# Cosmic Ray Propagation


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# Outline

- Transport equation for cosmic rays. Elementary theory of diffusion.
- Basic galactic model of cosmic ray origin.

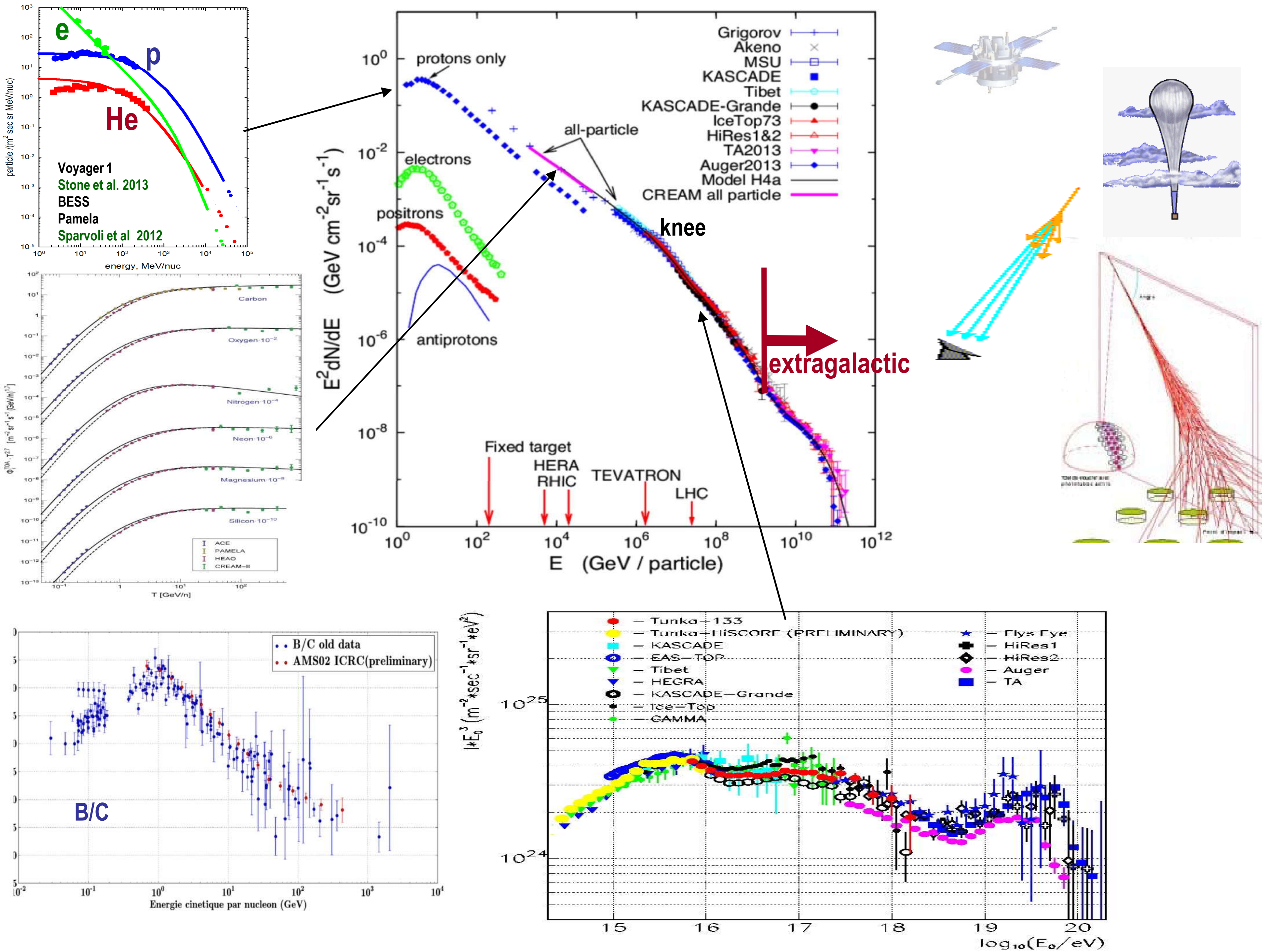
  Primary and secondary nuclei. Electrons and positrons. Anisotropy. Fluctuations.

- Nature of the knee.
- Cosmic rays of extragalactic origin.

  GZK cutoff. Data interpretation.

- Collective effects of cosmic rays.

  Streaming instability. Parker instability. Galactic wind model.


e
p
He
Voyager 1
Stone et al. 2013
BESS
Pamela
Sparvoli et al 2012
energy, MeV/nuc
particle /(m² sec sr MeV/nuc)
Carbon
Oxygen·10⁻²
Nitrogen·10⁻⁴
Neon·10⁻⁶
Magnesium·10⁻⁸
Silicon·10⁻¹⁰
ACE
PAMELA
HEAO
CREAM-II
T [GeV/n]
protons only
electrons
positrons
antiprotons
all-particle
knee
extragalactic
Fixed target
HERA
RHIC
TEVATRON
LHC
Grigorov
Akeno
MSU
KASCADE
Tibet
KASCADE-Grande
IceTop73
HiRes1&2
TA2013
Auger2013
Model H4a
CREAM all particle
E²dN/dE (GeV cm⁻² sr⁻¹ s⁻¹)
E (GeV / particle)
B/C old data
AMS02 ICRC(preliminary)
B/C
Energie cinetique par nucleon (GeV)
Tunka-133
Tunka-HiSCORE (PRELIMINARY)
KASCADE
EAS-TOP
Tibet
HEGRA
KASCADE-Grande
Ice-Top
GAMMA
Flys Eye
HiRes1
HiRes2
Auger
TA
log₁₀(E₀/eV)

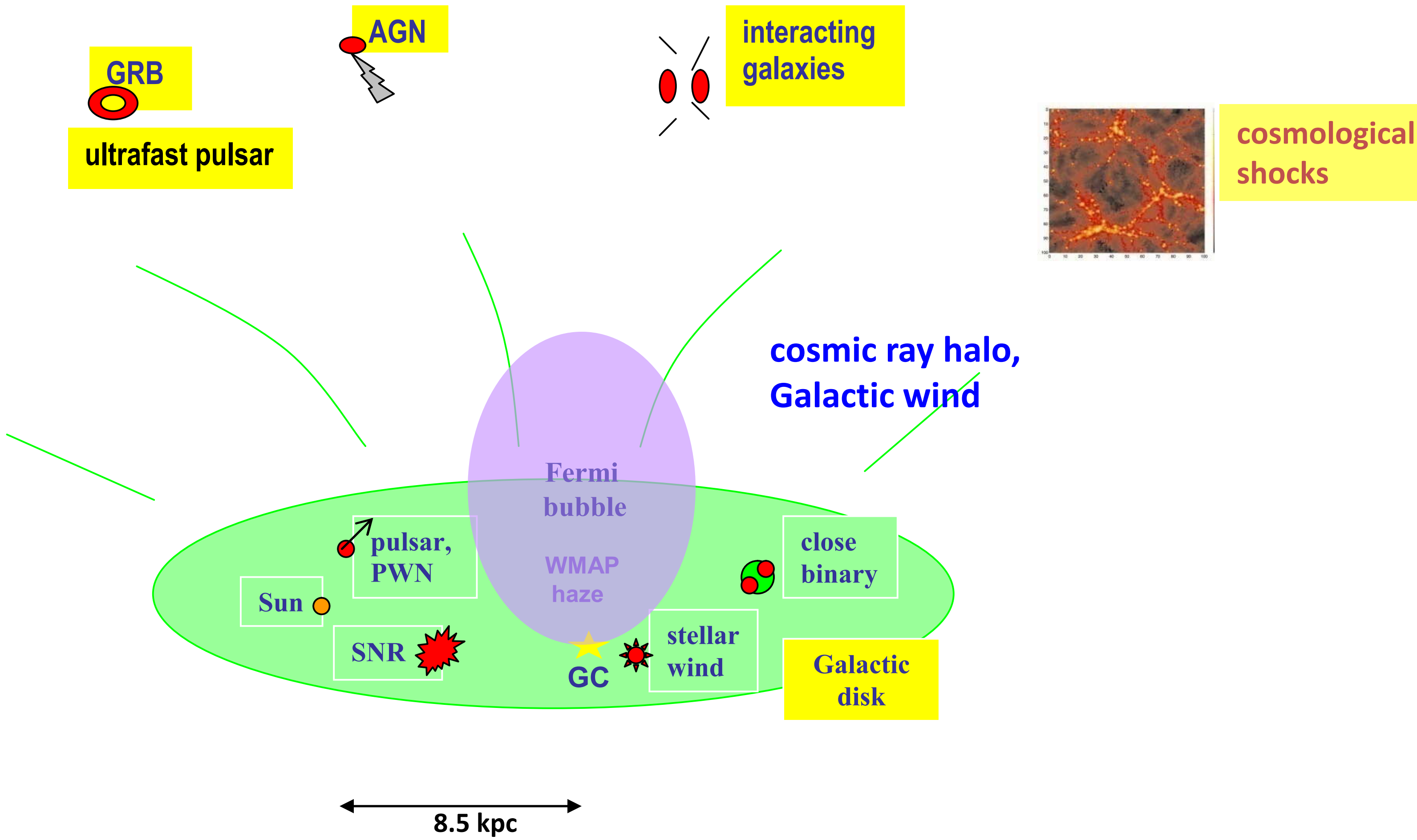
## “golden age” of new cosmic ray measurements

**Spacecrafts:** Voyagers, ACE, Pamela, Fermi/LAT, AMS

**Balloons:** BESS, ATIC, CREAM, TRACER

**Cherenkov telescopes:** HESS, MAGIC, VERITAS

**EAS detectors:** KASCADE/KASCADE-Grande, MILAGRO, ARGO-YBJ, TUNKA, EAS-TOP, IceCube/IceTop, Auger, Telescope Array




$N_{cr}$ ~ $10^{-10}$ $cm^{-3}$ - number density in the Galaxy

$w_{cr}$ ~ 1.5 $eV/cm^3$ - energy density

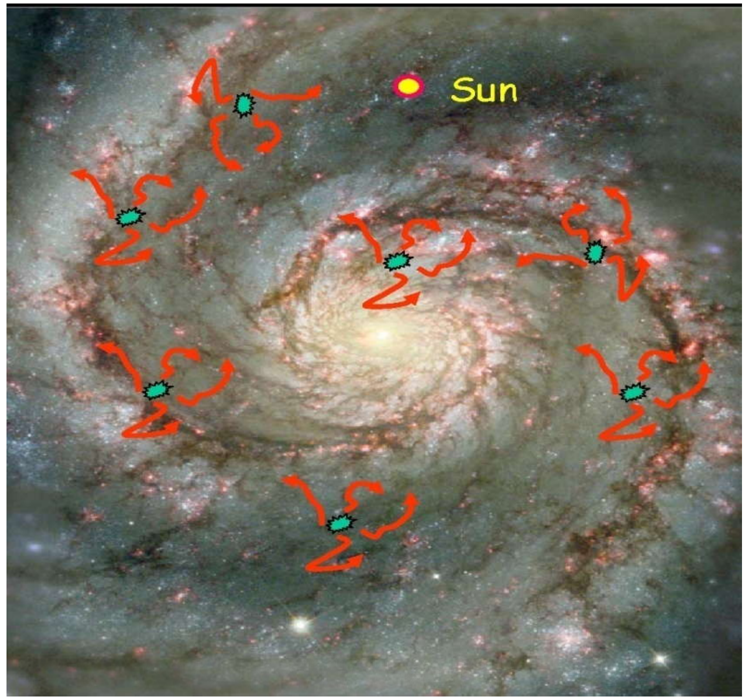
$L_{cr}$ ~ $10^{41}$ erg/s - total power of galactic sources

$E_{max}$ ~ $3x10^{20}$ eV - max. detected energy

$A_1$ ~ $10^{-3}$ – dipole anisotropy at 1 – 100 TeV

$r_g$ ~ $1 \times E/(Z \times 3 \times 10^{15}$ eV) pc - Larmor radius at $B=3x10^{-6}$ G

# Transport equation for cosmic rays. Elementary theory of diffusion.




energy balance: ~ 15% of SN kinetic energy go to cosmic rays to maintain observed cosmic ray density Ginzburg & Syrovatskii 1964

**steady state:**
(without energy losses and fragmentation)

$$E^{-2.7} \qquad E^{-2.1\ldots-2.4} \qquad E^{-0.6\ldots-0.3}$$

$$J_{cr}(E) = Q_{cr}(E) \times T(E)$$

- two power laws!

source — escape time from the Galaxy

# secondary nuclei and escape length

**secondary species:**

**Li, Be, B, d, $^3$He, $\bar{p}$ …**

$$\frac{J_2}{T} = \langle n_{ism} \rangle \mathrm{v} \sigma_{21} J_1$$

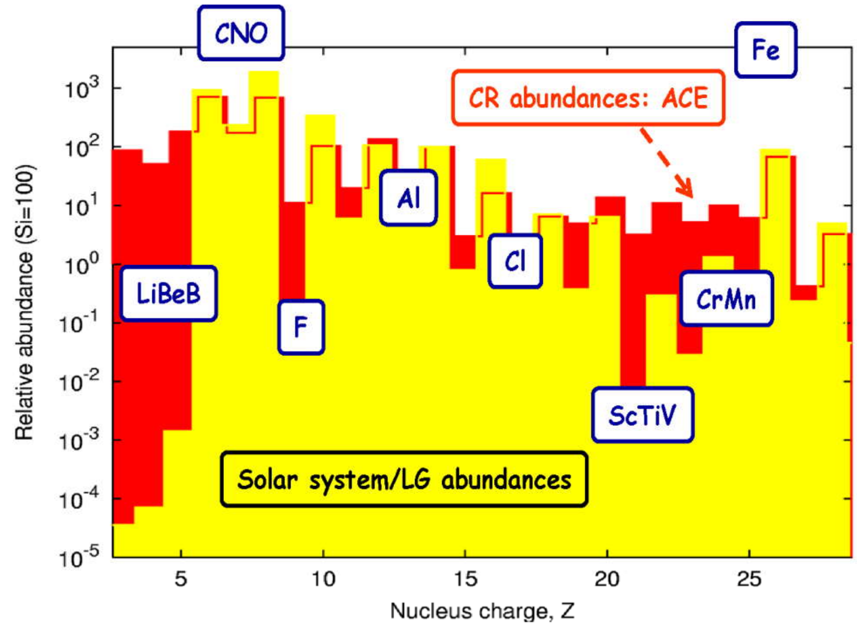
**cosmic ray escape length:**

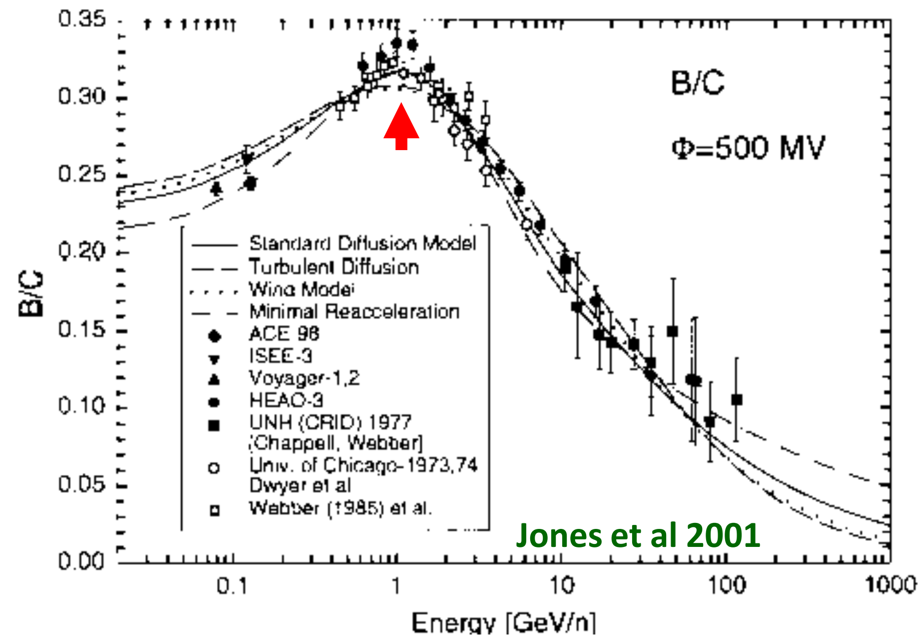
$$X = m_a \langle n_{ism} \rangle \mathrm{v} T$$

**~ 10 g/cm$^2$ at 1 GeV/nucleon**

**surface gas mass density of galactic disk ~ 2.4 mg/cm$^2$**

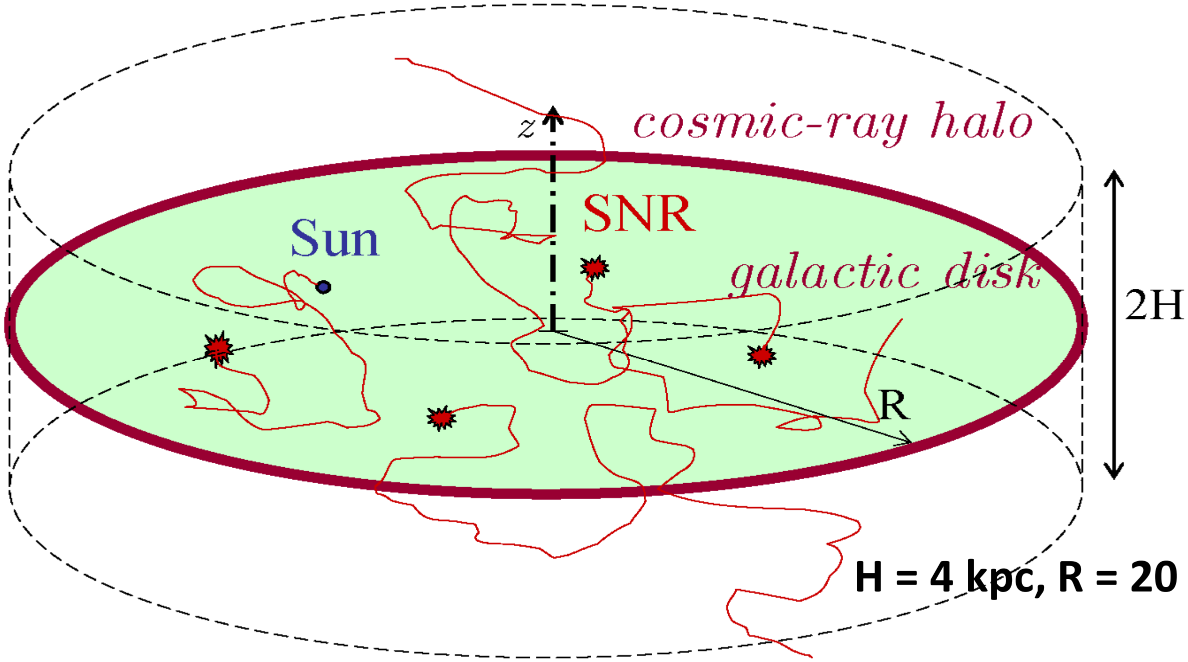
**- good confinement and intermixing of cosmic rays in the Galaxy**



# basic empirical diffusion model

Ginzburg & Ptuskin 1976, Berezinskii et al. 1990, Strong & Moskalenko 1998 (GALPROP: http://galprop.stanford.edu), Donato et al 2002, Ptuskin et al. 2006, Strong et al. 2007, Vladimirov et al. 2010, Bernardo et al. 2010, Maurin et al 2010, Putze et al 2010, Trotta et al 2011, Johannesson et al. 2016




H = 4 kpc, R = 20 kpc

empirical diffusion coefficient $D \sim 3\times10^{28}\ \mathrm{cm}^2/\mathrm{s}$ at $\sim 3$ GeV/n

diffusion mean free path $l \sim 1$ pc

diffusion is due to resonant scattering by random magnetic field $r_g \sim 1/k$

wave number

# transport equation for cosmic rays

$$\frac{\partial F}{\partial t} - \nabla(\mathbf{D}\nabla F) + \nabla(\mathbf{u}F) + \frac{\partial}{\partial p}\left(\left(\frac{dp}{dt}\right)_{\text{loss}} F\right) - \frac{\nabla\mathbf{u}}{3}\frac{\partial}{\partial p}(pF) - \frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial}{\partial p}\frac{F}{p^2}\right) + \frac{F}{\tau_{\text{cat}}} = Q$$

diffusion

convection

energy loss, <0

adiabatic change of momentum $\dot{p} = -\frac{\nabla\mathbf{u}}{3}p$

stochastic acceleration

catastrophic losses (fragmentation, decay …)

source term

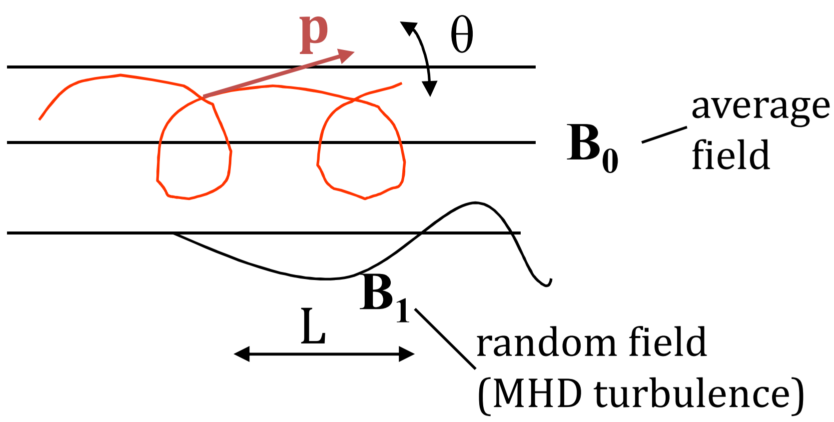
$F(p,\mathbf{r},t)$ is the cosmic ray number density per unit of total particle momentum,
total number density $N(\mathbf{r},t) = \int dpF(p,\mathbf{r},t)$,

$Fdp = 4\pi\, p^2 fdp$ in terms of phase-space density $f(p,\mathbf{r},t)$,

intensity $J(E) = F(p)/4\pi = p^2 f(p)$.

$$\frac{\partial f}{\partial t} - \nabla\mathbf{D}\nabla f + u\nabla f - \frac{\nabla u}{3}p\frac{\partial f}{\partial p} - \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial}{\partial p}f\right) + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\left(\frac{dp}{dt}\right)_{\text{loss}} f\right) + \frac{f}{\tau_{cat}} = q.$$

# microscopic theory of diffusion




Larmor radius

$$r_g = \frac{p_t c}{ZeB_0} \approx 3.3\times10^{12} P_{GV} / B_{\mu G} \text{ cm}$$

effect of random field:

a) large-scale field: $kr_g \ll 1$ - adiabatic motion, $\sin^2\vartheta / B = const$

($k = 2\pi / \lambda$ - wave number)

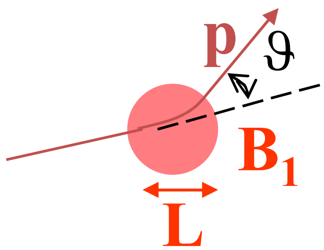
b) small-scale field: $kr_g \geq 1$ scattering,

cyclotron resonance $\omega - k_z v_z = n\omega_B$, $n = 0, \pm1, \pm2, ...$

$\omega \approx kV_a \ll kv$, $n = \pm1$, $k_z = \pm(r_g \cos\vartheta)^{-1}$ → $r_g = 1/k$

resonance condition

# elementary theory



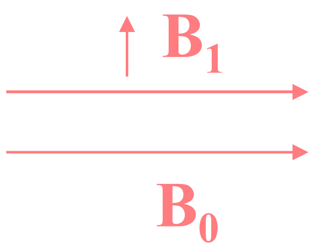

particle scattering by weak magnetic field perturbations, $r_{g1} >> L$ :

$\vartheta = L / r_{g1}$

diffusion on pitch-angle:

$$D_\vartheta \sim \vartheta^2 / \delta t \sim (\mathrm{v} / r_g) \times (B_{1,res} / B_0)^2, \text{ at } k_{res} = 1 / r_g$$

(with estimates $L \sim r_g$, $\delta t \sim r_g / \mathrm{v}$)




scattering back $\Delta\vartheta \sim 1$ at $\tau \sim 1 / D_\vartheta \sim (r_g / \mathrm{v}) \times (B_0 / B_{1,res})^2$

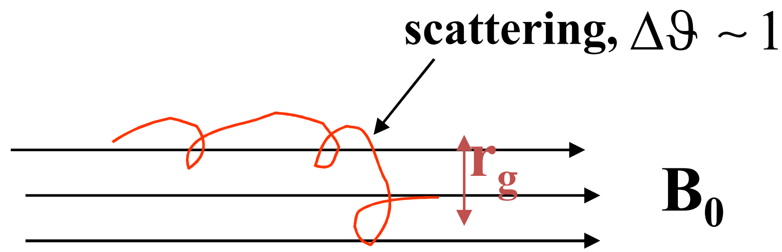
mean free path $l \sim \mathrm{v}\tau \sim r_g (B_0 / B_{1,res})^2$

diffusion coefficient $$D_{||} \approx \frac{\mathrm{v} r_g}{3} \times \left( \frac{B_0}{B_{1,res}} \right)^2$$

Bohm diffusion $$D_{Bohm} = \mathrm{v} r_g / 3$$

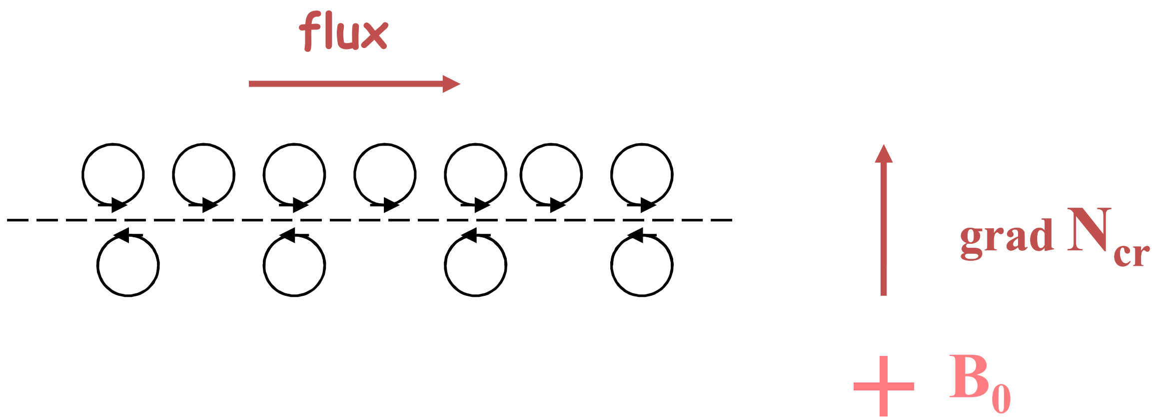
perpendicular diffusion

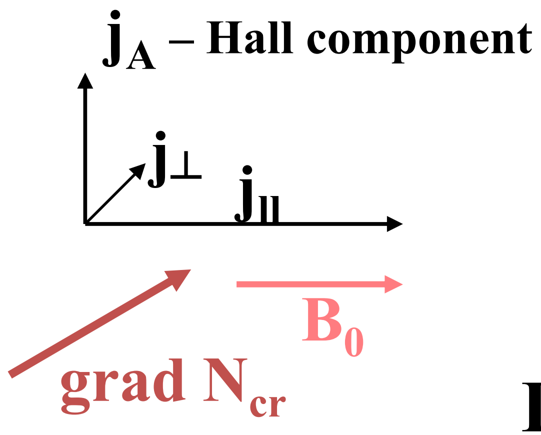
$$D_\perp \sim r_g^2/\tau \sim v r_g (B_{1,res}/B_0)^2$$




Hall diffusion

$$D_A \sim v r_g$$






$A_{res} = B_{1,res}/B_0 << 1$

$K_{res} = 1/r_g$

## diffusion tensor

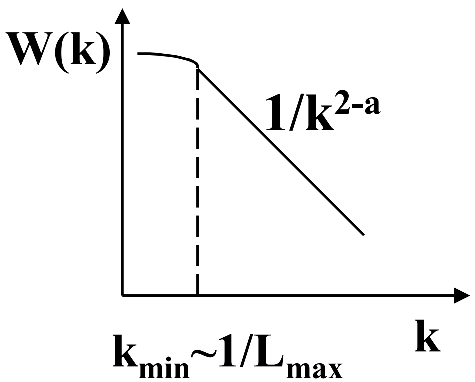
$$\mathbf{D} = \begin{pmatrix} D_\perp & D_A & 0 \\ -D_A & D_\perp & 0 \\ 0 & 0 & D_{\parallel} \end{pmatrix}, \qquad z \parallel \mathbf{B_0}$$

$$D_{\parallel} = \frac{v r_g}{3 A_{res}^2}, \quad D_\perp = \frac{v r_g A_{res}^2}{3}, \quad D_A = \frac{v r_g}{3}$$

$$\frac{\partial J}{\partial t} - \nabla_s D_{sm} \nabla_m J = 0, \quad j_s = -D_{sm} \nabla_m J$$

spectrum of random field $B_{res}^2 = \int_{k_{res}} W(k)\,dk$

$W(k)dk \sim k^{-2+a}dk$, $D_{ll} \sim vr_g{}^a$



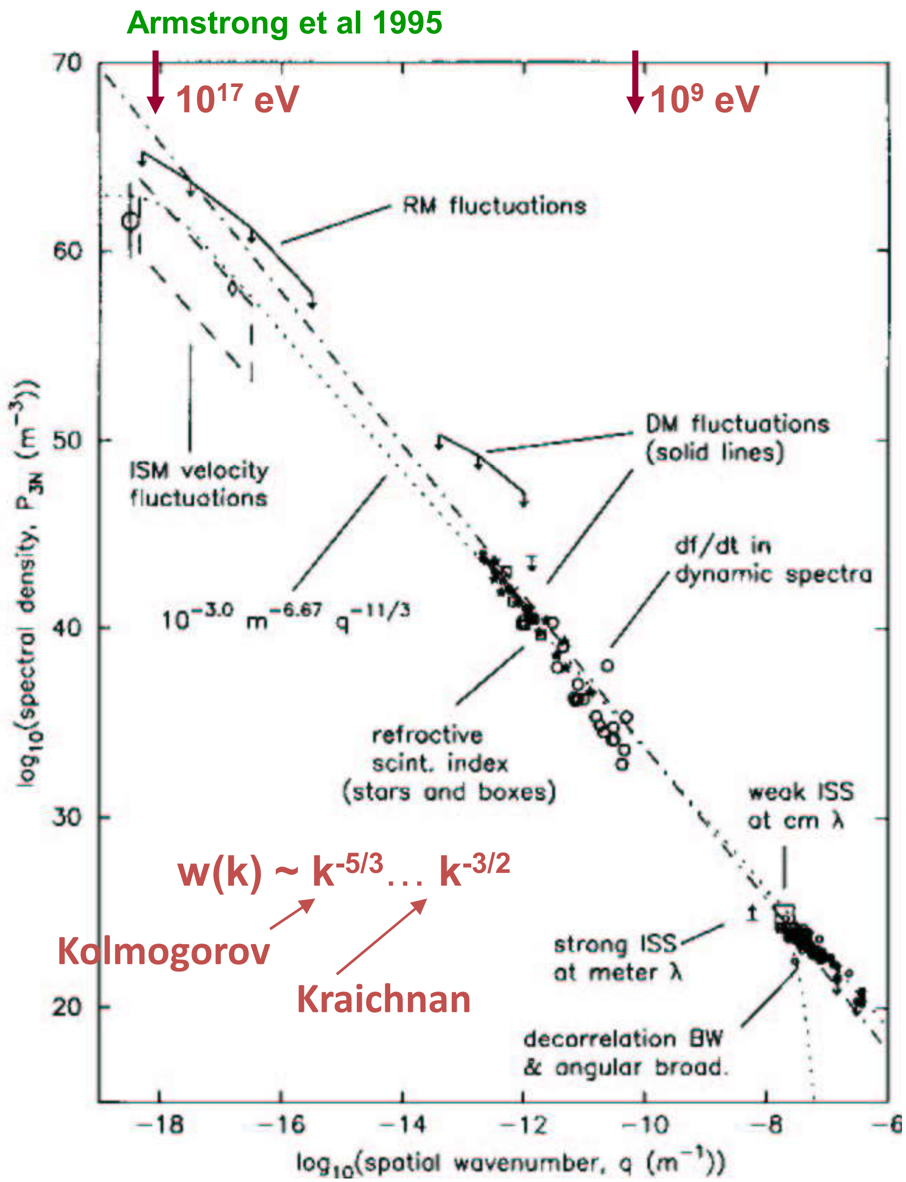

$a = 1/3$ Kolmogorov spectrum

$a = 1/2$ Kraichnan spectrum

$a = 0$ random discontinuities

$a = 1$ white noise (leads to Bohm diffusion scaling $D_B = vr_g/3$)

# interstellar turbulence




observations:

Kolmogorov-type spectrum

$L_{\max} \sim 100$ pc, $B \sim 3$ μG,

$A_L = (\delta B / B)_L \sim 1,$

$a = 2 - 5/3 = 1/3$

estimate of diffusion coefficient:

$$D \sim 4 \times 10^{27} \beta \left(\frac{p}{Z}\right)_{\mathrm{GV}}^{1/3} \mathrm{cm}^2/\mathrm{s}$$

up to $E \sim 10^{17} Z$ eV

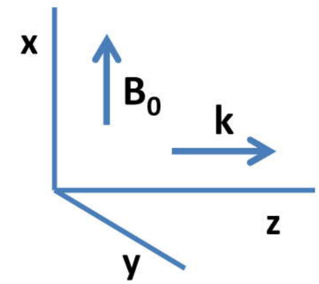
alternative: Kraichnan type spectrum

$a = 2 - 3/2 = 1/2$

$$A_L \sim 0.3, \quad D \propto \beta \left(\frac{p}{Z}\right)^{1/2}$$

# problem: structure of interstellar mhd turbulence

- anisotropic quasi-Alfvenic Kolmogorov turbulence where Alfvenic eddies are stretched along magnetic field, $k_{\|} = k_{\perp}^{2/3} L^{-1/3}$, can not provide required diffusion coefficient since a particle traverses many uncorrelated during one gyro-rotation Shebalin et al. 1983, Higdon 1984, Montgomery & Matthaeus 1995, Goldreich & Shridhar 1995, Chandran 2000, Yan & Lazarian et al. 2002, Bereznyak et al 2010, Yan 2013



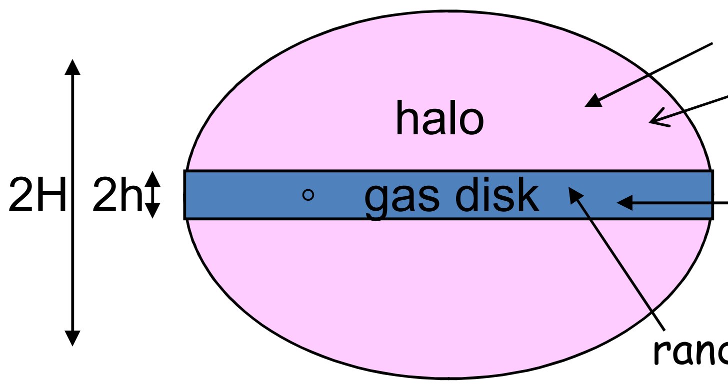

$$\mathbf{A}:\ \omega = k_z V_a$$
$$u_y, \delta B_y \neq 0,$$
$$\mathbf{MS}:\ \omega = k V_{\pm}$$
$$\delta\rho, u_x, u_y, \delta B_x \neq 0.$$

- fast magnetosonic waves can be isotropic (via independent acoustic-type cascade Cho & Lazarian 2003) and effectively scatter cosmic rays but they may not provide needed diffusion coefficient in galactic disk because of strong dissipation in warm plasma via Landau damping Barnes & Scargle 1967, ... Spanier & Schlickeiser 2005

→ "sandwich" model

$D_h,\ \rho = 0$

magnetosonic waves

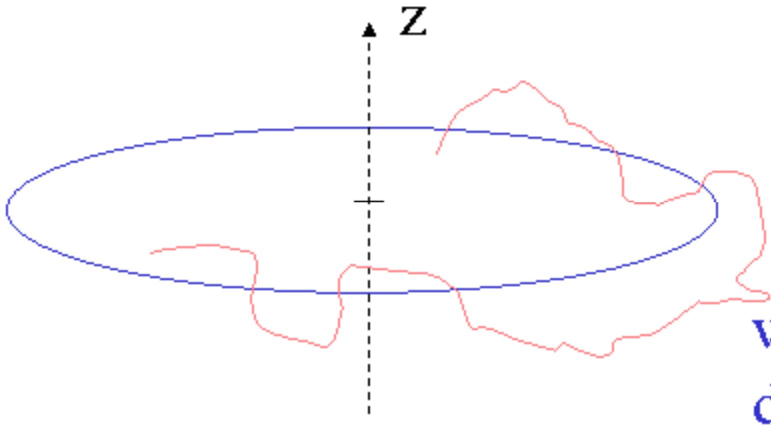


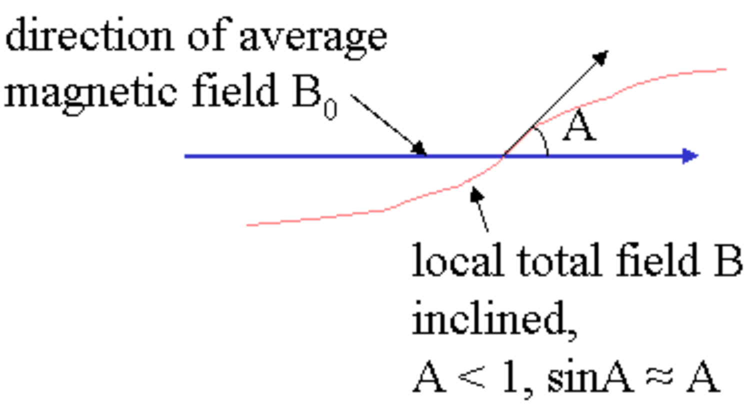

$D_g \gg D_h,\ \rho_g$

random shocks Bykov & Toptygin 1987

# anomalous perpendicular diffusion

- cosmic ray particles are strongly magnetized: $r_g/l \sim 10^{-6}$ at 1 GeV;
- average Galactic magnetic field is almost pure azimuthal (in the disk): $B_{0\varphi} : B_{0r} : B_{0z} = 1 : 0.2 : 0.003$
- large-scale random field is large: $\delta B \sim B_0$ at $L \approx 100$ pc




what is efficient perpendicular diffusion coefficient in static random field if locally $D_\perp = D_A = 0$ ?

"natural" answer: $D^{ef}_\perp = \langle A^2 \rangle D_{ll}$ is wrong !

the right answer is $D^{ef}_\perp = 0$ !!

## Static random magnetic field, diffusion along magnetic field lines

random component — average

$$\mathbf{B} = \mathbf{B_0} + \mathbf{B_1(r)}, \quad \langle \mathbf{B_1} \rangle = 0, \qquad \mathbf{A} = \mathbf{B_1}/B_0 \ll 1$$

$$\partial N/\partial t - \nabla_i D_{ij} \nabla_j N = 0,$$ - diffusion equation for cosmic ray density

$$D_{ij} = D_{ll} B_i B_j / B^2$$ - parallel diffusion

$$D_{ij} = \langle D_{ij} \rangle + \delta D_{ij}(\mathbf{r}), \quad \langle \delta D_{ij} \rangle = 0.$$

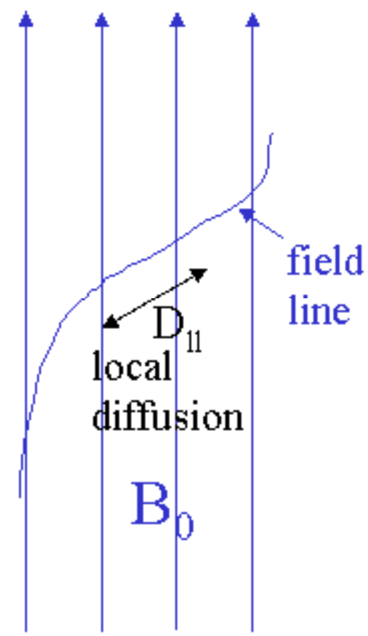
hense $N = \langle N \rangle + \delta N, \quad \langle \delta N \rangle = 0$

after averaging: stochastic nonlinearity

$$\partial \langle N \rangle / \partial t - \nabla_i \langle D_{ij} \rangle \nabla_j N - \nabla_i \langle \delta D_{ij} \nabla_j \delta N \rangle = 0,$$

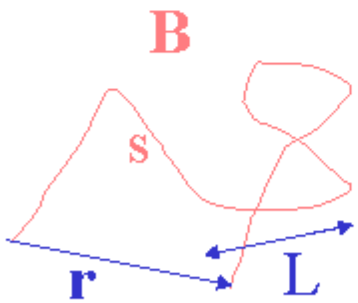
$$\partial \langle N \rangle / \partial t - \nabla_i D_{ij}^{ef} \nabla_j \langle N \rangle = 0,$$

where $D_{ll}^{ef} = (1 - g\langle \mathbf{A}^2 \rangle) D_{ll}, \quad D_{\perp}^{ef} = 0$ no perpendicular diffusion on average !



# compaund diffuison

$D_{ll} \neq 0$, $D_\perp = D_A = 0$, isotropic random large-scale magnetic field



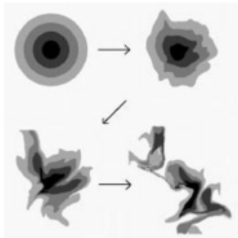

random walk of magnetic field line: $r^2 \sim Ls$

particle diffusion along the line: $s^2 \sim D_{ll}t$

displacement of particle in **r** space: $r^2 \sim L(D_{ll}t)^{1/2}$

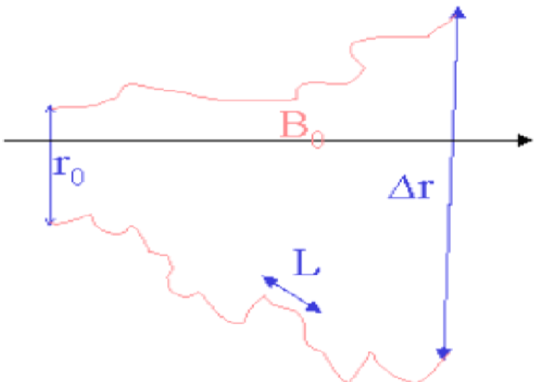
hence $D^{ef} = r^2/t \rightarrow 0$ at $t \rightarrow \infty$

*Compound diffusion works only for degenerate static diffusion tensor. Finite $D_\perp$, $D_A$ or fluctuations in time destroy compound diffusion. Particle loses correlation with initial magnetic field line and "forget" information on previous trajectory*



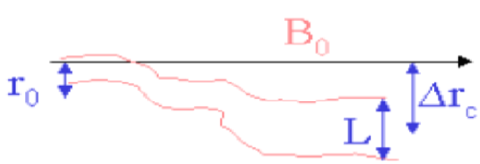
# spreading of magnetic field lines and cosmic ray diffusion

weak static large-scale random field $A << 1$




independent random walk, $r_0 >> L$

$\Delta r^2 \sim A^2 L s$ - separation of two lines




$r_0 << L$, two close lines, remain correlated up to $s \sim s_c$

$s_c \sim L \times (\ln(L/r_0))/A^2$ - decorrelation length

for flat spectrum of random magnetic field $W(k) \sim k^{-2+a}$, $a > 0$:

$s_c \sim L/A^2$ (with no $r_0$!)

Particle diffusion in this field $D^{ef}_{\perp} \sim (\Delta r_c)^2/\Delta t_c$

displacement $\Delta r_c \sim (A^2 L s_c)^{1/2}$

"memory" time $\Delta t_c \sim s_c^2/D_{||}$

$D^{ef}_{\perp} \sim A^4 D_{||}$ - anomalous perpendicular diffuson (perturbation on $A^2(D_{||}/D_{\perp})^{1/2}$)

# some results for strong static random field

1D case:

$$D^{ef} = (\langle D^{-1} \rangle)^{-1}$$

2D case:


isotropic random field $\langle \mathbf{B} \rangle = 0$

$$D^{ef} = (D_{\perp} D_{ll})^{1/2} \sim v r_g / 3$$

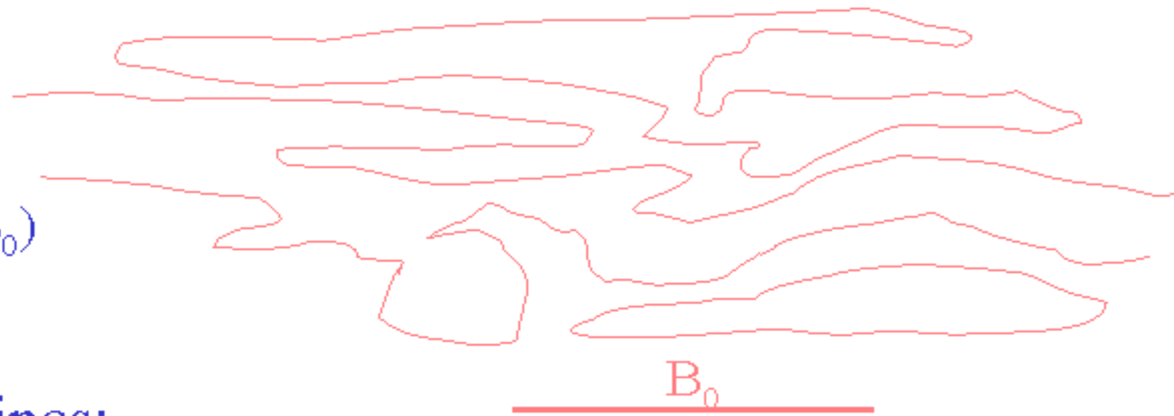
coinsides with Bohm diffusion coefficient

Remark: in the case of random Alfven waves, the approximation of static field works if

$$V_a < A^2 D_{ll}/L$$

## 3D case:

($B_1$ – large-scale random field, $B_1 \lesssim B_0$)




percolation of field lines:

$$D^{ef}_{ll} \sim D_{ll}, \quad D^{ef}_{\perp} \sim (B_1/B_0)^4 D_{ll}$$ ($D^{ef} \sim D_{ll}$ at $B_1 \sim B_0$)

no percolation of field lines:

$$D^{ef}_{ll} \sim (B_0/B_1)^2 v r_g, \quad D^{ef}_{\perp} \sim (B_1/B_0)^2 v r_g$$ (Bohm dif. $D_{ef} \sim v r_g$ at $B_1 \sim B_0$)

**thus, two general types of scaling: $D^{ef} \sim D_{ll}$ and $D^{ef} \sim D_B$**

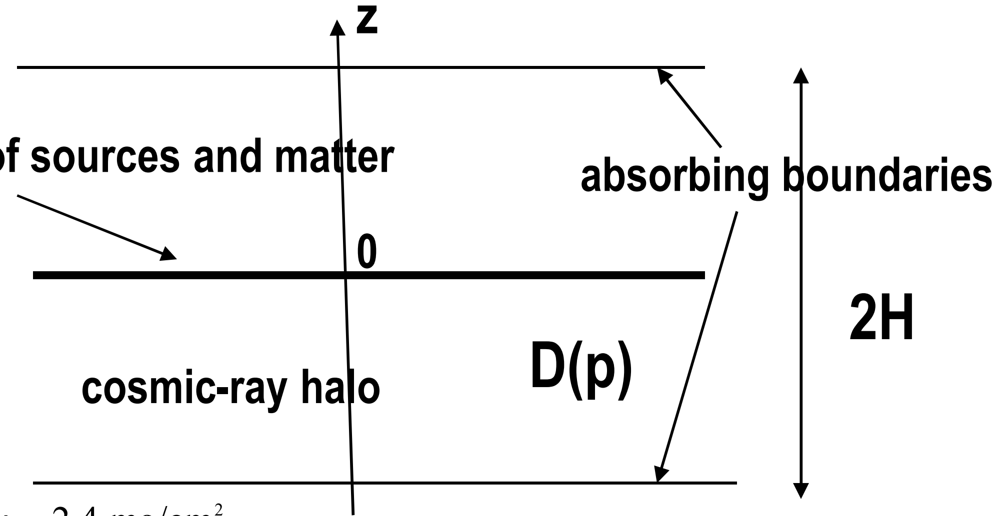

literature: "Handbook of Plasma Physics", ed. R.N. Sudan, A.A. Galeev, 1981, North-Holland; M. B. Isichenko 1992, Rev. Mod. Phys. 64, 961; L. Chuvilgin, V. Ptuskin 1993, Astron. Astrophys. 279, 278 ; R. Balescu 1995, Phys. Rev. E, 51, 4807, Casse et al 2002

# Basic galactic model of cosmic ray propagation.

**Primary and secondary nuclei. Electrons and positrons. Anisotropy. Fluctuations.**

# Stable nuclei

## leaky box approximation
**for 1D diffusion model with galactic disk of infinitesimal thickness**




**surface gas density** $\mu \approx 2.4\ \mathrm{mg/cm^2}$

**surface source density**

$$-D(p)\frac{\partial^2 f(p,z)}{\partial z^2}+\frac{v\sigma\mu}{m}\delta(z)f(p,z)+\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\left(\frac{dp}{dt}\right)_{ion}f(p,z)\right]=s(p)\delta(z)\quad (*)$$

$$\left(dp/dt\right)_{ion}=b_0(p)\mu\delta(z)/m<0$$

solution of (*) at $z\neq 0$: $f=f_0\left(1-\frac{|z|}{H}\right)$, where $f_0(p)=f(p,z=0)$ (**)

integration of (*) $\lim\int_{-\varepsilon}^{\varepsilon}\ldots dz$ at $\varepsilon\to 0$ gives the boundary condition at $z=0+\varepsilon$:

$$-2D\frac{\partial f_0}{\partial z}+\frac{v\sigma\mu}{m}f_0+\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\frac{b_0\mu}{m}f_0\right]=s(p)\quad (***)$$

calculating $-2D\frac{\partial f_0}{\partial z}$ from (**) and substituting it in (***) gives eq. for $J_0(E)$:

**leaky box Eq.**

$$\frac{J_0}{X}+\frac{d}{dE}\left[\left(\frac{dE}{dx}\right)_{ion}J_0\right]+\frac{\sigma}{m}J_0=\frac{s(p)p^2}{\mu v},$$

$$X=\frac{\mu v H}{2D}$$ - escape length; in $\mathrm{g/cm^2}$.

# physical explanations of peak in sec./prim. ratio:

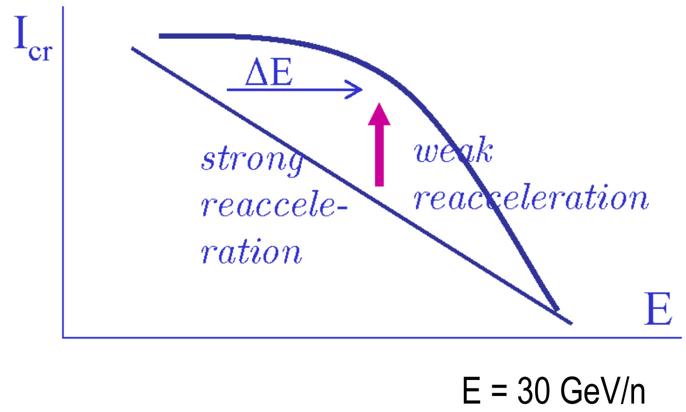
## •distributed reacceleration

Simon et al. 1986; Seo & Ptuskin 1994

$D_{pp} \sim p^2 V_a^2 / D, \quad D \sim vR^{1/3}$

- Kolmogorov spectrum of turbulence

sources spectrum $q \sim R^{-2.4}$
(more flat at R < 3 GV)




problem: low flux of secondary antiprotons

steep source spectrum

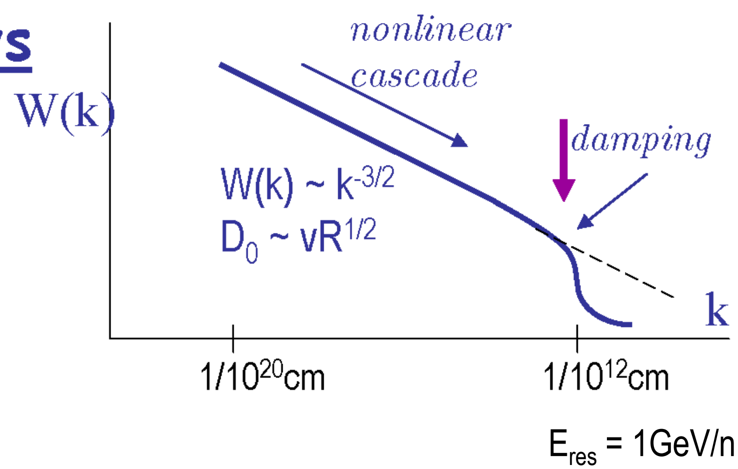
## •wave damping on cosmic rays

VSP, Moskalenko et al. 2006

Iroshnikov - Kraichnan cascade

$D_0 \sim vR^{1/2}$

sources spectrum $q \sim R^{-2.2}$
(more steep at R < 40 GV)




problem: large anisotropy

# diffusion models

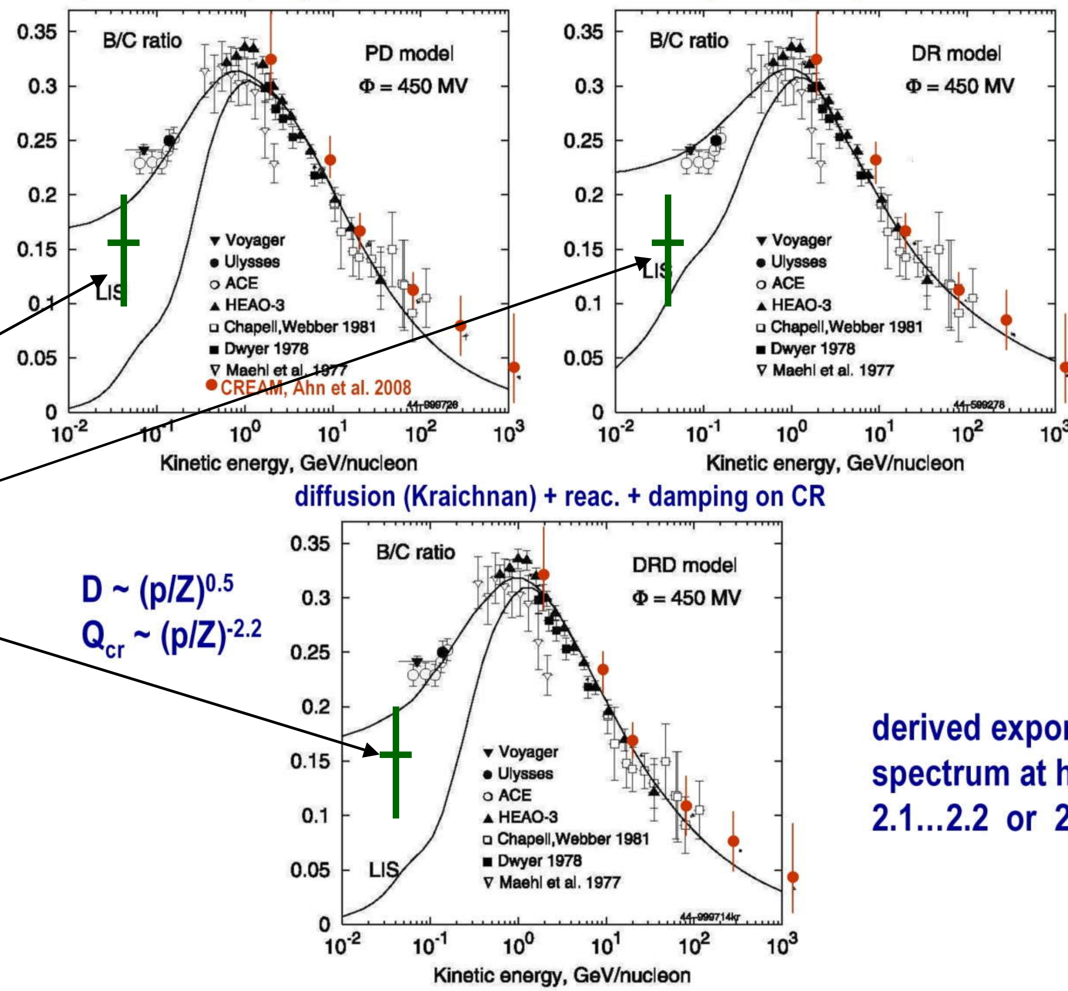
## B/C ratio in three models of cosmic ray propagation

plain diffusion, “unphysical” break

diffusion (Kolmogorov) + reacceleration

$D \sim (p/Z)^{0.6}$
$Q_{cr} \sim (p/Z)^{-2.1}$

$D \sim (p/Z)^{0.3}$
$Q_{cr} \sim (p/Z)^{-2.4}$




**Voyager 1**
**LIS**
**2013**

diffusion (Kraichnan) + reac. + damping on CR

$D \sim (p/Z)^{0.5}$
$Q_{cr} \sim (p/Z)^{-2.2}$

derived exponent of source spectrum at high energies 2.1…2.2 or 2.4

# cosmic ray energy spectra below the knee

## low-energy spectra and composition;

**Voyager 1** space probe, (Cummings et al 2016)

## deviations from the plain power laws at 10 to$10^5$ Gev/n:

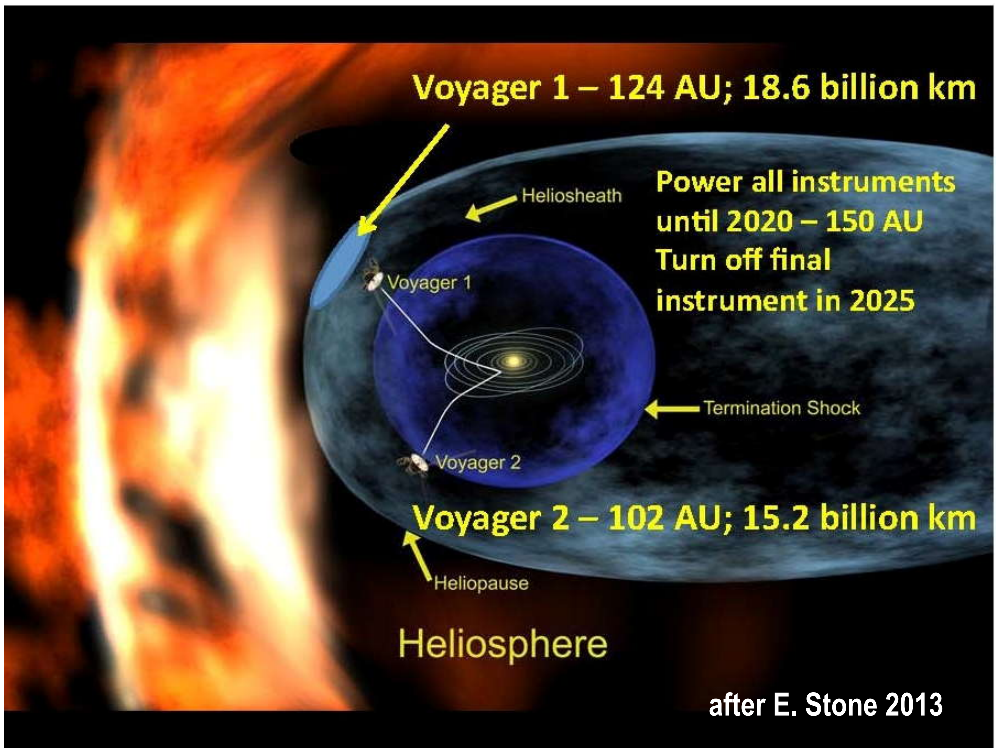
**ATIC-2** , Advanced Thin Ionization Calorimeter, balloon-borne instrument (Panov et al. 2009),

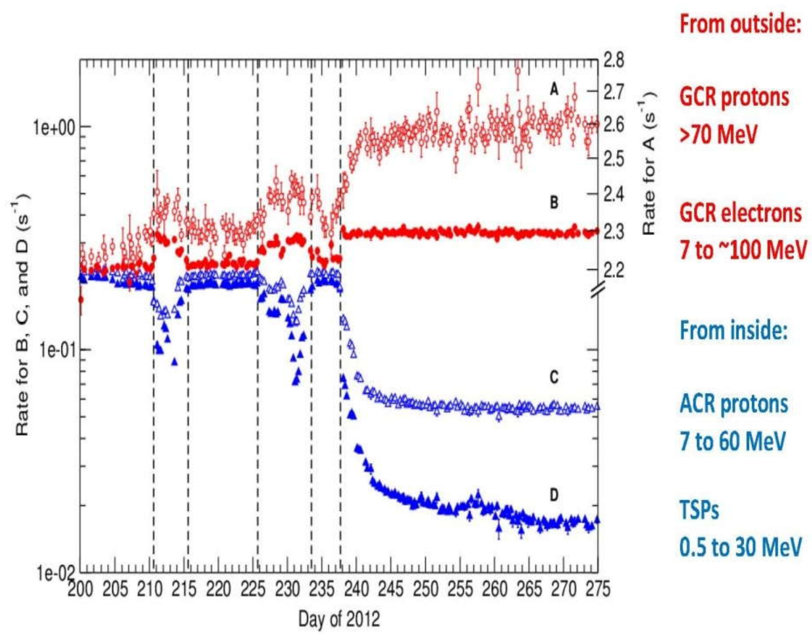
**CREAM**, Cosmic Ray Energetics and Mass, ionization calorimeter, balloon instrument (Yoon et al 2011),

**PAMELA**, Payload for Antimeater Matter Exploration and Light-nuclei Astrophysics, magneic spectrometer, satellite-based experiment (Adriani et al. 2011),

**AMS-02**, Alpha Magnetic Spectrometer, International Space Station (Aguilar et al 2015), …

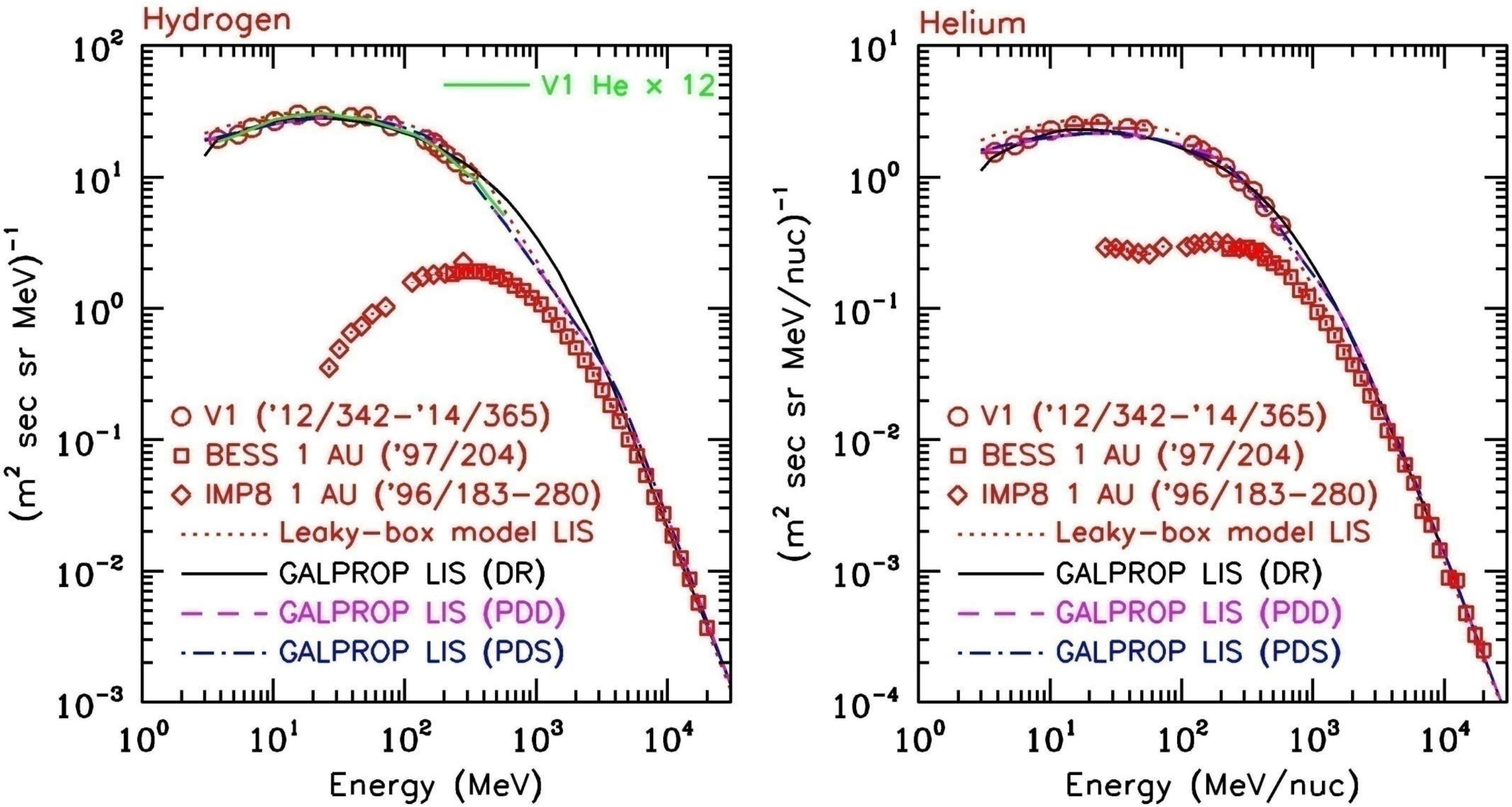
# Voyager 1 in the interstellar space



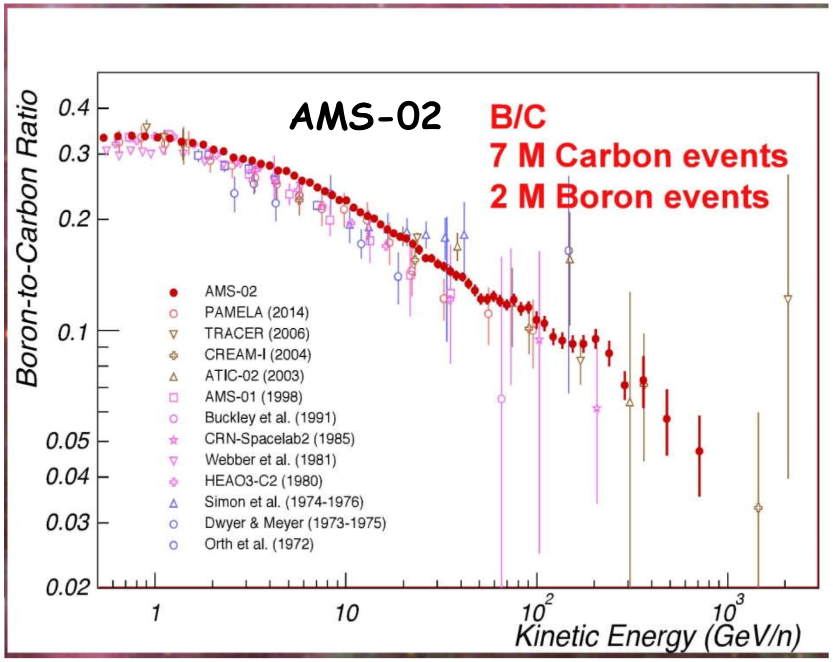



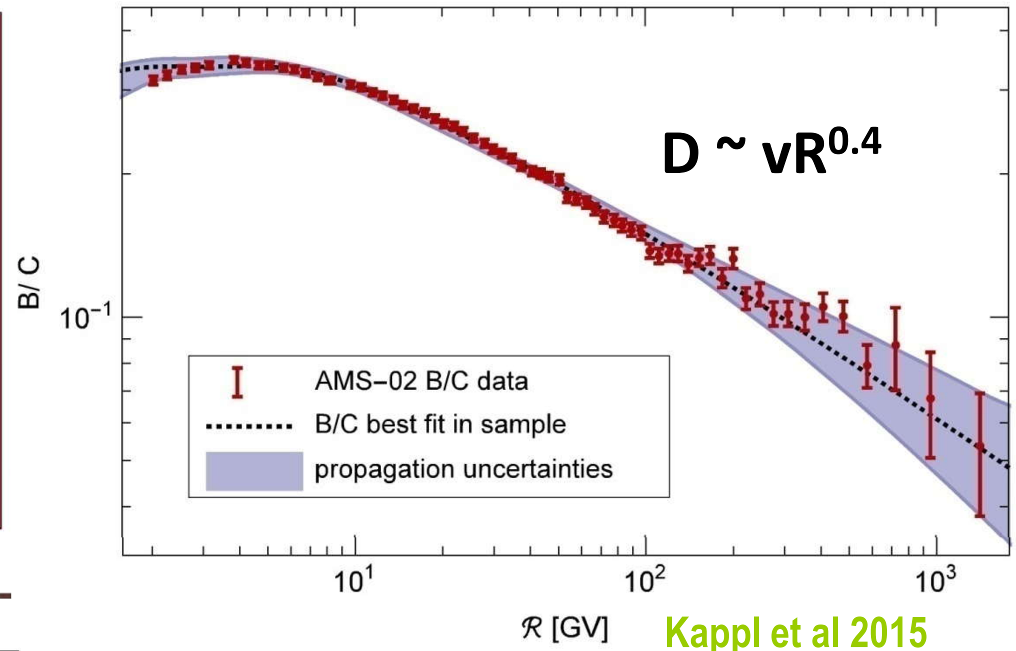

d = 138 AU at present

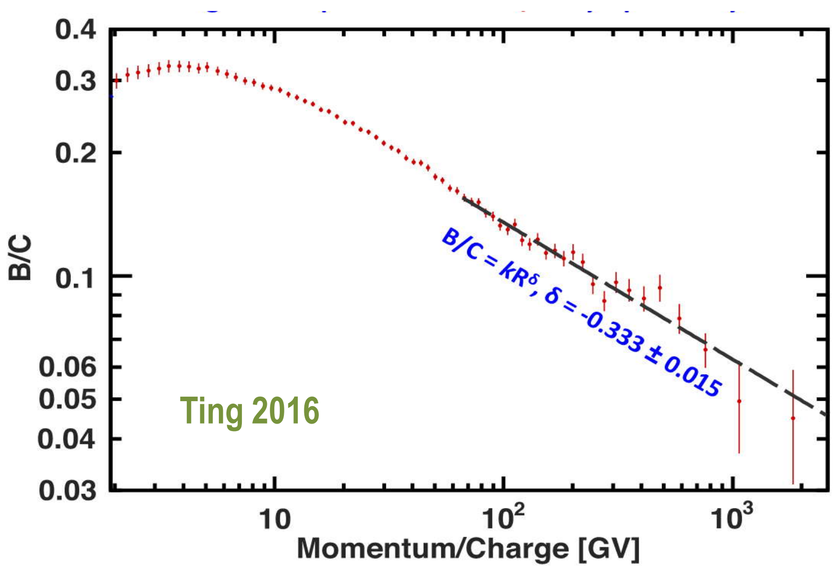
**Galactic cosmic ray nuclei from H to Ni down to 3 MeV/n; electrons to 2.7 MeV** Cummings et al 2013

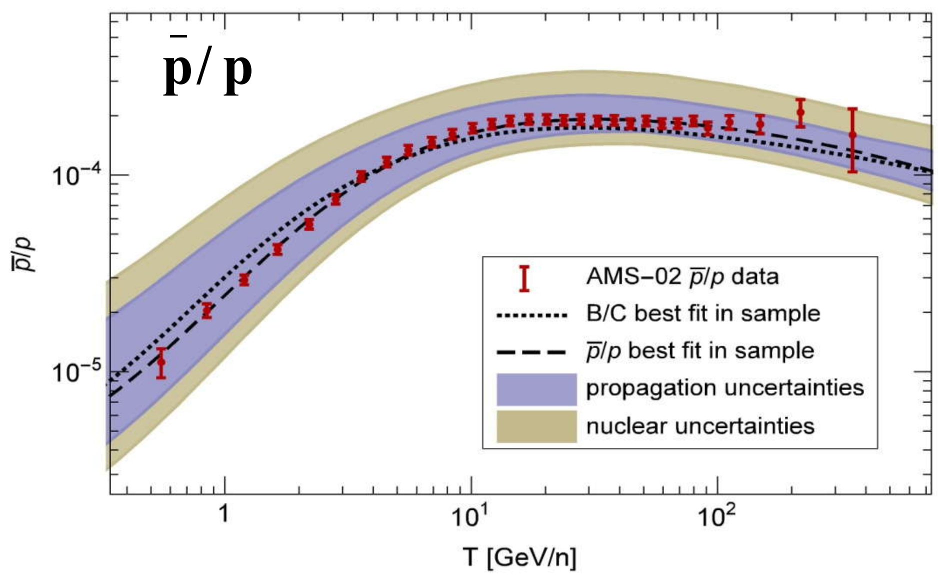


# secondary nuclei



# flat component of secondary nuclei produced by strong SNR shocks

Wandel et al. 1987, Berezhko et al. 2003, Aloisio et al. 2015

## production by primaries inside SNRs

grammage gained in SNR

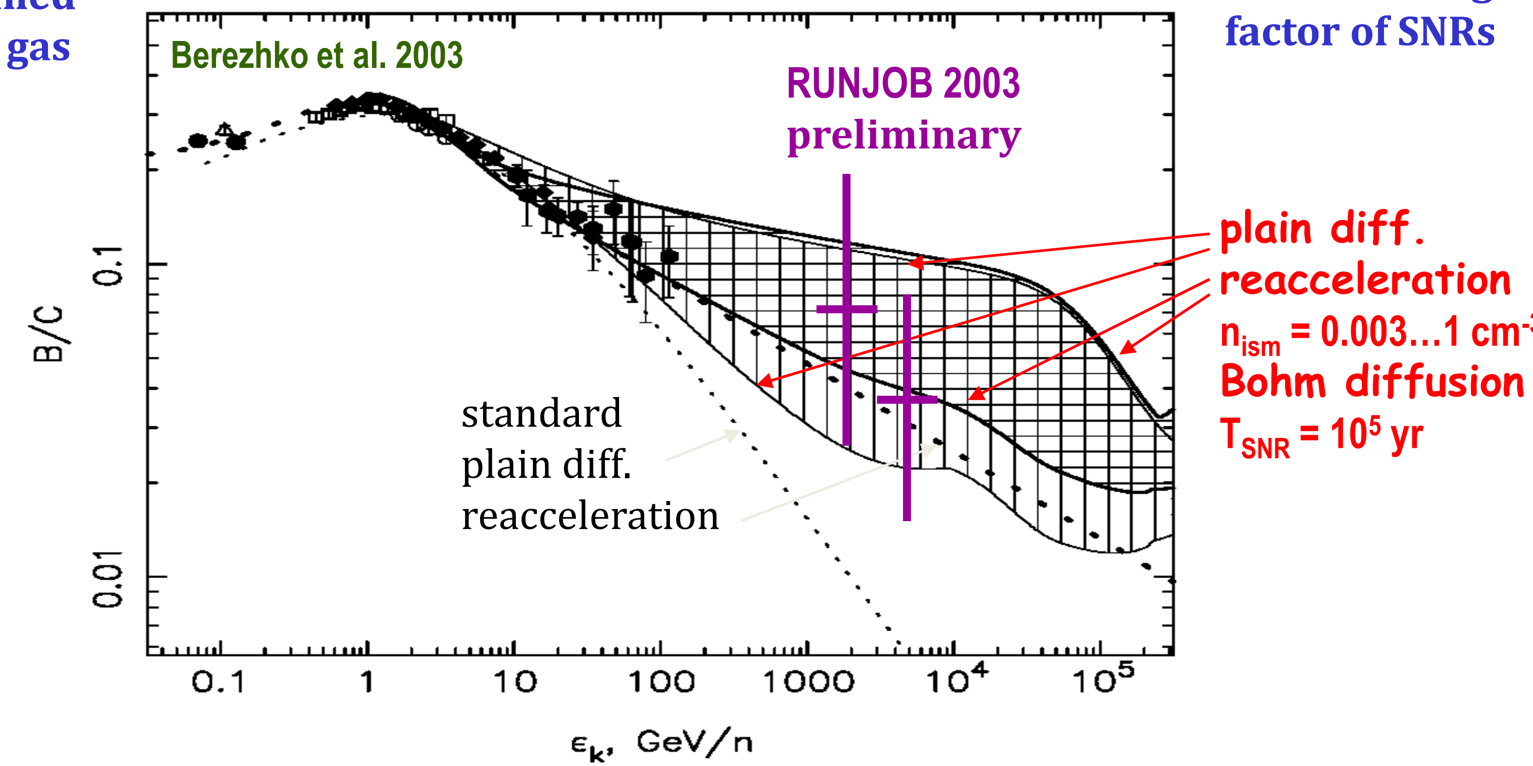
$$\frac{N_{2,\mathrm{flat}}}{N_{2,\mathrm{stand}}} \sim \frac{X_{\mathrm{SNR}}}{X_{\mathrm{ISM}}} \sim 0.02$$

grammage gained in interstellar gas

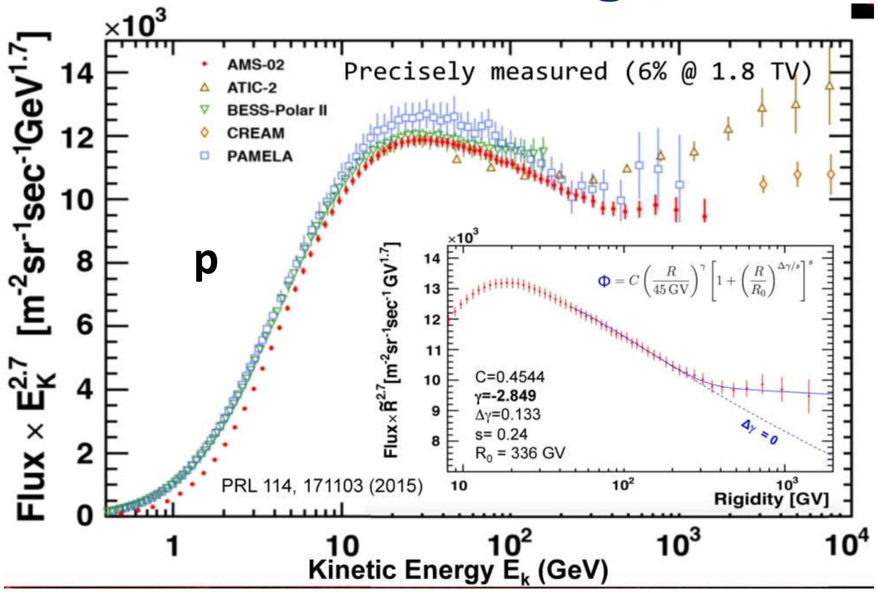
## reacceleration in ISM by strong shocks

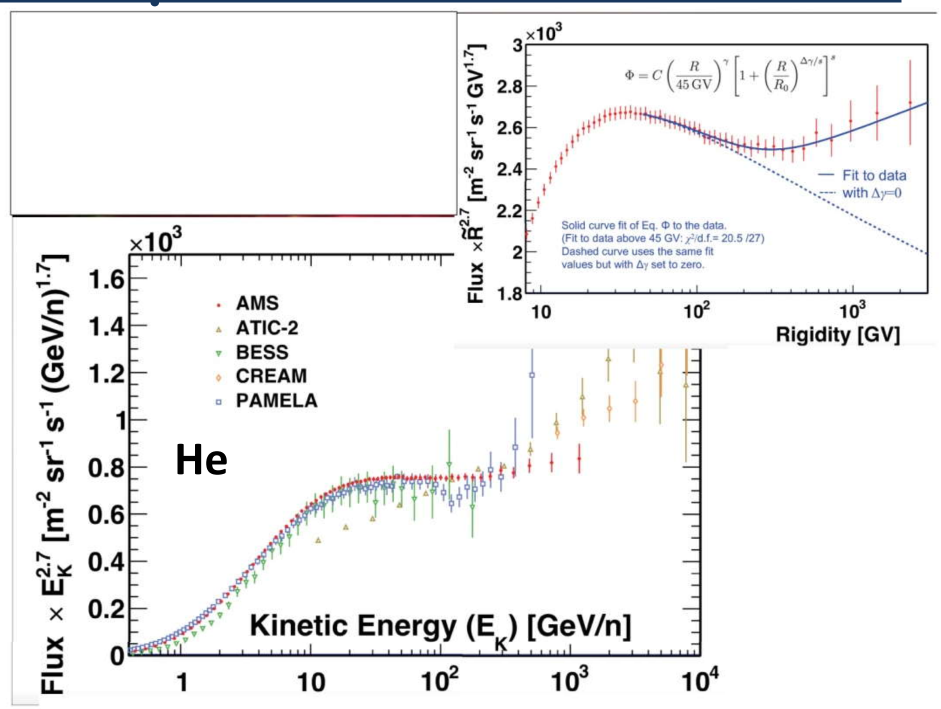
$$\frac{N_{2,\mathrm{flat}}}{N_{2,\mathrm{stand}}} \sim \frac{X_{\mathrm{ISM}}}{X_{\mathrm{SNR}}} f_{\mathrm{SNR}} \sim 0.2$$

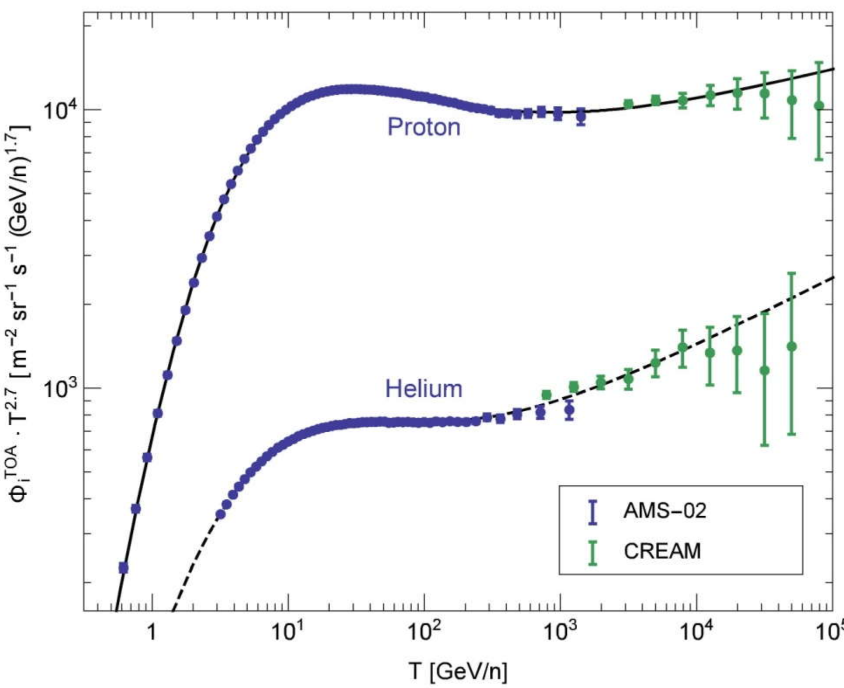
volume filling factor of SNRs



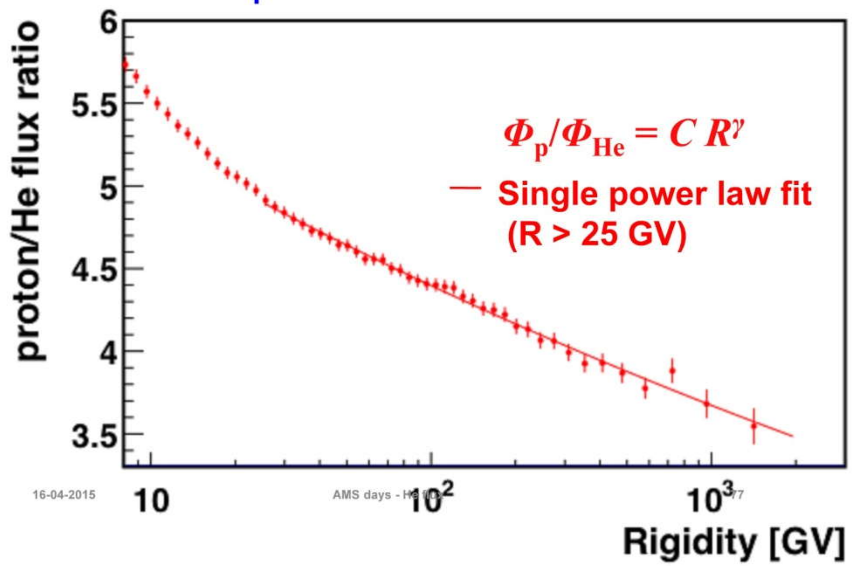
# hardening of H & He spectra above ~ 300 GV




proton/He flux ratio



# some suggested explanations of features in H & He spectra:

## hardening above 300 GeV/nucleon

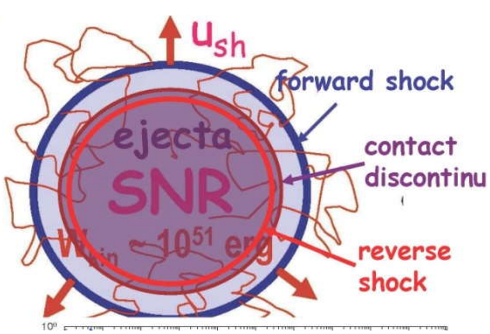
spectrum produced by superposition of sources Vladimirov et al 2012, Zatsepin & Sokolskaya 2006;
reacceleration by SNR shocks VP, Zirakashvili, Seo 2011, Thoundam & Hoerandal 2015;
effect of local sources Erlykin & Wolfendale 2011, Bernard et al 2013, Liu et al 2015;
streaming instability below 300 GV Blasi, Amato. 2012;
different turbulence in halo and disk Tomassetti 2012.

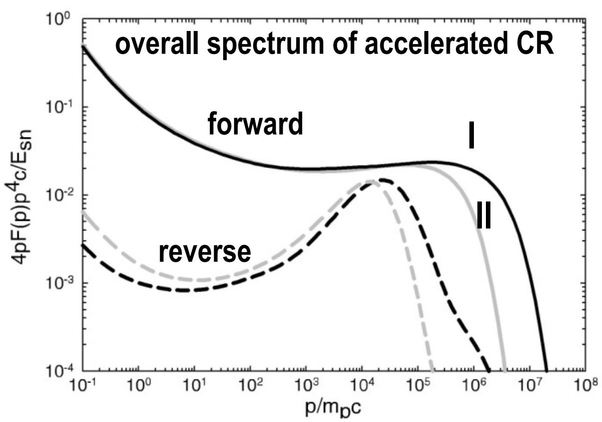
## spectra of p and He are different

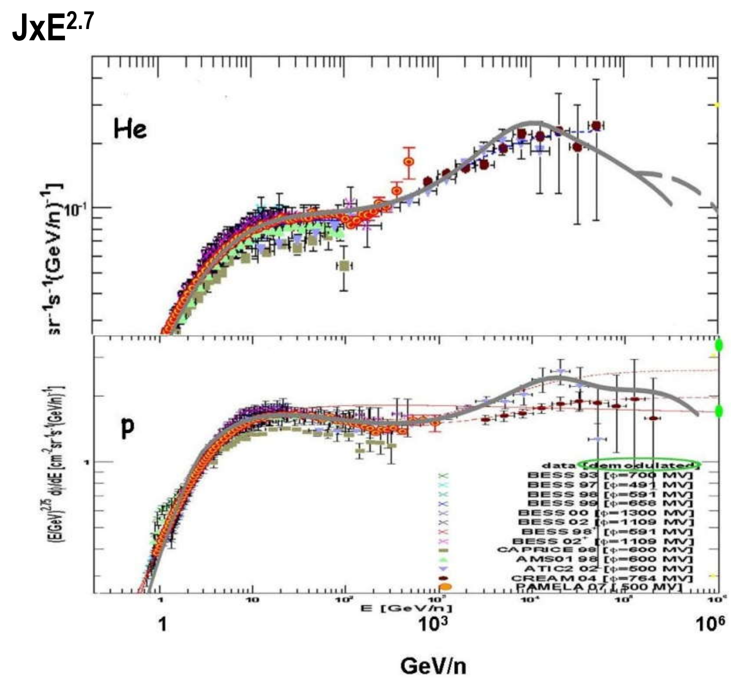
shock goes through material enriched in He:
bubble Ohira & Ioka 2011 or variable (ionized) He/p concentration Drury 2011;
p,He injection for varying $M_A$ Malkov et al 2012.

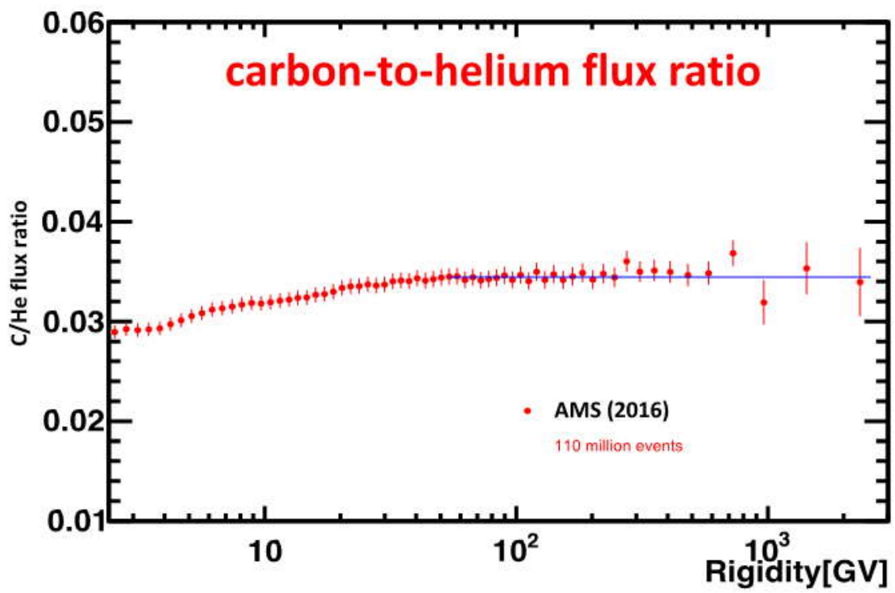
## possible explanation of both features:

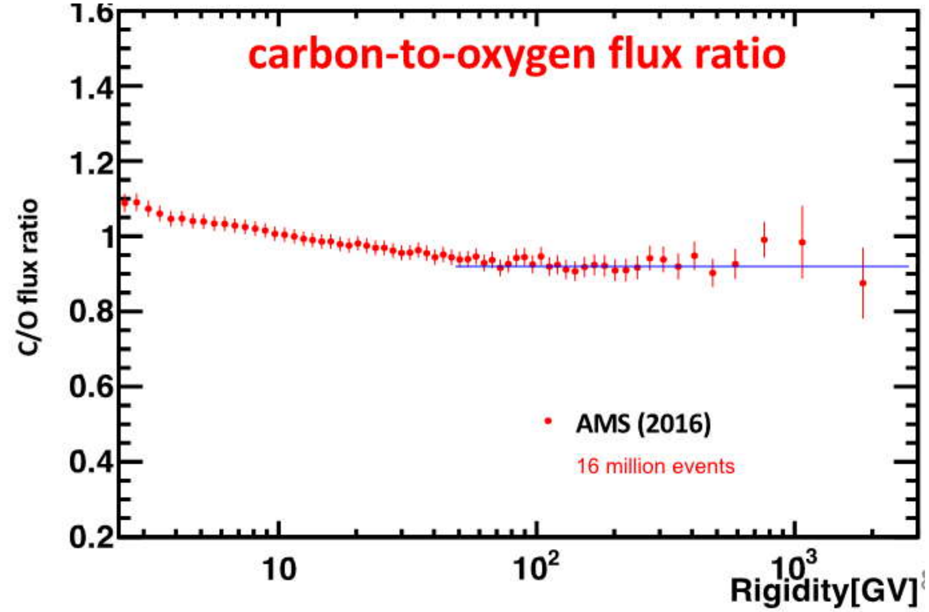
concave spectrum and contribution of
reversed SNR shock VP, Zirakashvili, Seol 2013



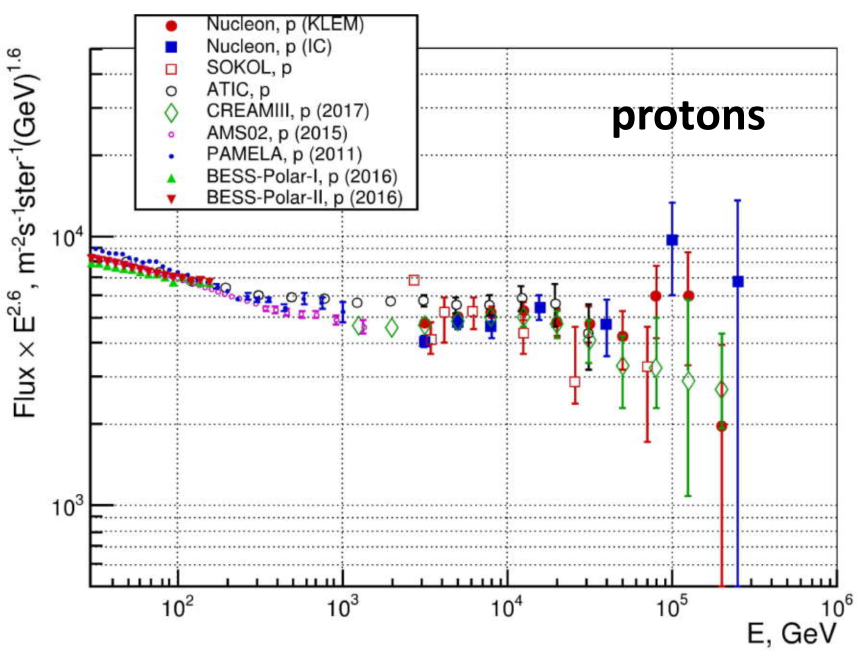






## direct measurements at higher energies



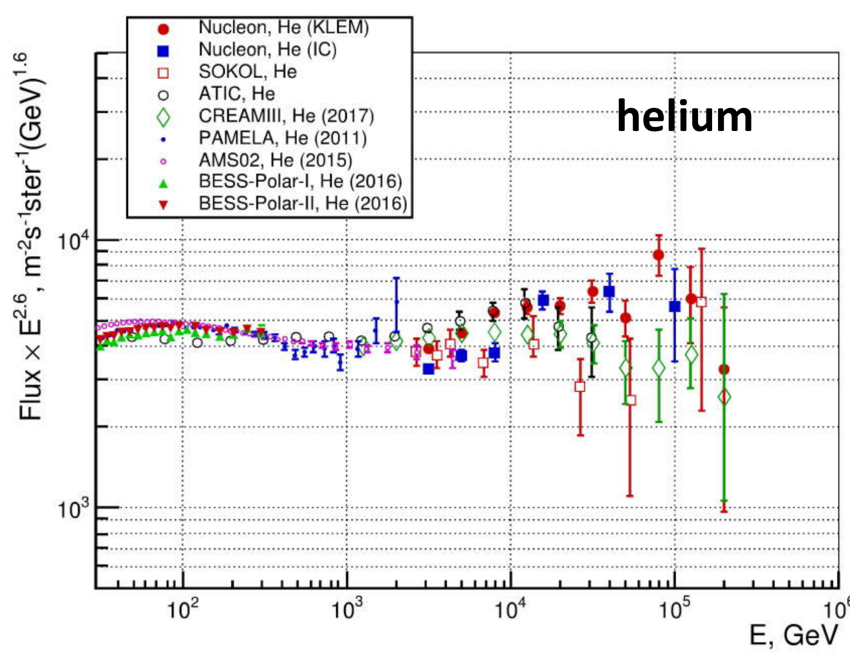



Table 11

Propagation Model Parameters

| Parameter | DR | PD1 | PD2 |
|---|---|---|---|
| $D_0$ ($10^{28}$ cm$^2$ s$^{-1}$)[a] | 14.60 ± 0.20 | 12.20 ± 0.46 | 12.3 ± 1.6 |
| $\delta_1$ | 0.3268 ± 0.0051 | −0.631 ± 0.023 | −0.641 ± 0.042 |
| $\delta_2$ | ⋯ | 0.570 ± 0.022 | 0.578 ± 0.073 |
| $\rho_d$ (GV) | ⋯ | 4.886 ± 0.060 | 4.84 ± 0.10 |
| $v_A$ (km s$^{-1}$) | 42.20 ± 0.61 | ⋯ | ⋯ |
| $z_h$ (kpc) | 4 | 4 | 4 |
| $r_h$ (kpc) | 25 | 25 | 25 |
| $\chi^2$ | 394.3 | 437.1 | 400.4 |

**Note.**
[a] Normalization at 10 GV.

# Model of cosmic ray propagation at 3 MeV/n to 100 GeV/n

Cummings et al 2016

in all models three different cosmic-ray injection spectra are used for protons, He, and heavier elements Z > 2; breaks in source spectra and diffusion coefficient

Table 12

Injection Model Parameters and Modulation Potentials

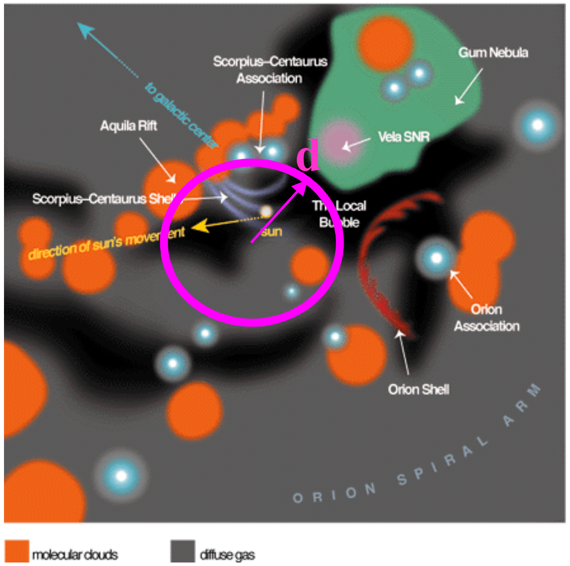
| CR Species | Parameter | DR | PD1 | PD2 |
|---|---|---|---|---|
| $p$ ($Z = 1$) | $\gamma_0$ | −0.6 ± 3.7 | 1.183 ± 0.025 | 1.186 ± 0.024 |
| | $\gamma_1$ | 1.935 ± 0.011 | 2.945 ± 0.021 | 2.947 ± 0.024 |
| | $\gamma_2$ | 2.4742 ± 0.0090 | 2.2283 ± 0.0042 | 2.2225 ± 0.0061 |
| | $\rho_{q,1}$ (GV) | 0.117 ± 0.028 | 1.251 ± 0.031 | 1.244 ± 0.031 |
| | $\rho_{q,2}$ (GV) | 18.0 ± 1.8 | 6.62 ± 0.15 | 6.50 ± 0.18 |
| | $X_p$ at 10 GV[a] | $2.41 \times 10^4$ | $2.48 \times 10^4$ | $2.53 \times 10^4$ |
| | $N_p$ ($10^{-3}$ cm$^{-2}$ sr$^{-1}$ s$^{-1}$ GeV$^{-1}$) at 10 GeV | 2.363 ± 0.010 | 2.2739 ± 0.0043 | 2.2818 ± 0.0043 |
| He ($Z = 2$) | $\gamma_0$ | 0.9 ± 2.5 | 1.507 ± 0.021 | 1.514 ± 0.022 |
| | $\gamma_1$ | 1.9667 ± 0.0051 | 3.018 ± 0.068 | 3.02 ± 0.15 |
| | $\gamma_2$ | 2.4432 ± 0.0085 | 2.2431 ± 0.0052 | 2.2356 ± 0.0042 |
| | $\rho_{q,1}$ (GV) | 0.26 ± 0.10 | 2.457 ± 0.045 | 2.457 ± 0.073 |
| | $\rho_{q,2}$ (GV) | 21.7850 ± 0.0044 | 4.51 ± 0.15 | 4.49 ± 0.21 |
| | $X_{He}$ at 10 GV[a] | 8463 ± 52 | $1.02 \times 10^4$ | $1.03 \times 10^4$ |
| | $\Phi_{PAMELA}$ (MV) | 472.1 ± 6.8 | 468.5 ± 8.4 | 467.6 ± 9.9 |
| | $\chi^2 (Z \leqslant 2)$ | 522.9 | 614.8 | 602.3 |
| $Z > 2$ | $\gamma_0$ | 1.338 ± 0.024 | 1.329 ± 0.031 | 0.88 ± 0.16 |
| | $\gamma_1$ | 2.2076 ± 0.0085 | 2.349 ± 0.015 | 1.63 ± 0.16 |
| | $\gamma_2$ | 2.657 ± 0.032 | ⋯ | 2.3266 ± 0.0025 |
| | $\rho_{q,1}$ (GV) | 2.017 ± 0.027 | 2.047 ± 0.038 | 0.8666 ± 0.0019 |
| | $\rho_{q,2}$ (GV) | 18.62 ± 0.50 | ⋯ | 2.28 ± 0.41 |
| | $\Phi_{HEAO-3}$ (MV) | 889 ± 11 | 785 ± 15 | 755 ± 62 |
| | $\Phi_{ACE-CRIS}$ (MV) | 520.0 ± 5.1 | 485.1 ± 7.0 | 453 ± 23 |

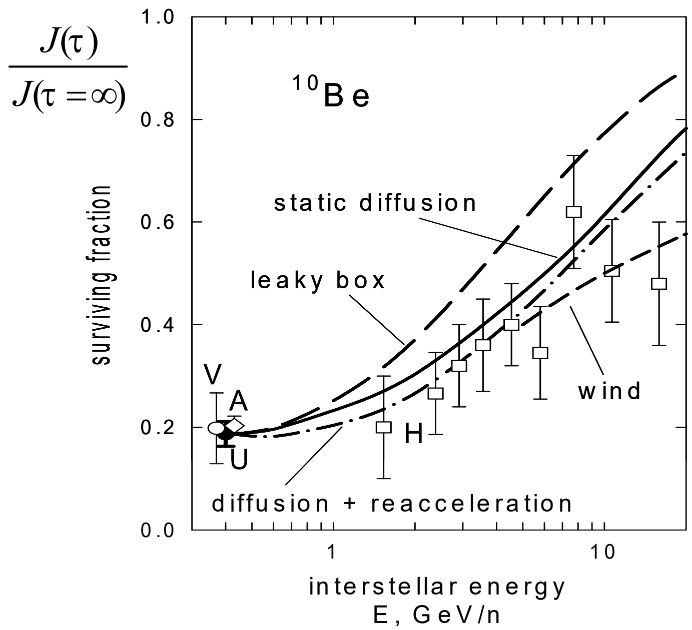
**Note.**
[a] Relative to Si, $X_{Si} = 100$, see Equation (14).

# radioactive secondaries

decay time at rest

$^{10}Be$ (2.3 Myr) $^{26}Al$ (1.3 Myr) $^{36}Cl$ (0.43 Myr) $^{54}Mn$ (0.9 Myr) $^{14}C$ (0.0082 Myr)




$$d = 180\sqrt{D_{28}\tau}\ \text{pc}$$

$$-\nabla D \nabla J_2 + \frac{J_2}{\tau} + \ldots = n(r) v \sigma_{12} J_1$$

elementary model at $\tau \ll H^2/4D$: $\frac{J_2(\tau)}{J_2(\tau = \infty)} \approx \frac{\tau}{\sqrt{\frac{H^2}{D}\cdot\tau}}$

leaky box approximation $\frac{J_2}{T} + \frac{J_2}{\tau} = \langle n \rangle v \sigma_{12} J_1$ gives $\ldots \approx \frac{\tau}{T} \Rightarrow$ does not work!

**$D = (2 \ldots 5)\times 10^{28}\ cm^2/s$**

**at 0.5 GeV/n**

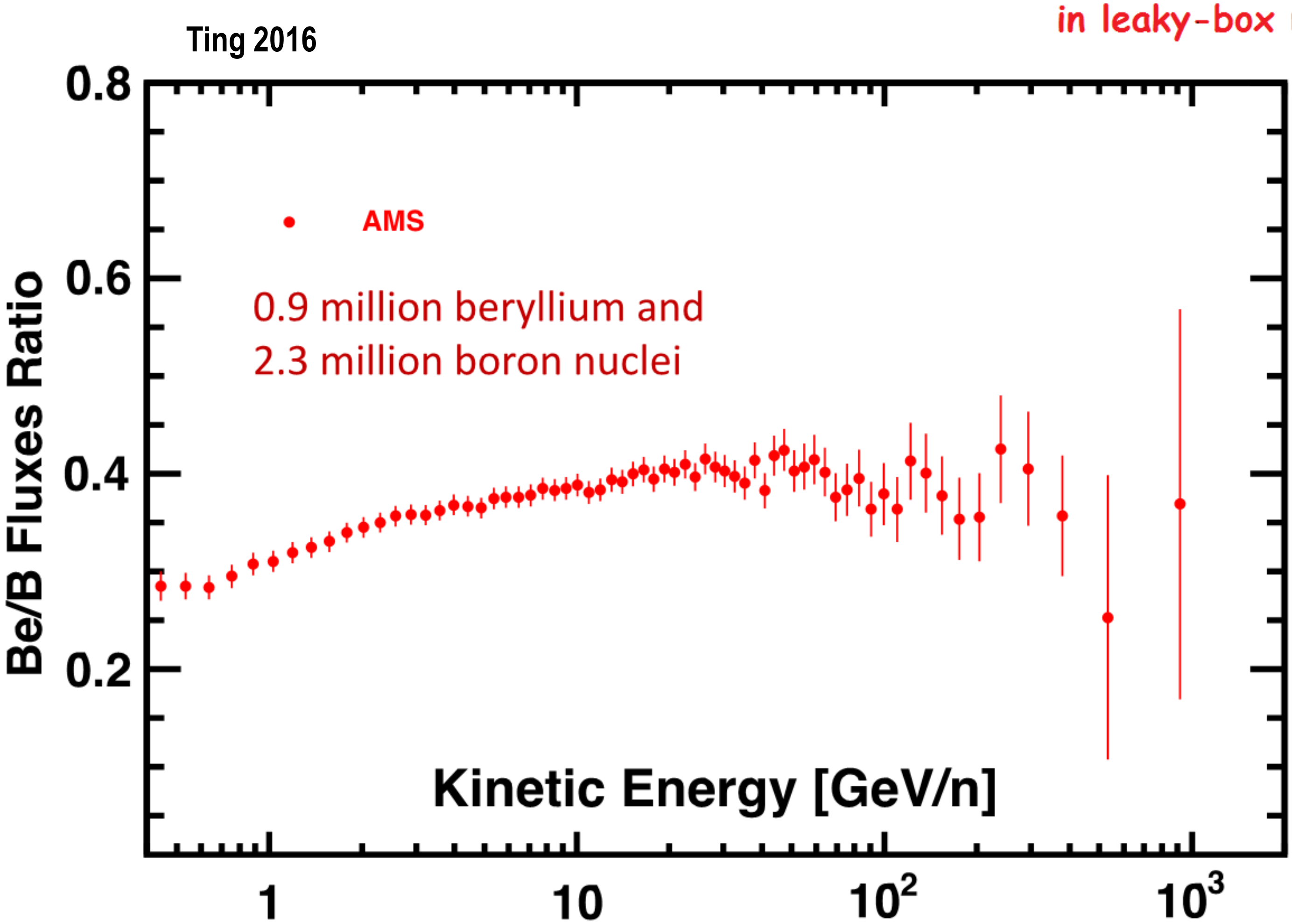
**H ~ 4 kpc, $T_{esc} = H^2/2D$ ~ $7.10^7$ yr**

VP & Soutoul 1998

# radioactive secondary Be

**The beryllium-to-boron (Be/B) flux ratio increases with energy due to time dilation of the decaying Be. The age of cosmic rays in the galaxy is ~12 million years.**

in leaky-box model

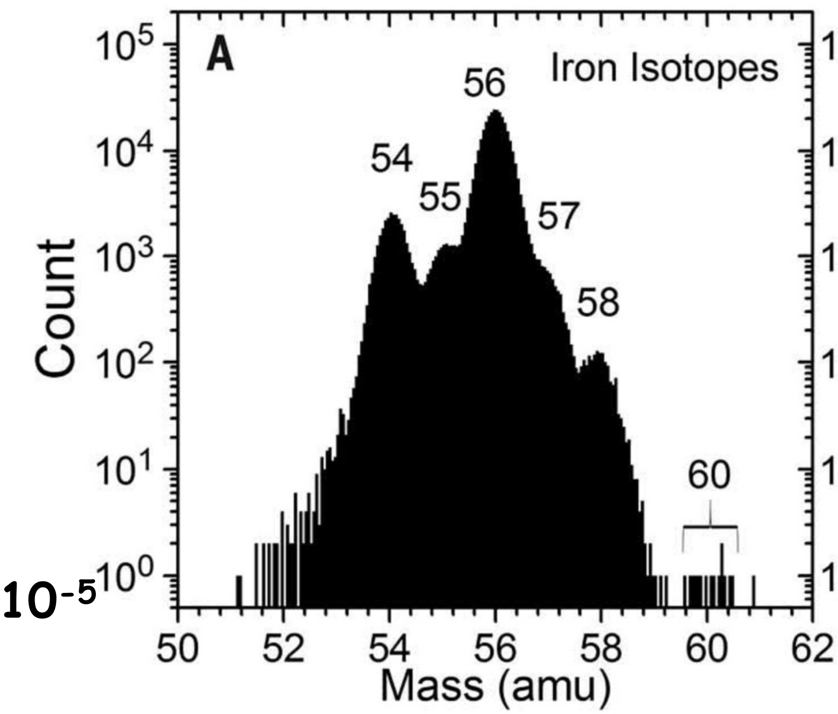


# $^{60}Fe$ nucleosynthesis-clock isotope in Galactic cosmic rays

beta-decay $t_{1/2}$ = 2.6 $10^6$ yr
primary cosmic-ray clock

ACE – CRIS instrument
(Si solid-state detectors)

Binns et al 2016




17 yr of data collection at 195 – 500 MeV/n

3.55 $10^5$ Fe nuclei 15 $^{60}Fe$ nuclei

average source ratio $^{60}Fe/^{56}Fe$ = (7.5+-2.9) $10^{-5}$
ratio ejected by massive star ~ 4 $10^{-4}$

time between nucleosynthesis and acceleration: $10^5$ yr < T < 2 $10^6$ yr

$^{59}Ni$ $^{60}Fe$

distance to the source (SNR) < 600 pc

# electrons and positrons in cosmic rays

inverse Compton and synchrotron losses

$$\frac{dE}{dt} = -bE^2,\ b = (1.2\text{-}1.6)\times 10^{-16}\ (\text{GeV s})^{-1},$$

$$t_{\max}(1\ \text{TeV}) \sim 2\times 10^5\ \text{yr}$$

diffusion distance

$$r_{\max} = \sqrt{6Dt_{\max}},\ D = 3\times 10^{28} E_{\text{GeV}}^{a}\ \text{cm}^2/\text{s}$$

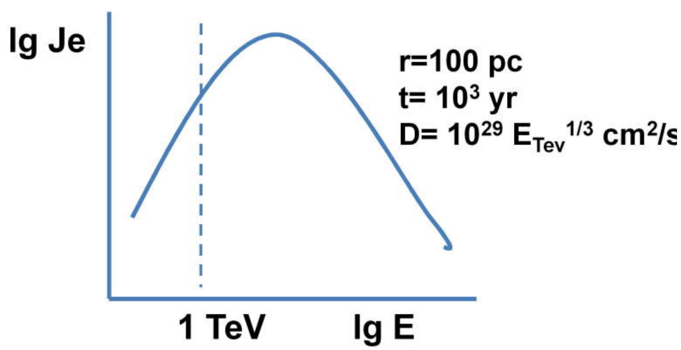
$$r_{\max} \approx \frac{10}{\left(E_{\text{GeV}}\right)^{(1-a)/2}}\ \text{kpc}$$

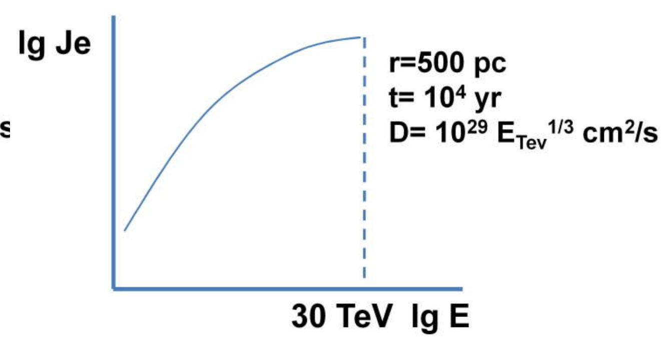
**solution for point instant source in infinite medium** Syrovatsky 1959

$$\frac{\partial G}{\partial t} - D(E)\Delta G - \frac{\partial}{\partial E}(bE^2G) = \frac{\delta(r)}{4\pi r^2}\cdot\delta(E-E_0)\cdot\delta(t);\quad \Delta(\ldots) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2(\ldots)\right)$$

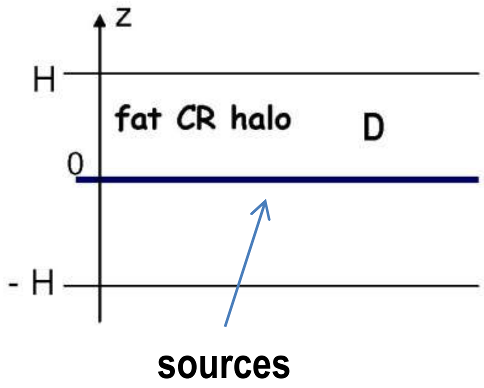
introduce new function $\varphi = bE^2G$ and new variables $y = t - \frac{1}{bE} - t_0 + \frac{1}{bE_0}$ and $\lambda(E,E_0) = \int_E^{E_0} dE_1 \frac{D(E_1)}{bE_1^{\,2}}$.

$$\Rightarrow\ \frac{\partial\varphi}{\partial\lambda} - \Delta\varphi = \frac{\delta(r)}{4\pi r^2}\delta(\lambda)\delta(y) \Rightarrow G = \frac{\exp\left(\frac{-r^2}{4\lambda}\right)}{(4\pi\lambda)^{3/2}}\cdot\delta\left(E_0 - \frac{E}{1-b\cdot(t-t_0)\cdot E}\right),\ \lambda = \frac{D_0E^{a-1}}{(1-a)bE_0^{\,a}}\ \text{for } D = D_0E^a.$$



# solution for homoginious source distribution in Galactic disk




$$-D(E)\frac{\partial^2 J}{\partial z^2}-\frac{\partial}{\partial E}\left(bE^2 J\right)=s\delta(z)$$

$$J_e(E)\approx\frac{s(E)H}{2D(E)}\propto E^{-(\gamma_s+a)}$$

at $H^2/D(E)\ll 1/((1-a)bE)$, low energies ($\leq 3$ GeV);

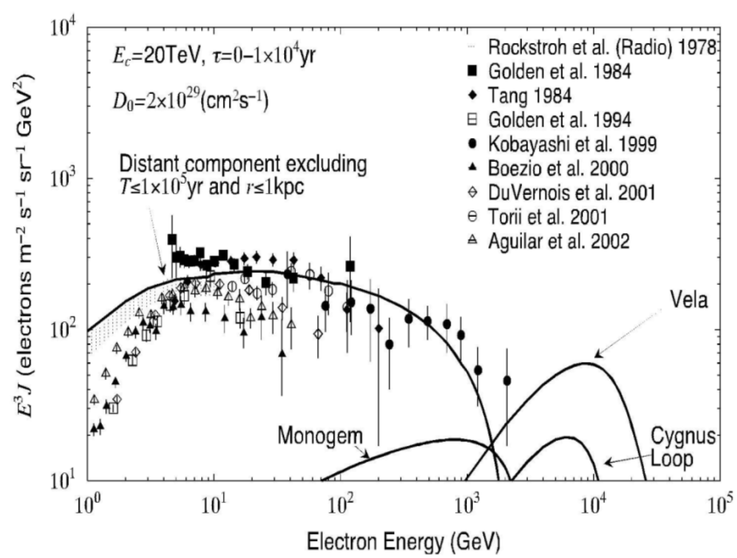
$$J_e(E)\approx\frac{2s(E)}{\sqrt{\pi(1-a)bED(E)}}\propto E^{-\left(\gamma_s+\frac{1+a}{2}\right)}$$

at $H^2/D(E)\gg 1/((1-a)bE)$, high energies (3 ... 100 GeV).

NB: if source region has finite thickness $2h$ then $J_e(E)\propto E^{-(\gamma_s+1)}$ at $h^2/D\gg 1/((1-a)bE)$

$\frac{1+a}{2}=0.7...0.8$ above few GeV $\Rightarrow$ expected primary electron spectrum $J_e\propto E^{-\gamma_{e-}}$, $\gamma_{e-}\approx 3$

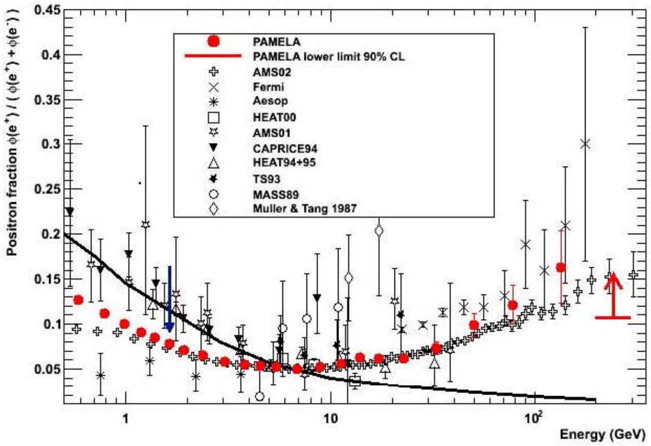
expected secondary positron spectrum $J_e\propto E^{-\gamma_{e+}}$, $\gamma_{e+}\approx 3.5$



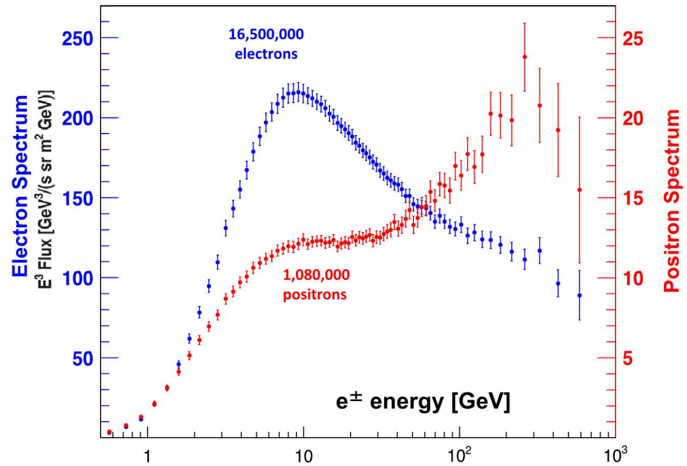

Kobayashi et al 2004

# data on positrons

collection of data Mitchell 2013

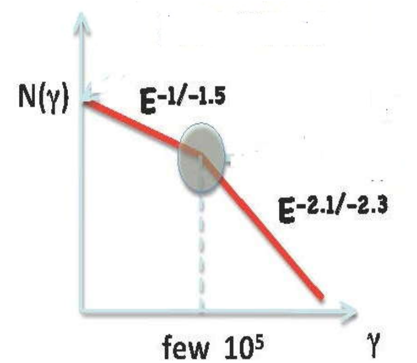



AMS 02



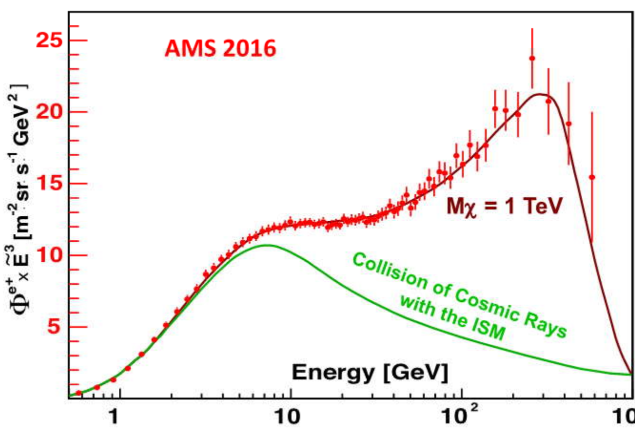

## anti-matter factory

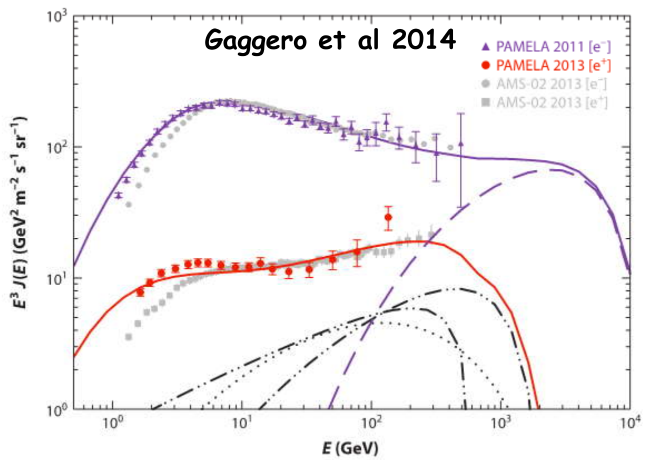

$L_{pairs}$ = 20-30% $L_{psr}$




after Amato

Positron Spectrum




contribution of nearby pulsars and SNRs

- pulasar/PWN origin Harding, Ramaty 1987, Aharonian et al. 1995, Hooper et al. 2008, Malyshev et al. 2009, Blasi, Amato 2011, Di Mauro et al. 2014
- reverse shock in radioactive ejecta Ellison et al 1990, Zirakashvili, Aharonian 2011
- annihilation and decay of dark matter Tylka 1989, Fan et al 2011

# Cosmic ray anisotropy

assuming $f = f_0 + f_1 \cdot \cos\vartheta,\ f_0 = \frac{1}{4\pi}\int d\Omega f,\ f_1 \ll f_0,$ z axis is in direction of maximum intensity.

particle flux $j_z = \int J_1 \cos^2\theta \sin\theta\, d\theta\, d\varphi = \frac{4\pi}{3} J_1.$

diffusion approximation $j_z = -D_{zz}\frac{\partial f_0}{\partial z} \Rightarrow f_1 = -\frac{3D_{zz}}{\mathrm{v}}\frac{\partial f_0}{\partial z}.$

degree of anisotropy
(first angular harmonic)

$$A_z = \frac{f_1}{f_0} = -\frac{3D_{zz}}{\mathrm{v} f_0}\frac{\partial f_0}{\partial z}.$$

**Compton-Getting effect:** cosmic rays are isotropic in a flow with velocity **u**

$$f(\mathbf{p}) = f'(\mathbf{p}')\ \ (= f_0(p'))\ \text{ - Lorentz - invariance}$$

f(**p**)
lab ref frame

$$u \ll c:\ \ \mathbf{p}' \approx \mathbf{p} - \frac{\mathbf{u}}{\mathrm{v}}p,\ \ p' \approx p - \frac{\mathbf{up}}{\mathrm{v}},$$

$$f(\mathbf{p}) \approx f_0\left(p - \frac{\mathbf{up}}{\mathrm{v}}\right) \approx f_0(p) - \frac{\mathbf{up}}{\mathrm{v}}\frac{\partial f_0}{\partial p},$$

**u**
f'(**p'**)=$f_0$(p')
CR are isotropic

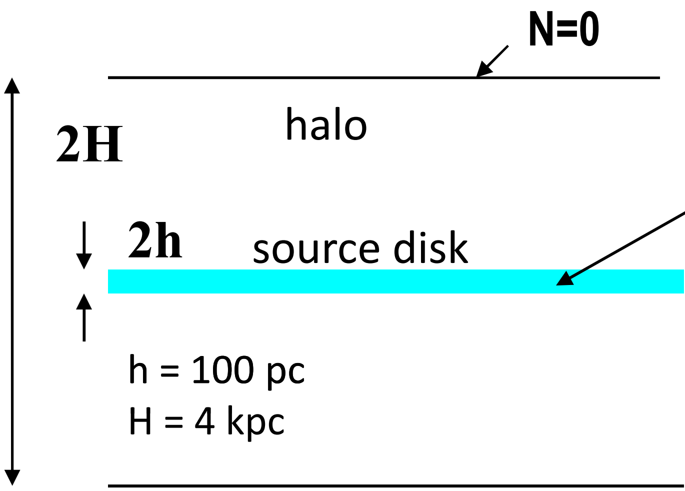
$$\mathbf{j}_{CG}(p) = \int \frac{d\Omega}{4\pi}\mathbf{v} f(p) = -\frac{\mathbf{u}p}{3}\frac{\partial f_0(p)}{\partial p}\ \text{ - Compton-Getting flux,}\ \ \int d^3p\,\mathbf{j}_{CG} = \mathbf{u}N_{cr},$$

anisotropy

$$\mathbf{A}_{CG} = -\frac{\mathbf{u}p}{\mathrm{v}f_0}\frac{\partial f_0}{\partial p}$$

# leakage from the Galaxy

## anisotropy perpendicular to galactic disk:



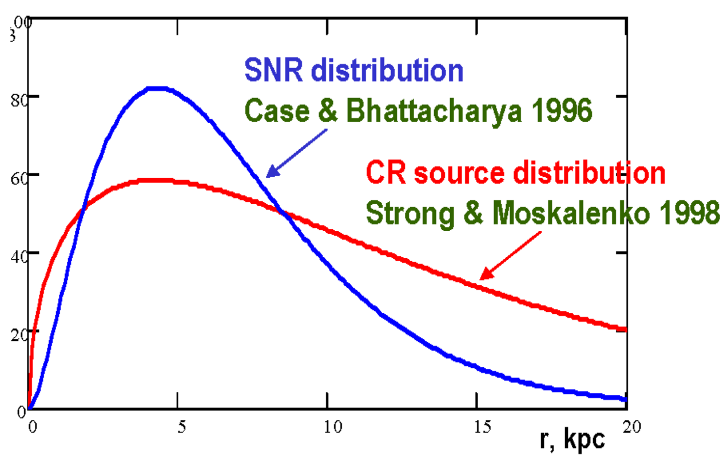

$$A_z = \frac{3D}{\beta c H}\frac{z}{h} = \frac{3\mu}{2X}\frac{z}{h}$$

distance from the midplane ~ 20 pc

$A_d = 1.1\times10^{-3}E_{TeV}^{0.54}$ too large at ~ 100 TeV

$A_a = 6.2\times10^{-4}E_{TeV}^{0.3}$

## radial anisotropy:




$$A_r = \frac{3D}{\beta c}\frac{1}{N}\frac{dN}{dr}$$

$1/L_r$

$L_r$ = 19 kpc

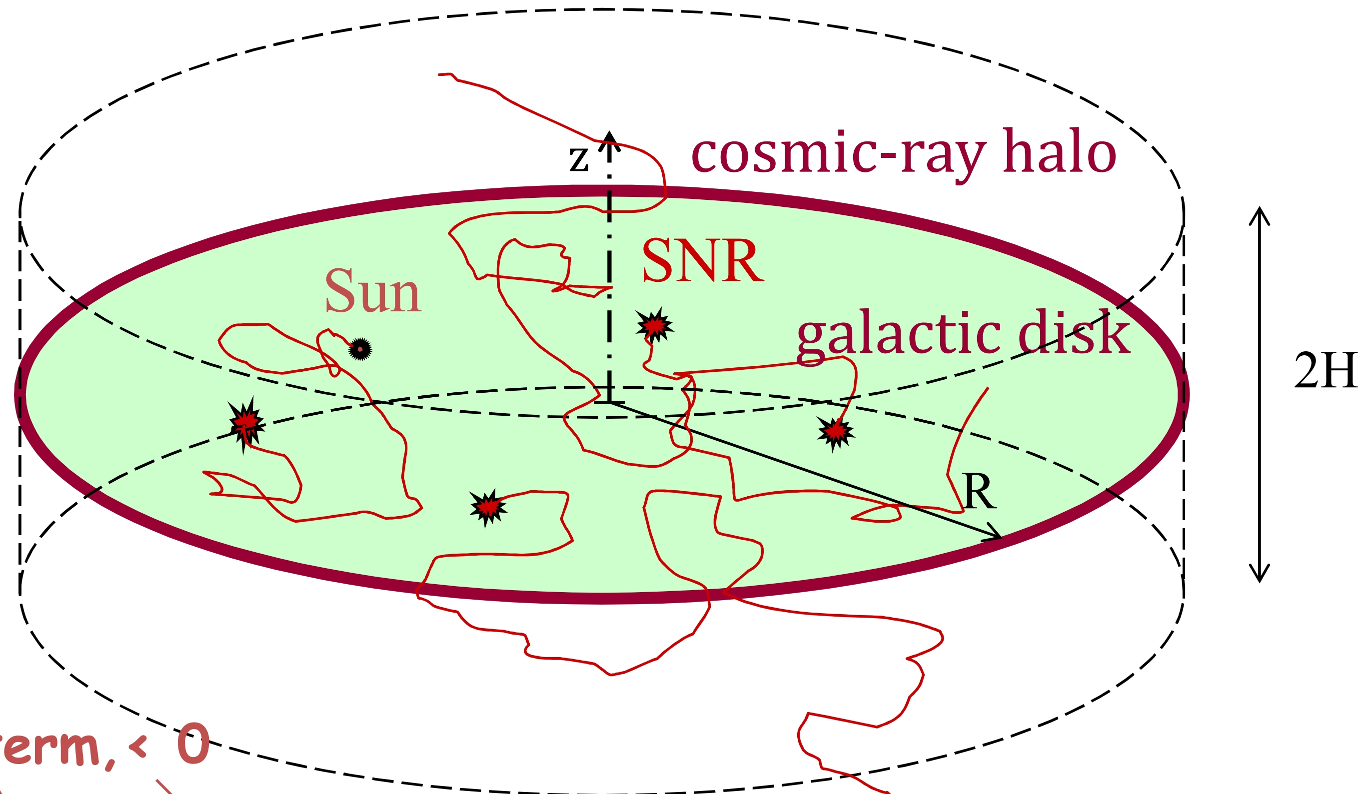
$A_d = 1.5\times10^{-3}E_{TeV}^{0.54}$ too large at ~ 100 TeV

$A_a = 8.2\times10^{-4}E_{TeV}^{0.3}$

wrong phase !

# “statistical mechanics of supernovae”

Jones 1969, Lee 1979, Berezinskii et al. 1990, Lagutin & Nikulin 1995, Taillet et al. 2004, Büsching et al. 2005, Ptuskin et al. 2005, Sveshnikova et al. Blasi & Amato 2012, Sveshnikova et al 2013, Mertsch & Funk 2015




energy loss term, < 0
(for electrons)

$$\frac{\partial N}{\partial t} - D\Delta N + \frac{\partial}{\partial E}\left(\frac{dE}{dt}N\right) = \sum_i S(E)\delta(x-x_i)\delta(y-y_i)\delta(z)\delta(t-t_i)$$

number of particles accelerated in one burst "typical" fluctuations:

**proton density fluctuation**

$$\frac{\delta N}{N} \approx \frac{1}{2\pi^{3/4} H}\left(\frac{D(E)}{\sigma_{sn}}\right)^{1/4} \propto E^{\frac{a}{4}},$$

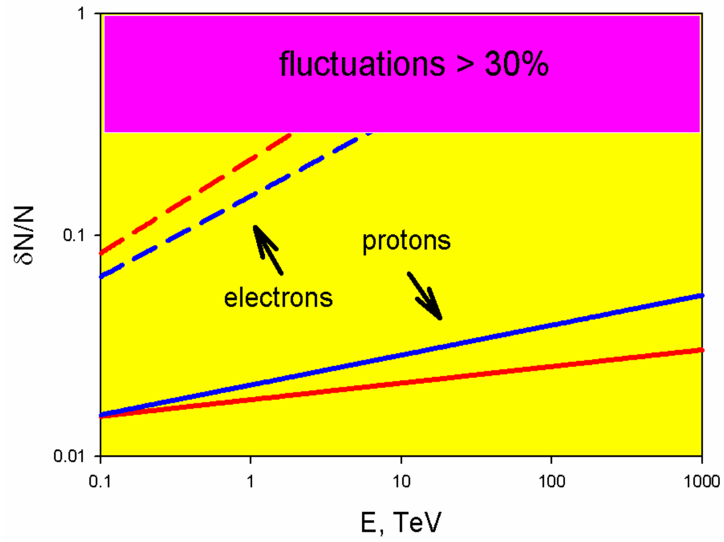
**electron density fluctuations**

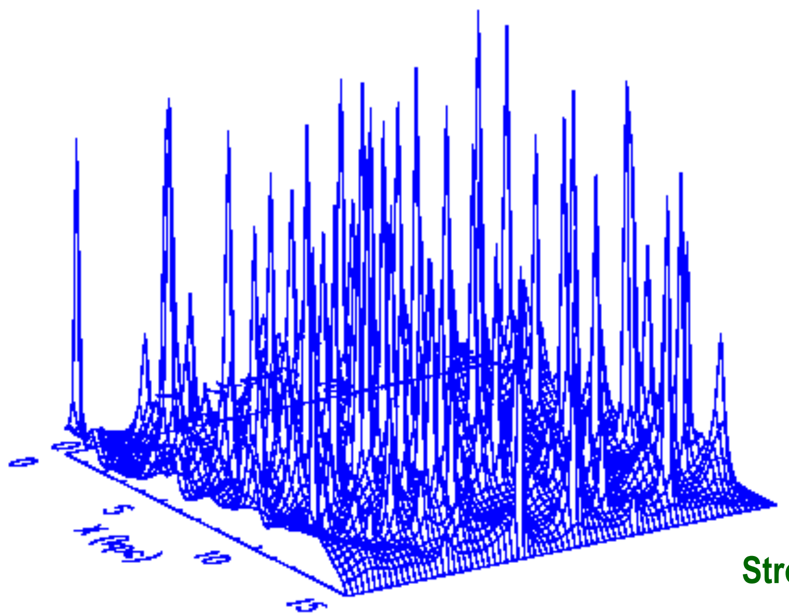
$$\frac{\delta N_e}{N_e} \approx \frac{\left((1-a)bE\right)^{1/2}}{2\left(\pi\sigma_{sn} D(E)\right)^{1/4}} \propto E^{\frac{2-a}{4}}.$$

**fluctuation anisotropy of protons**

$$\delta_{\text{fluct}} \approx \frac{3\sqrt{2}D(E)}{4cH} \propto E^{a}$$

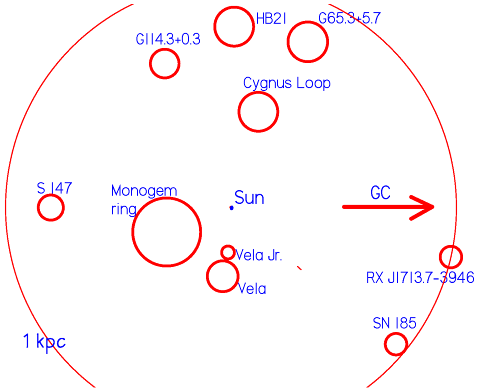
(~ $A_z$ !)

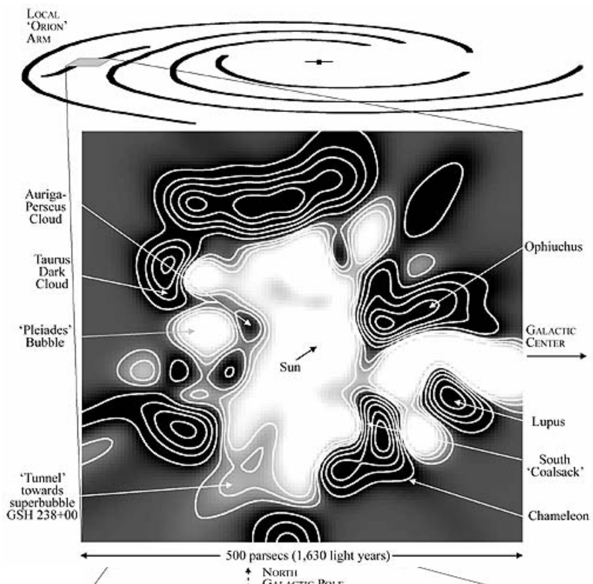


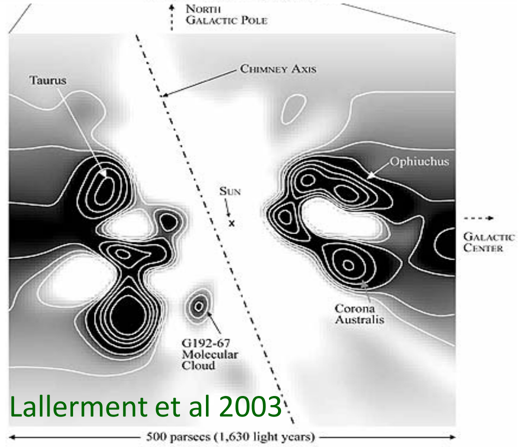



electrons

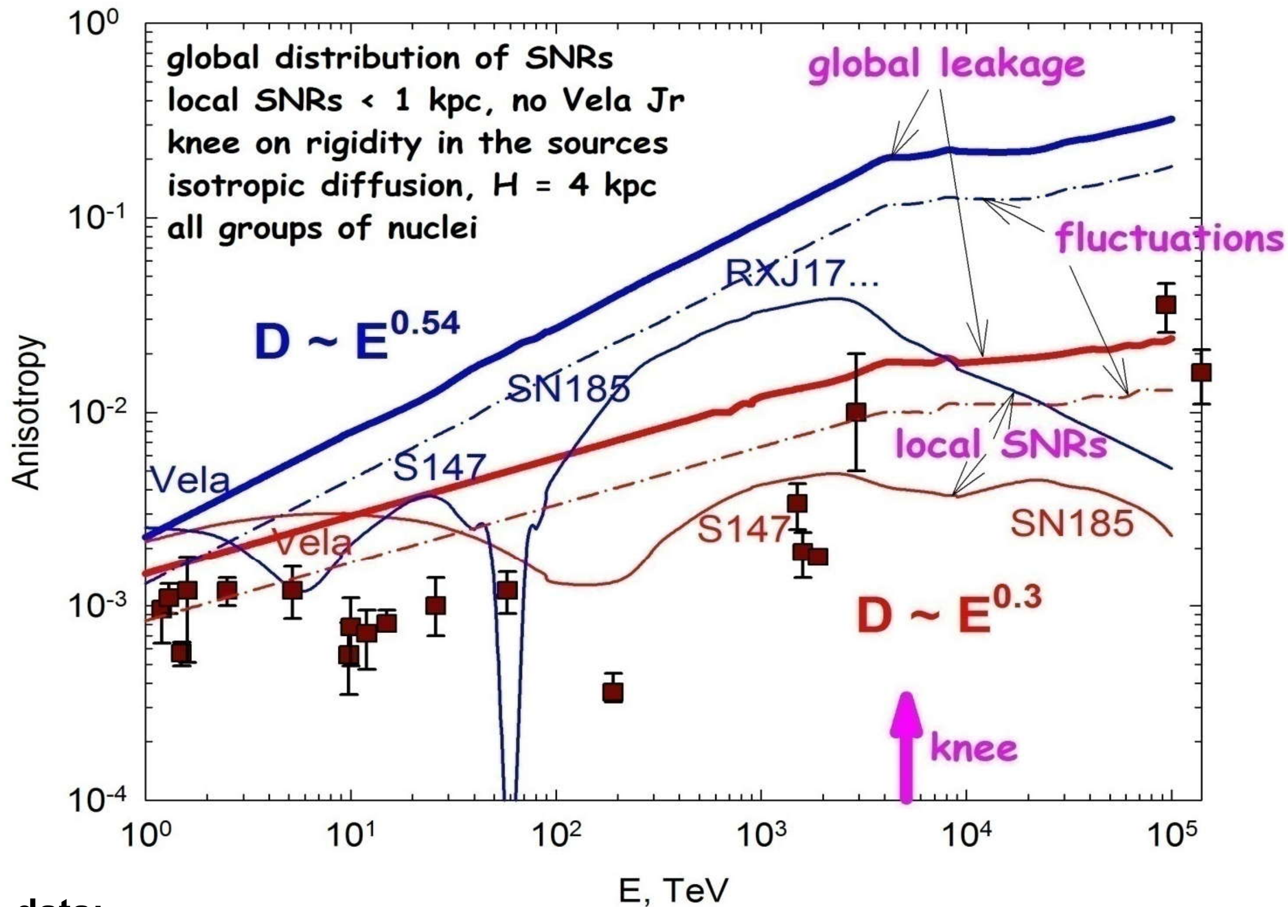
Strong & Moskalenko 2001




Lallerment et al 2003




**data:**

Gomboshi et al. 1975, Linsley & Watson 1977, Lloyd-Evans 1982, Kifune et al. 1986, Lee & Ng 1987, Bird et al. 1989, Nagashima et al. 1989, Andreev et al. 1991, Cutler & Groom 1991, Fenton et al. 1995, Mori et al. 1995, Aglietta et al. 1996, Efimov et al. 1997, Munakata et al. 1999, Ambrosio et al. 2003

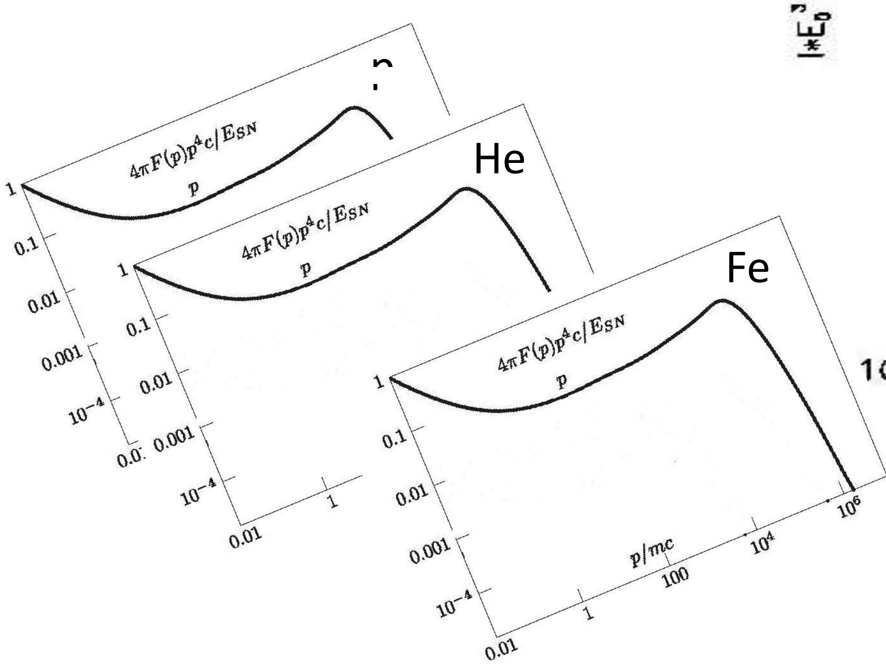
direction of magnetic field based on: l = 185°, b = -38°
observed direction of CR anisotropy

# Nature of the knee

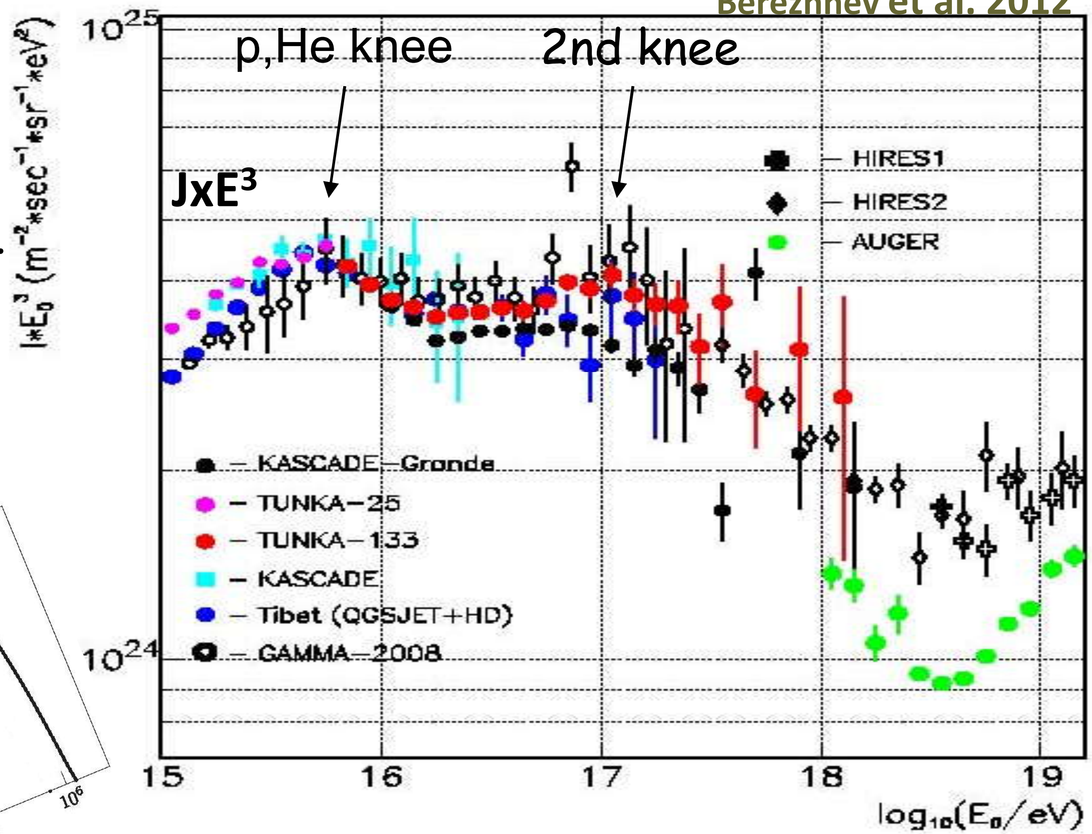
# knee and beyond

## structure above the knee

different types of nuclei, $E_{knee}$ ~ Z
different types of SN
transition to extragalactic component

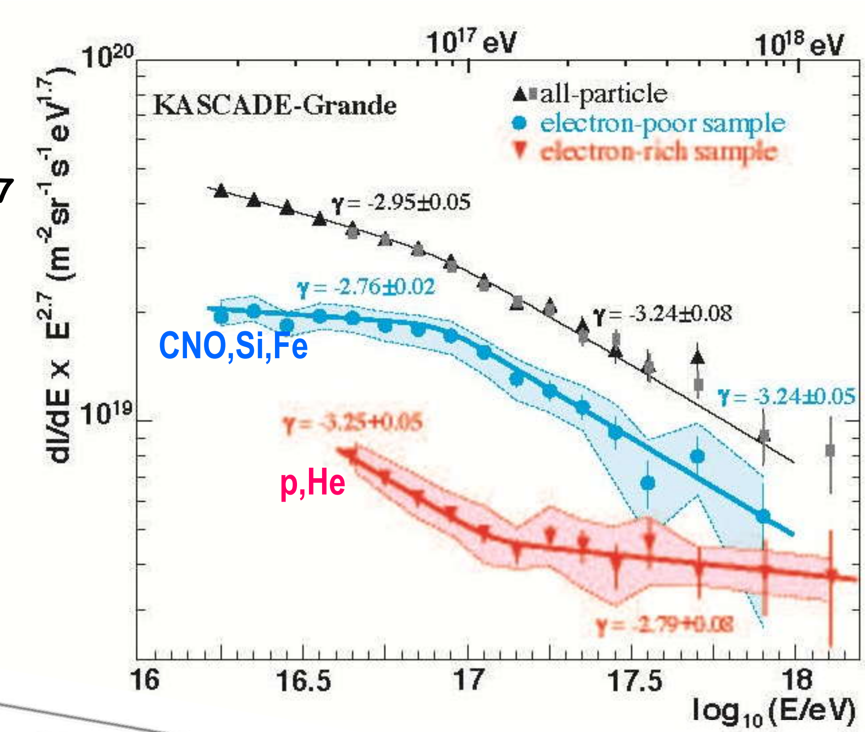



Berezhnev et al. 2012



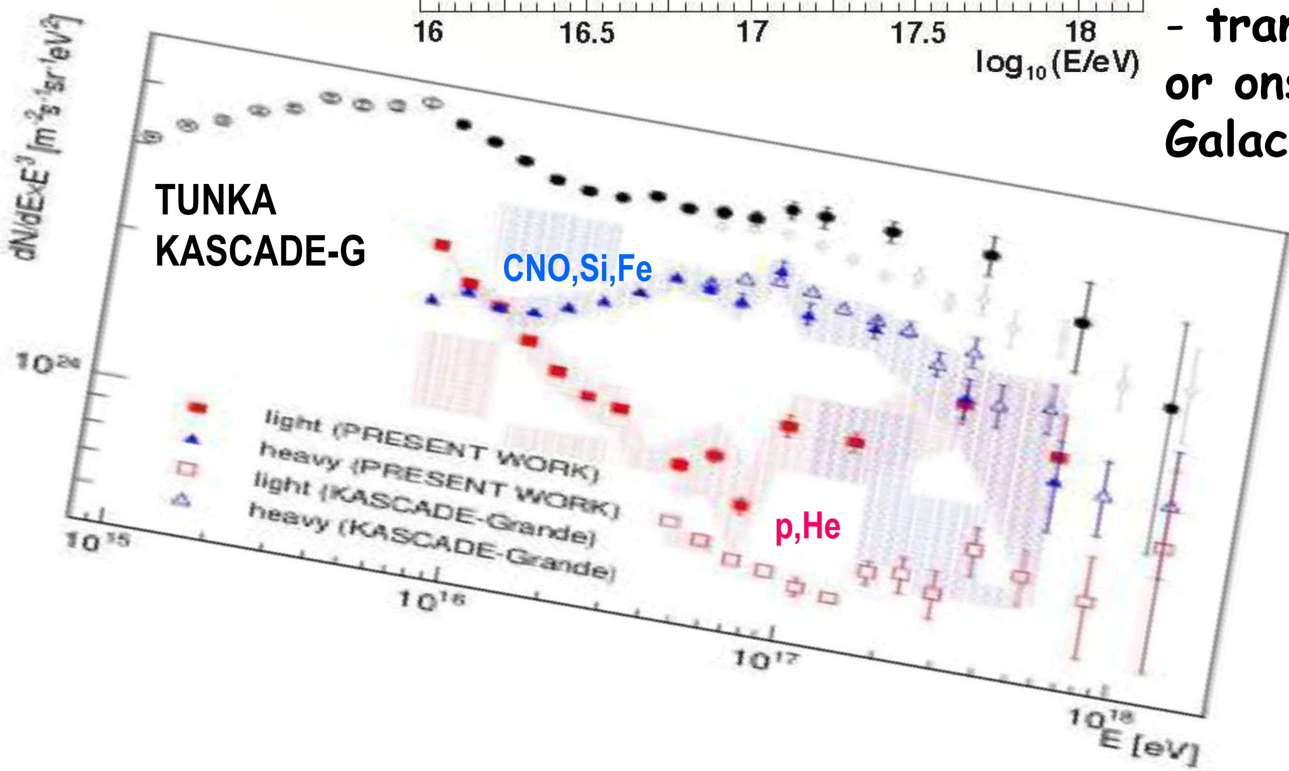

Sveshnikova et al 2014: composition at 1PeV: H 17%, He 46%, CNO 8%, Fe 16%
+ EG component H 75%, He 25% as in Kotera & Lemoine 2008

JxE$^{2.7}$




Iron knee at
$8*10^{16}$ eV = $26*3*10^{15}$ eV

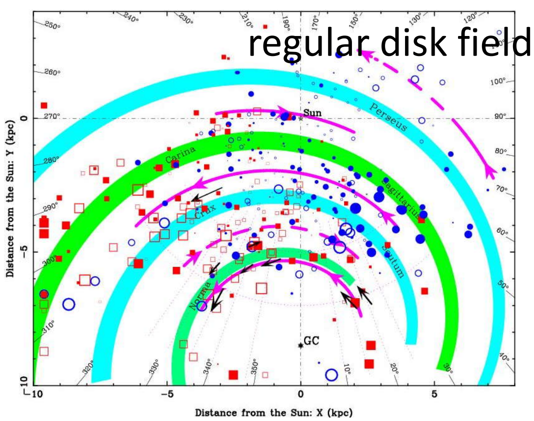
light ankle at $1.2*10^{17}$ eV
- transition to EG component
or onset of a new high energy
Galactic source population

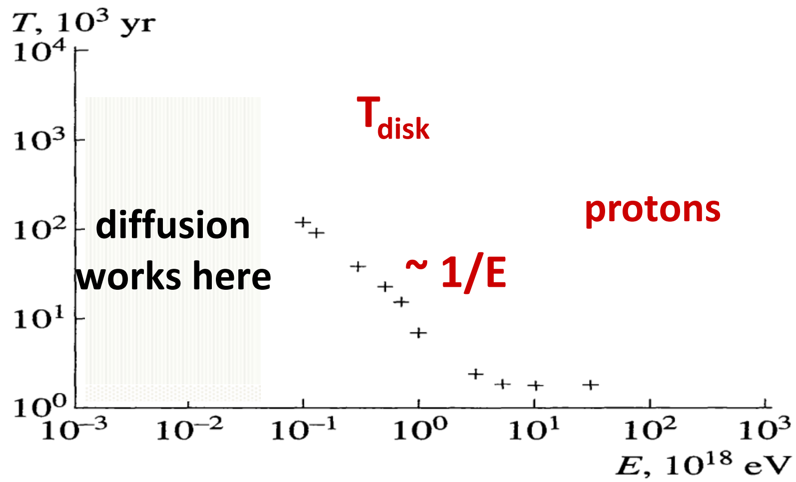


# extension of propagation model to higher energies: trajectory calculations

Syrovatsky 1971, Berezinsky et al. 1991, Gorchakov et al 1991, VP et al 1993, Lampard et al 1997, Zirakashvili et al 1998, Candia et al. 2003, Hörandel et al. 2005

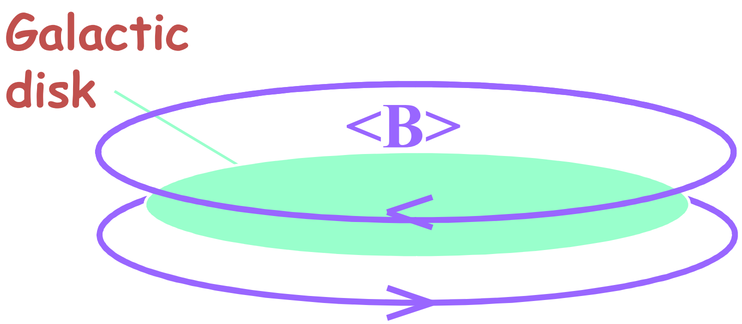
ld



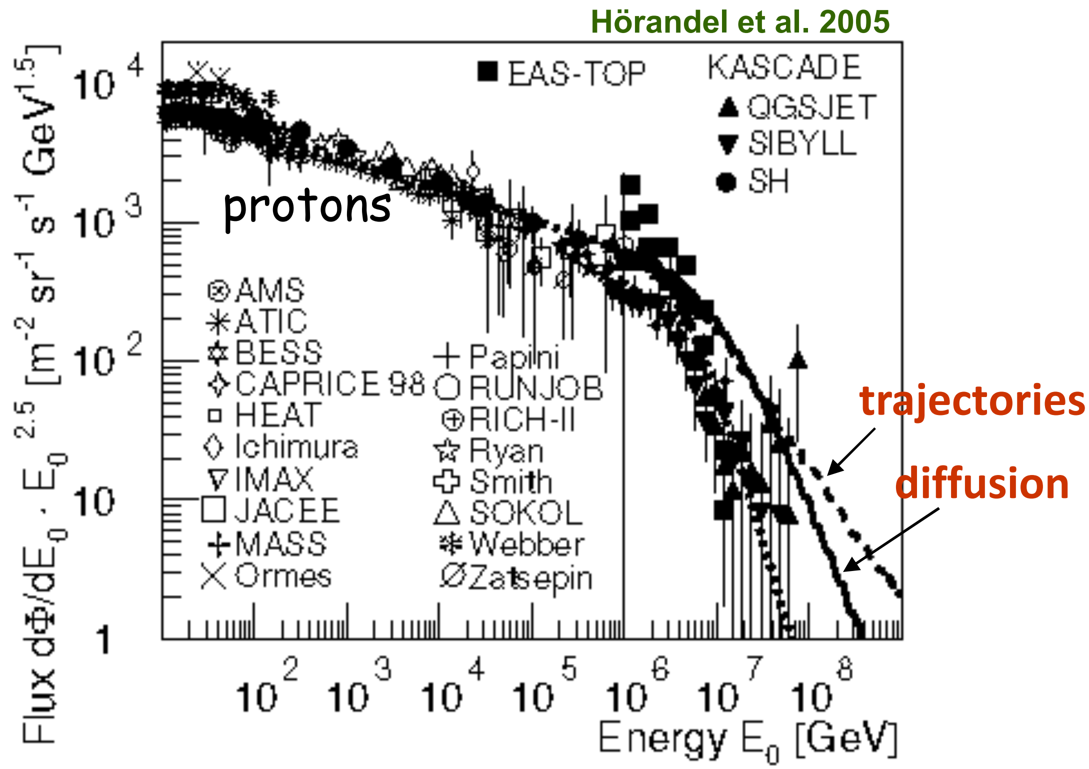



# alternative explanation of the knee:

knee as effect of Hall diffusion

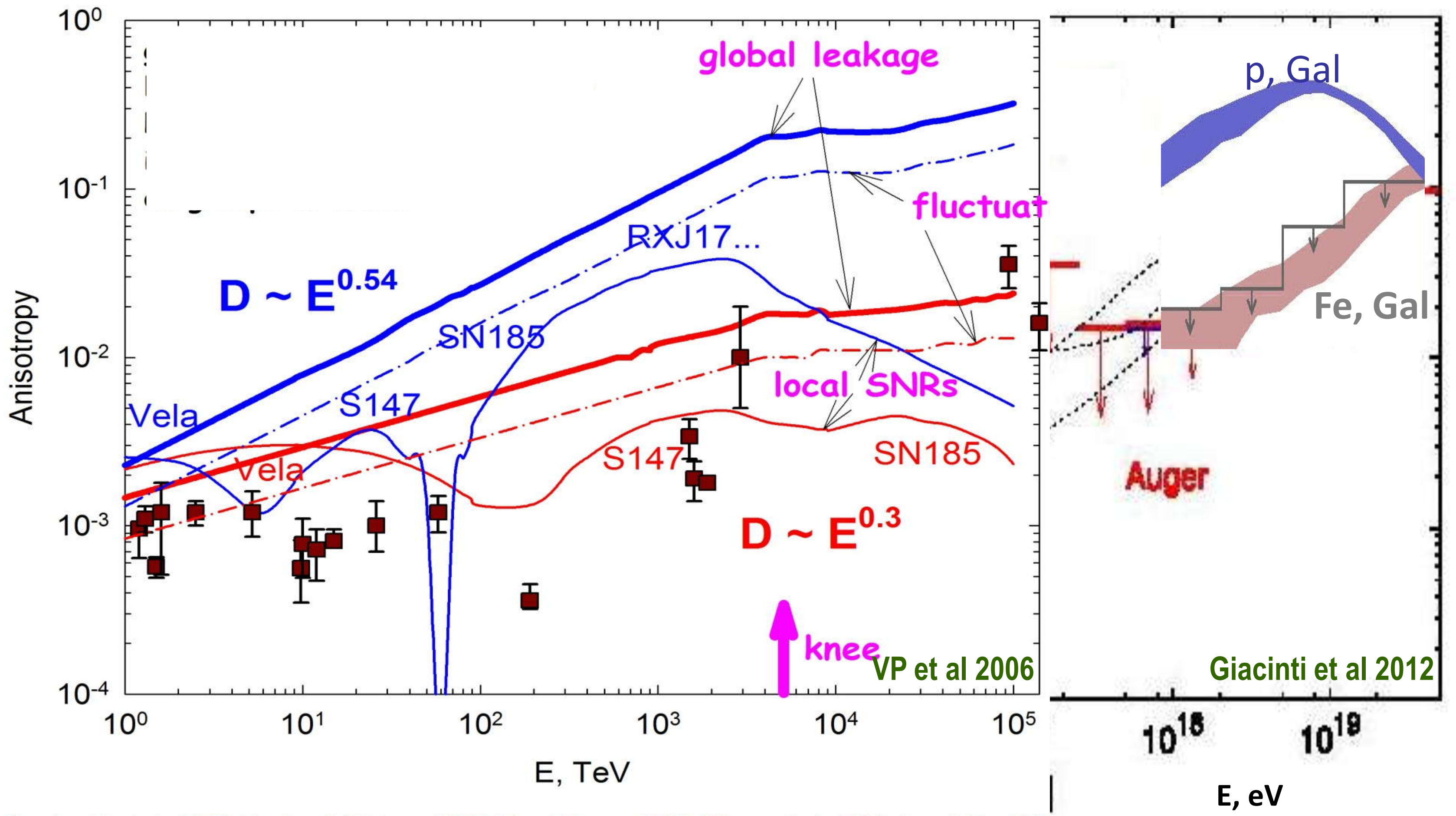



$D_\perp \sim R^{0.2}$, $D_A \sim R$




additional change of source spectrum is needed

# cosmic ray anisotropy, equatorial dipole amplitude

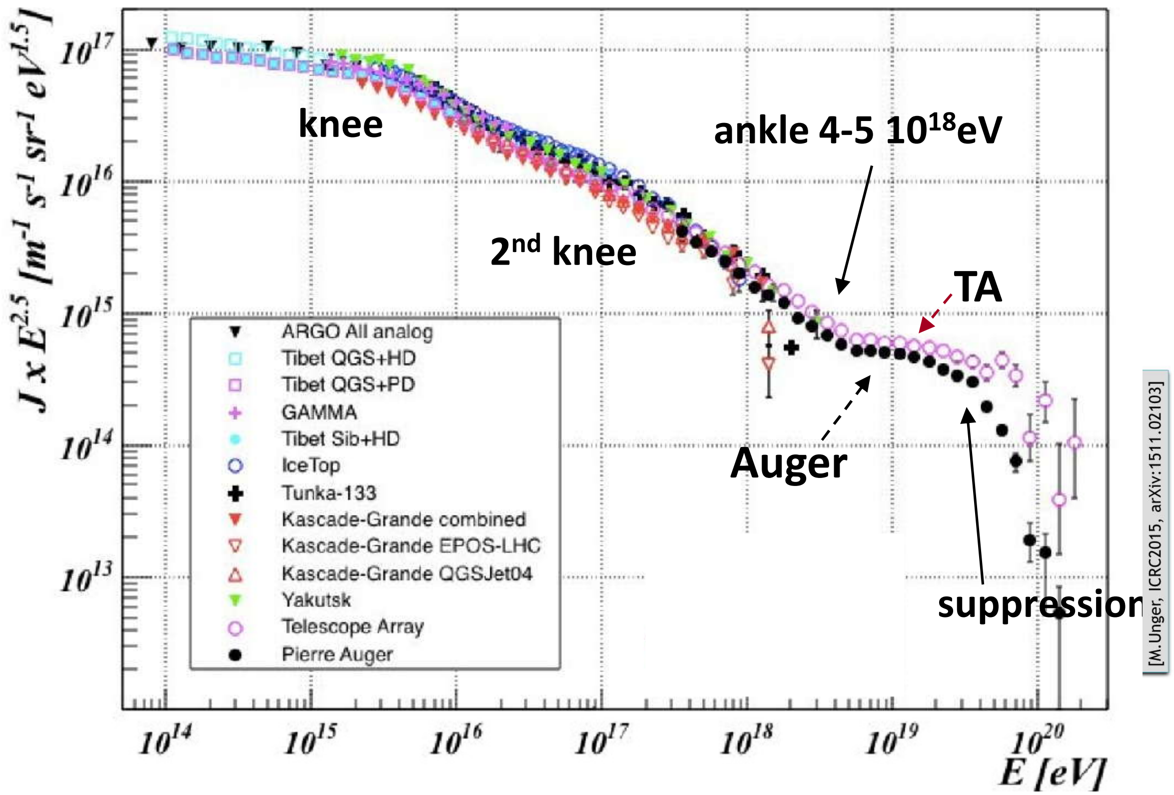



Gomboshi et al. 1975, Linsley & Watson 1977, Lloyd-Evans 1982, Kifune et al. 1986, Lee & Ng 1987
Bird et al. 1989, Nagashima et al. 1989, Andreev et al. 1991, Cutler & Groom 1991, Fenton et al. 19
Mori et al. 1995, Aglietta et al. 1996, Efimov et al. 1997, Munakata et al. 1999, Ambrosio et al. 2003

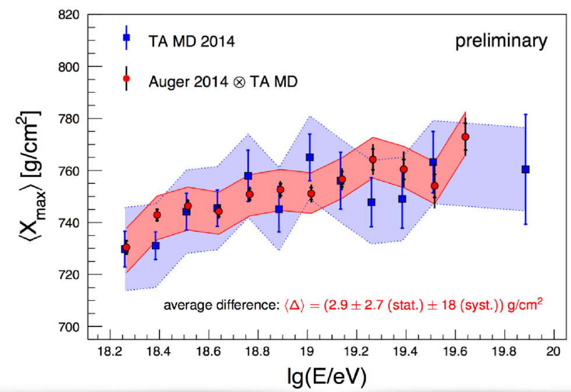
# Cosmic rays of extragalactic origin.

## GZK cutoff. Data interpretation.

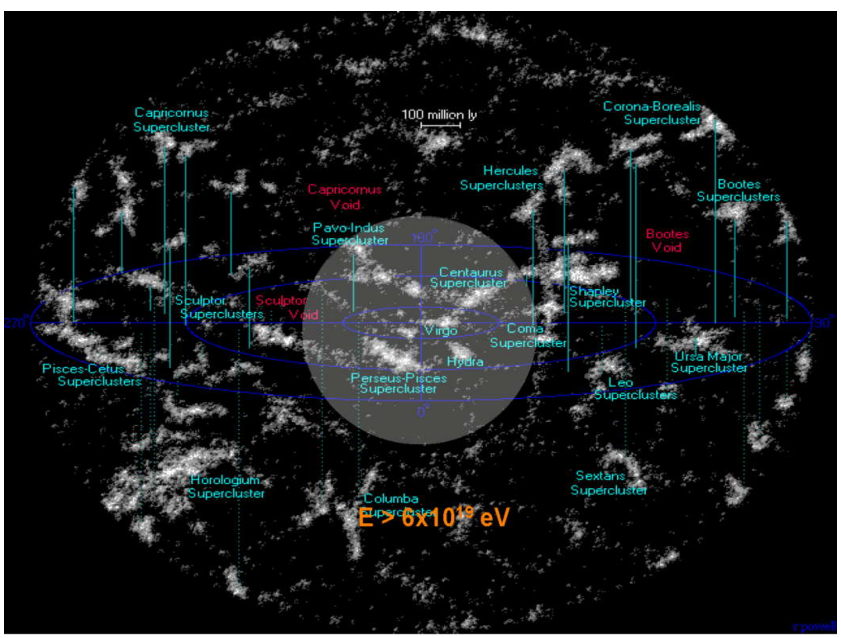
# Extragalactic cosmic rays




Mass composition - Auger vs TA




The two results are in good agreement within systematic uncertainties
TA cannot distinguish between pure proton or mixed composition
with the current level of uncertainty

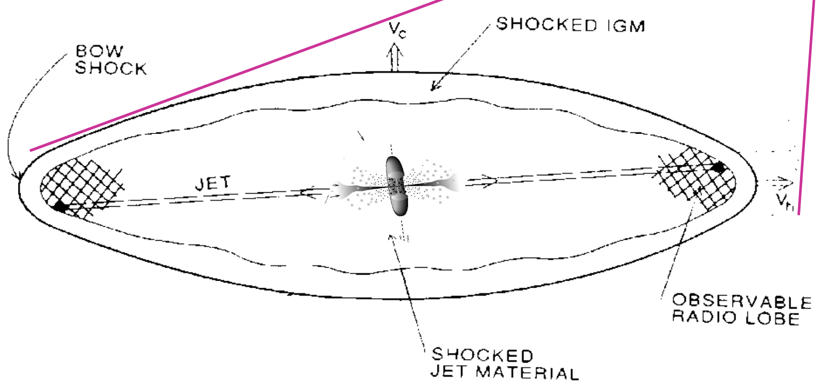



**Auger data on dispersion of <$X_{max}$> are also available**

# extragalactic sources of cosmic rays

energy release in units $10^{40}$ erg/(s Mpc$^3$)

| needed in CR at $E > 10^{19.5}$ eV | SN | AGN jets | GRB | newly born fast pulsars (< 5ms) | accretion on galaxy clusters |
|---|---|---|---|---|---|
| $3\ 10^{-4}$ (Auger) | $3\ 10^{-1}$ | 3 | $3\ 10^{-4}$ | $10^{-3}$ | 10 |
| | kin. | & $6\ 10^{-2}$ for | X/gamma | rotation | strong shocks |
| $8\ 10^{-3}$ for $E>10^9$ eV | | $L_{kin} > 10^{44}$ erg/s | | | |




Schematic diagram of overpressured cocoons around jets (Begelman & Cioffi 1989).

**AGN jets**

$$E_{max} \approx 10^{20} \times Z \times \beta^{1/2} \times \left(L_{jet} / 10^{45}\ erg/s\right)^{1/2}\ eV$$

Lovelace 1976, Biermann & Strittmatter 1987, Norman et al 1995, Lemoine & Waxman 2009

**fast new born pulsars**
**$B = 10^{12} ... 10^{13}$ G**

$$E_{max} \approx 10^{19} \times Z \times \left(\Omega / 10^4 sec\right)^2\ eV$$

Gunn & Ostriker 1969, Berezinsky et al. 1990, Arons 2003, Blasi et al 2000, Fang et al. 2013

# energy loss of ultra-high energy cosmic rays in extragalactic space

microwave & EBL photons

- pair production $p\gamma \rightarrow p e^+ e^-$
- pion production $p\gamma \rightarrow N\pi$

GZK cutoff at $E_{GZK} \sim 6\times10^{19}$ eV

Greisen 1966; Zatsepin & Kuzmin 1966

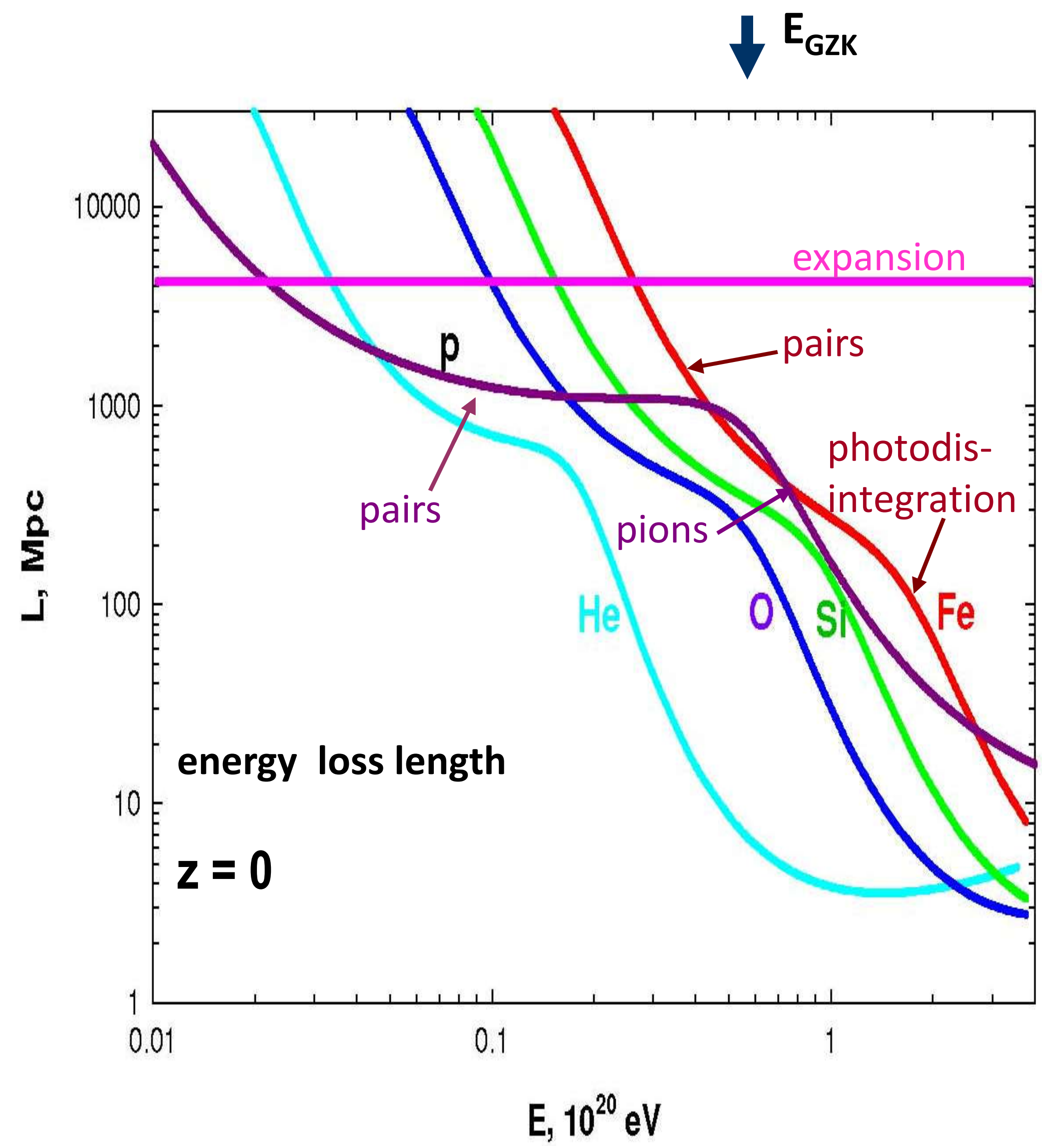
- photodisintegration of nuclei

Stecker 1969

- Universe expansion

$-(1/E)\,(dE/dt)_{adiabatic} = H$

$H_0$=100h km/(s Mpc), h=0.71



## cosmic ray nuclei in expanding Universe

**for homogeneous source distribution and arbitrary regime of cosmic-ray propagation:**

$$-H(z)(1+z)\frac{\partial}{\partial z}\left(\frac{F(A,\varepsilon,z)}{(1+z)^3}\right)-\frac{\partial}{\partial \varepsilon}\left(\varepsilon\left(\frac{H(z)}{(1+z)^3}+\frac{1}{\tau(A,\varepsilon,z)}\right)F(A,\varepsilon,z)\right)$$

$$+\nu(A,\varepsilon,z)F(A,\varepsilon,z)=\sum_{i=1,2...}\nu(A+i\rightarrow A,\varepsilon,z)F(A+i,\varepsilon,z)+<q(A,\varepsilon)>(1+z)^m$$

Ptuskin et al 1999

$F(A,\varepsilon,z)$ - cosmic ray distribution function, Z = 1 ... 26,

$\varepsilon = E/A$ - energy per nucleon,

z - redshift, $1+z=\frac{a_{now}}{a_{then}}$, $a$ is the cosmic scale factor,

$\frac{dr}{dt}=rH$, $\frac{dz}{dt}=-(1+z)H(z)$, $\frac{dE}{dt}=-HE$ at $E\approx pc$; $H_0$=100 km/(s$\cdot$Mpc), $h$=0.7,

$H(z)=H_0\sqrt{(1+z)^3\Omega_m+\Omega_\Lambda}$ - Hubble parameter (at $\Omega_m+\Omega_\Lambda\approx 0.3+0.7=1$),

Hubble scale $R_{mg}$= $c/H_0=3\times 10^3 h^{-1}$ Mpc,

$q(A,\varepsilon)$ - source term at $z=0$, $m$ describes source evolution,

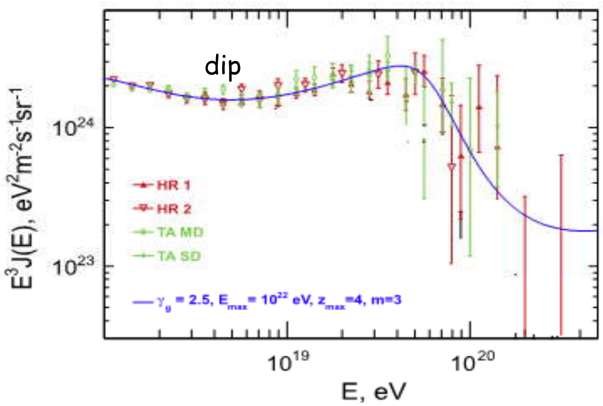
$\tau(A,\varepsilon,z)$ - energy loss time on $e^+e^-$ and $\pi^0$ photoproduction,

$\nu(A,\varepsilon,z)$ - photodisintegration rate, sources of Galactic cosmic rays: $E^{-2.2}$, H=92% , He=7.7% , C=0.27% , O=0.38%, Mg=0.067%, Si=0.07% , Fe=0.073%

# sources of extragalactic cosmic rays:

## TA data (pure protons?)

Berezinsky 2014




source spectrum

$E^{-2.5}$, $E_{max} \sim 10^{22}$ eV

* GZK suppression of proton spectrum

* ankle structure is due to e+e-production

## Auger data

at the source:

$E^{-0.96}$,

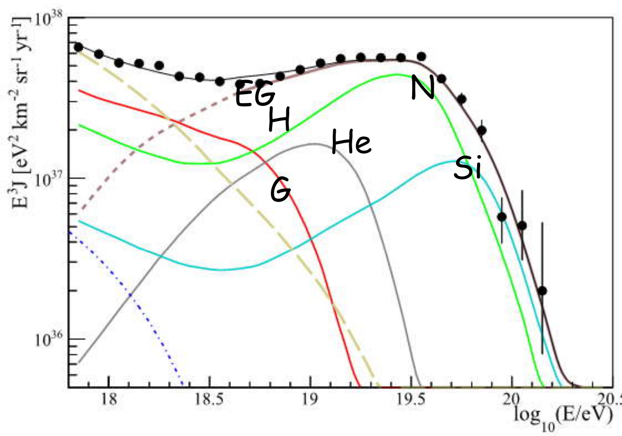
$E_{max}$ = 5 $10^{18}$ Z eV

H=0%, He=67%,

N=28%, Si=5%,

Fe=0%

Aab et al 2016




- source composition is highly enriched in medium and heavy nuclei;
- source spectrum is more hard than $E^{-1}$, $E_{max}$ ~ 5 $10^{18}$Z eV
- disappointing model for UHE neutrino production
- shape of the all-particle spectrum is likely due to concurrence of two effects: maximum energy reached at the sources and energy losses during propagation

# Collective effects of cosmic rays.

## Streaming instability. Parker instability. Galactic wind model.

# cosmic-ray streaming instability

Ginzburg 1965, Lerche 1971, Wentzel 1969, Kulsrud & Pearce 1969, Kulsrud & Cesarsky 1971, Skilling 1975, Holmes 1975, Bell 1978, Farmer & Goldreich 2004

motion of cosmic rays through background plasma with bulk velocity $u_{cr} > V_a$ generates Alfven waves

CR energy density

growth rate of waves amplitude

$$\frac{w_{cr}}{c^2} \cdot \frac{u_{cr} - V_a}{\tau} = 2\Gamma_{cr} \frac{w_{\delta B}}{V_a}$$

wave energy density $\delta B^2/4\pi$

after Wentzel; Blasi

**resonant scattering**

$$\tau \approx \frac{r_g}{v} \frac{B_0^{\,2}}{\delta B_{res}^2}$$

rate of momentum loss by particles

rate of momentum gain by waves

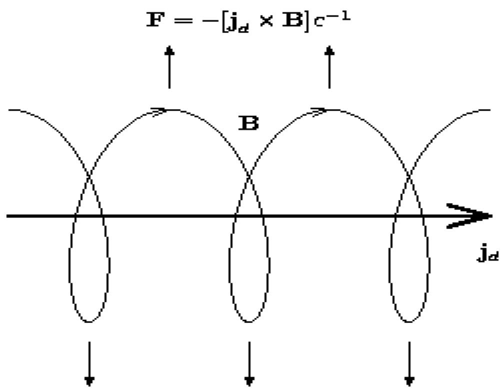
$$\Gamma_{cr} = \sqrt{\frac{4\pi}{\rho}} \frac{e u_{cr} N_{cr}}{c} \left(1 - \frac{V_a}{u_{cr}}\right), \quad \text{at } k_{res} = 1/r_g$$

**in diffusion approximation**

$$\Gamma_{cr}(k_{res}) = \frac{16\pi^2 V_a v p^4}{3 k_{res} W(k_{res})} \left|\frac{\partial f}{\partial x}\right|$$

weak turbulence $\delta B \ll B_0$ and $\Gamma_{cr} \ll \omega(k) \rightarrow w_{cr}(u_{cr}/c) \ll B_0^2/4\pi$

$$\omega = k_z V_a$$




strong instability $w_{cr}(u_{cr}/c) > B_0^2/4\pi$

Bell 2004

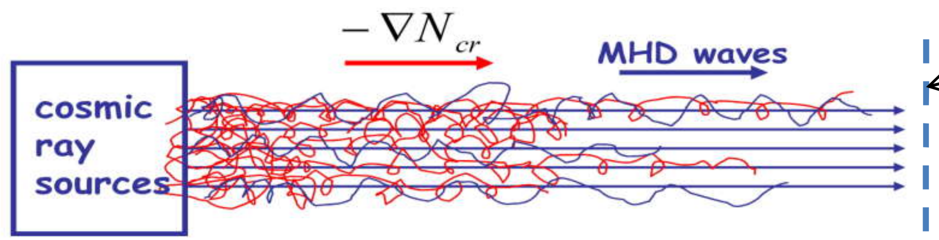
almost purely growing **non-resonant** mode

$$\Gamma_{cr} = \sqrt{\frac{4\pi}{\rho}\frac{e u_{cr} N_{cr}}{c}} \quad \text{at} \quad k_{\max} = k_{res}\frac{4\pi u_{cr} w_{cr}}{c B_0^2} > k_{res}$$

non-linear saturation at $\frac{B^2}{4\pi} \approx \frac{u_{cr}}{c} w_{cr}$

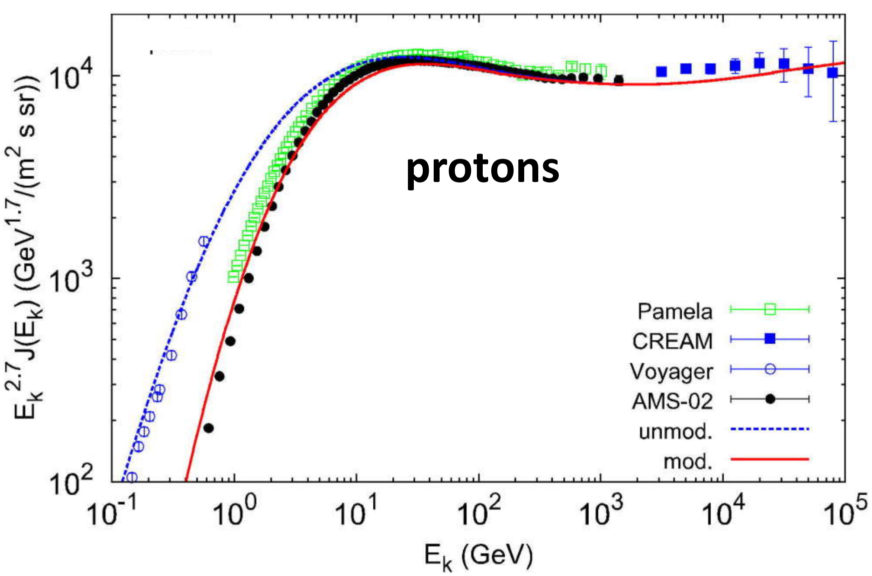
(not always reached because of finite shock age/size !)

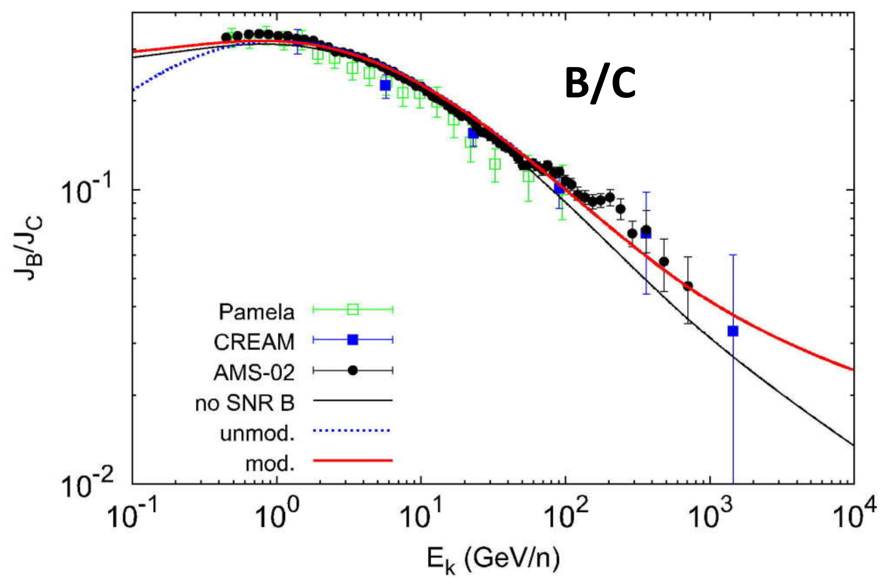
# nonlinear diffusion in the Galaxy




**cosmic ray leakage from the Galaxy is regulated by streaming instability which is substituted by diffusion on background interstellar Kolmogorov - type turbulence above ~300 GeV/n**

**Aloisio et al 2015**

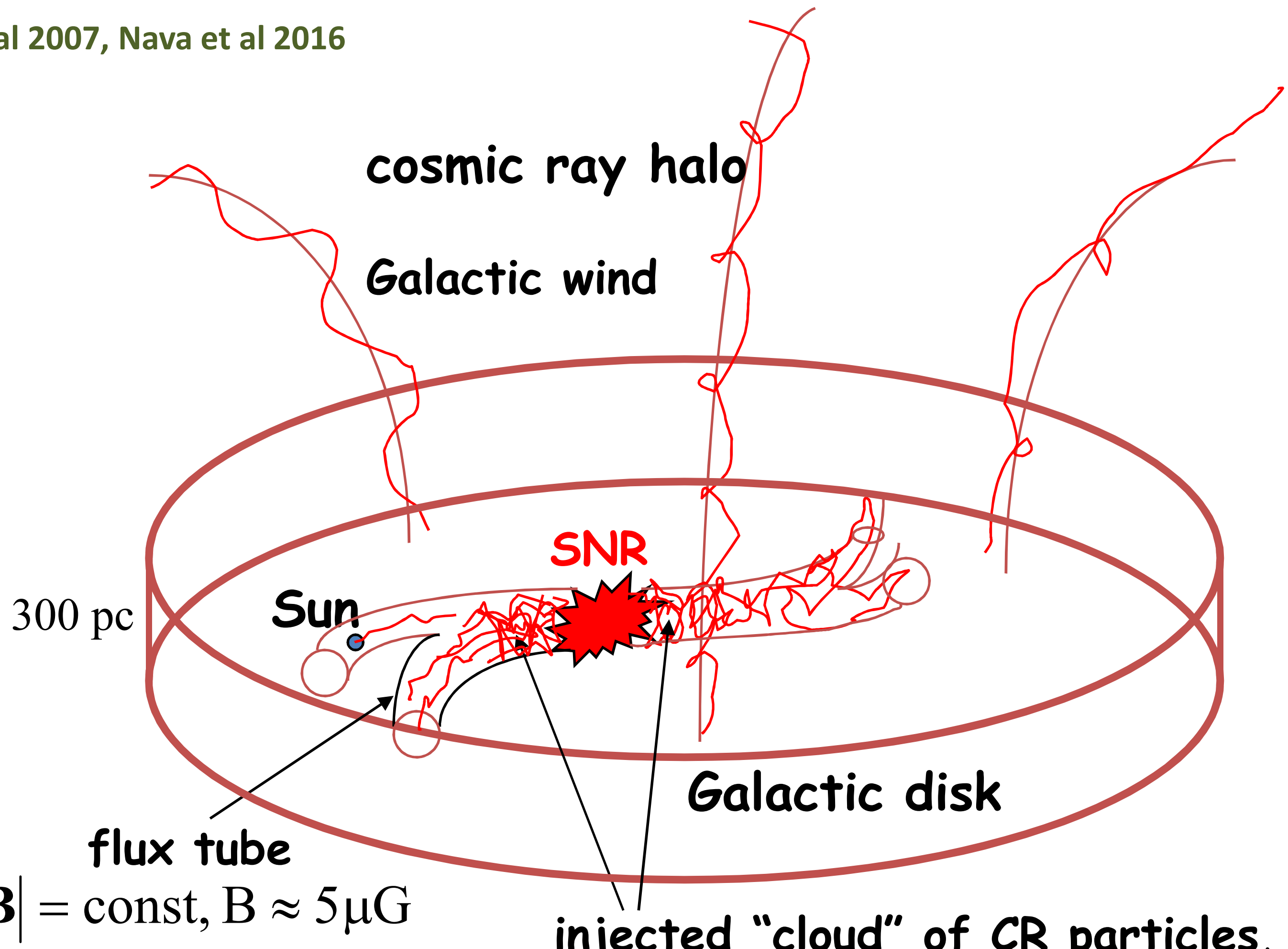



**$X_{SNR}$=0.17 g/cm$^2$ ($T_{SNR}$=2 10$^4$ yr) added**

# non-linear evolution of cosmic ray "cloud" injected from SNR

VP et al 2007, Nava et al 2016




$|\mathbf{B}| = \text{const}, \mathrm{B} \approx 5\mu\mathrm{G}$

$S_{\text{total}} = \text{const}$

$\mathbf{N(p) \sim p^{-2.1}, W_{cr} = 10^{50}\ erg}$

$$\frac{\partial f}{\partial t} - \frac{\partial}{\partial x} D \frac{\partial f}{\partial x} = 0,$$

where $$D = \frac{\kappa}{\left|\frac{\partial f}{\partial x}\right|^{2/3}}, \qquad \kappa = \frac{(\nu r_g)^{1/3} B^{4/3}}{2^{5/3} 3^{1/3} \pi^{7/3} C_K p^{8/3}}$$

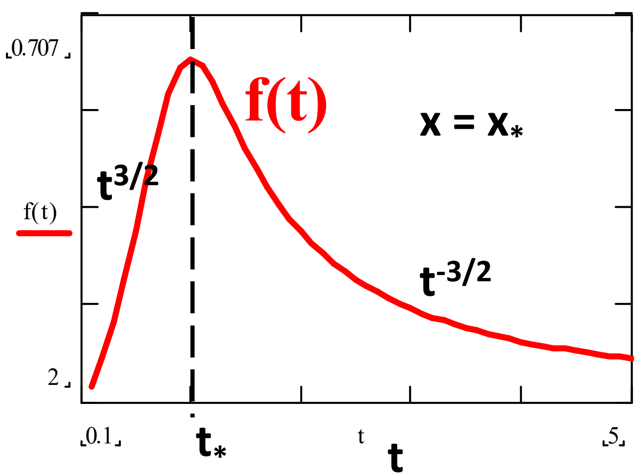
the nonlinear wave dissipation is assumed $$\Gamma_{dis} = (2C_K)^{-3/2} k V_a \sqrt{\frac{kW(k)}{B^2}}, \; C_K \approx 3.6$$

**solution**

$$f(p,x,t) = \frac{1}{4\pi p^2 S} \left( \frac{4\pi^6 \kappa^3 S^2 p^4 t^3}{9N(p)^4} + \frac{9x^4}{2^8 \pi^2 \kappa^3 S^2 p^4 t^3} \right)^{-1/2}$$

$$n_{\rm cr} \equiv 4\pi p^3 \cdot f(p,x,t) =$$

$$= \frac{10^{-10}\,{\rm cm}^{-3}}{\sqrt{1.7\cdot 10^{-3}\left(10^{50}\,{\rm erg}/W_{\rm cr}\right)^4 \beta\, P_{\rm GV}^{1.8}\, t_{\rm Myr}^3 + 3.0\cdot 10^2\, \beta^{-1} P_{\rm GV}\, x_{\rm kpc}^4 / t_{\rm Myr}^3}},$$




$$x_* \approx 79\left(W_{\rm cr}/10^{50}\,{\rm erg}\right) P_{\rm GV}^{0.2}\, t_{*\rm Myr}^{3/2}\ {\rm pc}$$

$$n_{\rm cr*} \approx 7\cdot 10^{-11} \frac{\left(W_{\rm cr}/10^{50}\,{\rm erg}\right)}{P_{\rm GV}^{0.7}\, x_{*\rm kpc}}\ {\rm cm}^{-3}$$

compare to test particle diffusion

$$f \propto \frac{\exp(-x^2/4Dt)}{2(\pi D t)^{1/2}}$$

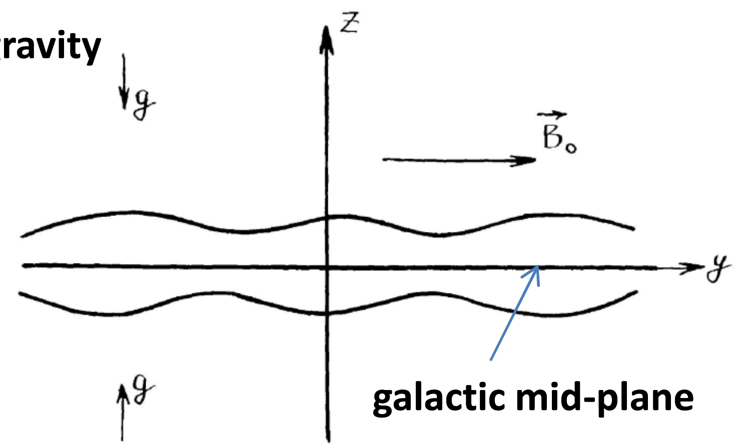
**Application: gamma-rays from nearby dense gas material of W28, IC443**

Aharonian & Atoyan 1996, Gabici et al 2009, Ohira et al 2010

# Parker instability

Parker 1966, 1992et, Kuznetsov & Ptuskin 1983, Hanasz & Lesch 2000, Kuwabara & Ko 2006, Lo et al. 2006, Rodrigues et al. 2015

gravity




galactic mid-plane

equilibrium:

$$\frac{\partial}{\partial z}\left(P_{gas}+P_{cr}+\frac{B_0^2}{8\pi}+\frac{B_{rand}^2}{24\pi}\right)=-\rho(z)g(z),$$

$$P_{gas}\propto\rho^{\gamma}$$

height scale of equilibrium distribution ~ few kpc with non-exponential tail

unstable if the polytropic index $\gamma<\gamma_*$, $\gamma_*=1+\frac{P_{m0}}{P_{gas}}\cdot\frac{0.5P_{gas}+P_{m0}+P_{cr}}{P_g+1.5P_{m0}+P_{m,rand}+P_{cr}}$

instability develops during $10^7$ to $10^8$ yr

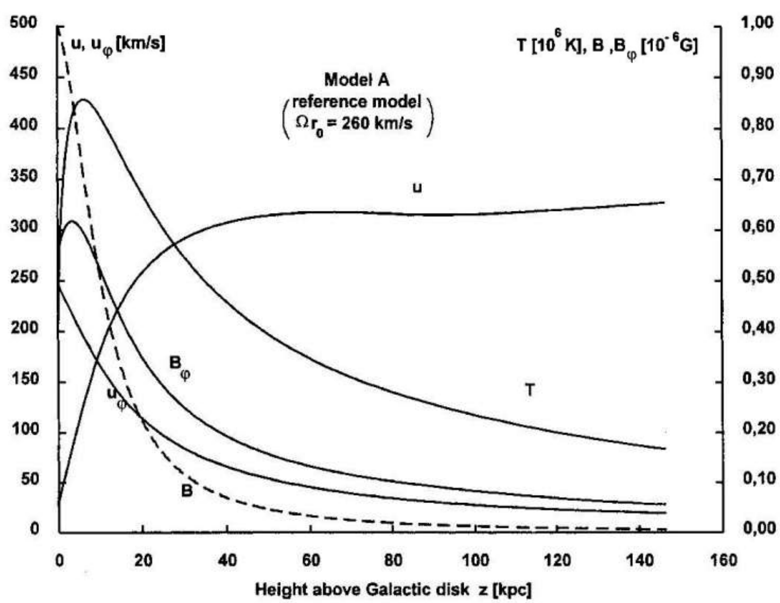
Eq. for cosmic ray energy density:

$$\frac{\partial w_{cr}}{\partial t}-\nabla D\nabla w_{cr}+\mathbf{u}\nabla w_{cr}+(w_{cr}+P_{cr})\nabla\mathbf{u}=S$$

# Galactic wind driven by cosmic rays

Cosmic rays are produced in the galactic disk. Thermal gas is confined by gravity and cosmic rays are not. Cosmic ray scale height is larger then the scale height of thermal gas and cosmic ray pressure gradient drives the wind flow.

Ipavich 1975, Breitschwerdt et al. 1991, 1993, …




The steady state MHD equations have the form

Zirakashvili et al 1996

$$\mathrm{div}\,(\rho \boldsymbol{u}) = 0\,, \qquad (23)$$

$$\rho(\boldsymbol{u}\nabla)\boldsymbol{u} = -\nabla(P_g + P_c) + \rho\nabla\Phi + \frac{1}{4\pi}(\mathrm{rot}\boldsymbol{B}\times\boldsymbol{B})\,, \qquad (24)$$

$$\nabla\left[\rho\boldsymbol{u}\left(\frac{|\boldsymbol{u}|^2}{2} + \frac{\gamma_g}{\gamma_g - 1}\frac{P_g}{\rho} - \Phi\right) + \frac{1}{4\pi}(\boldsymbol{B}\times(\boldsymbol{u}\times\boldsymbol{B}))\right]$$

$$= -\boldsymbol{u}\nabla P_c + H - \Lambda \qquad (25)$$

$$\mathrm{rot}\,(\boldsymbol{u}\times\boldsymbol{B}) = 0\,, \qquad (26)$$

$$\mathrm{div}\,\boldsymbol{B} = 0\,, \qquad (27)$$

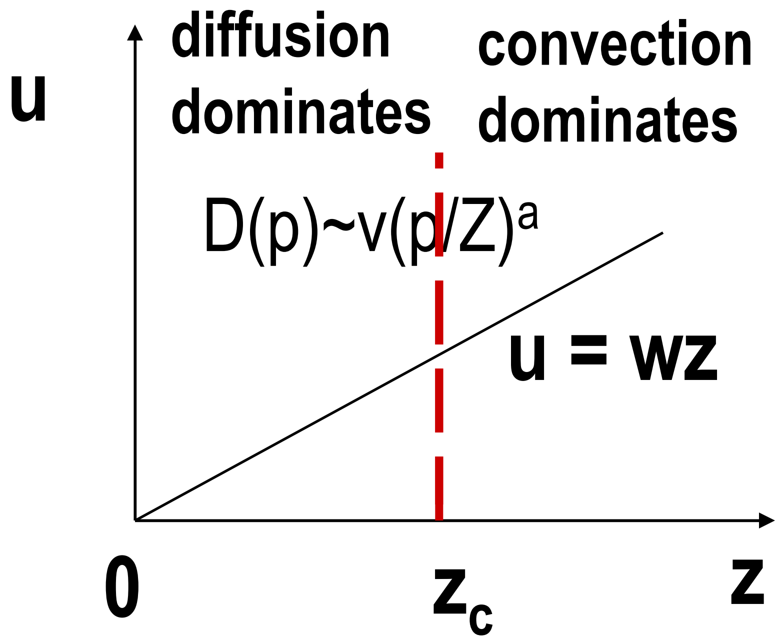
$$\nabla_i\left(\frac{\gamma_c}{\gamma_c - 1}(u_i + V_{\mathrm{ai}})P_c - \frac{D_{ij}\nabla_j P_c}{\gamma_c - 1}\right) = (u_i + V_{ai})\nabla_i P_c\,. \qquad (28)$$

Here $H$, and $\Lambda$, $P_g$, and $\gamma_g$ denote the heating, and energy loss rates of the thermal gas (in the following we shall only consider heating by wave damping), its (thermal) pressure, and its adiabatic index, respectively, whereas $\boldsymbol{V}_a = \boldsymbol{B}/\sqrt{4\pi\rho}$ is the vector of the Alfvén velocity. The term $\boldsymbol{u}\nabla P_c$ in Eq. (25) describes the mechanical work done by the cosmic ray pressure $P_c$ on the volume element of gas. Equation (28) for the cosmic ray pressure contains the cosmic ray diffusion tensor $D_{ij}$.



more on wind model Recchia et al 2017, Uhlig et al 2012, Everett et al 2008, Girichidis et al 2916, Ruszkowski et al 2016

# diffusion – convection transport in galactic halo




effect of boundary layer

**stable nuclei**

$$\frac{z_c}{u(z_c)} \approx \frac{z_c^2}{D(p,z_c)} \quad \Rightarrow \quad z_c(p) = \frac{D(p,z_c)}{u(z_c)}$$

$$z_c(p) = \sqrt{D(p)/w} \quad \text{if } \; u = wz$$

$$X \approx \frac{\mu v z_c(p)}{2D(p)} = \frac{\mu v}{2\sqrt{wD(p)}} \propto v^{1/2} \left(p/Z\right)^{-a/2}$$

$$\Rightarrow \; X \propto p^{-0.5} \;\; \text{requiers} \;\; D \propto p$$

# cosmic ray streaming instability with nonlinear saturation

Zirakashvili et al. 1996, 2002 Ptuskin et al. 1997

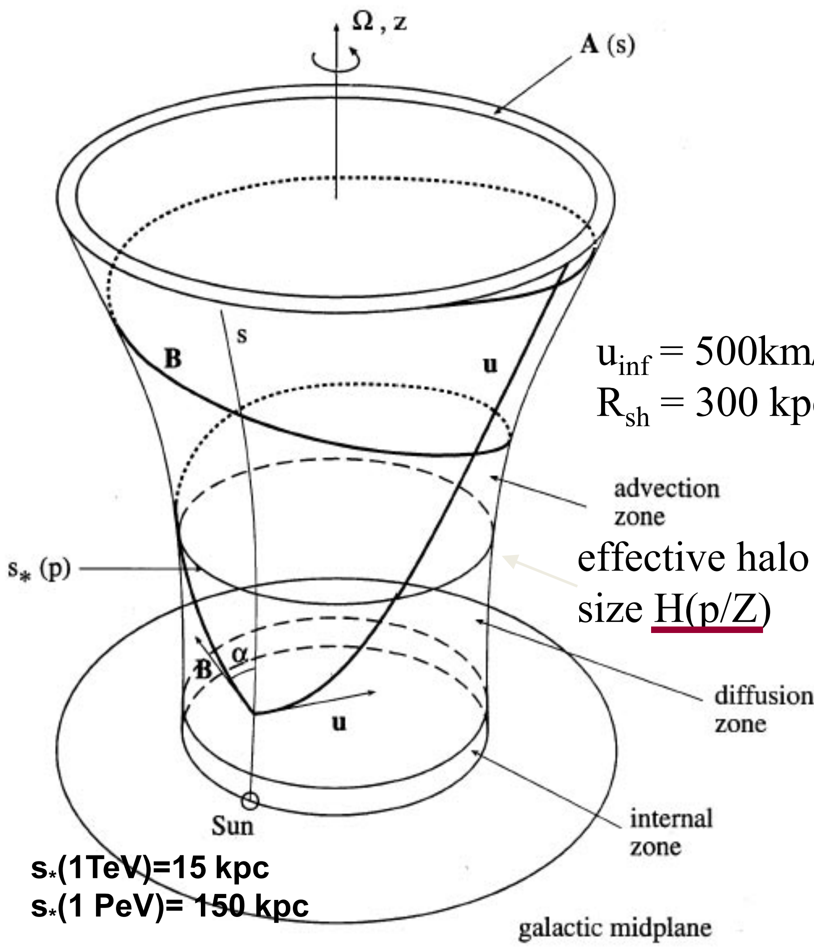



Fig. 1. The structure of the galactic wind flow for a flux tube originating at the position of the Sun. The boundary between the diffusion and advection zones is moving up with energy of the cosmic-ray particle.

$u_{inf} = 500\mathrm{km/s}$
$R_{sh} = 300$ kpc

CR emissivity of Galactic disk per unit area

$$D \approx C \frac{\beta_{th}^{1/2} c^2}{\omega_{Bi}} \frac{B^2}{\varepsilon_{cr}} \left(\frac{p}{Z m_p c}\right)^{\gamma_s - 1} \approx 10^{27} \left(\frac{p}{Z m_p c}\right)^{1.1} \mathrm{cm}^2/s,$$

$$\gamma = (3\gamma_s - 1)/2 \approx 2.7, \qquad \gamma_s = 2.1$$

## stable secondaries:

$$X \propto \beta R^{-(\gamma_s - 1)/2}$$

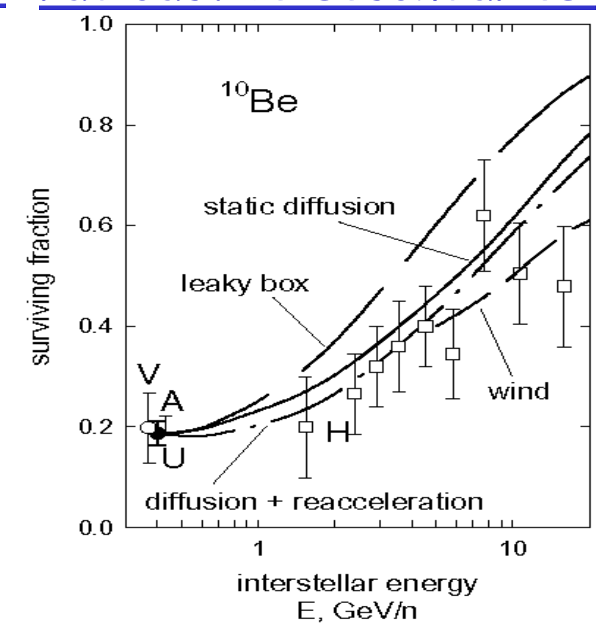
$$= \beta R^{-0.55}$$

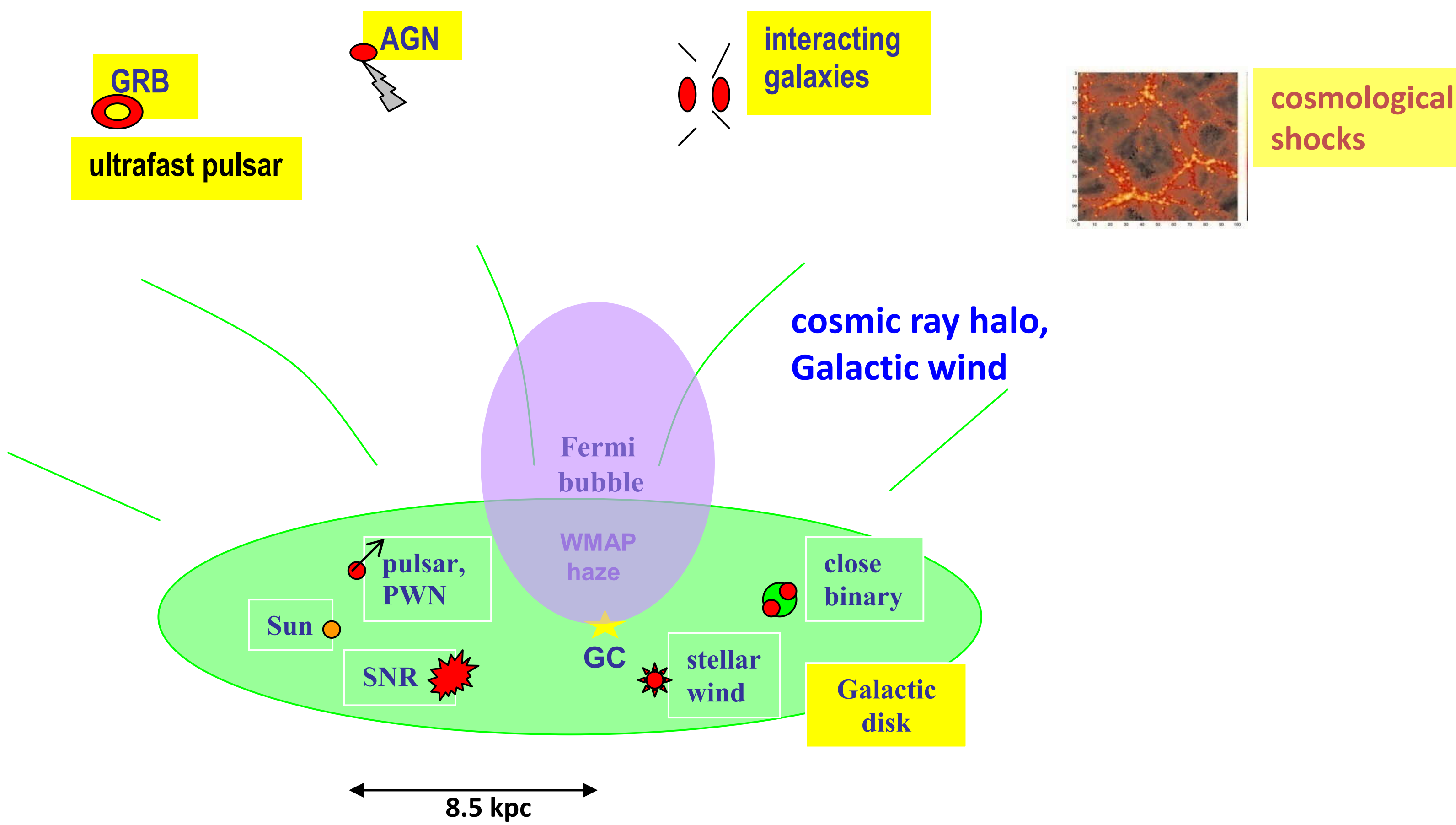
$$\text{at } E > 1\,\mathrm{GeV/n},$$

$$\propto \beta\mu c / V_a$$

at low energies.

## radioactive secondaries:






$N_{cr} \sim 10^{-10}$ cm$^{-3}$ - number density in the Galaxy

$w_{cr} \sim 1.5$ eV/cm$^3$ - energy density

$L_{cr} \sim 10^{41}$ erg/s - total power of galactic sources

$E_{max} \sim 3\times10^{20}$ eV - max. detected energy

$A_1 \sim 10^{-3}$ – dipole anisotropy at 1 – 100 TeV

$r_g \sim 1\times E/(Z\times3\times10^{15}$ eV) pc - Larmor radius at $B=3\times10^{-6}$ G