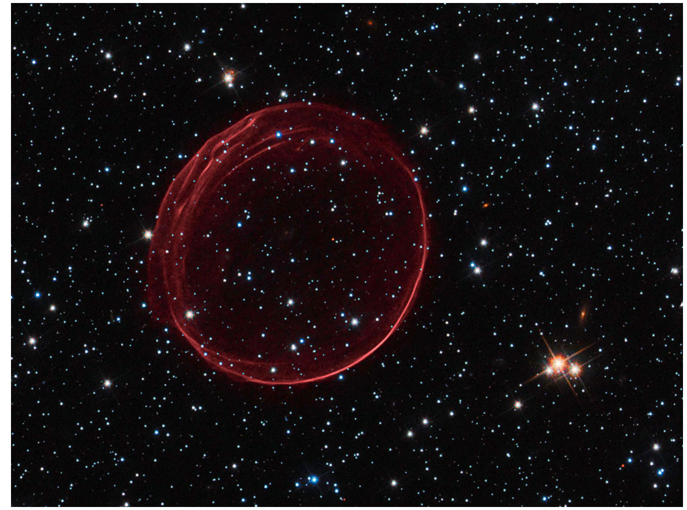
Incoherent emission $$S(Z,\omega) = Z$$

This relations can be proved through:

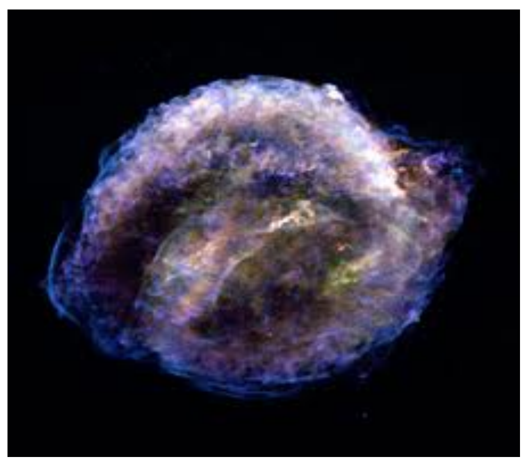
$$S(Z,\omega) = \sum_{j=1}^{Z}\sum_{i=1}^{Z}\left\{\cos\left[\frac{\omega}{\omega_B}(\alpha_j - \alpha_k)\right] + i\sin\left[\frac{\omega}{\omega_B}(\alpha_j - \alpha_k)\right]\right\}$$

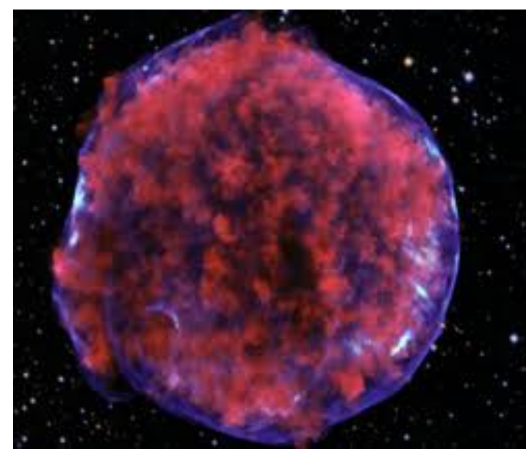






X-ray filaments observed in SNRs and produced by synchrotron emission of high energy electrons. Typical size of the observed filaments is at the level of 10^{-2} parsec. These observations lead to the conclusion of an amplified magnetic field in the SNR environment at the level of 100 μG. (Important insight for models of CR acceleration in SNR).





Inverse Compton Scattering

Thomson scattering

This is realized when low energy photons, with $\varepsilon \ll m_e c^2$, scatter on electrons at rest. The process is elastic and incident photons can be treated as a continuous E.M. wave. The cross section per solid angle of photon's emission is given by:

$$\epsilon_1 = \epsilon \qquad \frac{d\sigma_T}{d\Omega} = \frac{1}{2} r_0^2 (1 + \cos^2\theta) \qquad \sigma_T = \frac{8\pi}{3} r_0^2 \qquad r_0 = \frac{e^2}{mc^2}$$

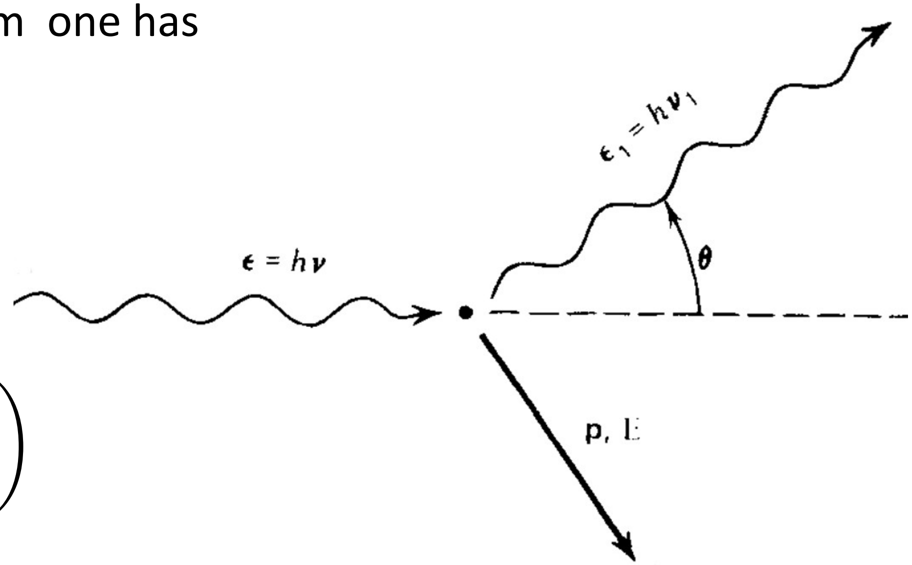
Compton scattering

Releasing the hypothesis of low energy photons, if $\varepsilon \geq m_e c^2$ the process is not anymore elastic and should be treated as a particle scattering (photon on electron at rest). Using the conservation of energy and momentum one has

$$\epsilon_1 = \frac{\epsilon}{1 + \frac{\epsilon}{mc^2}(1 - \cos\theta)}$$

$$\lambda_1 - \lambda = \lambda_c (1 - \cos\theta) \qquad \lambda_c = \frac{h}{mc}$$

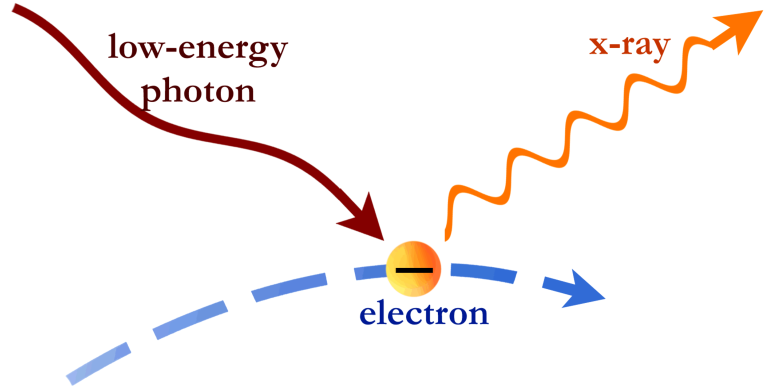
$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} \frac{\epsilon_1^2}{\epsilon^2} \left(\frac{\epsilon}{\epsilon_1} + \frac{\epsilon_1}{\epsilon} - \sin^2\theta \right)$$



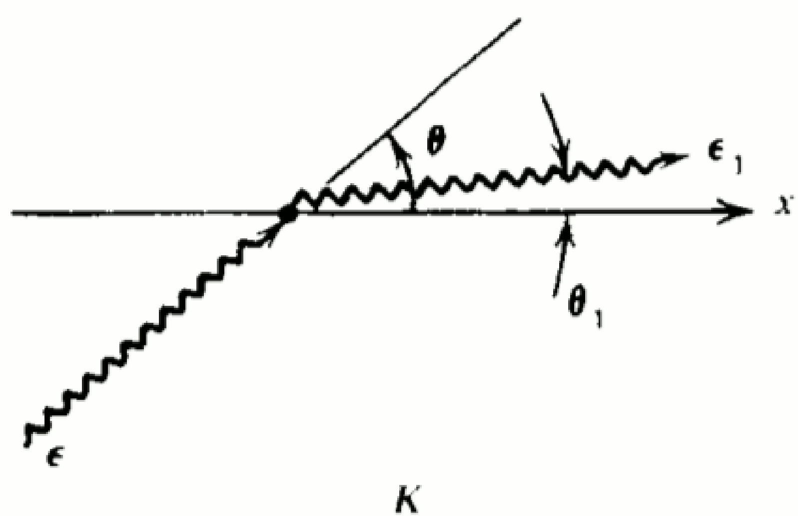
Klein-Nishina cross section.

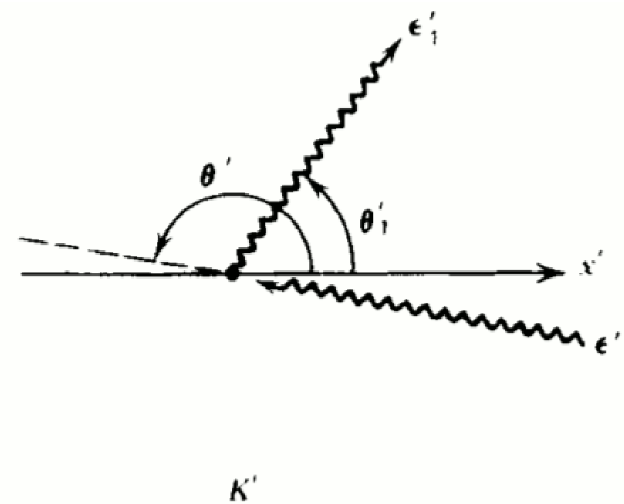
The Inverse Compton Scattering (ICS) is a process for which a moving electron has sufficiently kinetic energy that can be transferred to a low energy photon considerably increasing its energy. Here we will always assume that the energy of the photon in the rest system of the electron is much less than the rest mass of the electron

$$\gamma\epsilon \ll m_e c^2$$



This condition corresponds to the possibility of treating the scattering as in the case of Thomson in the rest frame of the electron.
Let us call K the LAB frame, in which the electron has high energy and the photon low energy, and K' the reference frame in which the electron is at rest.





Recalling the Lorentz transformations of the Doppler shift formula and for angles:

$$\epsilon' = \epsilon\gamma(1 - \beta\cos\theta) \qquad \cos(\theta') = \frac{\cos(\theta) + \beta}{1 + \beta\cos(\theta)}$$

$$\epsilon_1 = \epsilon_1'\gamma(1 - \beta\cos\theta_1') \qquad \sin(\theta') = \frac{\sin(\theta)}{\gamma(1 + \beta\cos(\theta))}$$

$$\epsilon_1 = \epsilon\frac{1 - \beta\cos\theta}{1 - \beta\cos\theta_1}$$

NOTE: the angles θ' and θ'_1 are typically of the order of $\pi/2$. The maximum value that the energy of the final photon can acquire corresponds to $\theta=\pi$ and $\theta_1=0$ the photon is scattered along the electron velocity vector (head-on) and

$$\epsilon_1 = \epsilon\frac{1 + \beta}{1 - \beta} = \epsilon\gamma^2(1 + \beta)^2 \simeq 4\gamma^2\epsilon$$

where the last equality holds in the relativistic limit $\beta\approx1$. The minimum value of ε_1 is in the case of $\theta=0$ and $\theta_1=\pi$ when the photon scatters from behind on the electron (tail-on). The minimum energy of the scattered photon will be

$$\epsilon_1 = \epsilon\frac{1 - \beta}{1 + \beta}$$

NOTE: The ICS converts low energy photons to high energy ones by a (large) factor γ^2. Kinematical effects alone limit the energy attainable, from the conservation of energy we can write $\varepsilon_1<\gamma mc^2 + \varepsilon$. Fixing ε and letting γ become large, we see that photon energies larger than γmc^2 cannot be obtained.

Power for single scattering

Before we considered a single photon scattering off a single electron. Here we consider the case of a given isotropic distribution of photons scattering off a given isotropic distribution of electrons.

Let n(p) the photons phase space distribution, which is a Lorentz invariant. Let v(ε)dε the density of photons having energy in the range (ε,ε+dε), v and n are related by

$$v(\epsilon)d\epsilon = n(p)d^3p$$

Recalling that d^3p transforms as energy under Lorentz, it follows that v(ε)dε/ε is a Lorentz invariant

$$\frac{v(\epsilon)d\epsilon}{\epsilon} = \frac{v'(\epsilon')d\epsilon'}{\epsilon'}$$

The total power scattered in the electron rest frame can be found using the Thomson cross section as

$$\frac{dE_1'}{dt'} = c\sigma_T \int \epsilon_1' v' d\epsilon'$$

Being v'dε' the number density of incident photons. Using the hypothesis $\gamma\epsilon << mc^2$ we can neglect the energy change of the photon in the electron's rest frame assuming $\epsilon'_1=\epsilon'$.
Moreover, we also know that the power emitted is a Lorentz invariant $dE_1/dt=dE'_1/dt'$

$$\frac{dE_1}{dt} = \frac{dE_1'}{dt'} = c\sigma_T \int \epsilon'^2 \frac{v' d\epsilon'}{\epsilon'} = c\sigma_T \int \epsilon'^2 \frac{v d\epsilon}{\epsilon}$$

Since $\epsilon' = \epsilon\gamma(1 - \beta\cos\theta)$ we can rewrite the expression above with all quantities written in the LAB frame

$$\frac{dE_1}{dt} = c\sigma_T\gamma^2 \int (1 - \beta\cos\theta)^2 \epsilon v d\epsilon$$

In the case of an isotropic distribution of photons one has

$$\langle(1 - \beta\cos\theta)^2\rangle = 1 + \frac{1}{3}\beta^2 \qquad \langle\cos\theta\rangle = 0 \qquad \langle\cos^2\theta\rangle = \frac{1}{3}$$

$$\frac{dE_1}{dt} = c\sigma_T\gamma^2(1 + \frac{1}{3}\beta^2)U_{ph} \qquad U_{ph} = \int \epsilon v d\epsilon$$

NOTE: U_{ph} is the energy density of initial photons (before scattering).

dE_1/dt is the power contained in the scattered radiation. To calculate the energy loss rate of the electrons, we have to subtract the initial power of the radiation before being scattered

$$\frac{dE_{in}}{dt} = c\sigma_T \int \epsilon v d\epsilon = c\sigma_T U_{ph}$$

Therefore the net power lost by electrons and thereby converted into radiation is

$$\left(\frac{dE}{dt}\right)_{ICS} = \frac{dE_1}{dt} - \frac{dE_{in}}{dt} = c\sigma_T U_{ph}\left[\gamma^2\left(1 + \frac{\beta^2}{3}\right) - 1\right]$$

Being $\gamma^2 - 1 = \gamma^2\beta^2$

$$\left(\frac{dE}{dt}\right)_{ICS} = \frac{4}{3} c\sigma_T \gamma^2 \beta^2 U_{ph}$$

NOTE: when the energy transfer in the electron's rest frame cannot be neglected (Klein-Nishina regime) the Eq. above becomes (Blumenthal and Gould 1970)

$$\left(\frac{dE}{dt}\right)_{ICS} = \frac{4}{3} c\sigma_T \gamma^2 \beta^2 U_{ph} \left[1 - \frac{63}{10} \frac{\gamma\langle\epsilon^2\rangle}{mc^2\langle\epsilon\rangle}\right]$$

$<\varepsilon^2>$ and $<\varepsilon>$ mean values integrated over U_{ph}.

NOTE: recalling the total power emitted by synchrotron

$$\left(\frac{dE}{dt}\right)_{Syn} = \frac{4}{3} \sigma_T c \beta^2 \gamma^2 U_B$$

one has

$$\frac{\left(\frac{dE}{dt}\right)_{ICS}}{\left(\frac{dE}{dt}\right)_{Syn}} = \frac{U_B}{U_{ph}}$$

the radiation losses due to synchrotron emission and ICS are in the same ratio as the magnetic field energy density and photons energy density.

NOTE: the synchrotron emission can be interpreted as an ICS on virtual photons of the magnetic field, just interpreting U_B as the energy density of such virtual photons.

NOTE: in the case of a distribution of electrons $N(\gamma)$ the total power emitted by ICS is

$$\left(\frac{dE}{dt}\right)_{ICS}^{Tot} = \int \left(\frac{dE}{dt}\right)_{ICS} N(\gamma) d\gamma$$

Inverse Compton Scattering Spectra

The spectrum of photons emitted by ICS depends on both the initial photon spectrum and electrons distribution. However, it is only necessary to determine the spectrum for the scattering of photons of a given energy ε_0 off electrons of a given energy γmc^2, because the general spectrum can then be found by averaging over the actual distributions of photons and electrons. We consider here cases in which both electrons and photons have isotropic distributions; the scattered photons are then also isotropically distributed, and it only remains to find their energy spectrum.

As before we consider the case of Thomson scattering in the rest frame of the electron ($\gamma\varepsilon \ll mc^2$). In addition we assume that the scattering in the electron's rest frame is also isotropic, i.e.

$$\frac{d\sigma'}{d\Omega'} = \frac{1}{4\pi}\sigma_T = \frac{2}{3}r_0^2$$

It is useful the use of the flux I(ε) based on photon number rather than energy. The number of photons crossing an area dA in the time dt within the solid angle dΩ in the energy range (ε,ε+dε) is

$$dN = I(\epsilon)dAdtd\Omega d\epsilon$$

Suppose that the isotropic incident photons are mono-energetic

$$I(\epsilon) = F_0\delta(\epsilon - \epsilon_0)$$

Let us determine the scattering of a beam of electrons with density N, energy γmc^2 traveling along the x-axis. The incident intensity in the rest frame K' is

$$I'(\epsilon') = F_0 \left(\frac{\epsilon'}{\epsilon}\right)^2 \delta(\epsilon - \epsilon_0)$$

It follows from the invariance

$$\frac{I'(\epsilon')}{\epsilon'^2} = \frac{I(\epsilon)}{\epsilon^2}$$

Recalling the transformations

$$\epsilon = \epsilon'\gamma(1 + \beta\cos\theta') \qquad \epsilon' = \epsilon\gamma(1 - \beta\cos\theta)$$

defining $\mu = \cos\theta \quad \mu' = \cos\theta'$ we can rewrite

$$I'(\epsilon', \mu') = \left(\frac{\epsilon'}{\epsilon_0}\right)^2 F_0\delta(\gamma\epsilon'(1 + \beta\mu') - \epsilon_0) = \left(\frac{\epsilon'}{\epsilon_0}\right)^2 \frac{F_0}{\gamma\beta\epsilon'}\delta\left(\mu' - \frac{\epsilon_0 - \gamma\epsilon'}{\gamma\beta\epsilon'}\right)$$

The emission function or emissivity in K', i.e. number of emitted photons per unit volume, energy and steradian is given by

$$J'(\epsilon'_1) = N'\sigma_T \frac{1}{2}\int_{-1}^{+1} d\mu' I'(\epsilon'_1, \mu')$$

Where we have used the Thomson condition $\varepsilon'_1=\varepsilon'$. Using the expression for $I(\varepsilon,\mu)$ into the Eq. above one has

$$J'(\epsilon') = J'(\epsilon'_1) = \frac{N'\sigma_T \epsilon'_1 F_0}{2\epsilon_0^2\gamma\beta} \qquad \text{if} \qquad \frac{\epsilon_0}{\gamma(1+\beta)} < \epsilon'_1 < \frac{\epsilon_0}{\gamma(1-\beta)}$$

$J'(\epsilon') = J'(\epsilon'_1) = 0$ otherwise.

Let us now come back to the LAB frame. As for the intensity also J/ε^2 is an invariant, hence

$$J(\epsilon_1,\mu_1) = \left(\frac{\epsilon_1}{\epsilon_1'}\right)^2 J'(\epsilon_1',\mu_1') = \frac{N\sigma_T\epsilon_1 F_0}{2\epsilon_0^2\gamma^2\beta} \qquad \frac{\epsilon_0}{\gamma^2(1+\beta)(1-\beta\mu_1)} < \epsilon_1 < \frac{\epsilon_0}{\gamma(1-\beta)(1-\beta\mu_1)}$$

and J=0 otherwise, we have used the transformations

$$N = \gamma N' \qquad \epsilon' = \epsilon\gamma(1-\beta\mu) \qquad \epsilon = \epsilon'\gamma(1+\beta\mu')$$

The result above holds in the case of a beam of electrons. To obtain the more general result of an isotropic distribution of electrons we should integrate over the angle θ ($\mu=\cos(\theta)$) between the electron and photon momenta.

$$J(\epsilon_1) = \frac{1}{2}\int_{-1}^{1} J(\epsilon_1,\mu_1)d\mu_1$$

NOTE: $J(\varepsilon_1,\mu_1)$ depends on μ_1 only through the condition on ε_1 , being constant if the condition is realized or zero if not (see Eq. above). The number of emitted photons per unit volume and steradian for an isotropic distribution of electrons is given by

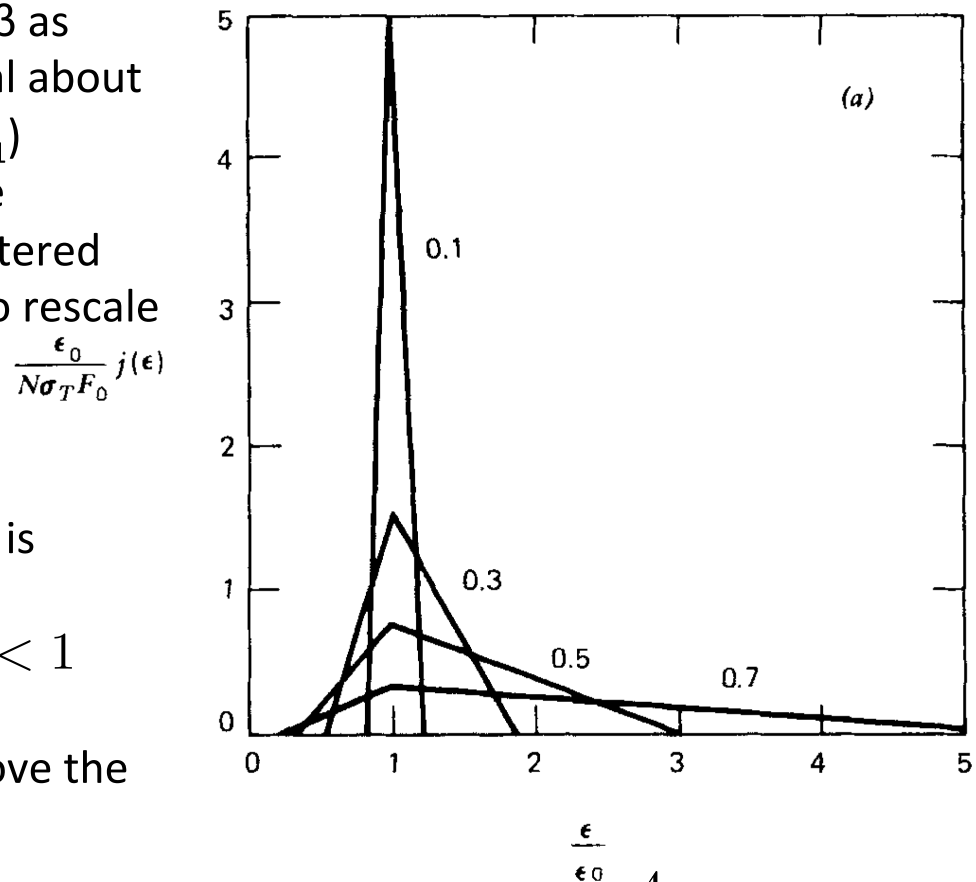
$$J(\epsilon_1) = \frac{N\sigma_T F_0}{4\epsilon_0\gamma^2\beta^2}(1+\beta)\frac{\epsilon_1}{\epsilon_0} - (1-\beta) \quad \text{if} \quad \frac{1-\beta}{1+\beta} < \frac{\epsilon_1}{\epsilon_0} < 1$$

$$J(\epsilon_1) = \frac{N\sigma_T F_0}{4\epsilon_0\gamma^2\beta^2}(1+\beta)\frac{\epsilon_1}{\epsilon_0} - (1-\beta) \quad \text{if} \quad 1 < \frac{\epsilon_1}{\epsilon_0} < \frac{1+\beta}{1-\beta}$$

and zero otherwise.

In figure J(ε_1) is plotted for different values of β as labeled. For small β the curves are symmetrical about the initial photon energy ε_0. As β increases J(ε_1) extend towards energies >> ε_0 , expressing the upward shift of the average energy of the scattered photons. For β near 1 (γ>>1) it is convenient to rescale the energy variable as

$$x = \frac{\epsilon_1}{4\gamma^2\epsilon_0}$$



The emission function (for isotropic photons) is

$$J(\epsilon_1) = \frac{3N\sigma_T F_0}{4\epsilon_0\gamma^2}\frac{2}{3}(1-x) \quad 0 < x < 1$$

Using the expression for J(ε_1) it is easy to prove the following relations

$$\int_0^\infty d\epsilon_1 J(\epsilon_1) = N\sigma_T F_0 \qquad \int_0^\infty d\epsilon_1(\epsilon_1 - \epsilon_0)J(\epsilon_1) = N\epsilon_0 F_0\frac{4}{3}\sigma_T\gamma^2\beta^2$$

The first result expresses the rate of photon scattering per unit volume and steradian, hence it is nothing but the conservation of the number of photons upon scattering. The second result is the average increase in photon energy per scattering, which is nothing but the energy lost by the electrons as determined before:

$$N\epsilon_0 F_0 = cU_{ph} \qquad \int_0^\infty d\epsilon_1(\epsilon_1 - \epsilon_0)J(\epsilon_1) = N\epsilon_0 F_0\frac{4}{3}\sigma_T\gamma^2\beta^2 = \frac{4}{3}c\sigma_T\gamma^2\beta^2 U_{ph}$$

Electrons distribution

Let us consider a more realistic case in which the distribution of electrons is a power law in Lorentz factor (energy)

$$N(\gamma) = N_0 \gamma^{-p}$$

Determining the total power emitted per unit volume and (photon) energy

$$\frac{dE}{dV\,dt\,d\epsilon_1} = 4\pi J(\epsilon_1)$$

Referring to the case of relativistic particles γ>>1 one has

$$\frac{dE}{dV\,dt\,d\epsilon_1} = 4\pi \int d\gamma \epsilon_1 J_\gamma(\epsilon_1) = 4\pi \frac{3N_0\sigma_T F_0}{4\epsilon_0} \int d\gamma \epsilon_1 \gamma^{-p-2} f\left(\frac{\epsilon_1}{4\gamma^2\epsilon_0}\right)$$

Changing integration variable

$$x = \frac{\epsilon_1}{4\epsilon_0\gamma^2} \quad \gamma = \left(\frac{\epsilon_1}{4\epsilon_0 x}\right)^{1/2} \quad d\gamma = -\frac{2\epsilon_0}{\epsilon_1}\left(\frac{\epsilon_1}{4\epsilon_0}\right)^{3/2} x^{-3/2} dx$$

$$\frac{dE}{dV\,dt\,d\epsilon_1} = \frac{3\pi N_0\sigma_T F_0}{4\epsilon_0}\epsilon_1 \frac{2\epsilon_0}{\epsilon_1}\left(\frac{\epsilon_1}{4\epsilon_0}\right)^{3/2}\left(\frac{\epsilon_1}{4\epsilon_0}\right)^{-\frac{p+2}{2}} \int_0^1 x^{3/2} x^{\frac{p+2}{2}} f(x) dx$$

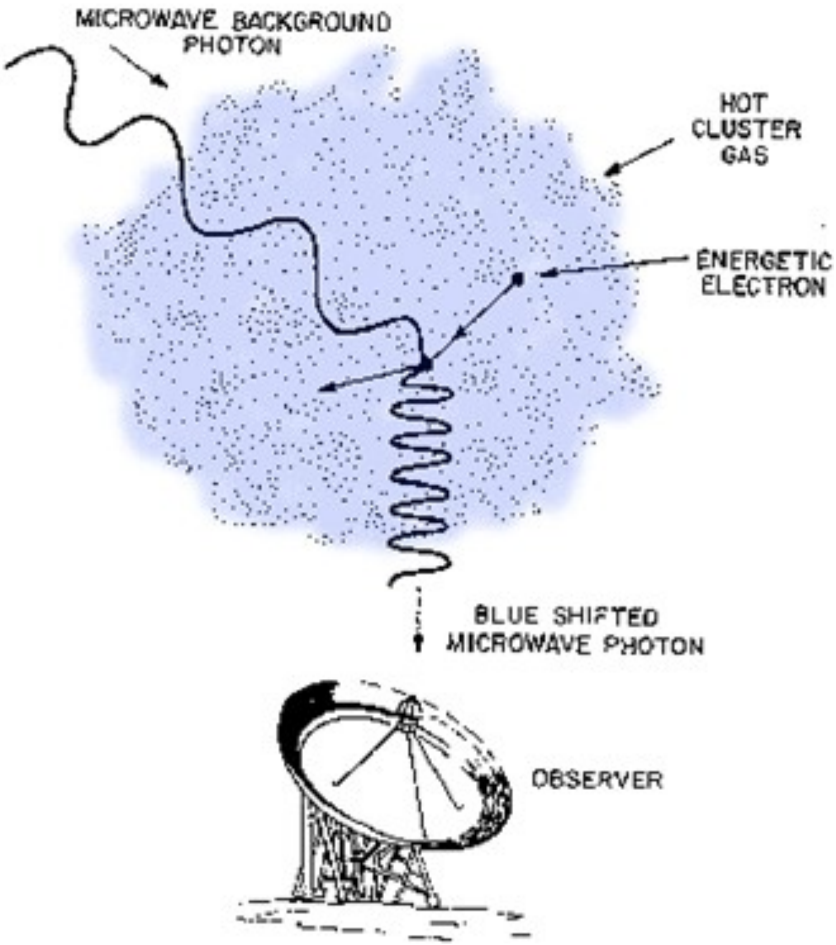
$$\frac{dE}{dV\,dt\,d\epsilon_1} \propto \epsilon_1^{-\frac{p-1}{2}}$$

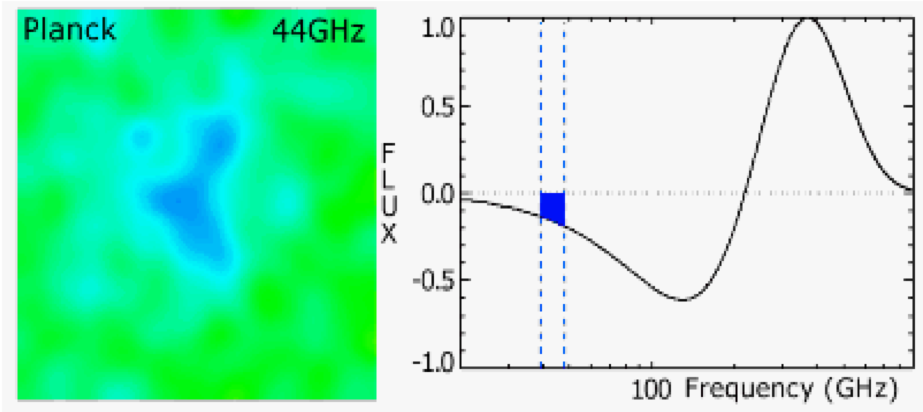
The same result of synchrotron emission.

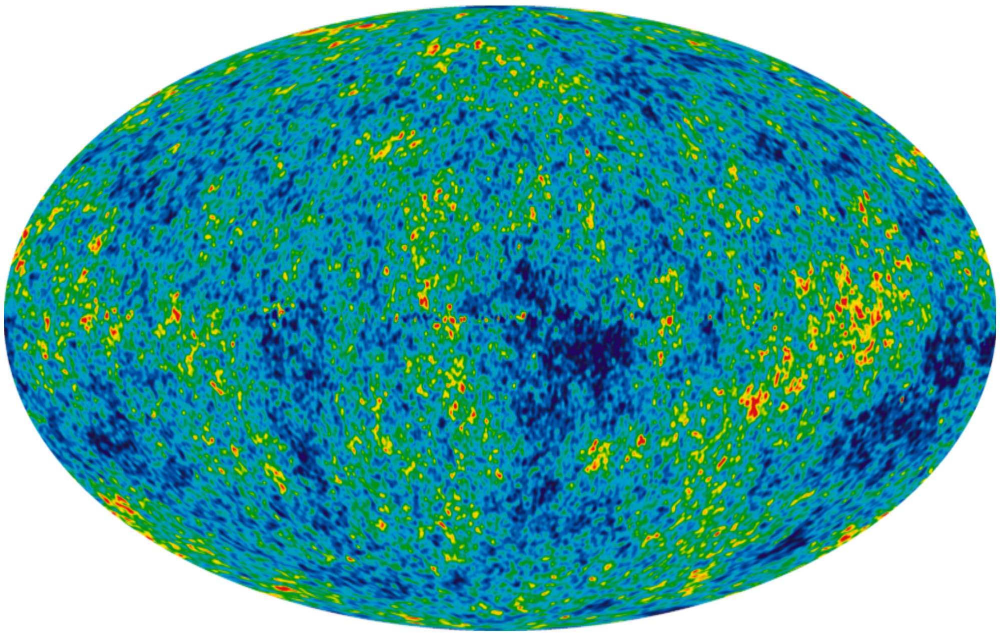
Synchrotron self Compton

If the energy density of photons emitted by synchrotron is comparable with the energy density associated to the magnetic field the mechanism of Synchrotron Self Compton (SSC) becomes efficient. This mechanism implies an ICS interaction of relativistic electrons with photons produced by the same electrons through synchrotron. The energy of photons emitted by SSC is typically in the γ-ray band.

Sunyaev Zel'dovich effect







Proton-Proton interactions

We discuss now the process of proton-proton interaction that gives rise to the production of neutral pions that in turn decay feeding the galactic gamma ray background. The process at hand is the interaction of cosmic rays protons with the protons of the interstellar medium that can be considered at rest and indicated with H

$$p + H \longrightarrow \pi^o + \text{ANYTHING}$$
$$\hookrightarrow 2\gamma$$

To fix the energetic scale of the problem let us start from the threshold for this reaction, that can be computed (using the definition) as the energy in the LAB frame of the incident proton needed to produce the final particles at rest in the CoM frame, i.e.

$$(E + m_p)^2 - p^2 = (M_{any} + m_\pi)^2 \quad \Rightarrow \quad E_{th}^{pp} = \frac{(M_{any} + m_\pi)^2 - 2m_\pi^2}{2m_p}$$

Being M_{any} the total rest mass of the final products apart from π^0 . The typical reaction of this kind implies the production of two protons in the final state in this case the threshold is:

$$p + H \to \pi^0 pp \qquad\qquad E_{th}^{pp} \simeq m_p + 2m_\pi$$

NOTE: Comparing the time scales for pp and diffusion it follows that the pp mechanism is irrelevant as energy losses mechanism for CR. Let us assume: constant pH cross-section $\sigma_{pH} \approx 30$ mb ($3\text{x}10^{-26}$ cm^2), number density for protons in the ISM n_H=1 cm^{-3}, constant diffusion coefficient D=$3\text{x}10^{28}$ cm^2/s, galactic halo and disk heights respectively H=3 Kpc and h=0.15 kpc.

Taking into account that the time spent in the disk is roughly h/H of the total diffusion time, one has

$$\tau_{pp} = \frac{H}{n_H h c \sigma_{pH}} \simeq 2 \times 10^{16}\mathrm{s} \qquad \tau_{diff} = \frac{H^2}{D} \simeq 2 \times 10^{15}\mathrm{s}$$

NOTE: pH interactions are important for the production of secondary gamma rays. Apart from the disk, pp is important in those regions with increased target density (and/or protons density) such as in molecular clouds or SNR.

The number of π^0 produced per unit volume, energy and steradian (emission function or emissivity) can be computed as

$$J_\pi(E_\pi) = 4\pi n_H \int_{E_\pi}^{\infty} dE_p \; I_p(E_p) \frac{d\sigma(E_\pi, E_p)}{dE_\pi}$$

where I_p is the flux of CR protons and $d\sigma/dE_\pi$ is the differential cross section for the production of a π^0 with energy E_π in the LAB frame.

NOTE: Eq. above holds in the case of an isotropic distribution of both bullets and targets.

The emissivity of secondary photons produced by the decay of pions is

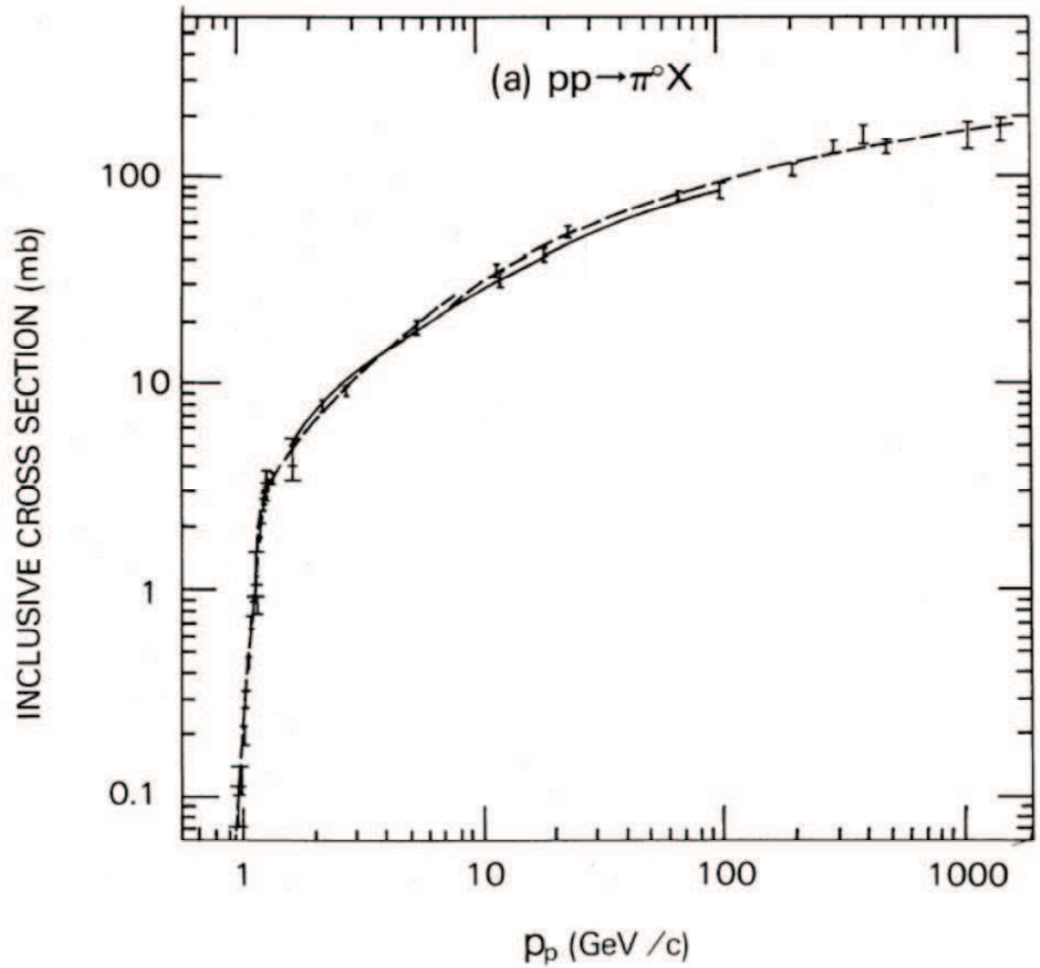
$$J_\gamma(\epsilon_\gamma) = 2 \int_{\epsilon_\gamma + m_\pi^2/4\epsilon_\gamma}^{\infty} dE_\pi \frac{J_\pi(E_\pi)}{\sqrt{E_\pi^2 - m_\pi^2}}$$

The cross section for π^0 production from p+p interactions can be written in terms of the total inclusive cross section associated to the reaction pp → π^0 + anything, it is given by

$$\frac{d\sigma(E_p, E_\pi)}{dE_\pi} = \sigma_{inc}(E_p)\frac{f(E_p, E_\pi)}{E_\pi}$$

The quantity f is an auxiliary function that, fitted on the experimental data, should fulfill the condition

$$\int \frac{dE_\pi}{E_\pi} f(E_p, E_\pi) = 1$$



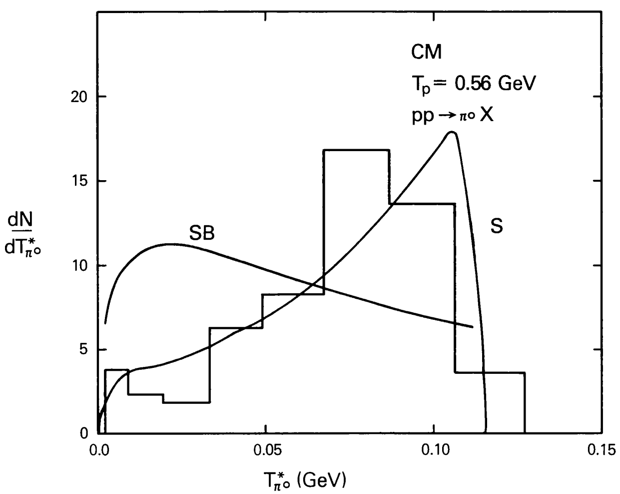
Low energies

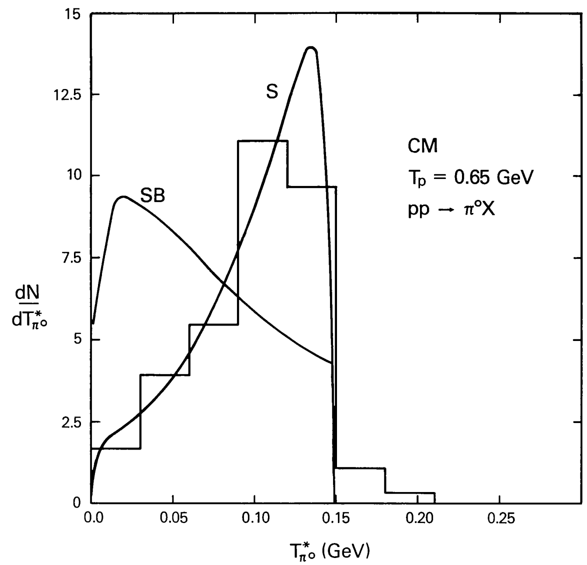
Near the pion production threshold at energies around 1 GeV the behavior of the cross section is model dependent, it suffers large experimental uncertainties.

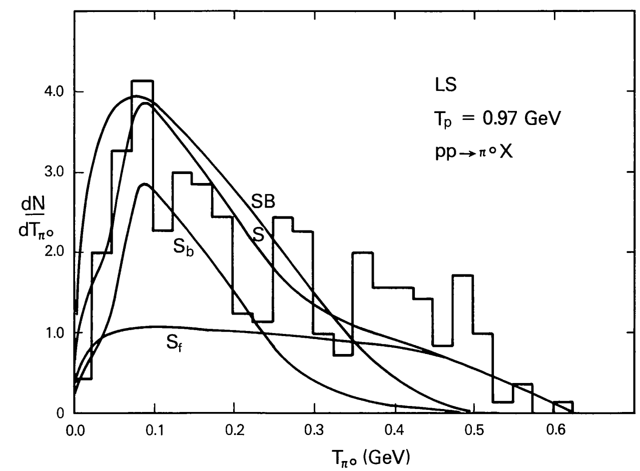
High energies

At energies larger than 10 GeV the behavior of the cross section is better understood with an almost constant (logarithmic) behavior and the function f satisfies the scaling f=f(x) with $x=E_\pi/E_p$.

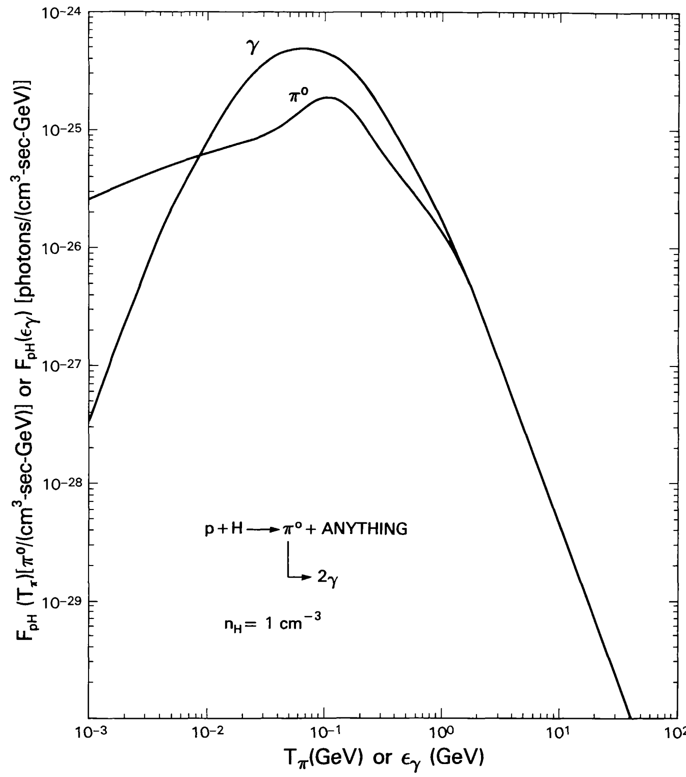
Dermer 1986







Comparison of the model prediction of the function $f(E_\pi, E_p)/E_\pi$ with experimental data. Photons and pions emissivity (C.D. Dermer, A&A (1986)).



NOTE: at high energies (>100 GeV) the CR spectrum as a power law behavior, which is reproduced by pions and gamma rays:

$$I_p(E) = I_0 \left(\frac{E}{E_0}\right)^{-\gamma}$$

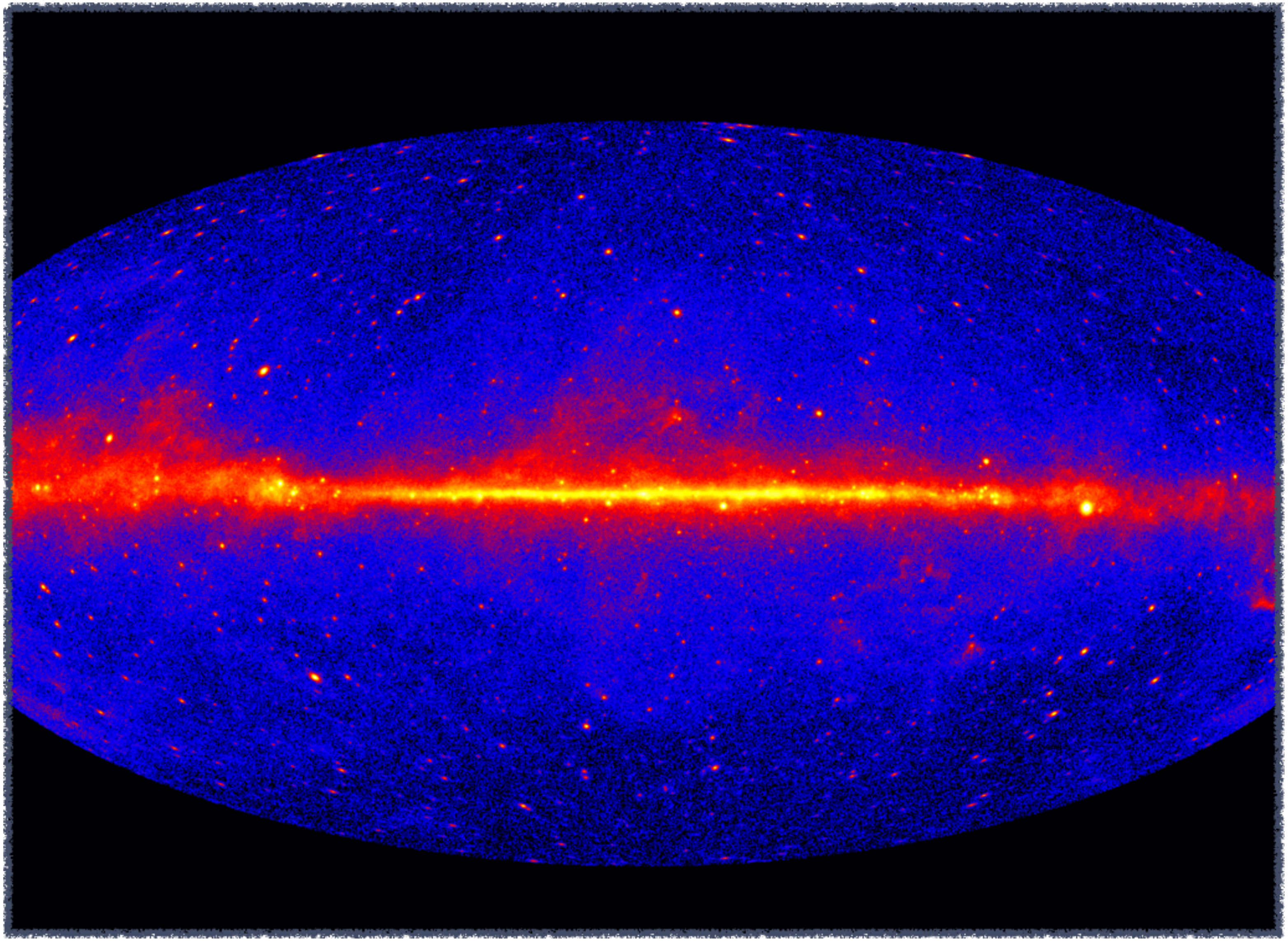
$$J_\pi(E_\pi) = 4\pi n_H I_0 E_0^\gamma E_\pi^{-\gamma} \sigma_0 \int_0^1 x^{\gamma-2} f(x) = 4\pi n_H \sigma_0 I_p(E_\pi)\Lambda_\gamma$$

$$J_\gamma(E_\gamma) = 2\int_{E_\gamma} \frac{dE_\pi}{E_\pi} J_\pi(E_\pi) = \frac{2}{\gamma} J_\pi(E_\gamma) = \frac{\gamma}{2} 4\pi n_H \sigma_0 I_p(E_\gamma)\Lambda_\gamma$$

with $\Lambda_\gamma = \int_0^1 dx f(x) x^{\gamma-2}$ a numerical constant.

Diffuse galactic gamma-ray background

The diffuse galactic gamma ray background as recently observed by Fermi-LAT is likely produced by pp scattering.



Fermi-LAT

Gamma-ray emission in SNR

The local environment of a Super Nova Remnant (SNR) is a likely place for both interactions: proton-proton and ICS. Gamma ray observations are of paramount importance in tagging the actual production mechanism.

Hadronic models

$$pp \to \pi^0 \to \gamma\gamma$$

γ rays emitted with the same spectrum of CR:

$$E^{-\gamma}$$

NOTE: this behavior of the gamma ray spectrum might be the smoking gun for CR acceleration. The flux of gamma rays should also exhibit the "pion bump" a peak in the flux due to pile up at the threshold $m_\pi/2$.

Leptonic models

$$e\gamma \to e\gamma$$

Gamma rays are produced by the inverse Compton scattering of relativistic electrons on local photon backgrounds. The spectrum of gamma rays emitted has a flatter behavior respect to CR

$$E^{-\frac{\gamma-1}{2}}$$

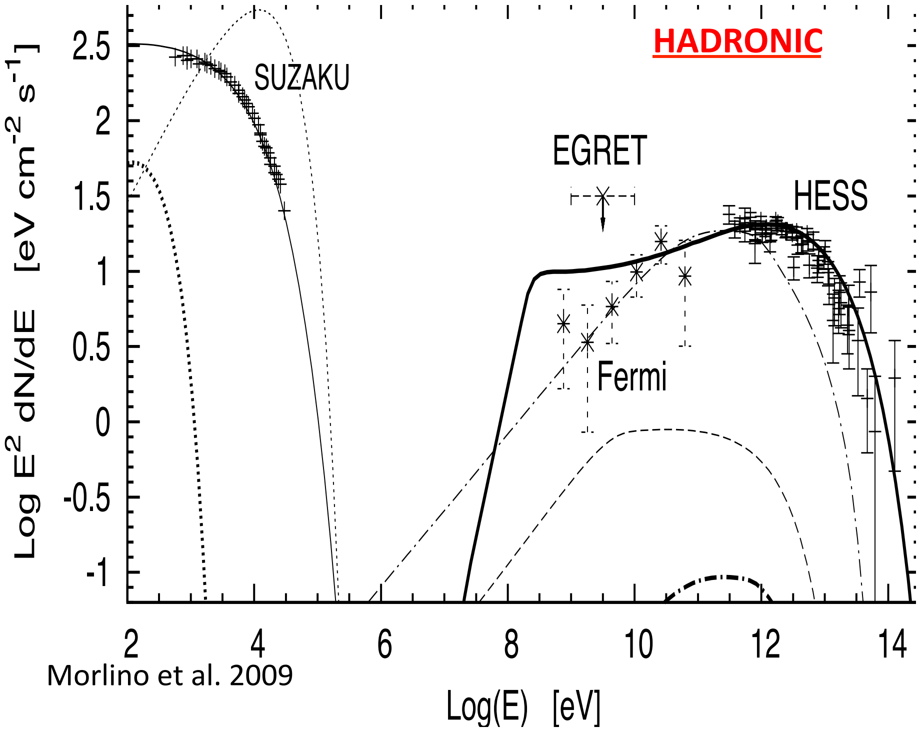
The case of RXJ1713

observed in keV, GeV and TeV range

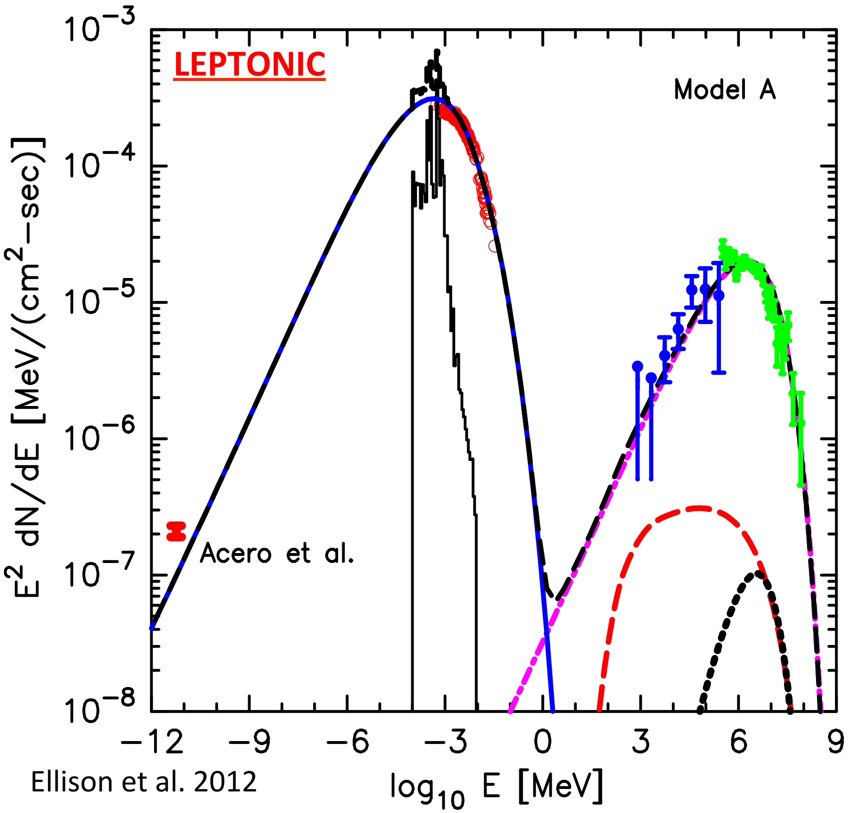
Bamba et al. (2009); Aharonian et al. (2004-2007); Abdo et al. (2011)

X-ray rims observed with B~160 μG are compatible with CR acceleration

hadronic origin of GeV-TeV emissions can be possible



Morlino et al. 2009



Ellison et al. 2012

No oxygen lines observed, very small target densities, less efficient pp interactions.

leptonic origin of GeV-TeV emissions requires high IR light (~20 times than observed) and too low B (if compared to X-ray emission).

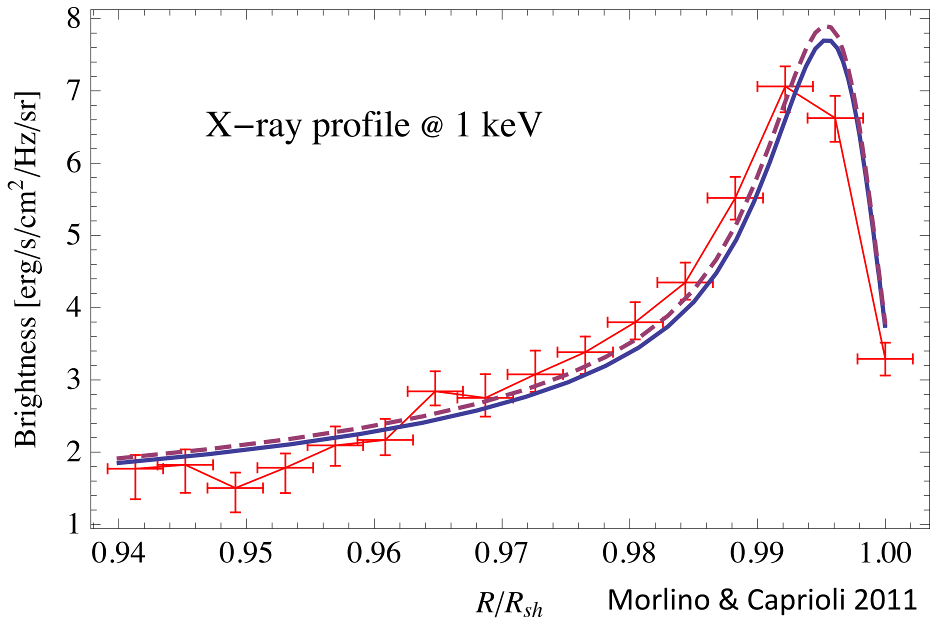
<u>complex environment, future high resolution gamma ray observations will distinguish different emitting regions</u>.

The case of Tycho

SNIa exploded in roughly homogeneous ISM (regular spherical shape)

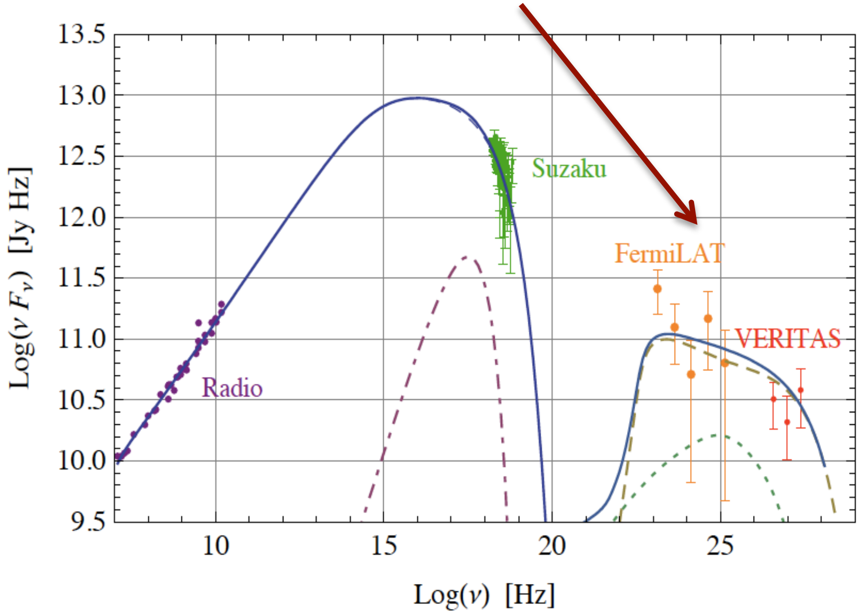
From X-ray observations B~300 μG

Maximum energy protons E_{max}~500 TeV



Morlino & Caprioli 2011

Steep spectrum hard to explain with leptons



Morlino & Caprioli 2011

Important example of the credibility level of theories based on DSA. <u>Space resolved gamma ray observations would test different theoretical hypothesis.</u>

Gamma-ray emission in SNR – quick summary

- The pion bump has not been seen so far (only in molecular clouds this feature seems observed, see later)

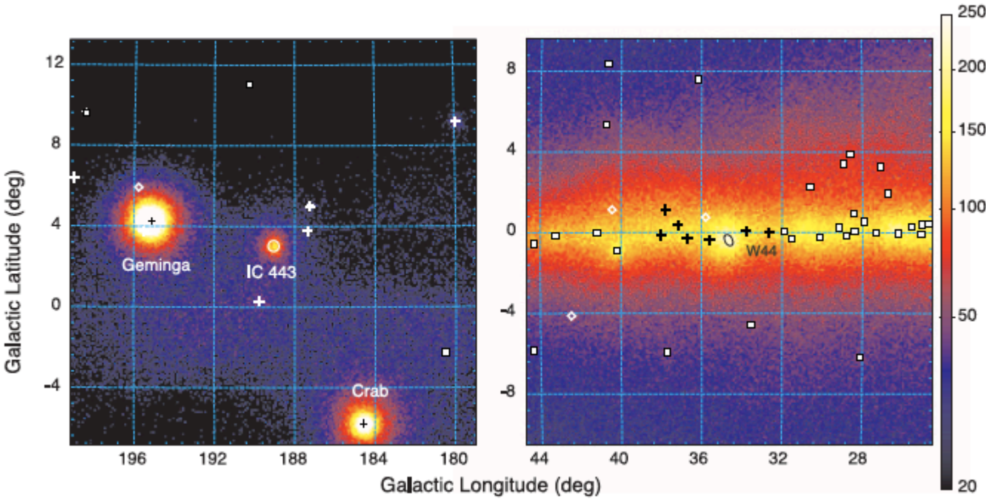
- The discrimination between leptonic models (ICS) and hadronic models (π^0 decay) can be achieved just observing the spectrum only with high angular resolution. Different parts of the SNR may have different spectra reflecting a different origin or/and the presence/absence of nearby targets (molecular clouds, see later). This may be the case of RXJ1713.

- Extension of the observations to high energies can provide an evidence of a cut-off in the PeV region (but low probability of finding a suitable SNR for this observations).

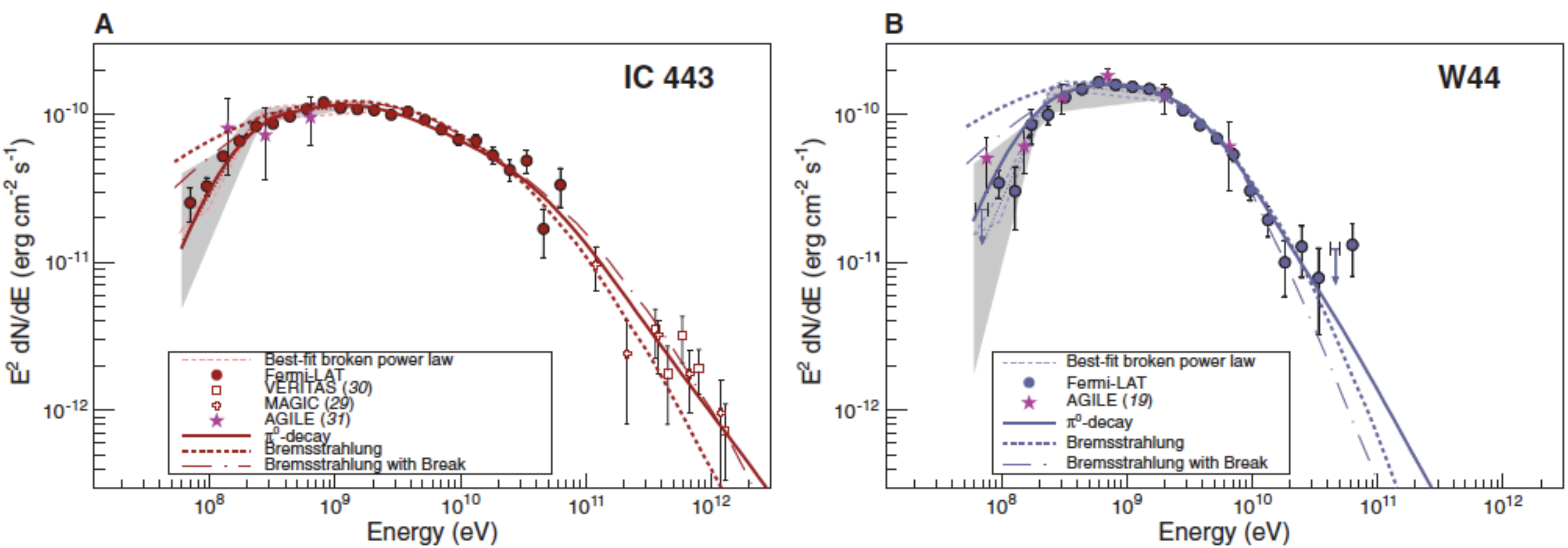
Gamma-ray emission from Molecular Clouds

- Observation of the pion bump directly linked to pion decay, i.e. a pile up of photons at the energy threshold $m_\pi/2$.

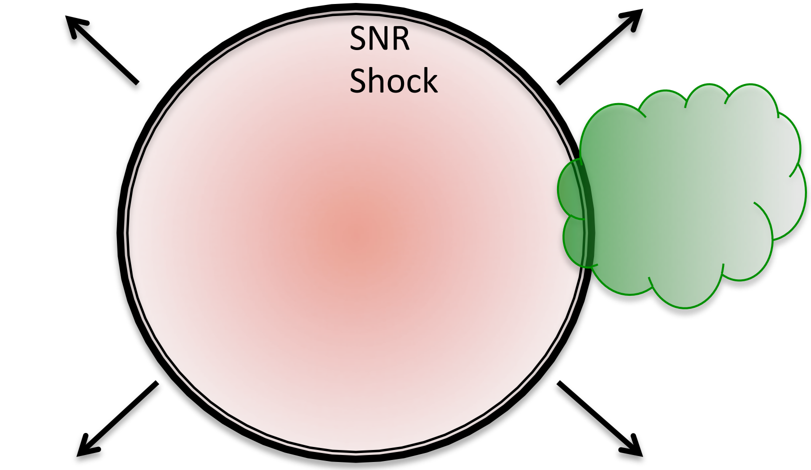
- SN close to molecular clouds are very interesting laboratories to investigate CR propagation around sources and escape from sources.



Ackermann et al. 2013



Shock inside the cloud

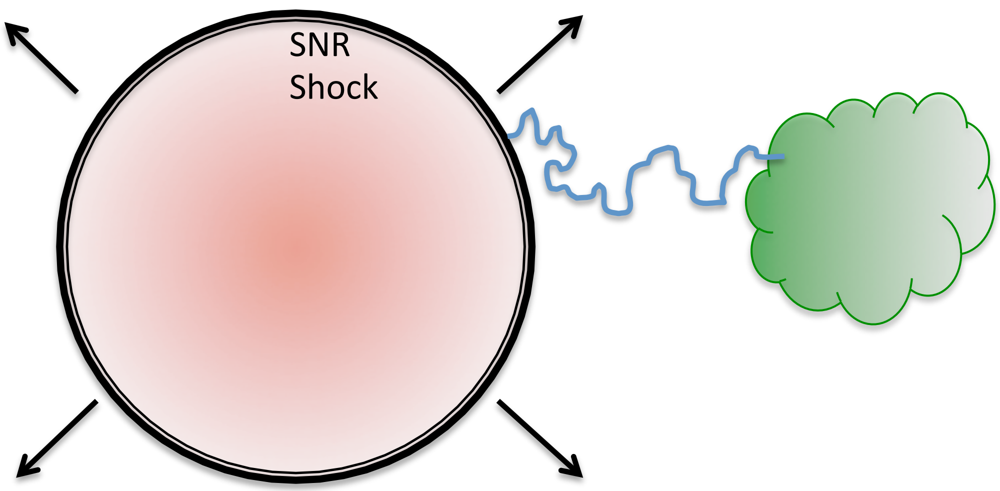


- the shock becomes collisional on scales

$$\lambda \approx \frac{1}{n_{cloud}\sigma_{mol}} \sim 10^{10}\left(\frac{n_{cloud}}{10^4 cm^{-3}}\right)^{-1}\left(\frac{\sigma_{mol}}{10^{-14}cm^2}\right)^{-1} cm$$

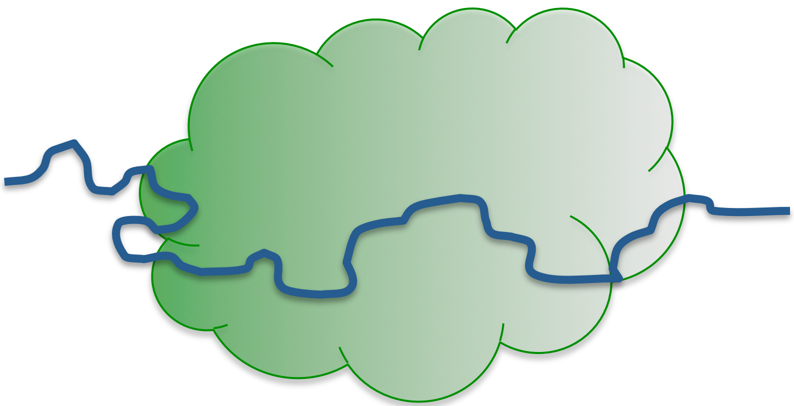
- It slows down since it feels the matter in the cloud, particle already accelerated escape streaming away and interacting with matter in the molecular cloud.

Shock outside the cloud

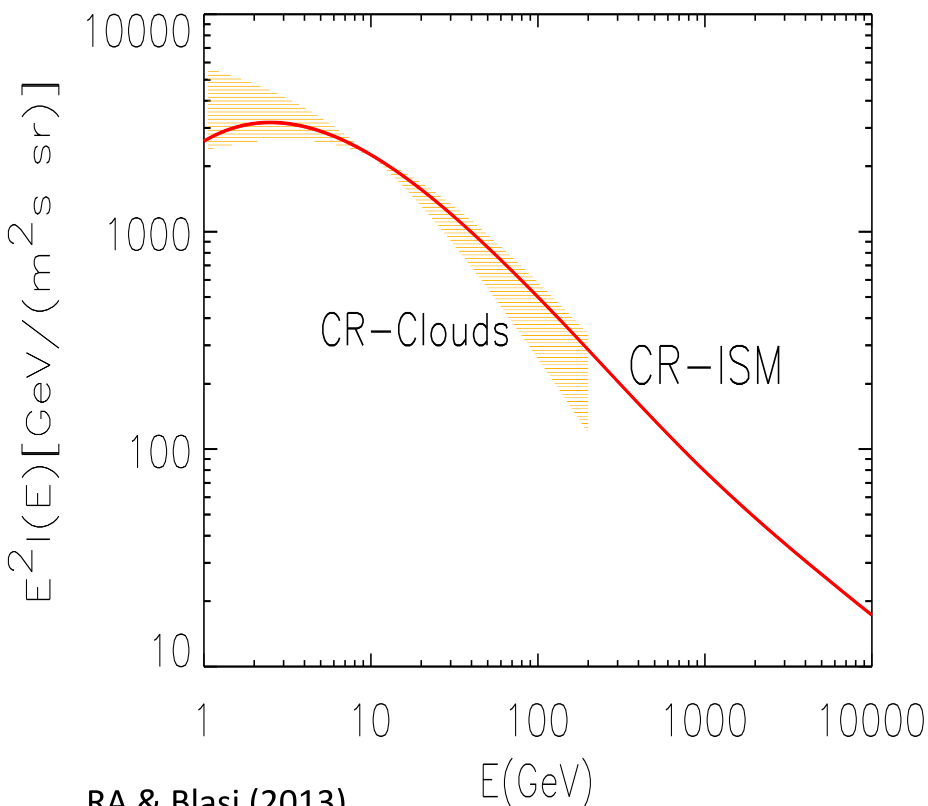


- γ-rays produced by CR reproduce the CR spectrum injected in ISM.
- γ-rays emission in this case could give direct information on the escaped flux of CR.

Gamma rays from isolated MC



This case is of particular importance in the study of the diffusive propagation of CR, offering a unique possibility of determining the CR spectrum unaffected by local effects such as the solar modulation. An interesting instance of these systems is represented by the γ-ray emission, already detected by COS-B, EGRET and more recently by Fermi, from the Gould Belt clouds, the nearest Giant Molecular Cloud (GMC).



RA & Blasi (2013)

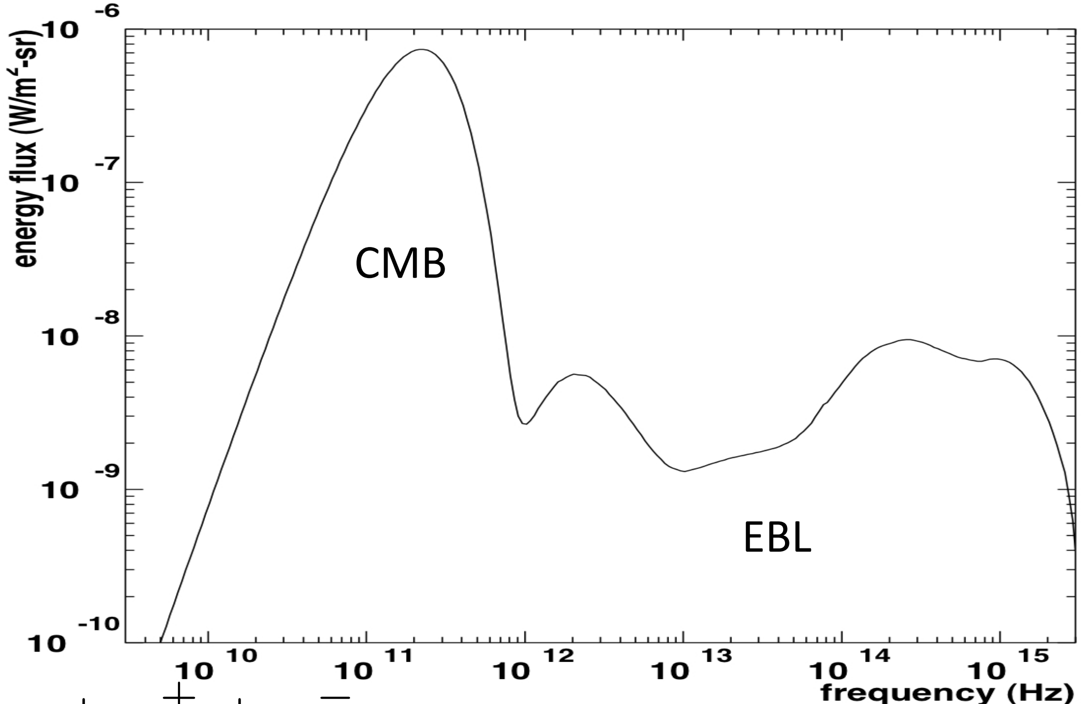
Observations of gamma rays from isolated Molecular Clouds can give important insights on the CR propagation models.
Possible confirmation of changes in the slope, non linear effects in propagation.

Gamma-ray emission in MC – quick summary

- Escape is the link between acceleration and CR observed at the Earth. High energy particles injected by the source are the sum of “escaped” and “released” particles.
- The two contributions to the injected spectrum (i.e. from escaped particles and particles released after the end of expansion) can be disentangled looking at the gamma ray emission from clouds.
- The study of these gamma emissions can also give important insights on the CR propagation inside clouds, most likely on self-generated turbulence, and on the diffusion topology.

Proton-Photon interactions

Let us consider now the interactions of high energy (relativistic) protons and astrophysical photon backgrounds. Relevant backgrounds are: the Cosmic Microwave Background (CMB) and the Extragalactic Background Light (EBL). Relevant processes are



Pair production

$$p + \gamma \rightarrow p + e^{+} + e^{-}$$

Photo-pion production

$$p + \gamma \rightarrow p + \pi_0 \qquad p + \gamma \rightarrow p + \pi^{+} + \pi^{-}$$

Both processes can be realized only if the proton has enough energy to produce the particles in the final state.

<u>Pair production</u>

$$E_{th} = \frac{(m_p + 2m_e)^2 - m_p^2}{2\epsilon(1 - cos\theta)} \simeq \frac{2m_e m_p}{\epsilon(1 - \cos\theta)}$$

CMB:
$\varepsilon = K_B T_0 \approx 2.3\text{x}10^{-4}$ eV

$$E_{th} \simeq 2 \times 10^{18} eV \qquad (\theta = \pi)$$

EBL:
$\varepsilon \approx 1$ eV

$$E_{th} \simeq 5 \times 10^{14} eV \qquad (\theta = \pi)$$

Photo pion production

$$E_{th} = \frac{(m_p + m_\pi)^2 - m_p^2}{2\epsilon(1 - cos\theta)} \simeq \frac{m_p m_\pi}{\epsilon(1 - \cos\theta)}$$

CMB:
$E = K_B T_0 \approx 2.3 \times 10^{-4}$ eV

$$E_{th} \simeq 2 \times 10^{20} eV \qquad (\theta = \pi)$$

EBL: $\varepsilon \approx 1$ eV

$$E_{th} \simeq 1 \times 10^{17} eV \qquad (\theta = \pi)$$

These reactions become important at very high energy, the regime of the so-called Ultra High Energy Cosmic Rays (UHECR) observed with energies up to 10^{20} eV.

NOTE: the photo pion production process implies a huge Lorentz boost between the LAB frame and the rest frame of the proton, at the level of 10^{11} .

The energy losses of UHE protons due to pair-production and photo pion production, i.e. energy lost per unit time, can be computed in a very general form through

$$\frac{1}{E}\frac{dE}{dt} = \frac{c}{2\Gamma^2}\int_{\epsilon'_{min}} d\epsilon' f(\epsilon')\sigma(\epsilon') \int_{\epsilon'/2\Gamma} d\epsilon \frac{n_\gamma(\epsilon)}{\epsilon^2}$$

being ε' and ε'_{min} the photon energy and the threshold energy in the reference frame in which the proton is at rest, $f(\varepsilon)$ is the so-called inelasticity of the process, i.e. the average fraction of energy lost by the proton in one interaction in the LAB reference frame, $\sigma(\varepsilon)$ the cross-section of the process, $n_\gamma(\varepsilon)$ the number of background photons per unit volume and energy, $\Gamma = E/m_p$ the Lorentz factor of the incident proton.

Taking into account only the CMB, which has a predominant role at energy >10^{17} eV, we can analytically workout the behavior of energy losses. Using

$$n_{CMB}(\epsilon) = \frac{1}{\pi^2(\hbar c)^2} \frac{\epsilon^2}{\exp\left(\frac{\epsilon}{K_B T_0}\right) - 1}$$

one has

$$\frac{1}{E}\frac{dE}{dt} = \frac{c K_B T_0}{2\pi^2 \Gamma^2 (\hbar c)^2} \int_{\epsilon'_{min}} d\epsilon' f(\epsilon') \sigma(\epsilon') \epsilon' \left\{ -\ln\left[1 - \exp\left(-\frac{\epsilon'}{2\Gamma K_B T_0}\right)\right]\right\}$$

Pair production

At energies below threshold < 2×10^{18} eV, the pair production process is realized only through the CMB photons on the tail of the Planckian distribution. In this regime the inelasticity f and the cross section σ can be estimated at their threshold value:

$$f(\epsilon') \simeq \frac{2m_e}{m_p} \qquad \sigma_{ee}(\epsilon') \simeq \frac{\pi}{12}\alpha r_0^2 \left(\frac{\epsilon'}{m_e} - 2\right)^3 \qquad \epsilon'_{min} = 2m_e\left(1 + \frac{m_e}{m_p}\right)$$

$$\frac{1}{E}\frac{dE}{dt} = \frac{16c}{\pi}\frac{m_e}{m_p}\alpha r_0^2 \left(\frac{K_B T_0}{c\hbar}\right)^3 \left(\frac{\Gamma K_B T_0}{m_e}\right)^2 \exp\left(-\frac{m_e}{K_B T_0}\right)$$

Exponentially suppressed at

$\Gamma K_B T_0 \ll m_e \quad E \ll 2\times10^{18} eV$

at energies higher than 2×10^{18} eV the pair production process becomes rapidly less efficient respect to photo pion production.

Photo pion production

Also in this case, at energies below threshold <2x10^{20} eV, being the interaction with the tail of the CMB distribution, we can determine f(ε) and σ(ε) at the threshold value

$$f(\epsilon') = \frac{\epsilon'}{m_p} \frac{1 + m_\pi^2/m_p\epsilon'}{1 + 2\epsilon'/m_p} \qquad \sigma(\epsilon') = \sigma_0 \left(\frac{\epsilon'}{\epsilon'_{min}} - 1 \right) \qquad \epsilon'_{min} = m_\pi \left(1 + \frac{m_\pi}{m_p} \right)$$

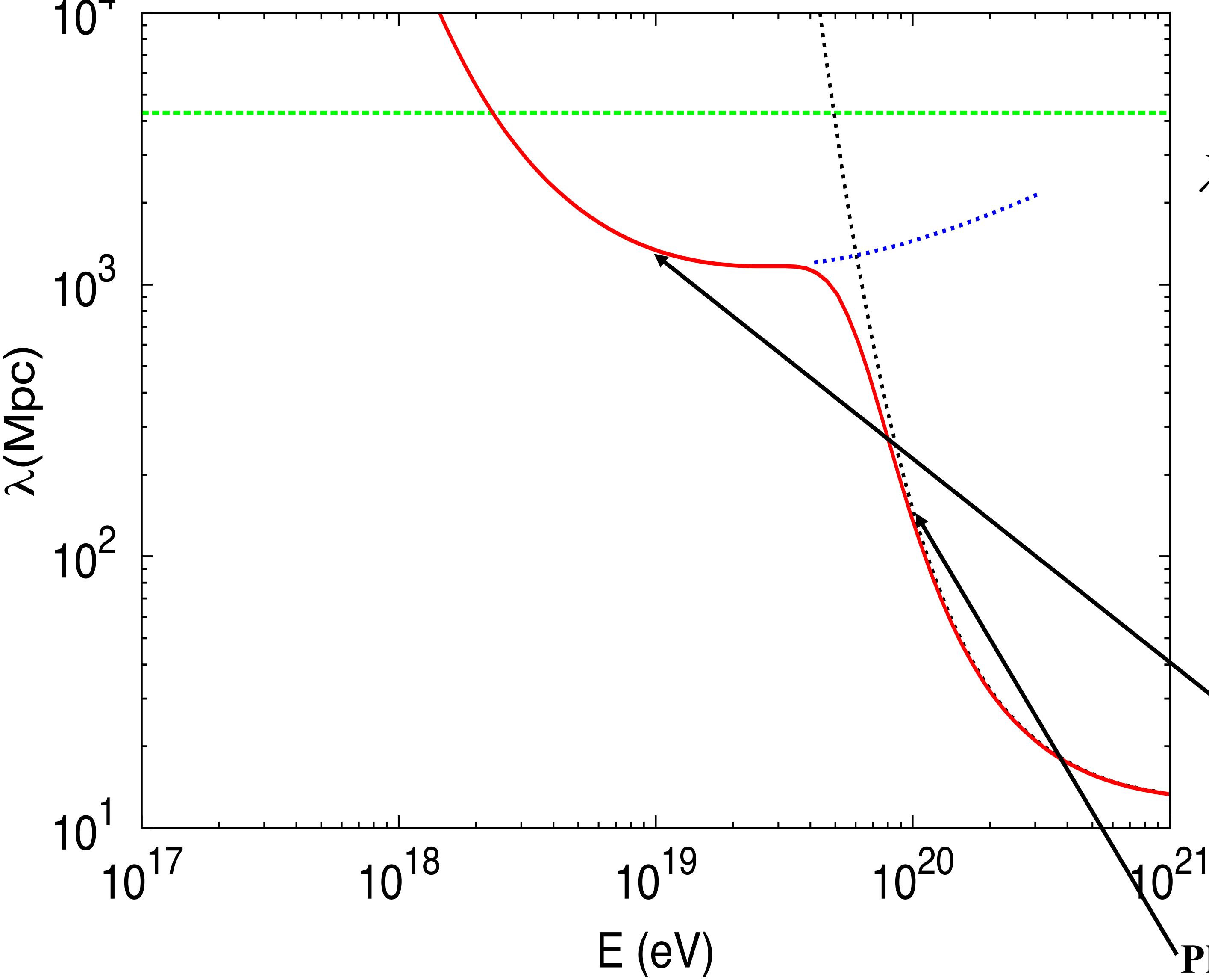
with σ_0 = 4x10^{-28} cm^2. The energy losses are

$$\frac{1}{E}\frac{dE}{dt} = \frac{2}{\pi^2} \left(\frac{K_B T_0}{c\hbar} \right)^3 \frac{\epsilon'_{min}}{m_p} \sigma_0 c \exp\left(-\frac{\epsilon'_{min}}{2\Gamma K_B T_0} \right) \left[1 + 4\frac{2\Gamma K_B T}{\epsilon'} \right]$$

At higher energies >2x10^{20} eV, the cross section becomes almost constant around 1x10^{-28} and the average fraction of energy lost by the proton (in the LAB frame) f≈1/2 and

$$\frac{1}{E}\frac{dE}{dt} = c f_0 \sigma_0 n_{CMB} = 1.8 \times 10^{-8} y^{-1}$$

Attenuation length of UHECR protons



$$\lambda = c\left(\frac{1}{E}\frac{dE}{dt}\right)^{-1}$$

Pair production

$p\,\gamma \rightarrow p\,e^{+}\,e^{-}$

Photopion production

$p\,\gamma \rightarrow N\,\pi$

Nucleus-Photon interactions

The interaction processes that involve high energy (>10^{17} eV) nuclei and astrophysical backgrounds (CMB and EBL) are: pair production and photo-disintegration.

$$(A,Z)+\gamma \to (A,Z)+e^{+}+e^{-}$$

$$(A,Z)+\gamma \to (A-n,Z-n)+nN$$

<u>Pair production</u> $E_{th} = AE_{th}^{p} \simeq A \times 2 \times 10^{18}\ \mathrm{eV}$

This is the same processes that involves protons. The only relevant background involved is the CMB. The type of nucleus (A,Z) remains unchanged, only the particle's energy changes because of the interaction. The rate of energy losses can be computed starting from the rate of protons and using the scaling

$$f^{A} = \frac{f^{p}}{A} \qquad \sigma^{A} = Z^{2}\sigma^{p} \qquad \left(\frac{1}{E}\frac{dE}{dt}\right)^{(A,Z)} = \frac{Z^{2}}{A}\left(\frac{1}{E}\frac{dE}{dt}\right)^{(p)}$$

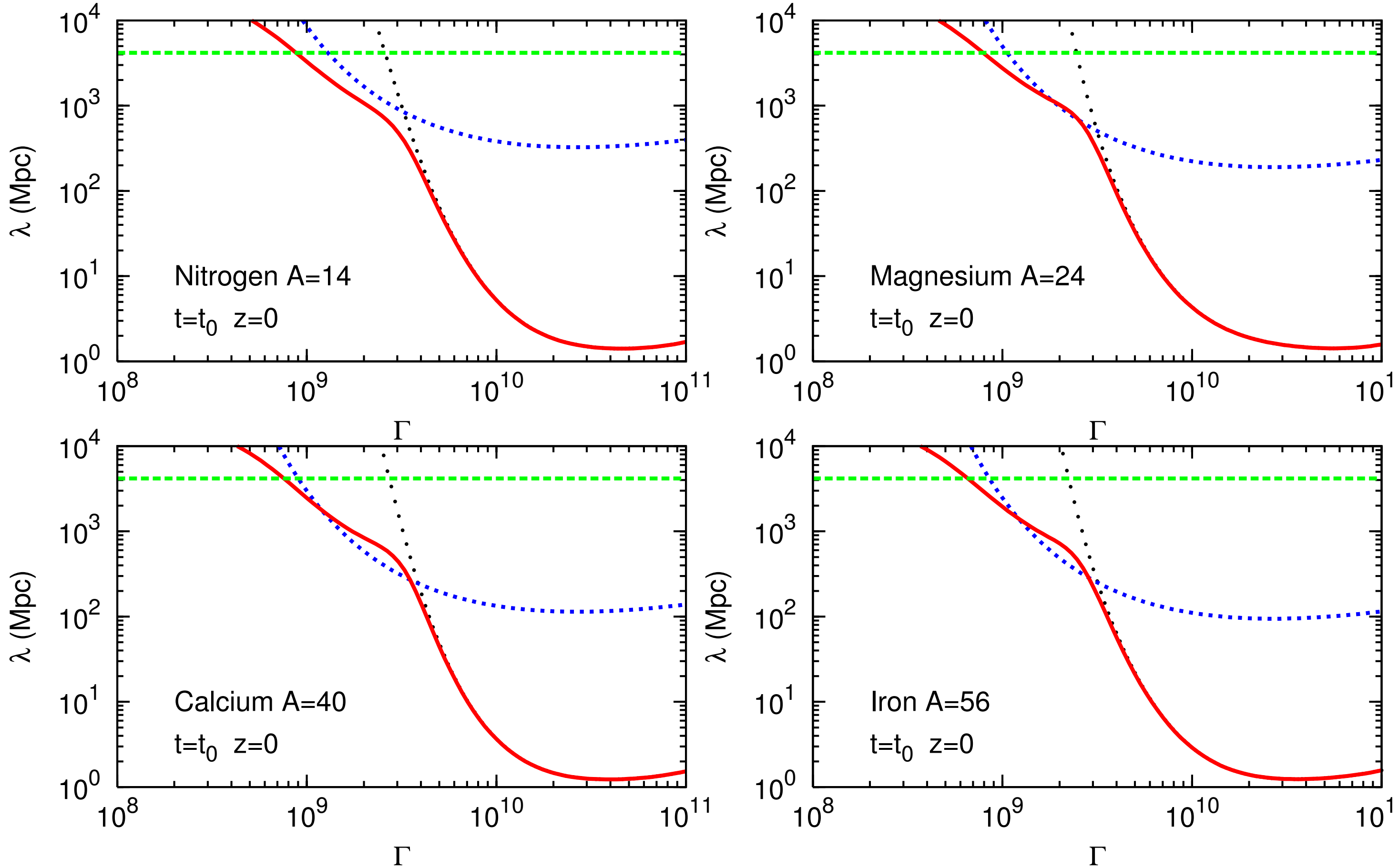
<u>Photo disintegration</u> $E_{th} = AE_{th}^{0} \simeq A \times (3 \div 5) \times 10^{18}\ \mathrm{eV}$

This process does not change the Lorentz factor of the nucleus, it changes only the type of nucleus that in the final state will be depleted by one or more nucleons. However, the dominant photo disintegration channel implies the emission of a single nucleon.
This process cannot be treated as a continuous decrease in energy of the propagating particle, because it implies a change in the particle type with the disappearence of the initial particle.

It can be treated as a "decaying" process with a typical "decaying time" given by

$$\frac{1}{\tau_A} = \frac{c}{2\Gamma^2} \int_{\epsilon'_{min}} d\epsilon' \sigma(\epsilon') \int_{\epsilon'/2\Gamma} d\epsilon \frac{n_\gamma(\epsilon)}{\epsilon^2}$$

NOTE: The propagation of UHE nuclei always implies the generation of secondary lighter UHE nuclei (the Lorentz factor is conserved!).



Just a little bit of Cosmology

We saw that UHE (>10^{17} eV) particles (both protons and nuclei) can travel over cosmological distance, hence Cosmology does matter!

The effect of the expansion of the universe is seen in the change in astrophysical backgrounds, in both density and radiation's energy, and in the UHE particle's energy.

CMB: it is the well known relic radiation of the big bang, with a Planckian distribution and the evolution with red shift through its temperature $T(z)=T_0(1+z)$.

EBL: composed of infrared, optical and ultraviolet photons produced by astrophysical sources and scattered by the ISM (dust) at present and past cosmological epochs. Has a less understood cosmological evolution, typically model dependent.

The rate of energy (adiabatically) lost by UHE particles (protons or nuclei) because of the expansion of the universe can be written as

$$\left(\frac{1}{\Gamma}\frac{d\Gamma}{dt}\right) = H(z) = H_0\sqrt{(1+z)^3\Omega_m + \Omega_\Lambda}$$

The rate of energy losses due to the processes discussed so far will get a red-shift dependence as follows:

$$n_\gamma(\epsilon) \to n_\gamma(\epsilon, z) \quad \epsilon \to \epsilon(1+z) \quad \Gamma \to \Gamma(1+z)$$

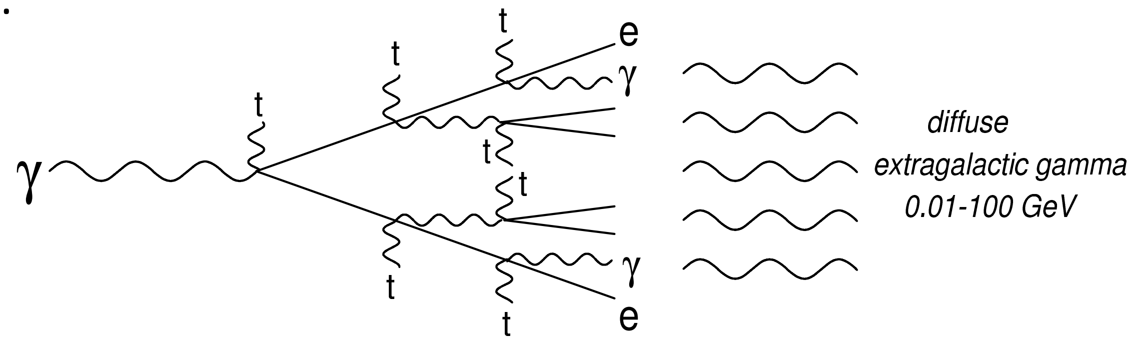
Secondary emission gamma rays and neutrinos

Proton and nuclei interactions with astrophysical backgrounds produce hadronic particles, mainly pion, that in turn decay giving rise to the production of secondary UHE photons, electron positron pairs and neutrinos.

$$p(A)\gamma \to e^{\pm}$$

$$p(A)\gamma \to \pi^0 \to \gamma\gamma$$

$$p(A)\gamma \to \pi^{\pm}\mu^{\pm} \to e^{\pm}\nu$$



Cascade develops on CMB and EBL through pair production and ICS, the typical threshold energy scales for pair production are

$$\mathcal{E}_{CMB} \simeq 2.5 \times 10^{14} eV \qquad \mathcal{E}_{EBL} \simeq 2.5 \times 10^{11} eV$$

The radiation left behind by the cascade is restricted to energies below the lowest threshold.

$$n_\gamma(E_\gamma) \propto \begin{cases} E_\gamma^{-3/2} & E_\gamma < \mathcal{E}_X \\ \\ E_\gamma^{-2} & \mathcal{E}_X \le E_\gamma \le \mathcal{E}_{EBL} \end{cases}$$

The cascade development has a universal behavior independent of the energy and spectrum of the initial photon/pair

$$\mathcal{E}_X = \frac{\mathcal{E}_{EBL}}{3}\frac{\epsilon_{CMB}}{\epsilon_{EBL}} \simeq 10^7 \text{ eV}$$

$\varepsilon_{CMB} = 3\text{x}10^{-4}$ eV and $\varepsilon_{EBL} = 1$ eV are the typical energies of the background photons.

The normalization of the cascade spectrum can be easily determined imposing energy conservation, i.e. the total energy of photons\pairs that started the cascades should be equal to the total energy of the cascading particles.

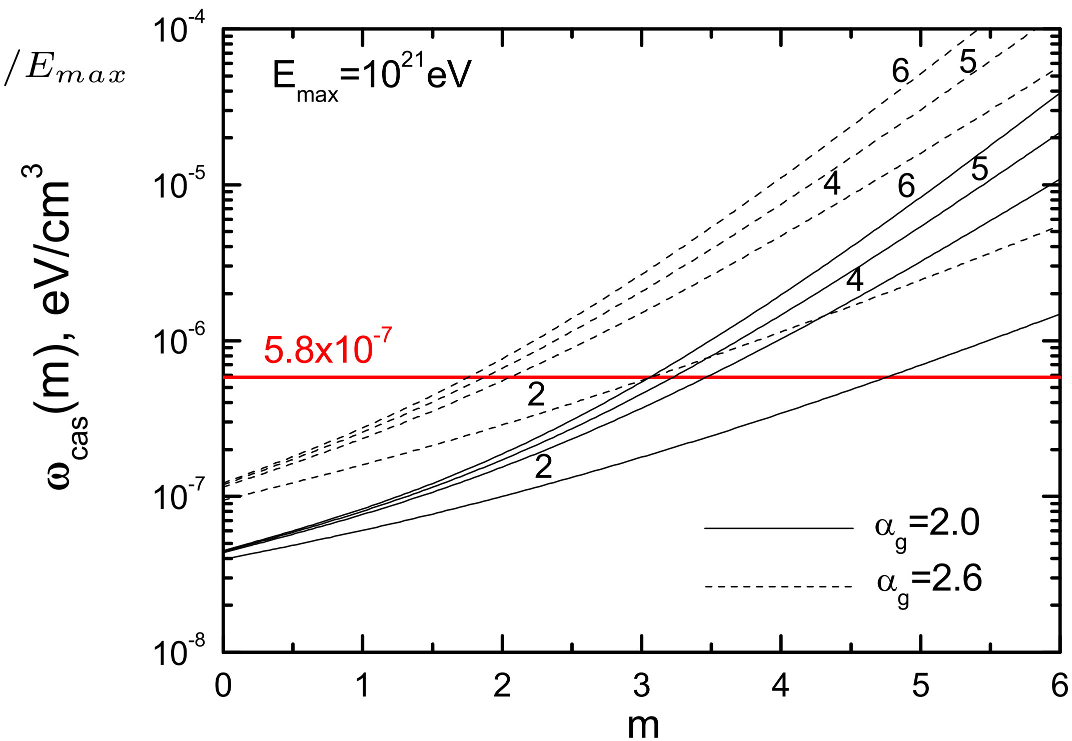
$$\omega_{cas}^{max} = \int dE_\gamma E_\gamma n_\gamma(E_\gamma)$$

Fermi-LAT data (ω_{cas}= 5.8x10^{-7} eV/cm^3) can be compared with the theoretical expectation above to constrain models of UHECR.

$$Q(E) = Q_0(1+z)^m \left(\frac{E}{E_0}\right)^{\alpha_g} e^{-E/E_{max}}$$

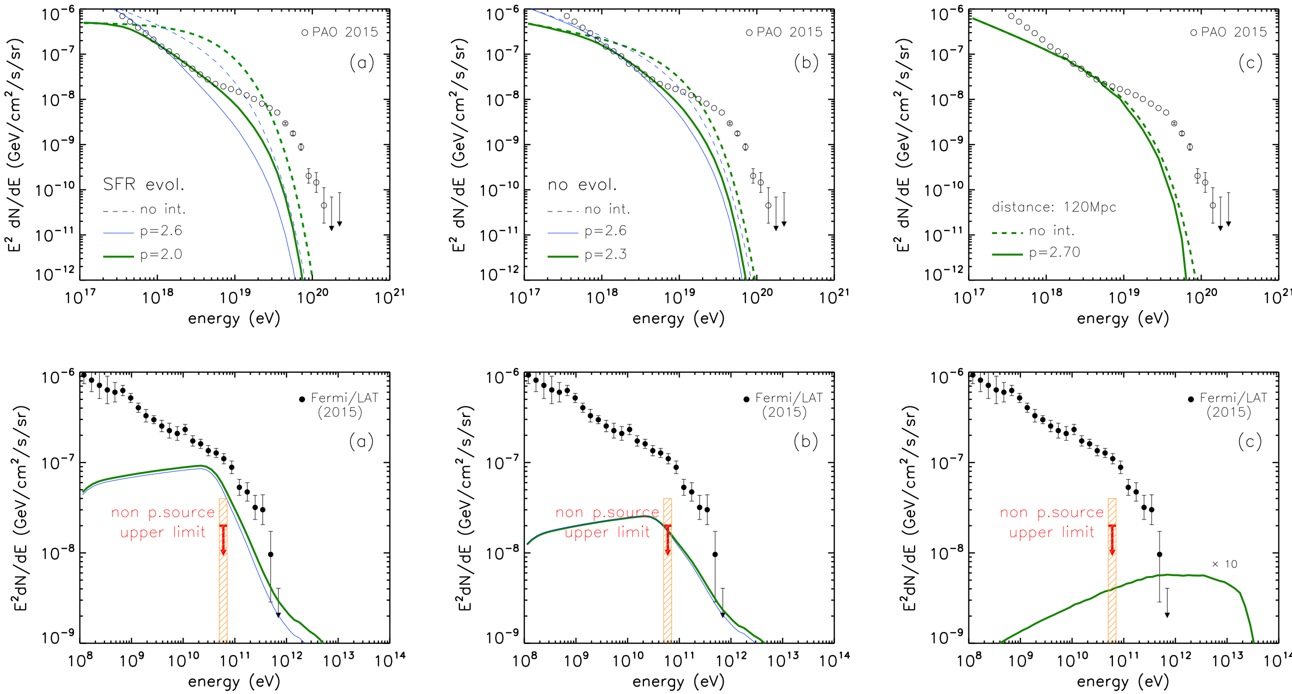
This comparison already fixes stringent limits on the cosmological evolution of sources of UHE protons.

NOTE: the production of secondary gamma rays and neutrinos is maximal in the case of a pure proton composition of UHECR.



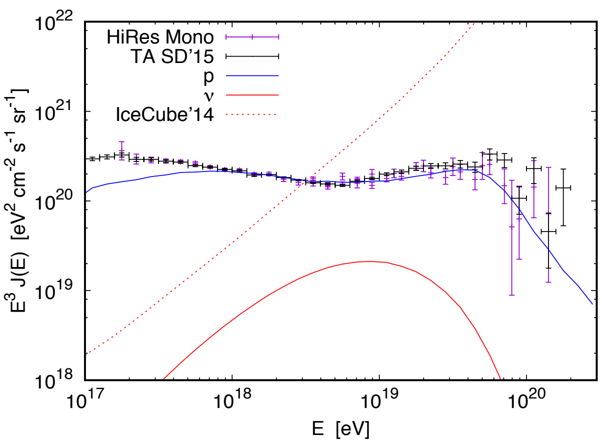
Berezinsky, Gazizov, Kachelriess, Ostapchenko (2011)

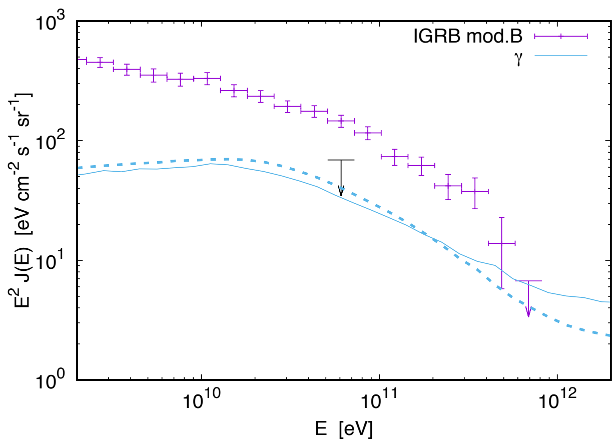
Diffuse gamma rays



Liu, Taylor, Wang, Aharonian (2016)

Diffuse extragalactic gamma-rays flux at E ~ 1 TeV is a very powerful observable to constrain the fraction of protons in the UHECR spectrum. With the available statistics, given the poor knowledge of the galactic diffuse foregrounds and EBL, it is impossible to exclude a pure proton composition at (1 – 40) EeV.





Berezinsky, Gazizov, Kalashev (2016)

The observation of the diffuse extra-galactic gamma-ray background will be one of the important tasks for the future CTA observatory.

Neutrinos

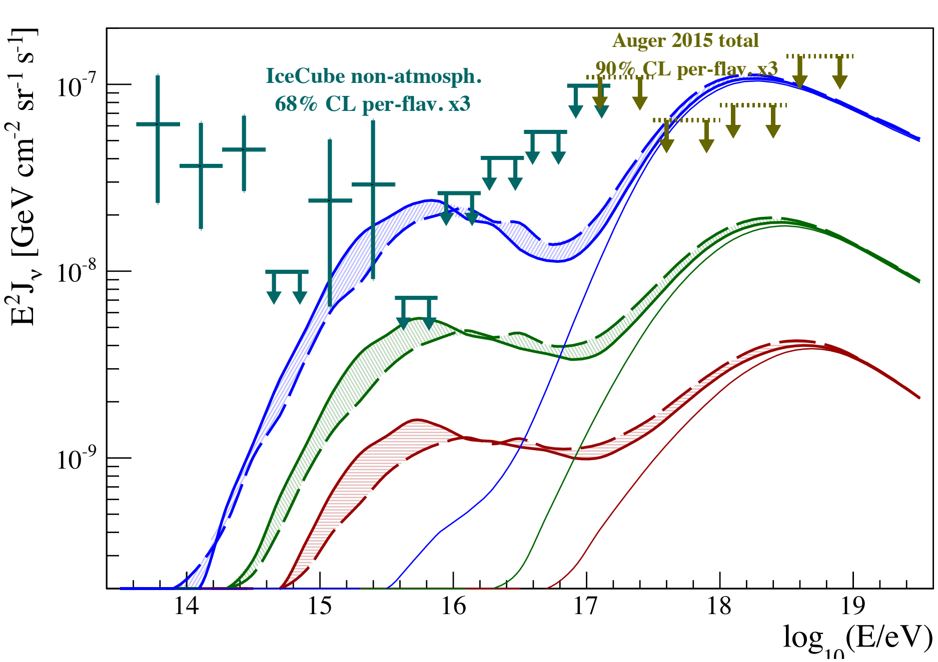
EeV neutrinos

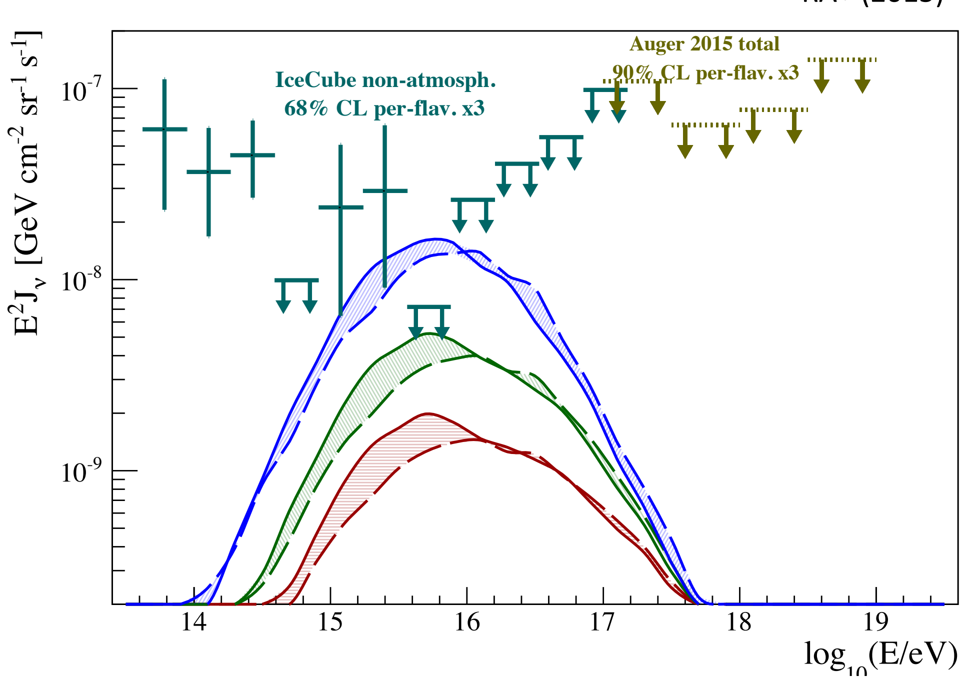
UHE nuclei suffer photo-pion production on CMB only for energies above AE_{GZK}. The production of EeV neutrinos strongly depends on the nuclei maximum energy. UHE neutrino production by nuclei practically disappears in models with maximum nuclei acceleration energy $E_{max} < 10^{21}$ eV.

PeV neutrinos

PeV neutrinos produced in the photo-pion production process of UHECR on the EBL radiation field The IceCube observations at PeV can be marginally reproduced in the case of strong cosmological evolution (AGN like).

RA+ (2015)





γ from distant AGN

The observed high energy gamma ray signal by distant blazars may be dominated by secondary gamma rays produced along the line of sight by the interaction of UHE protons with background photons. This hypothesis solves the problems connected with the flux observed by too distant AGN.

$$J_{\gamma,primary} \propto \frac{1}{d^2} exp^{-d/\lambda_\gamma}$$

$$J_{\gamma,secondary} \propto \frac{r_\gamma \lambda_\gamma}{4\pi d^2}\left[1 - e^{-d/\lambda_\gamma}\right]$$

NOTE: at large distances the contribution of secondary gammas dominates.

$$\Delta\theta \simeq 0.1^\circ \left(\frac{B}{10^{-14}G}\right)\left(\frac{4\times 10^7 GeV}{E}\right)\left(\frac{D}{1Gpc}\right)\left(\frac{l_c}{1Mpc}\right)$$

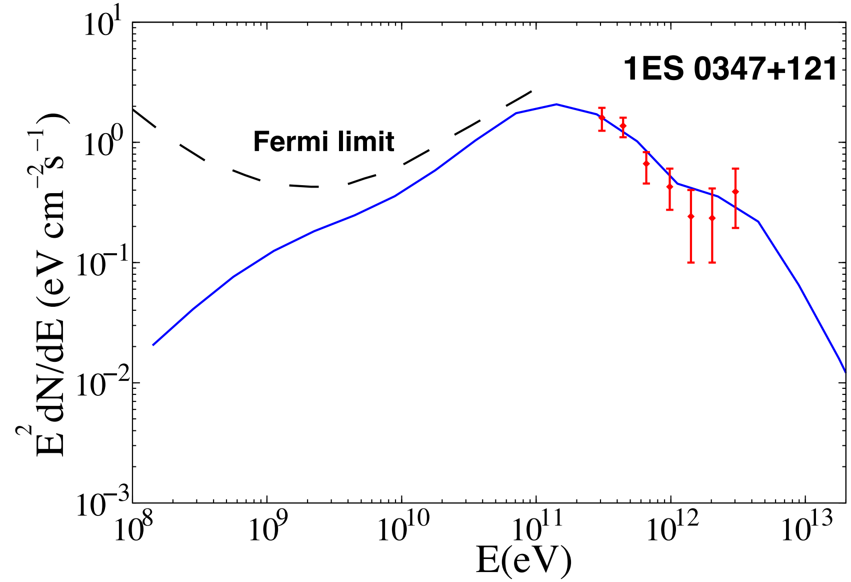
NOTE: this model requires low IMF at the level of femtogaus (10^{-15} G).

The spectrum of the final cascade is universal. The EM cascade behaves as a sort of calorimeter that redistribute the initial energy into gamma rays and neutrinos with a given spectrum, as discussed above.

Ferrigno, Blasi, De Marco (2004)   Essey, Kalashev, Kusenko, Beacom (2009-13)

The shape of the spectrum is fixed by the EBL, the overall height is proportional to the product of UHECR luminosity and the level of EBL.

The effect of different E_{max} is to change the relative contribution of the different reactions to the flux of secondaries. If E_{max} is large ($>10^{19}$ eV) interaction on CMB dominates, otherwise photo-pion production on EBL plays a role (provided that $E_{max}>10^{8}$ GeV).



Essey, Kalashev, Kusenko, Beacom (2011)