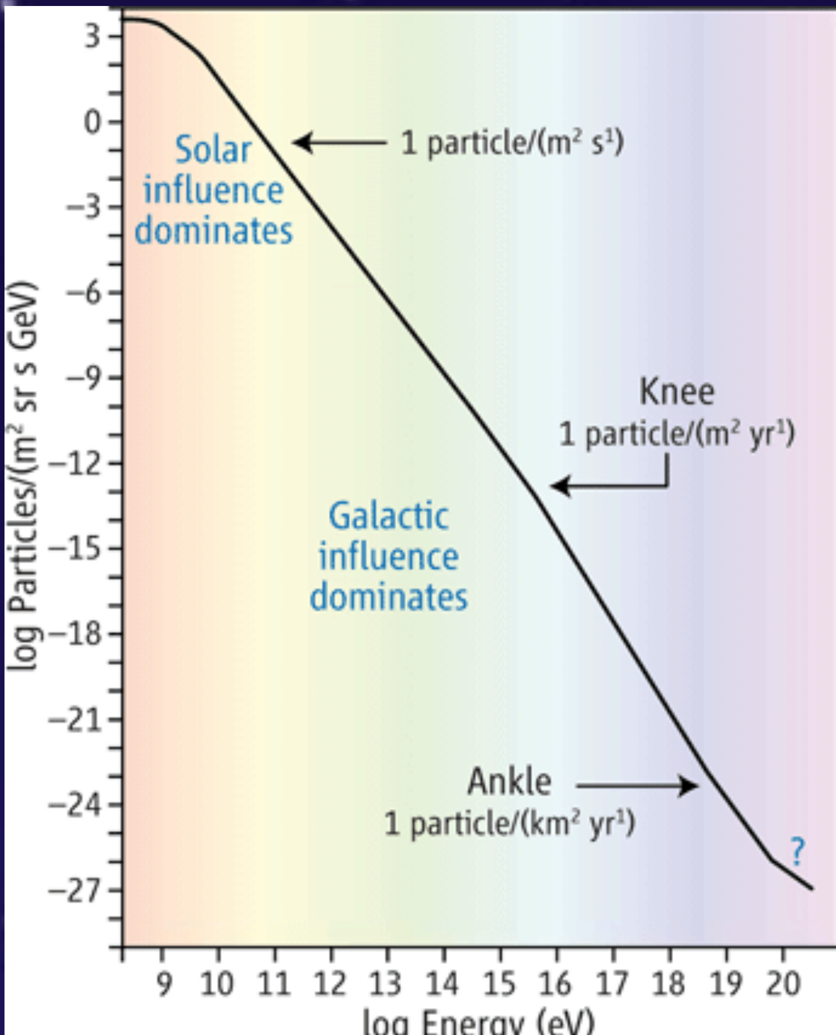
Particle transport in turbulence

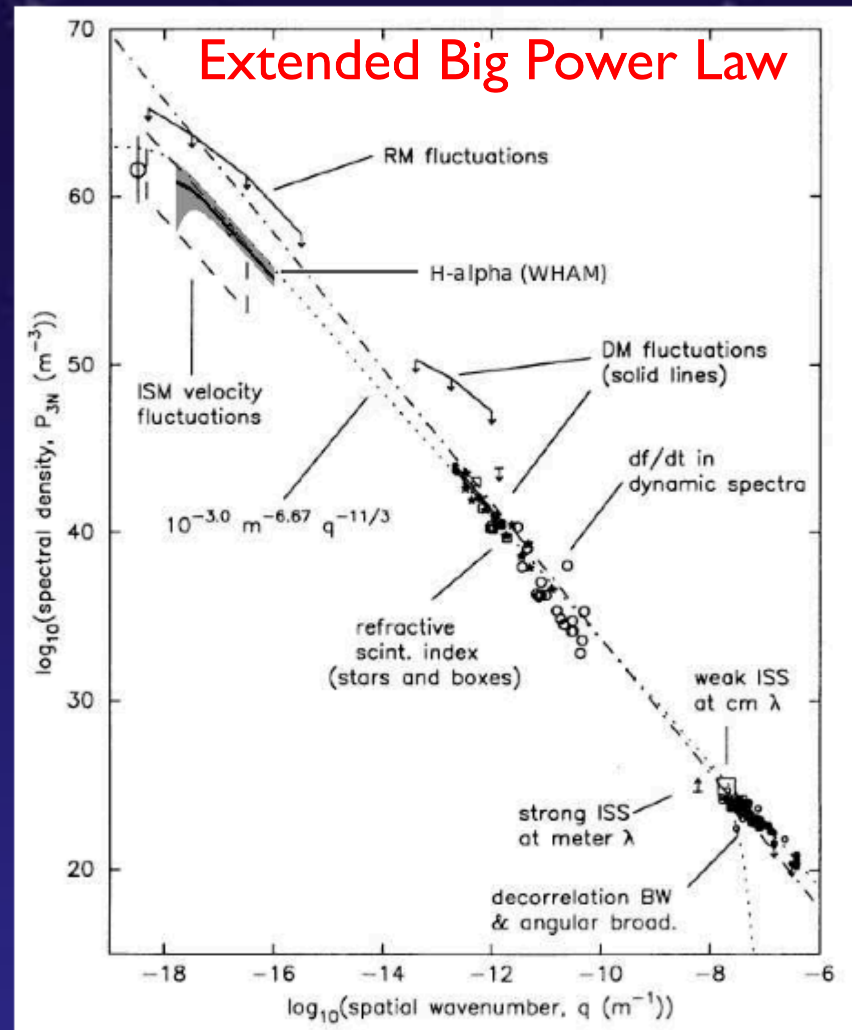
Huirong Yan

DESY & Uni Potsdam

Cosmic Rays and Turbulence



M. Duldig 2006



Armstrong et al. 1995, Chepurnov & Lazarian 2009

figure from Armstrong et al. (1995). The range of statistical errors is marked with the gray color.

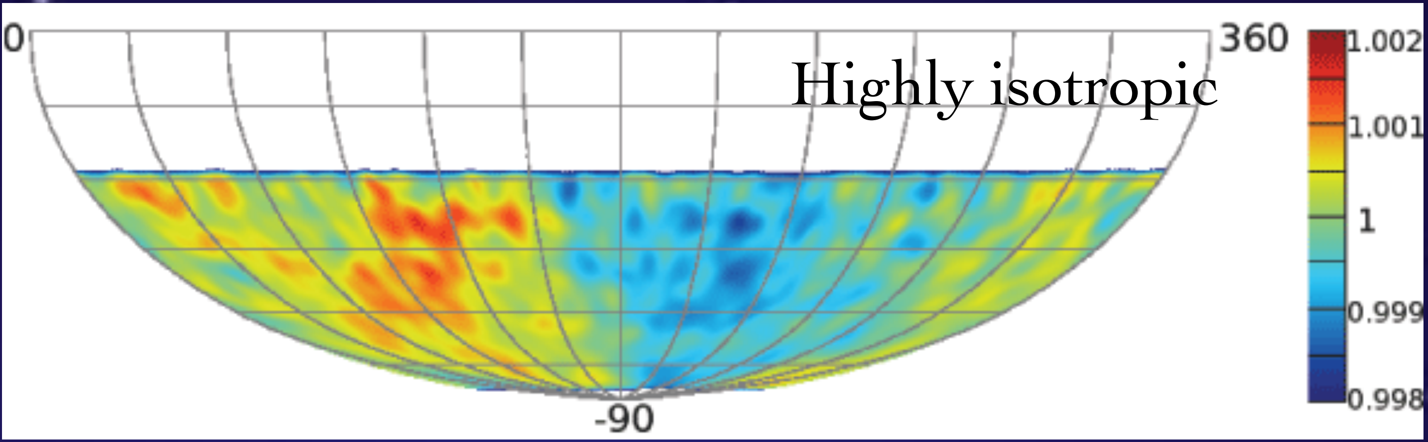
Basic numbers

- Flux of particles: $1/cm^2$ sec
- Isotropy: 10^{-4}
- Energy: 10^9eV-10^{20}eV
- Composition: H to Uranium
- Age: (from Li, Be, B)-$5g/cm^2$

$\Longrightarrow T \sim 3 \times 10^6$yrs

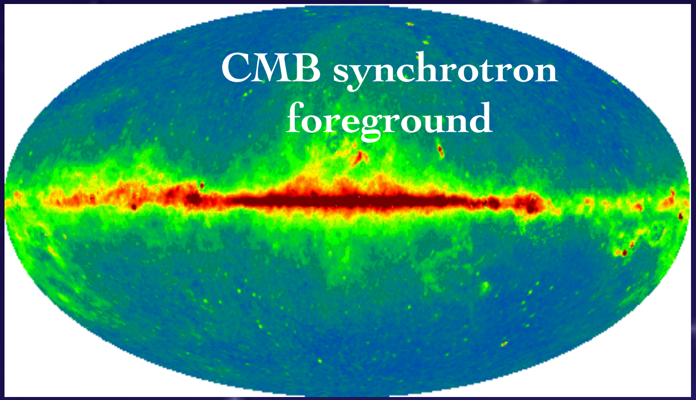
$\Longrightarrow E_{cr} \sim E_B \sim E_{th} \sim 1eV/cm^3$

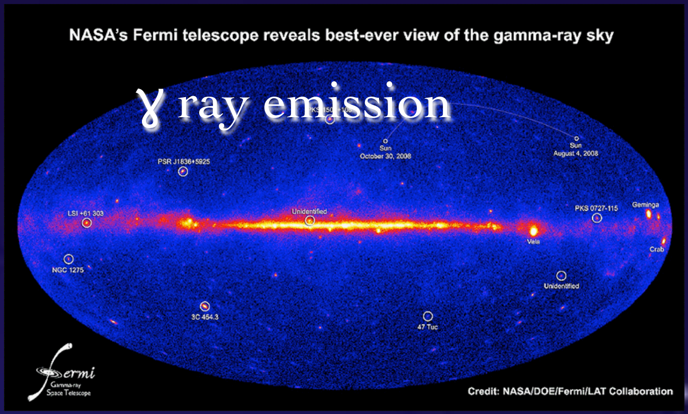
Pinpointing direct source is impossible!

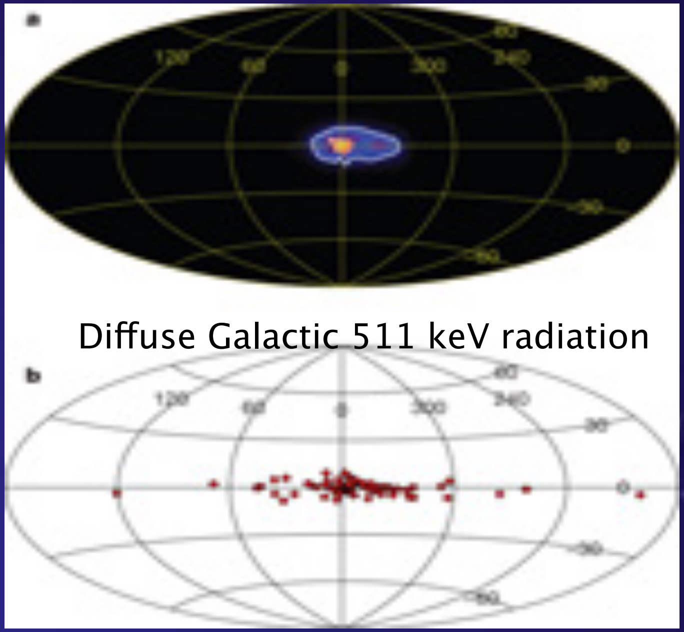


Icecube measurement

Importance I: Cosmic Ray (CR)Propagation

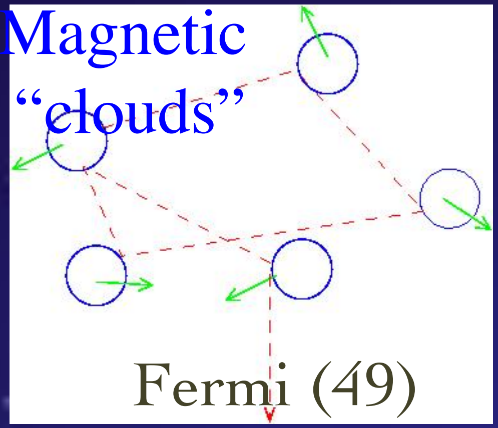


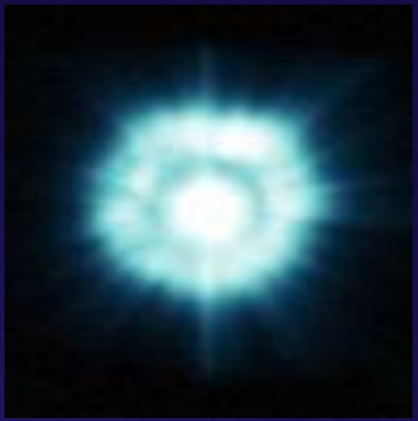




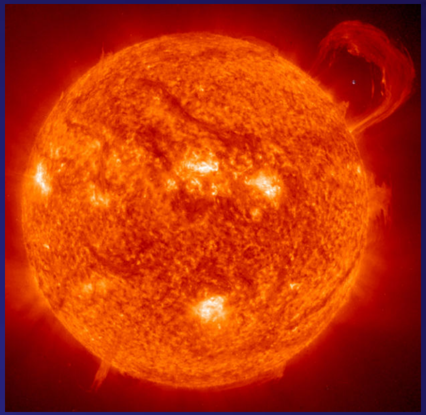
Importance of wave-particle interaction: Fermi II

Stochastic Acceleration:





Gamma ray burst



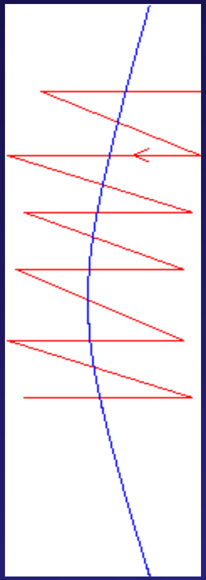
Solar Flare

Importance to Fermi I acceleration

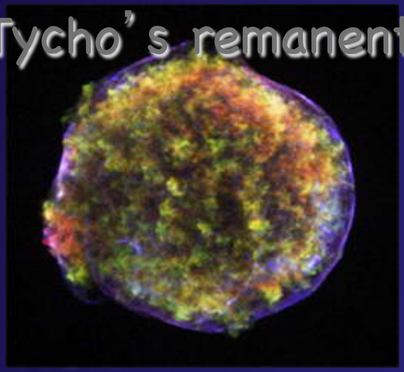
- Shock Acceleration

Shock front

Turbulence generated by streaming

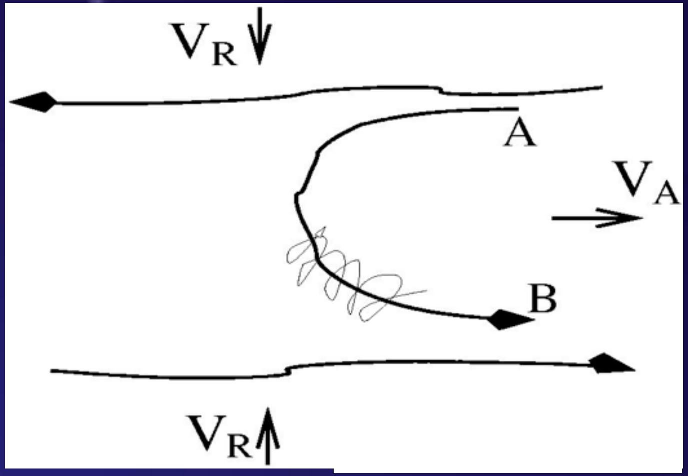


Turbulence generated by shock



Tycho's remanent

- Reconnection Acceleration



De Gouveia Dal Pino & Lazarian (2003)

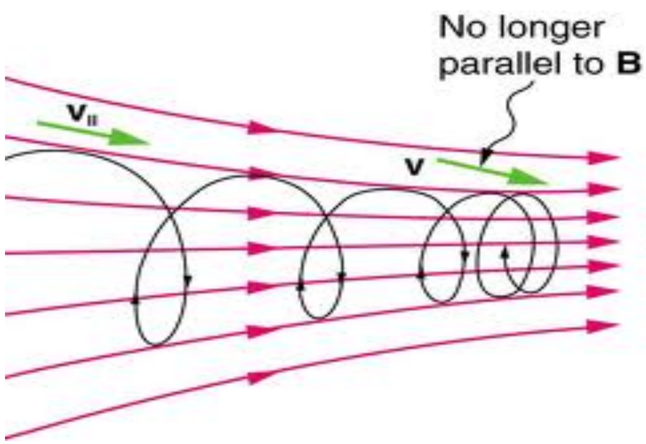
1st adiabatic invariant

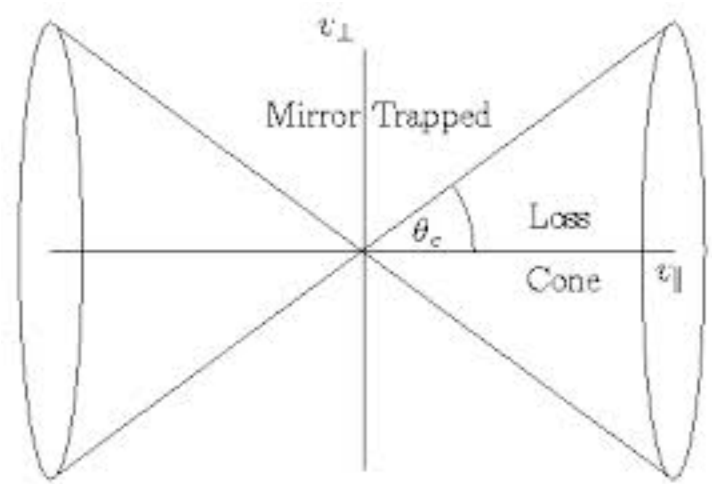
- magnetic moment $\vec{\mu} \equiv -\frac{W_\perp}{2B}\hat{b}$

(assignment: please use the force exerted by the $\nabla B_\parallel$ to prove)

Requirement: $|\frac{T_c}{B} \cdot \frac{\partial B}{\partial t}| \ll 1, |R_c \cdot \frac{\nabla B}{B}| \ll 1$

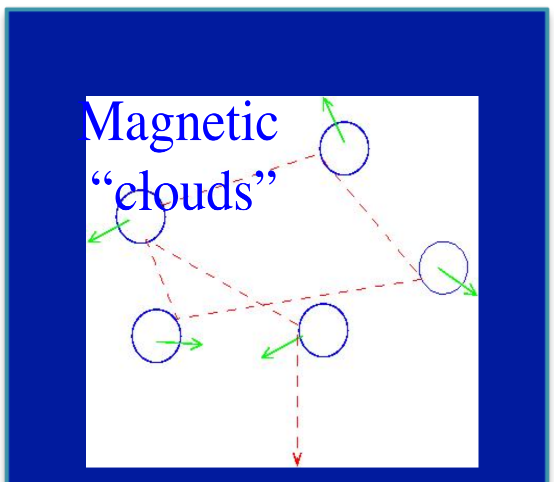
Application: magnetic mirror, $\sin^2 \theta_C = B/B_{max}$



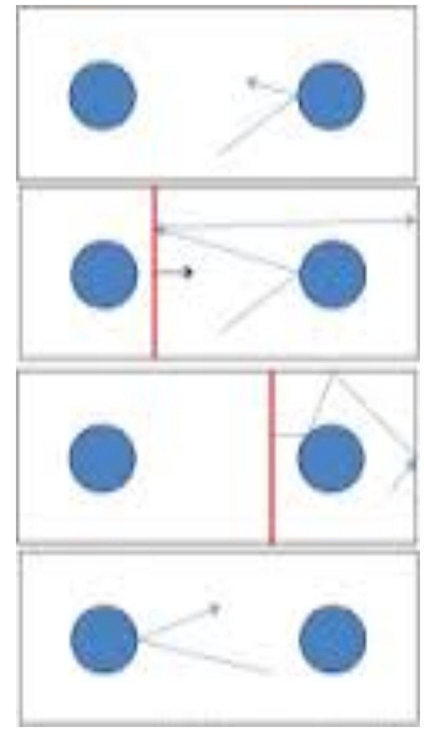


2^{nd} adiabatic invariant & Fermi acceleration

$$I = \oint p_{\|}\, dl \approx p_{\|}(t) 4l(t) = \text{constant}\ .$$



Fermi (1949)



Requirement: $$|\frac{T_l}{B} \cdot \frac{\partial B}{\partial t}| \ll 1$$

Much more stringent than the condition for the 1st adiabatic invariant!

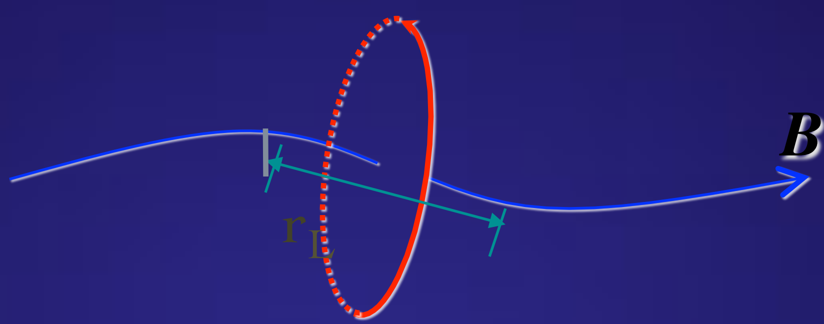
Resonance mechanism

<u>Gyroresonance</u>

$$\omega - k_{||}v_{||} = n\Omega, \ (n = \pm 1, \pm 2 \ldots),$$

Which states that the MHD wave frequency (Doppler shifted) is a multiple of gyrofrequency of particles ($v_{||}$ is particle speed parallel to **B**).

So, $k_{||,res} \sim \Omega/v = 1/r_L$



Transit Time Damping (TTD)

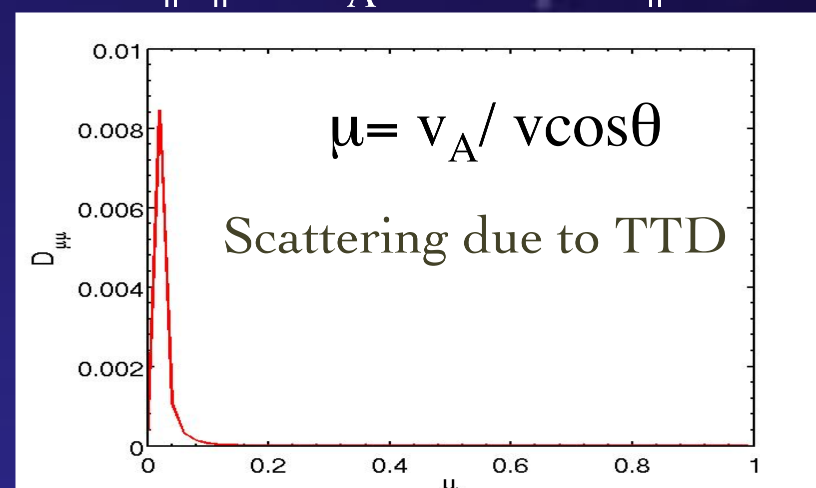
Transit time damping (TTD)

Compressibility of B field required!

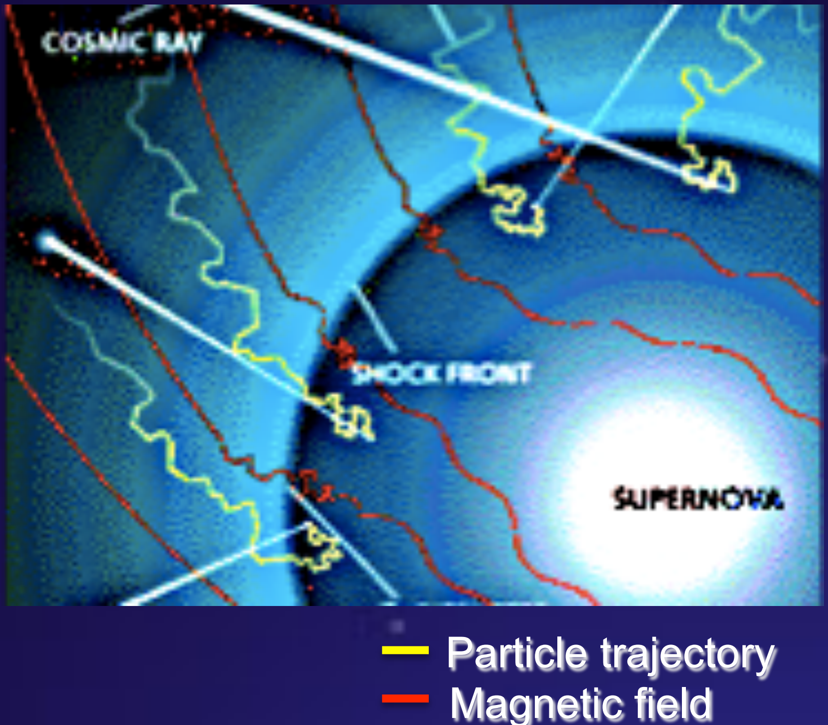
no resonant scale
All scales contribute

Landau resonance condition:

$\omega \approx k_{\parallel} v_{\parallel} \Rightarrow v_A = \omega/k \approx v_{\parallel} \cos\theta$



Long-standing problems of CR research



Before reaching the detector, CRs experience complicated propagation, determined by the interactions with the *magnetohydrodynamic (MHD) turbulence.*

- Ad hoc turbulence models
- Inadequate description (QLT) of the interactions between MHD perturbations and particles
- Perpendicular CR transport

Big simulation itself is not adequate

- big numerical simulations fit results due to the existence of "knobs" of free parameters (see, e.g., http://galprop.stanford.edu/).
- Self-consistent picture can be only achieved on the basis of theory with solid theoretical foundations and numerically tested.

Outline

a. Particle Scattering in tested model of MHD turbulence

b. Cross field transport in MHD turbulence

c. Instabilities and collisionless plasma

d. Turbulent reconnection model for γ ray burst (GRBs)

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Models of MHD turbulence

Earlier turbulence models

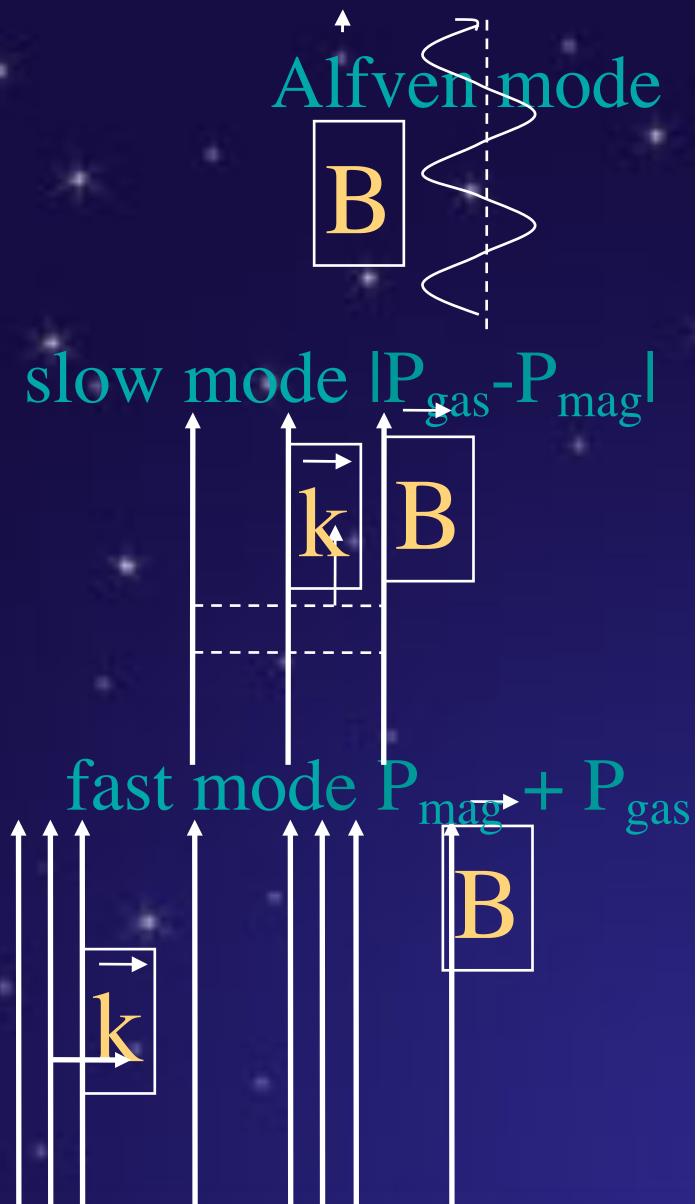
Slab model: Only MHD modes propagating along the magnetic field are counted.

Kolmogorov turbulence: isotropic, with 1D spectrum $E(k) \sim k^{-5/3}$

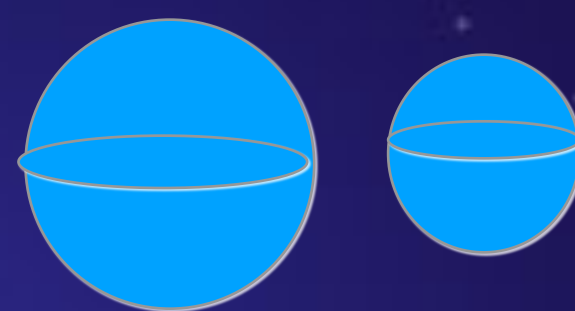
Tested models of MHD turbulence

1. Alfven and slow modes: Goldreich-Sridhar 95 scaling
2. Fast modes: isotropic, similar to acoustic turbulence

Contributions from turbulence can be separated



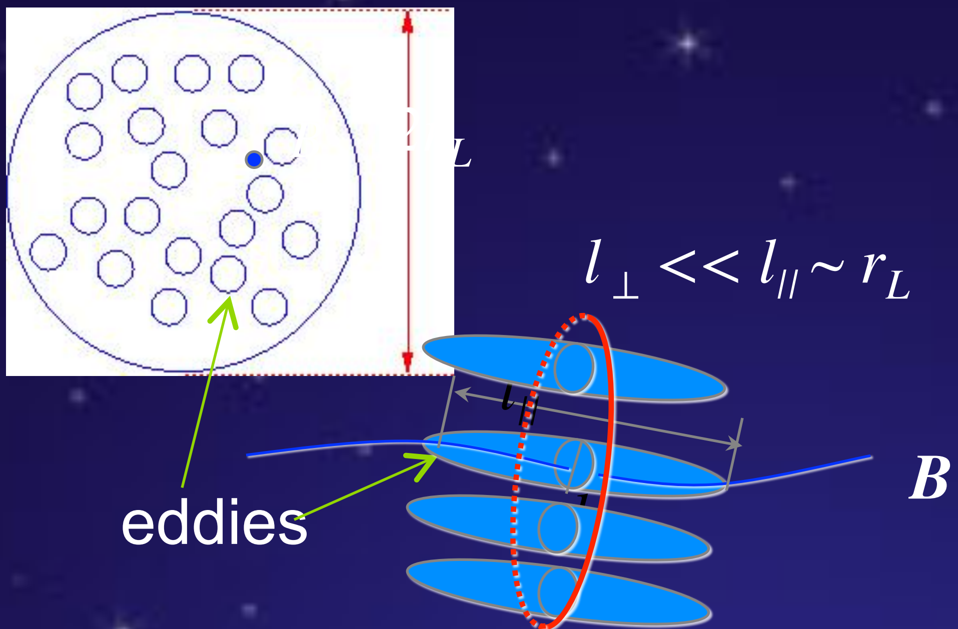
Goldreich & Sridhar 1995;
Lithwick & Goldreich 01



Cho & Lazarian 02

Contrary to common belief: Scattering in Alfvenic turbulence is negligible!

1. "random walk"



scattering efficiency is reduced

2. "steep spectrum"

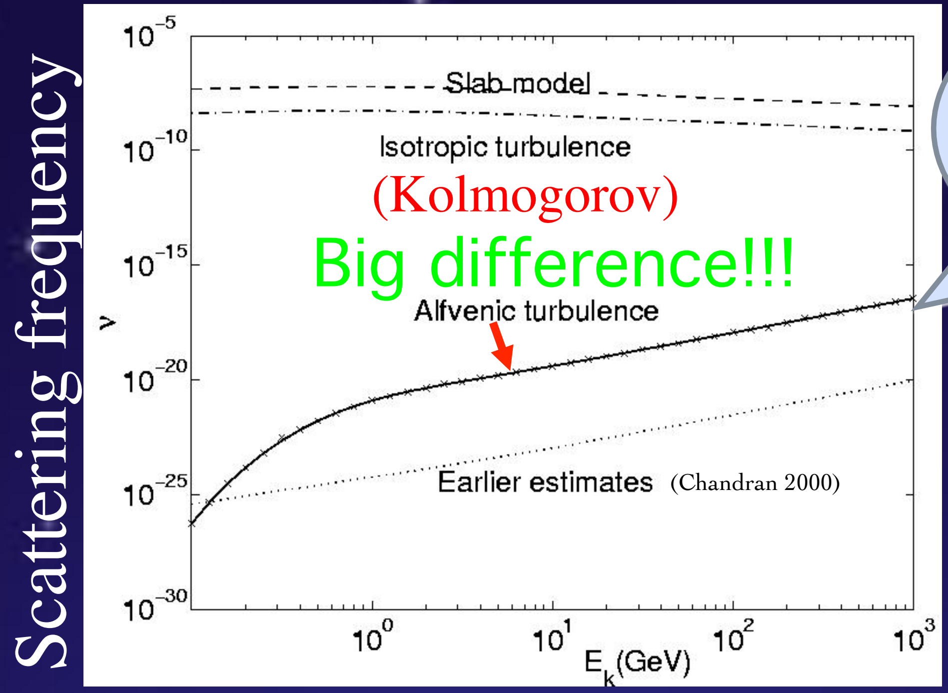
$$E(k_\perp) \sim k_\perp^{-5/3},\ k_\perp \sim L^{1/3} k_{||}^{3/2}$$

$$\Rightarrow E(k_{||}) \sim k_{||}^{-2}$$

steeper than Kolmogorov!
Less energy on resonant scale

Alvenic turbuelence cannot scatter cosmic rays!

Alfven modes



Kinetic energy

?
Remarkable isotropy δ~6x10^{-4} and long age 10^{7} yrs

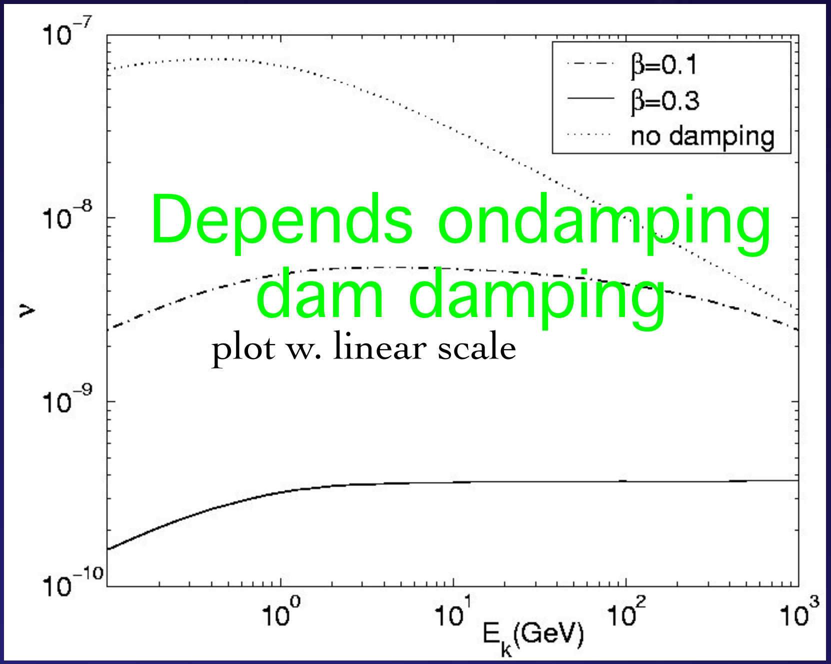
Total path length is ~ 10^{4} crossings at GeV from the primary to secondary ratio.

The often adopted Alfven modes are useless. Alternative solution is needed for CR scattering (Yan & Lazarian 02,04)?

FAST MODES ARE DOMINANT!

fast modes

Scattering frequency



Kinetic energy

Fast modes are identified as the dominate source for CR scattering (Yan & Lazarian 2002, 2004)!

DAMPING OF FAST WAVES

Increase with plasma $\beta = P_{gas}/P_{mag}$ and the angle θ between **k** and **B.**

Viscous damping (Braginskii 1965)

$$\Gamma_{\rm ion} = \begin{cases} k_\perp^2 \eta_0 / 6\rho_i, & \beta \ll 1, \\ k^2 \eta_0 (1 - 3\cos^2\theta)/6\rho_i, & \beta \gg 1. \end{cases}$$

Collisionless damping (Ginzburg 1961, Foote & Kulsrud 1979)

$\beta \ll 1$

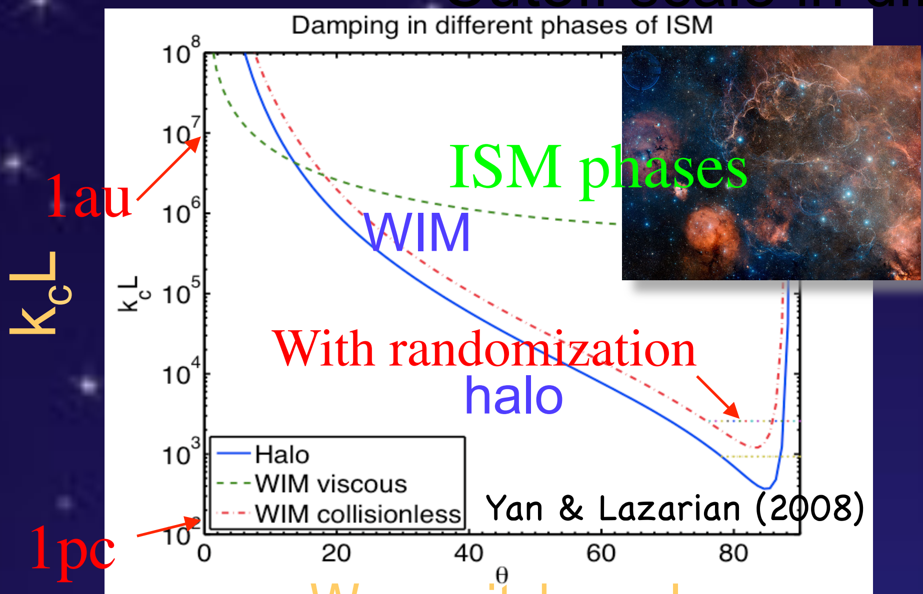
$$\Gamma_{\rm L} = \frac{\sqrt{\pi\beta}}{4}\omega\frac{\sin^2\theta}{\cos\theta}\left[\sqrt{\frac{m_e}{m_{\rm H}}}\exp\left(-\frac{m_e}{m_{\rm H}\beta\cos^2\theta}\right) + 5\exp\left(-\frac{1}{\beta\cos^2\theta}\right)\right],$$

$\beta \gg 1$

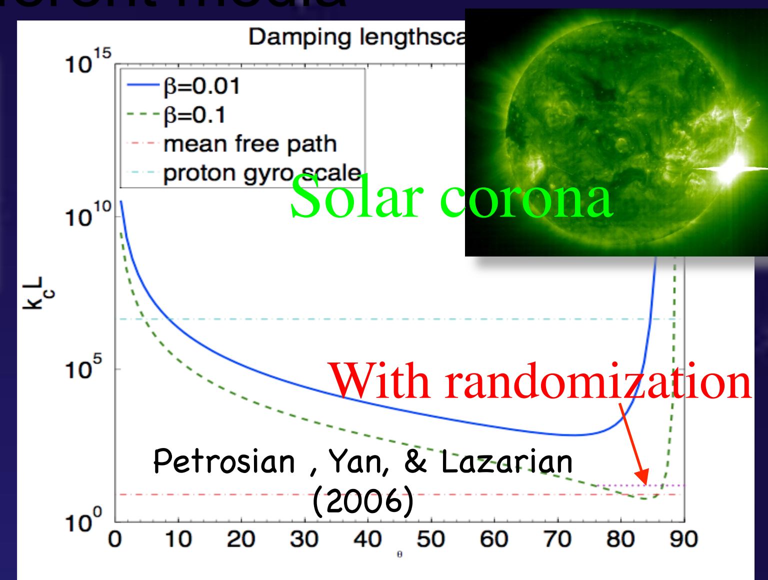
$$\Gamma_{\rm L} = \frac{\sin^2\theta}{\cos^3\theta}\begin{cases} 2\omega^2/\Omega_i, & k < \Omega_i/\beta V_{\rm A}, \\ 2\Omega_i/\beta, & k > \Omega_i/\beta V_{\rm A}, \end{cases}$$

Anisotropy of fast modes arising from damping

Cutoff scale in different media



Wave pitch angle



Wave pitch angle

- Damping depends on medium.
- Anisotropic damping results in quasi-slab geometry.
- Field line wandering should be accounted for.

Quasilinear theory is not adequate

- Long standing problem: 90 degree scattering
  $K_{res} = \Omega/v_{||} \rightarrow \infty$, the scale is below the dissipation scale of turbulence ⇒ No scattering at 90^o? ⇒ $\lambda_{||} \rightarrow \infty$?!

A key assumption in Quasilinear theory:

guiding center is unperturbed $Z_0 = v\mu t$;

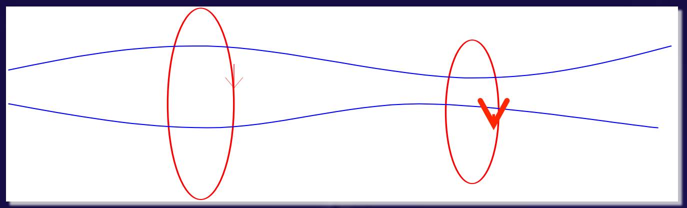
⇒

Nonlinear theory:

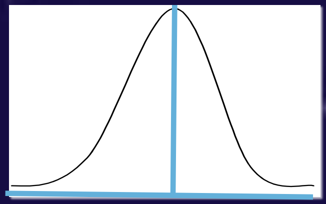
In reality, the guiding center is perturbed, especially on large scales,

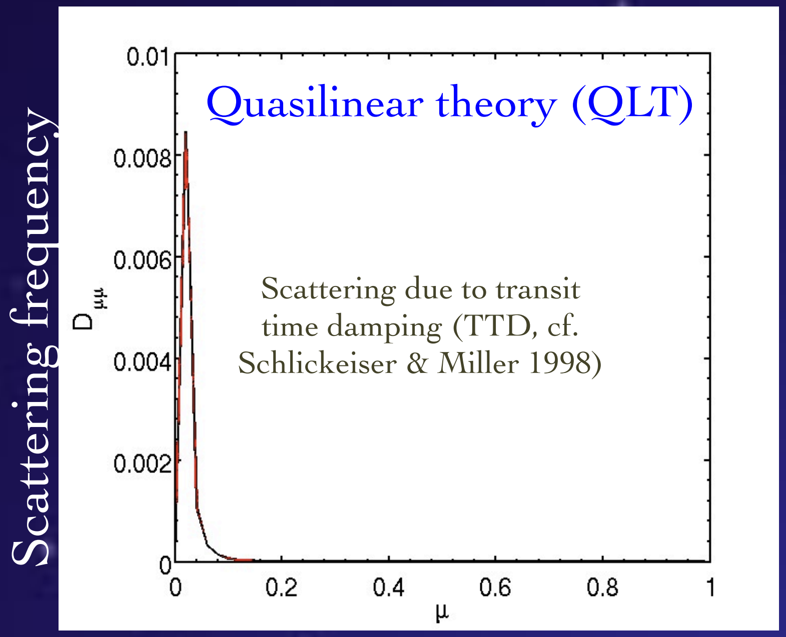
$z = (v\mu \pm \Delta v_{||})t.$

Nonlinear theory (NLT) solves the 90° problem!



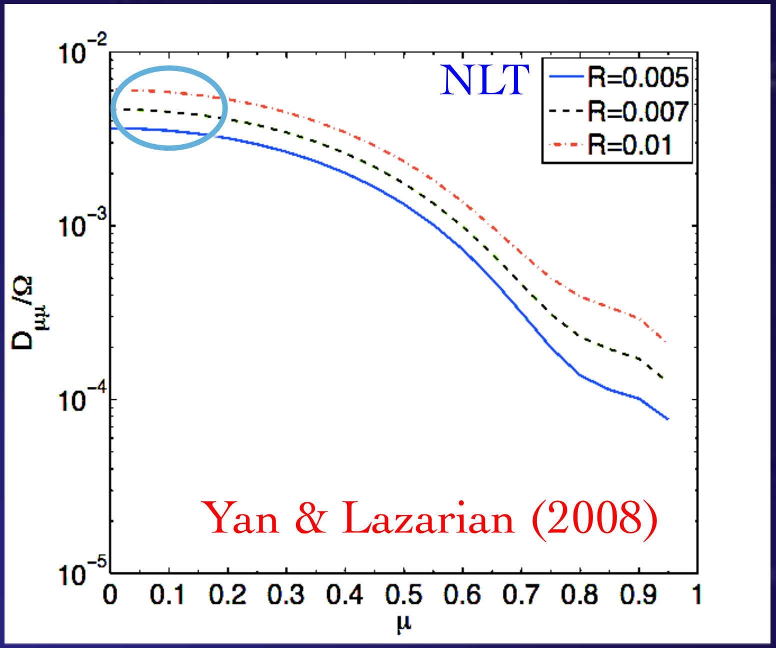
varying $v_\perp$ → varying $v_\parallel$





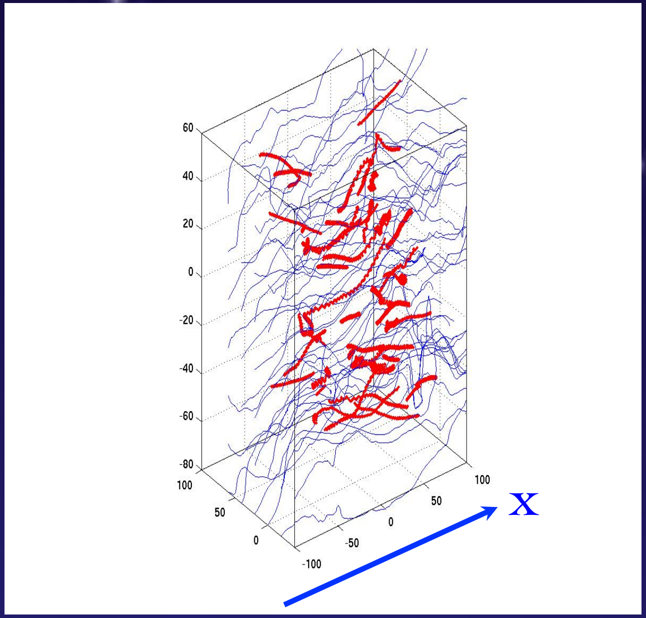
Pitch angle cosine

Broadened resonance



Pitch angle cosine

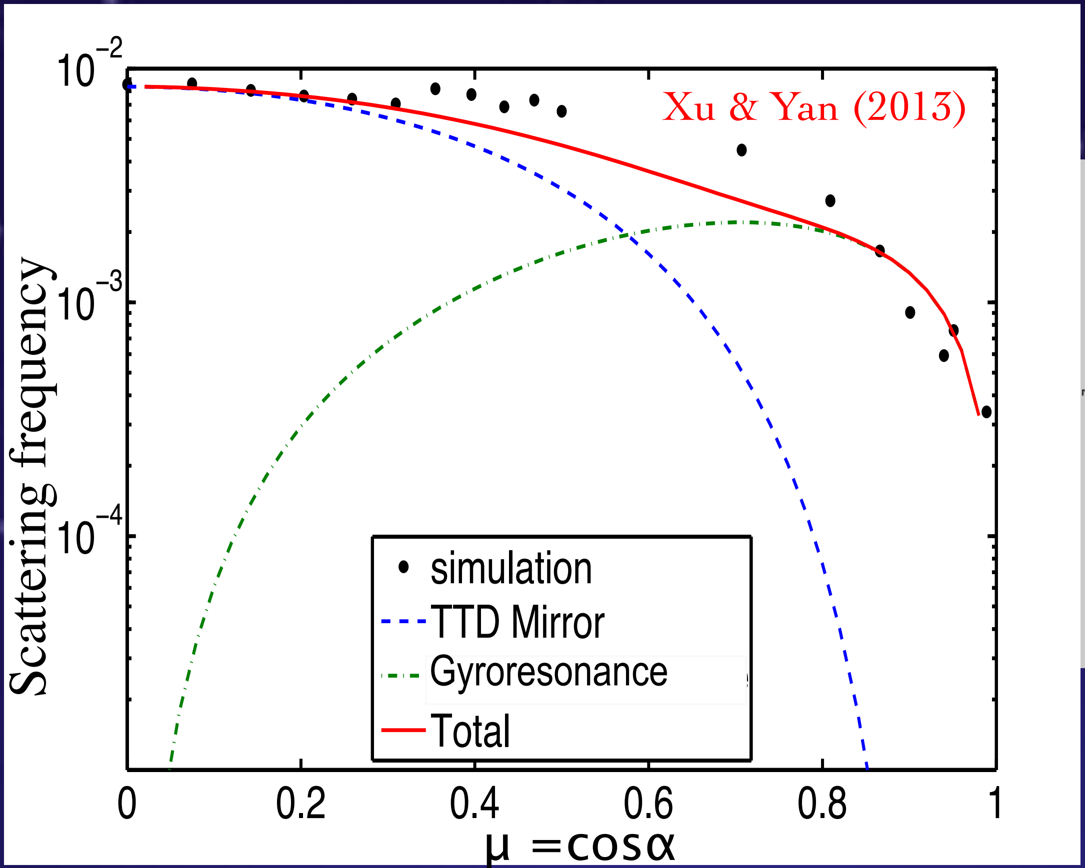
Comparison w. test particle simulation

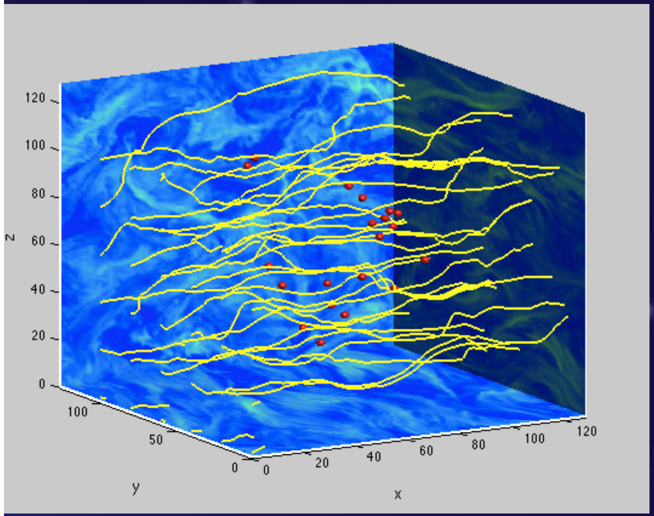


– Particle trajectory
— Magnetic field

- a realistic fluctuatating **B** fields from numerical simulations

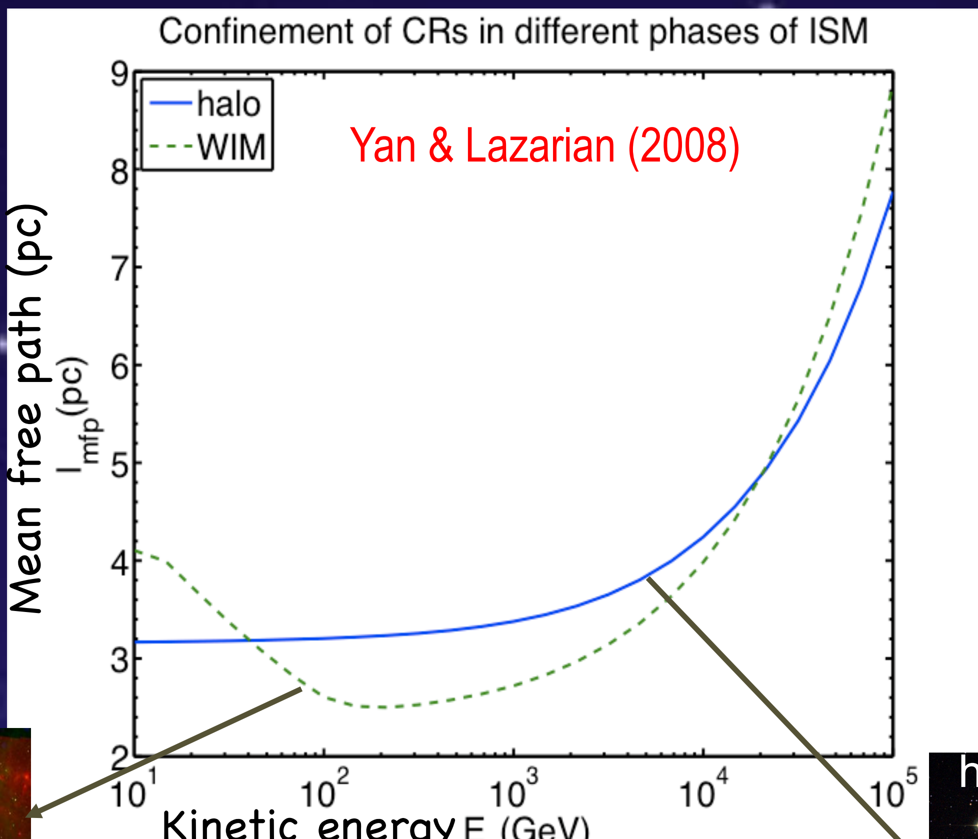
Prediction from NLT is confirmed by simulations





Mirror interaction dominates scattering at large pitch angles α *(including 90°)*, and gyroresonance with fast modes is dominant for small ones.

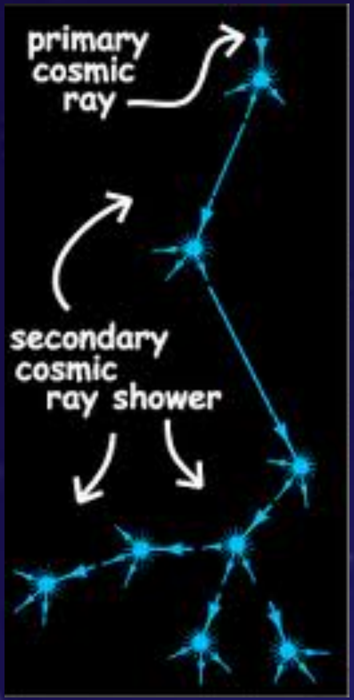
Major Implication: CR Transport varies from place to place!

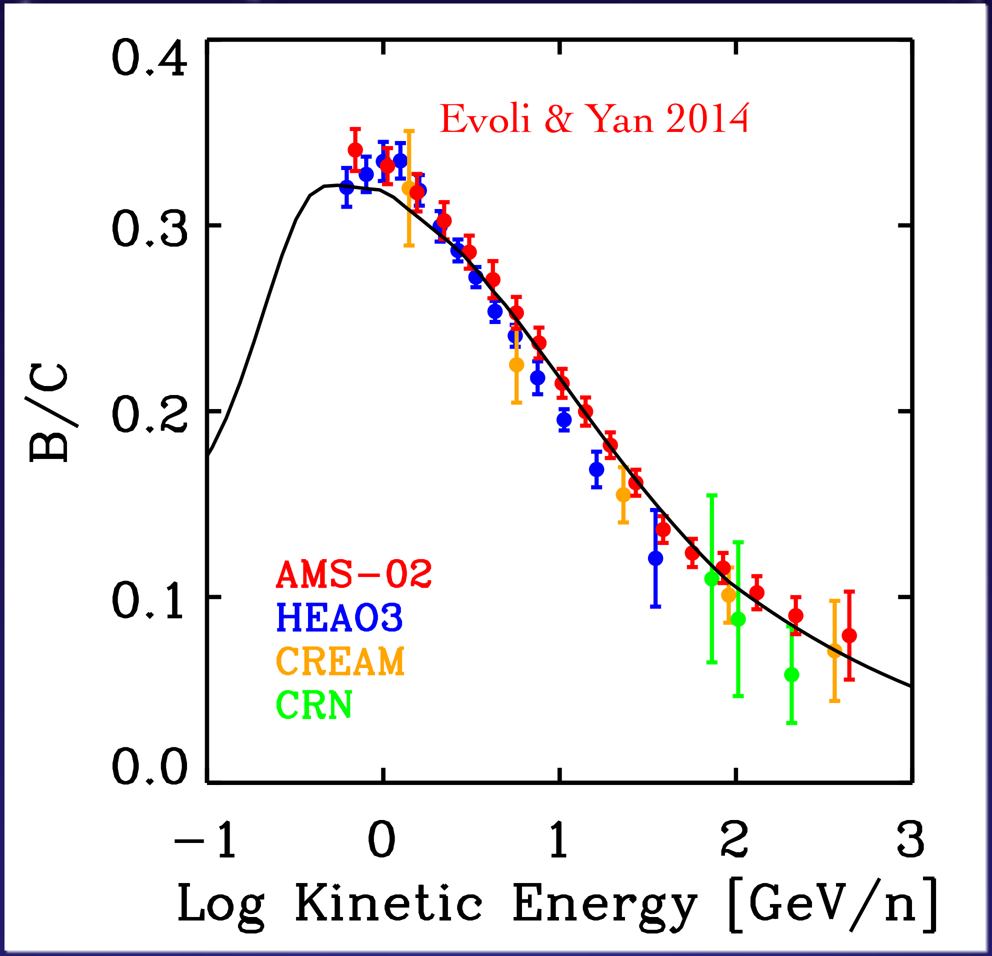


Observational support on nonuniform propagation of CRs (AMS 2010; Fermi-LAT 2011,2012; PAMELA 2011):

Cosmic ray spectrum;
Low energy positron excess;
Anisotropic distribution;
Diffuse Y ray emission

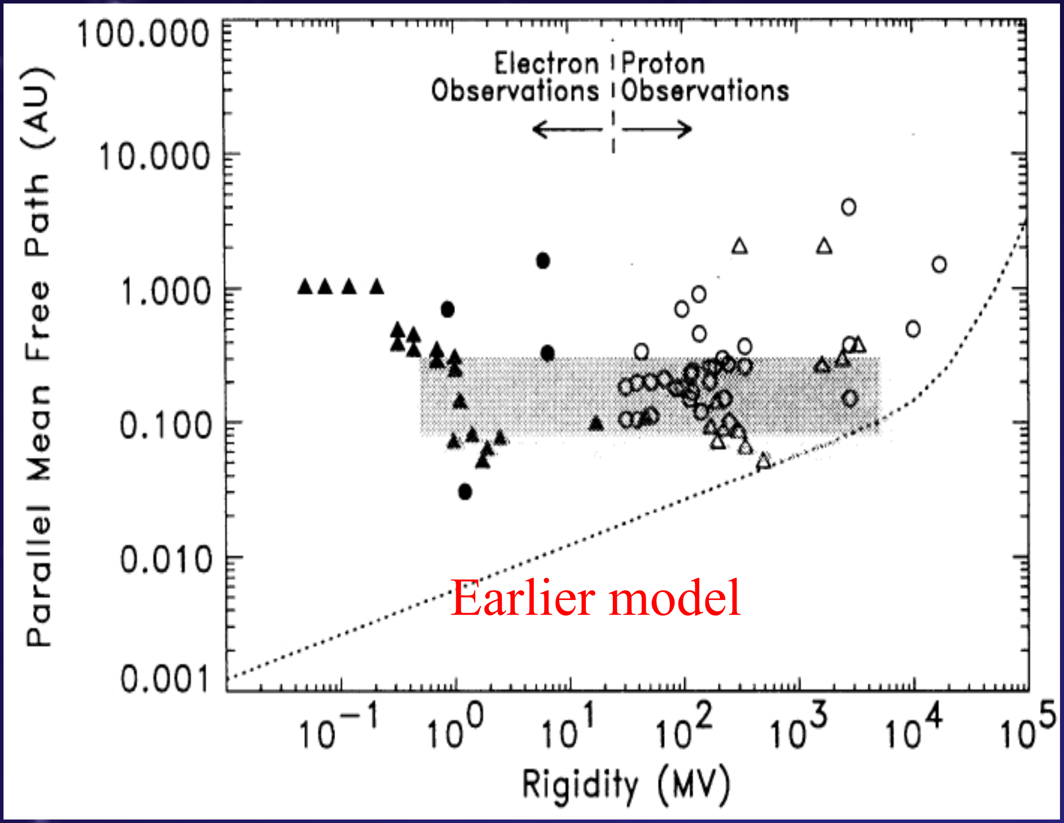
Ex. of implications: B/C ratio

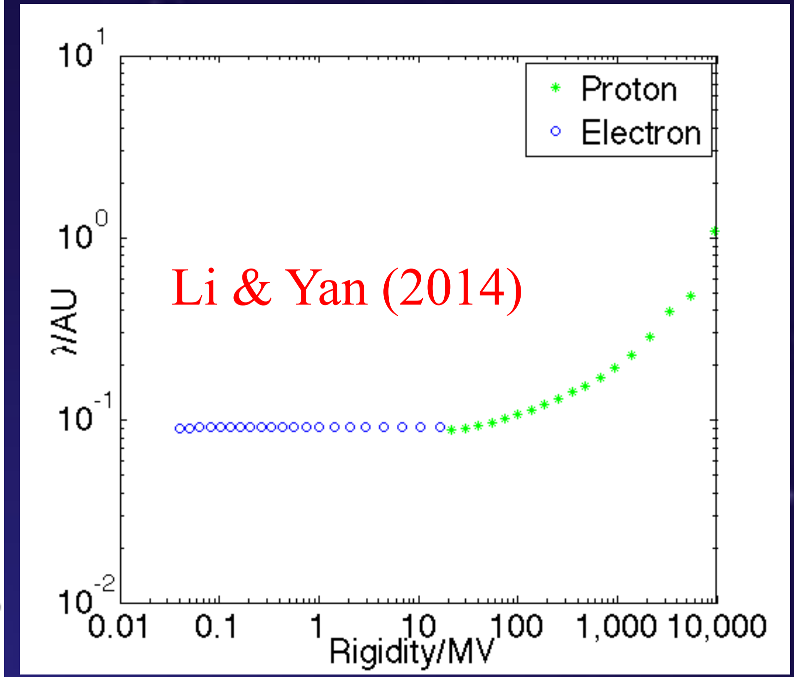




1GeV peak of B/C ratio can be produced without introducing the reacceleration!

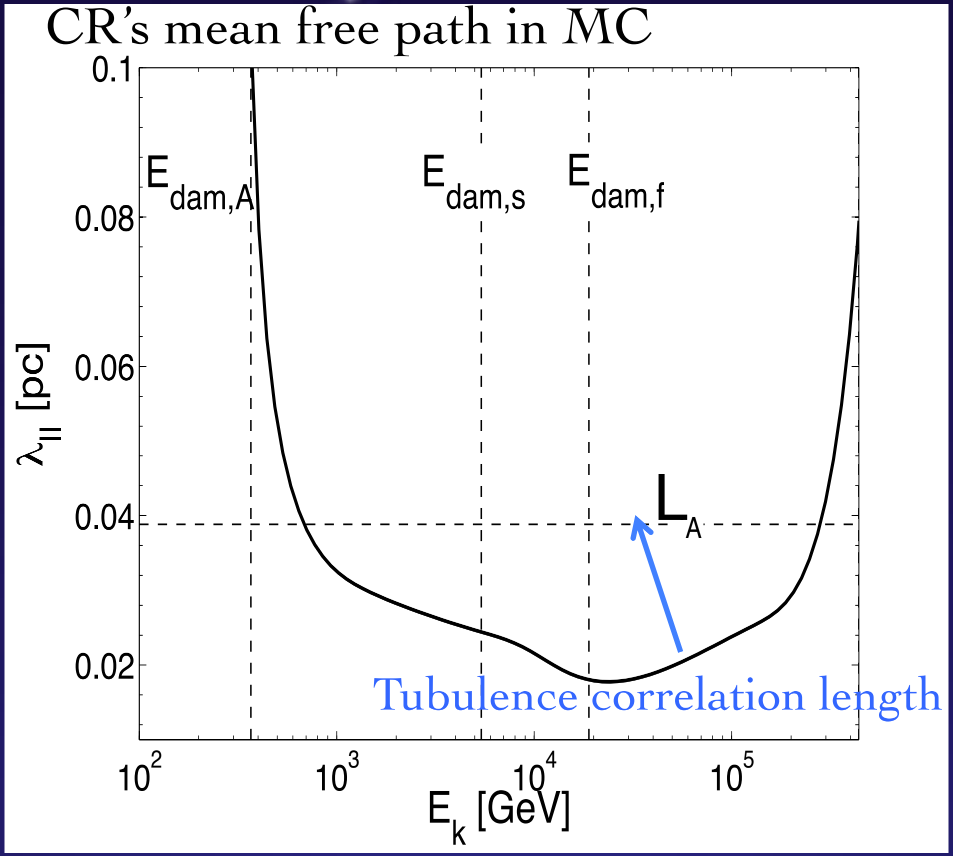
Ex. of implications: Palmer consensus explained!

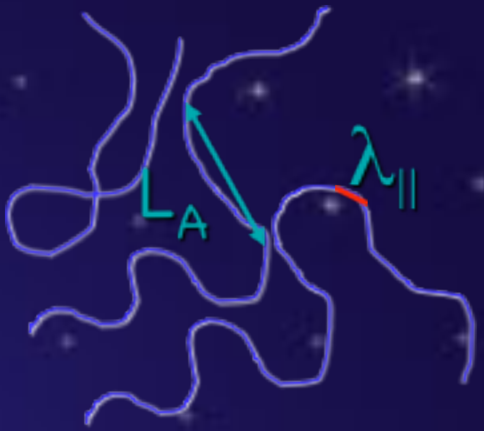




Flat dependence of mean free path can occur due to collisionless damping!

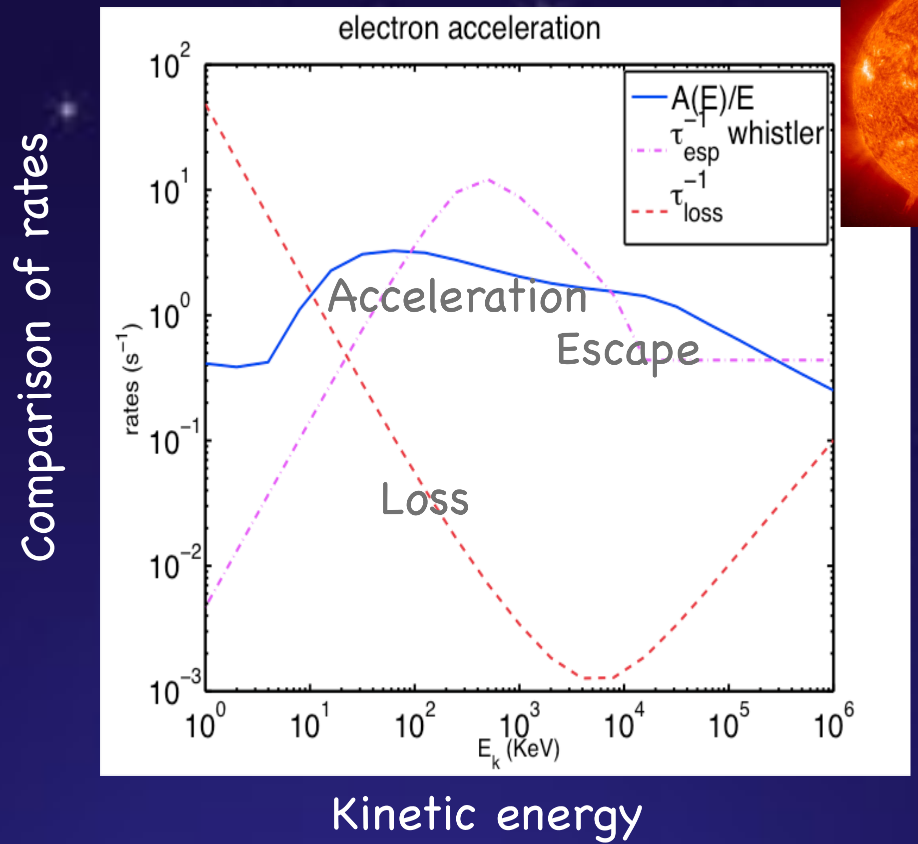
Propagation in partially ionized medium

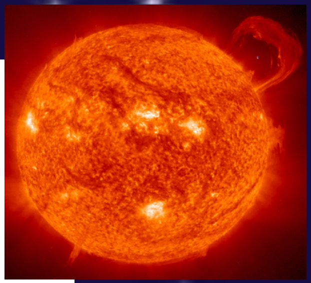




Xu, Yan & Lazarian 2016 ApJ

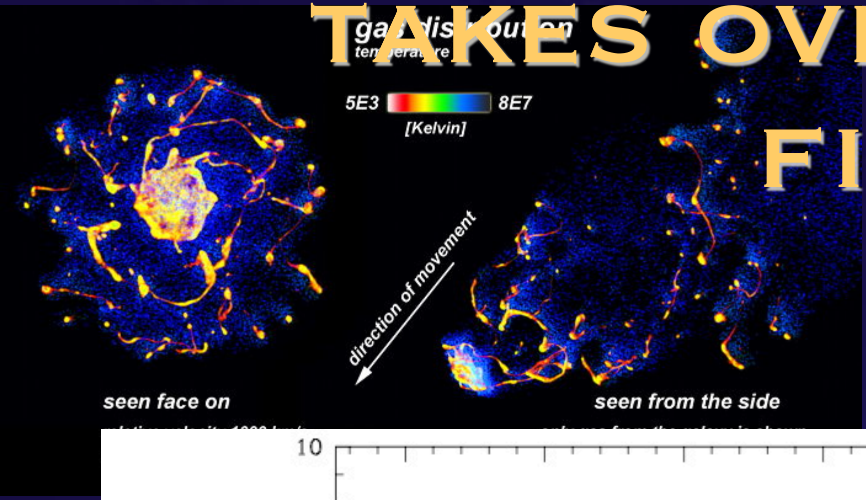
Acceleration by fast modes is an important mechanism for electrons!





Detailed Study of solar flare acceleration must include fast modes and their damping (Yan, Lazarian & Petrosian 2008).

Idea of fast modes takes over in other fields



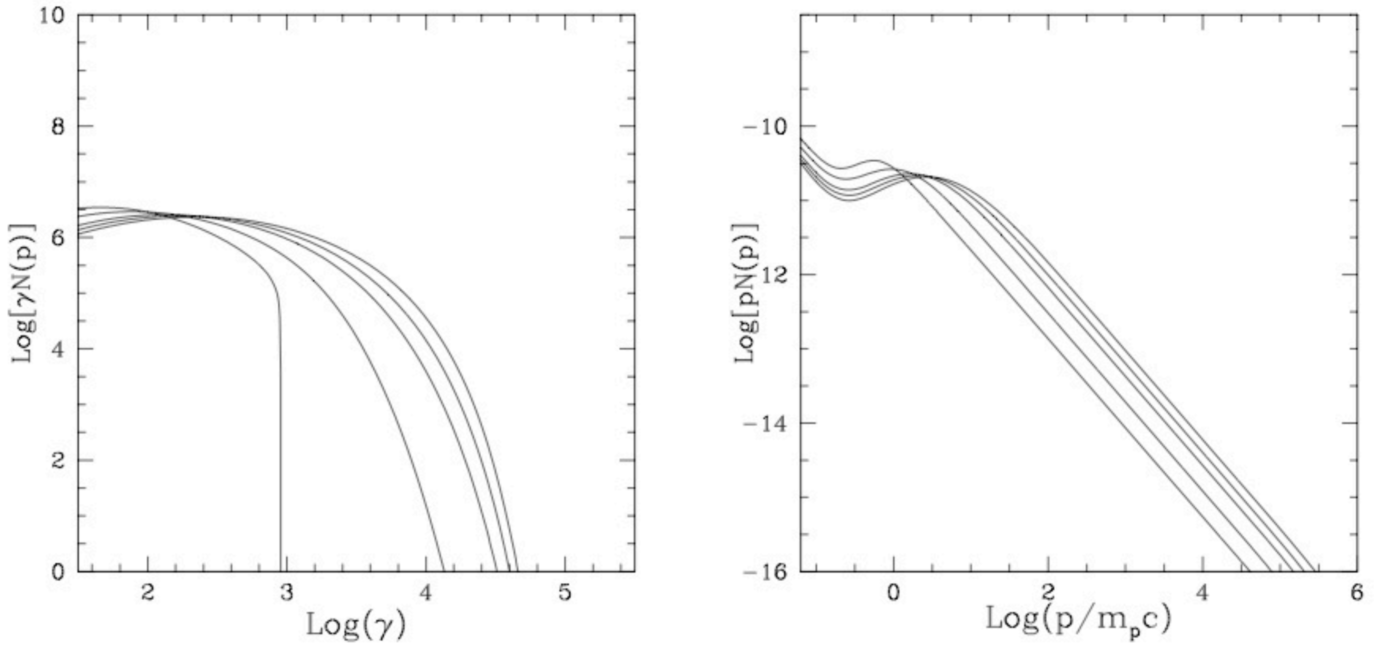
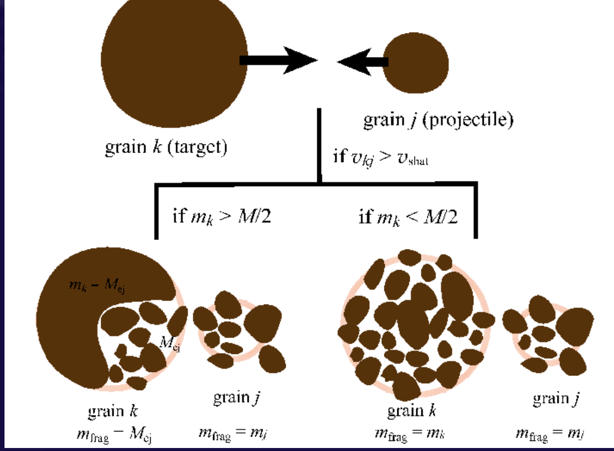
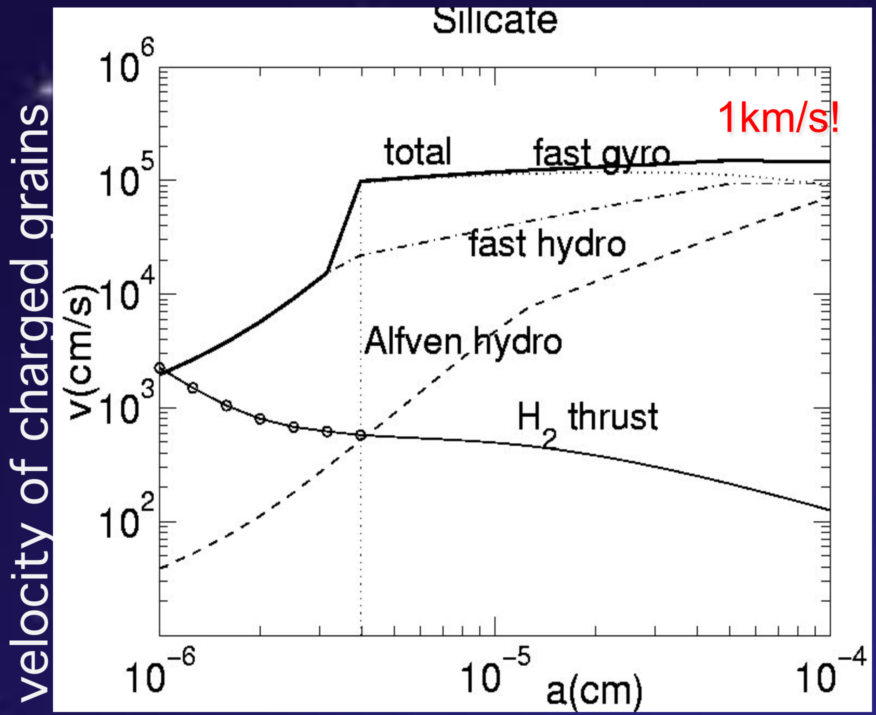


Figure 8. Left-hand panel: Time evolution of the spectrum of relativistic electrons as a function of the Lorentz factor. Right-hand panel: Time evolution of the spectrum of cosmic ray protons as a function of the particle momentum. In both panels calculations are reported for: $t = 0, 4 \times 10^{15}, 8 \times 10^{15}, 10^{16}, 1.2 \times 10^{16}$ s from the start of the re-acceleration phase. Calculations are performed assuming $(V_L/c_s)^2 = 0.18$, $L_o = 300$ kpc, $n_{th} = 10^{-3}$, $k_B T = 9$ keV, $B = 1\ \mu$G and redshift $z = 0.1$ (for IC losses).

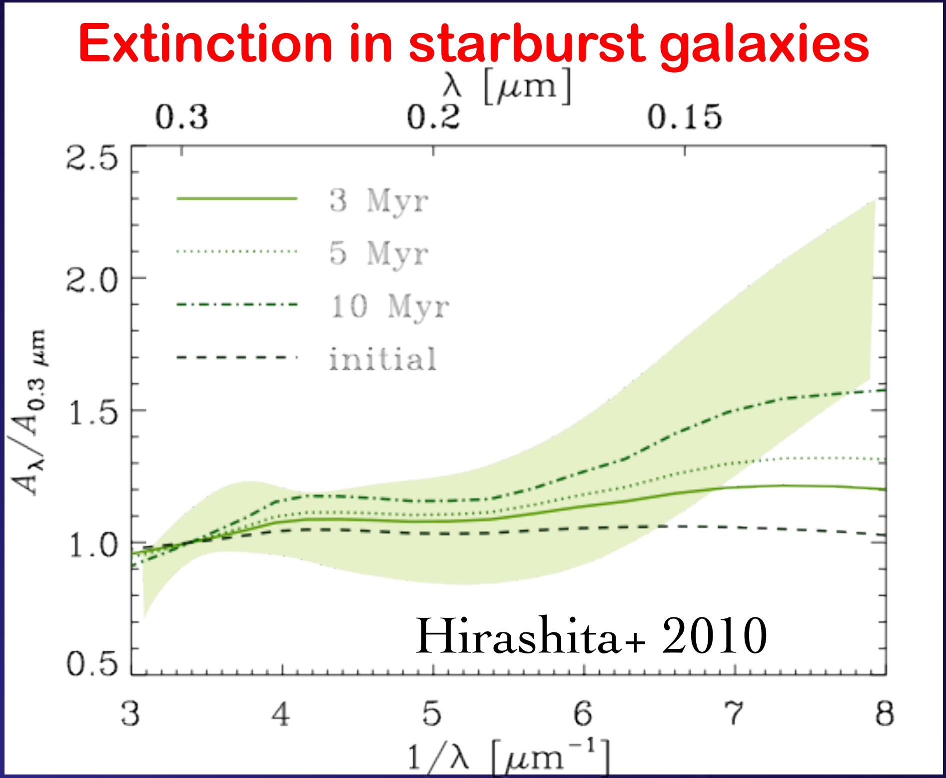
Brunetti & Lazarian (2007)

Dust dynamics is dominated by MHD turbulence!





Grain size



Grains can reach supersonic speed due to acceleration by turbulence and this results in more efficient shattering and adsorption of heavy elements (Yan & Lazarian 2003, Yan 2009).

Other Applications

a) Cosmic ray anisotropy (e.g., Giacinti & Sigl 2012, PRL)

[25] H. Yan and A. Lazarian, Phys. Rev. Lett. 89, 281102 (2002).
[25] H. Yan and A. Lazarian, Phys. Rev. Lett. 89, 281102 (2002).
[25] H. Yan and A. Lazarian, Phys. Rev. Lett. 89, 281102 (2002).

b) Dark matter & positron transport (e.g., Jean et al. 2009, A&A)

c) Radio galaxies (e.g., O'Sullivan et al. 2009, MNRAS)

d) Galactic center Sagittarius A* (e.g., Liu et al. 2006, ApJ)

e) ……

Outline

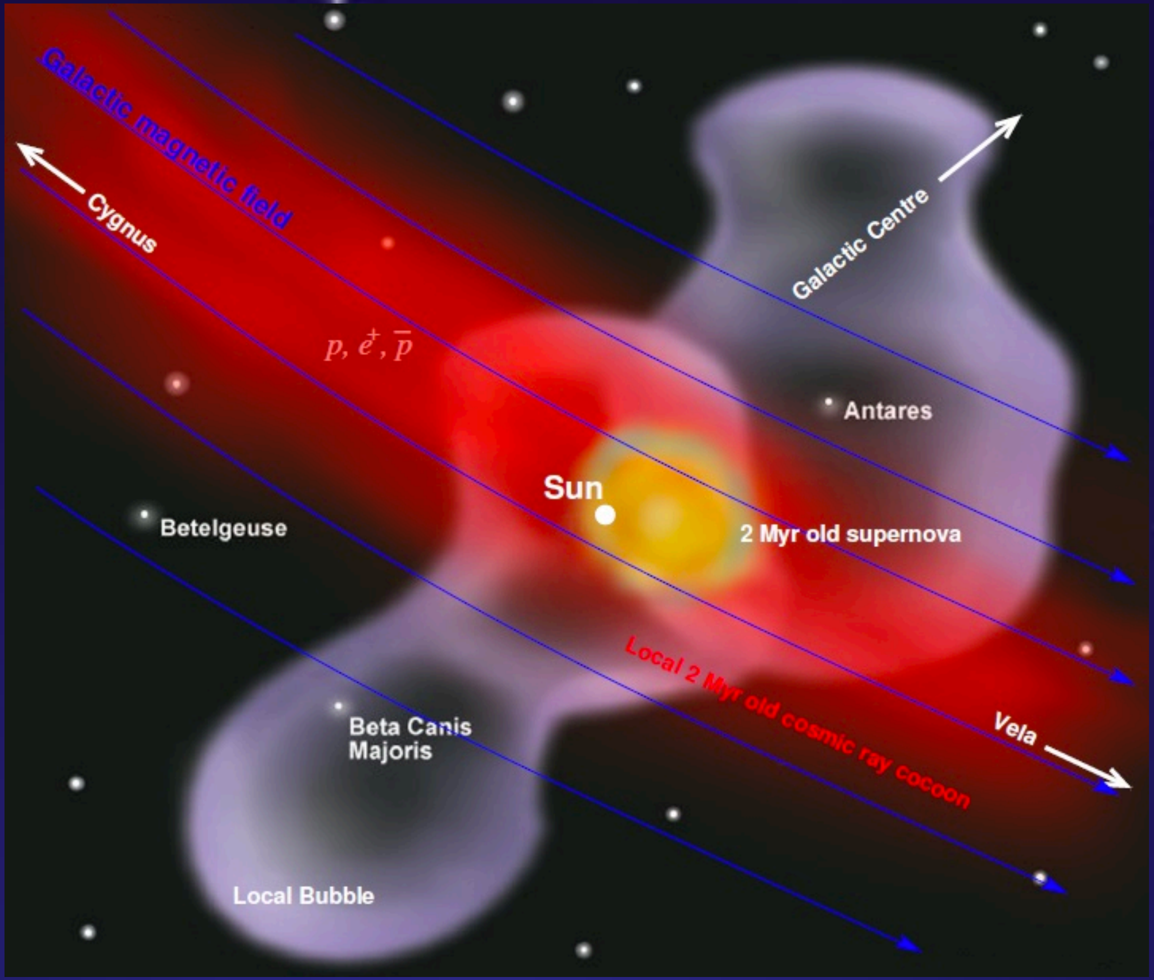
a. Particle Scattering in tested model of MHD turbulenc

b. Cross field transport in MHD turbulence

c. Instabilities and collisionless plasma

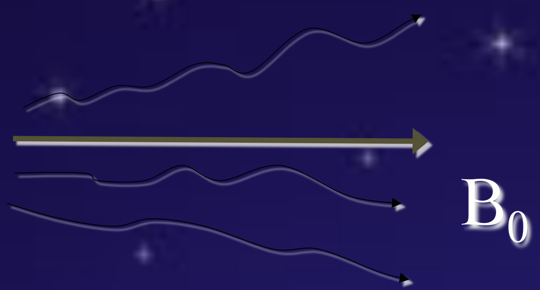
d. Turbulent reconnection model for γ ray burst (GRBs)

Perpendicular transport is critical for Galactic CRs



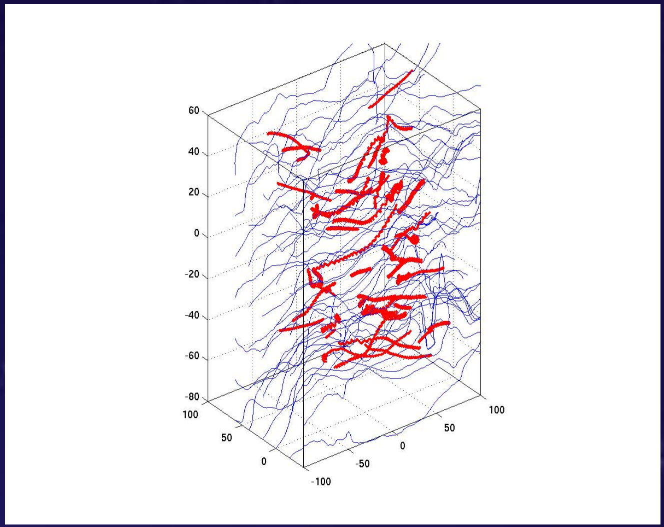
Perpendicular transport

- Dominated by field line wandering.



Intensive studies:

e.g., Jokipii & Parker 1969, Forman 74, Urch 77, Bieber & Matthaeus 97, Giacolone & Jokipii 99, Matthaeus et al 03, Shalchi et al. 04



Test particle simulations with realistic turbulence

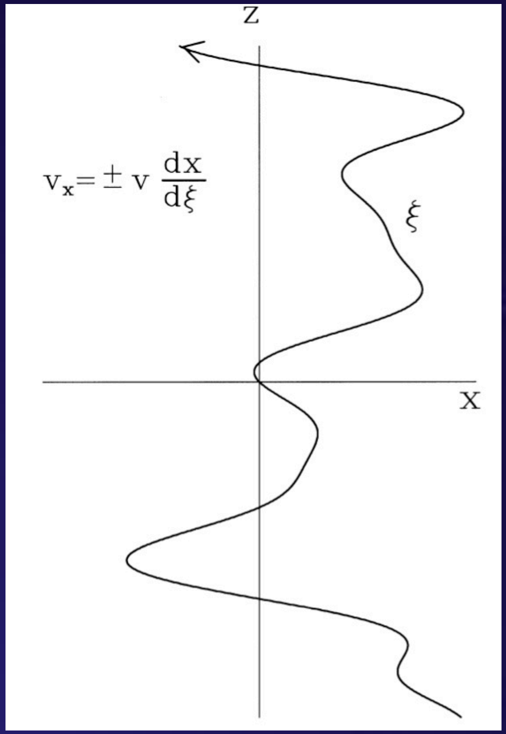
– Particle trajectory

— Magnetic field

What if we use the tested model of turbulence?

Is there subdiffusion ($\Delta x^2 \propto \Delta t^a$, $a<1$) ?

- Subdiffusion (or compound diffusion, Getmantsev 62, Lingenfelter et al 71, Fisk et al. 73, Webb et al 06) was observed in near-slab turbulence, which can occur on small scales due to instability.



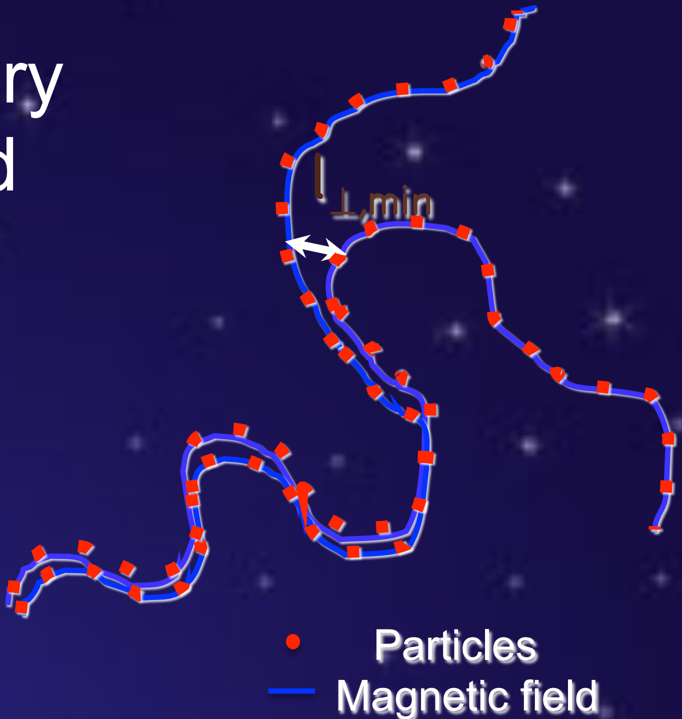
$$\Delta x^2 \propto \Delta z$$
$$\Delta z^2 \propto D_{\|} \Delta t$$

$$\boxed{\Delta x^2 \propto \sqrt{\Delta t}}$$

Diffusion is slow only if particles retrace their trajectories.

Subdiffusion is not typical!

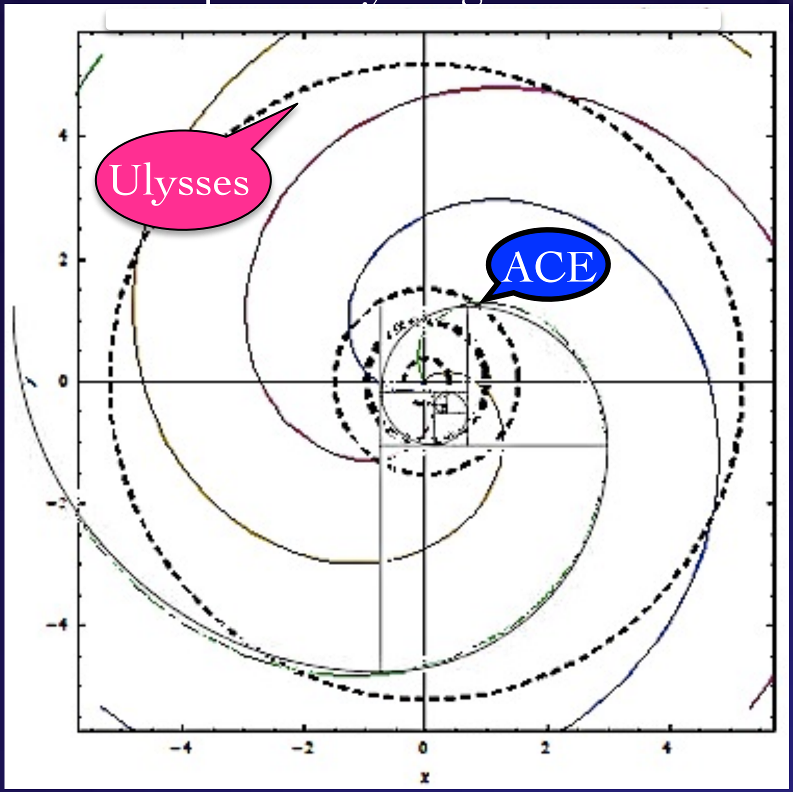
- In turbulence, particles' trajectory become independent when field lines are separated by the smallest eddy size, $l_{\perp,min}$.



- Subdiffusion only occurs below $l_{\perp,min}$. Beyond $l_{\perp,min}$, normal diffusion applies (Yan & Lazarian 2008).

Observational evidence from solar wind

Interplanetary magnetic field



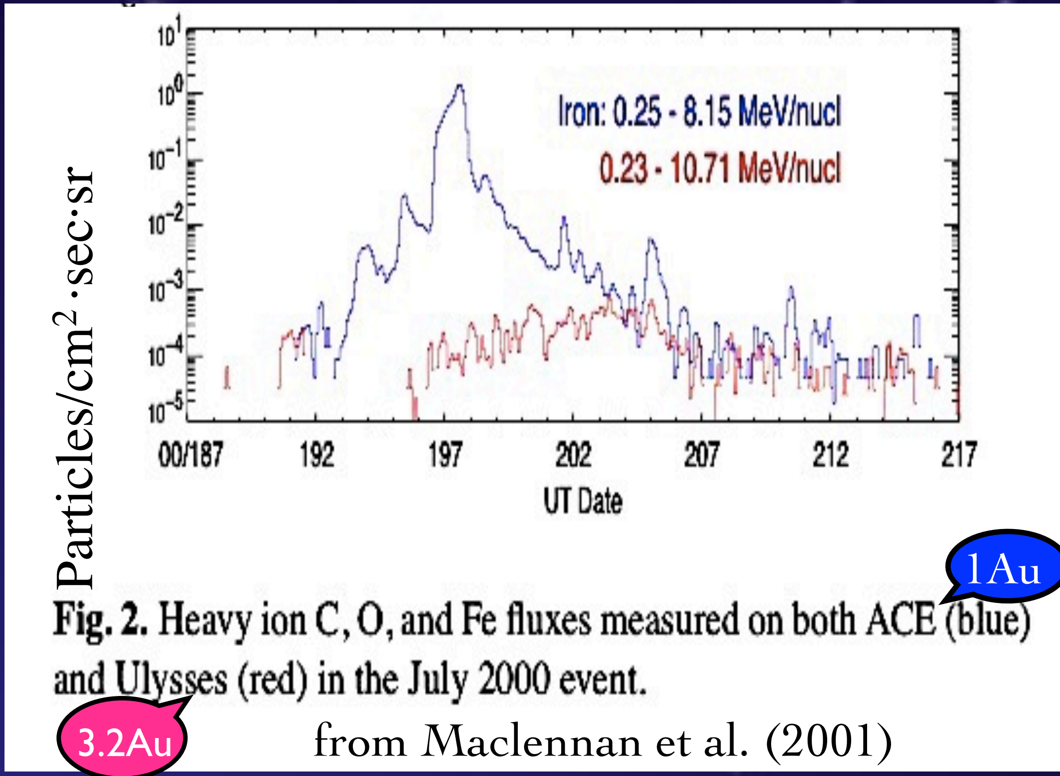
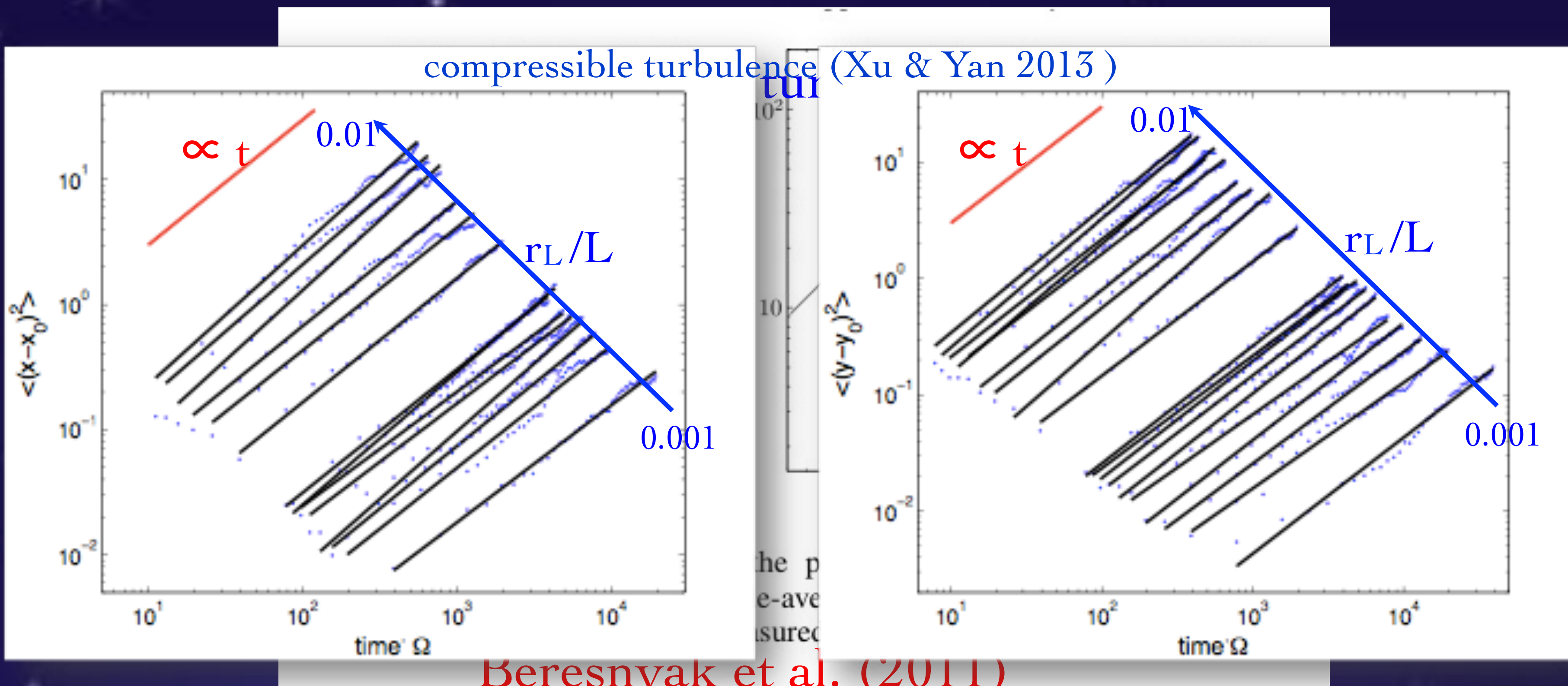


Fig. 2. Heavy ion C, O, and Fe fluxes measured on both ACE (blue) and Ulysses (red) in the July 2000 event.

from Maclennan et al. (2001)

Observations do not support the slow subdiffusion as discussed often in literatures (Getmantsev 62,Fisk et al. 73, Kó ta & Jokipii 2000; Mace et al. 2000; Qin at al. 2002;Webb et al 06).

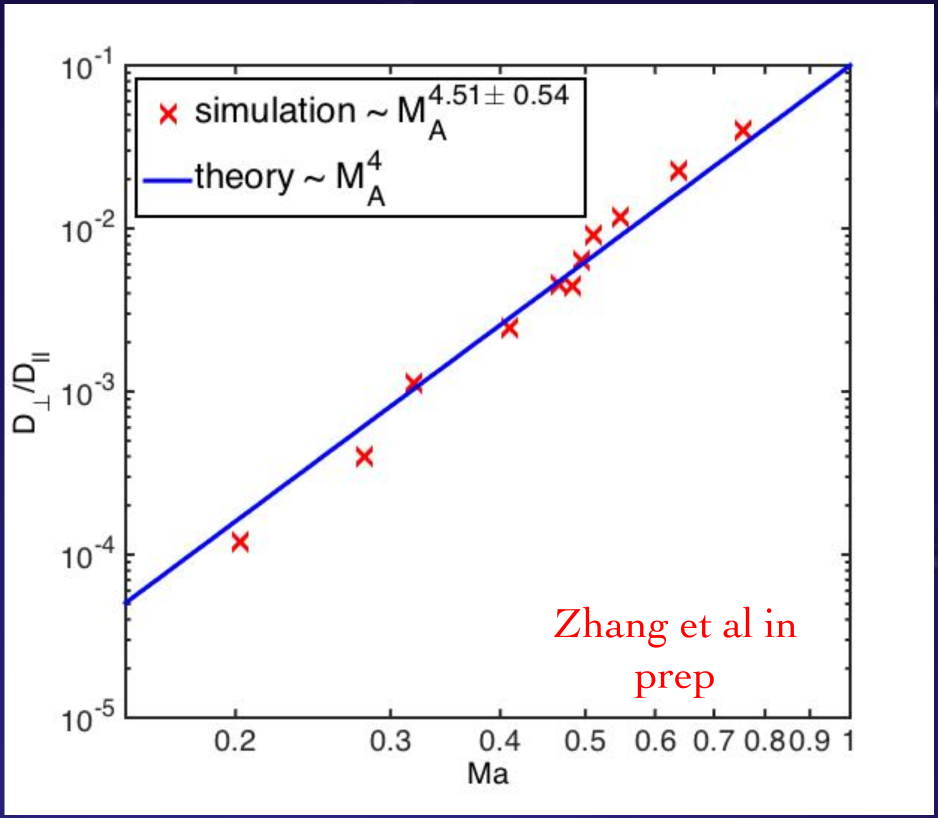
General Normal Diffusion is observed in simulations!



Cross field transport in 3D turbulence is in general a normal diffusion!

Cross field transport is normal diffusion on large scales ($\lambda_{||} < L$)

Numerical simulation:



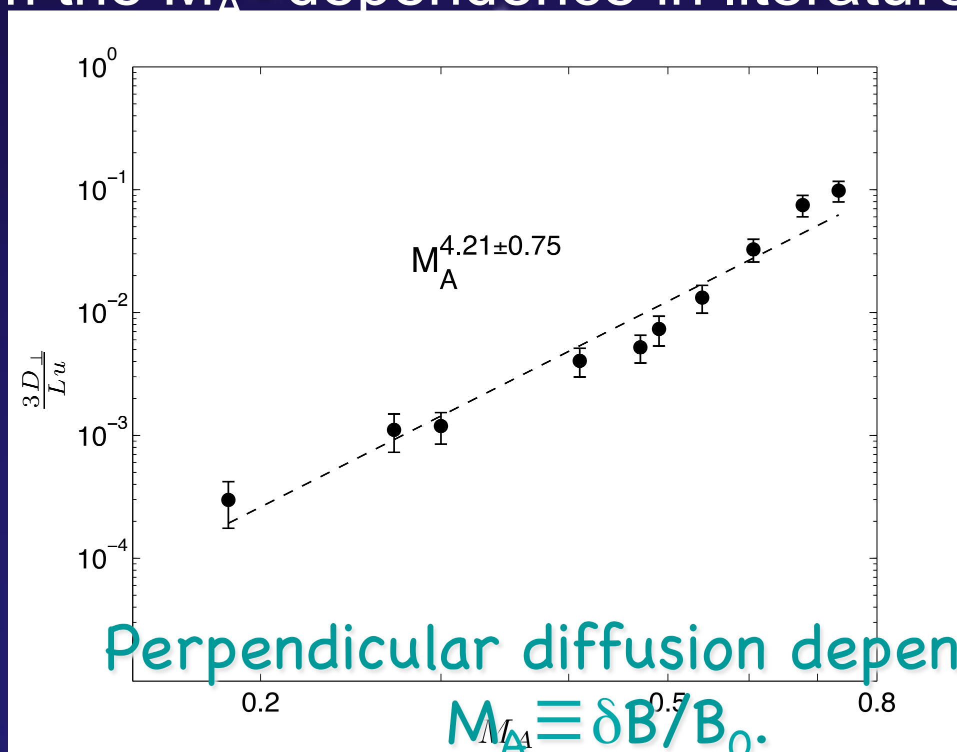
Cross field transport in 3D turbulence has M_A^4 dependence ($M_A \equiv \delta B/B$).

Prediction for perpendicular transport ($\lambda_{||} > L$)

- $M_A < 1$, CRs free stream over distance L, thus $\Delta t = (R/L\ M_A^2)^2\ L/v_{||}$,

$D_\perp = R^2/\Delta t = 1/3 Lv\ M_A^4$ (Yan & Lazarian 2008)

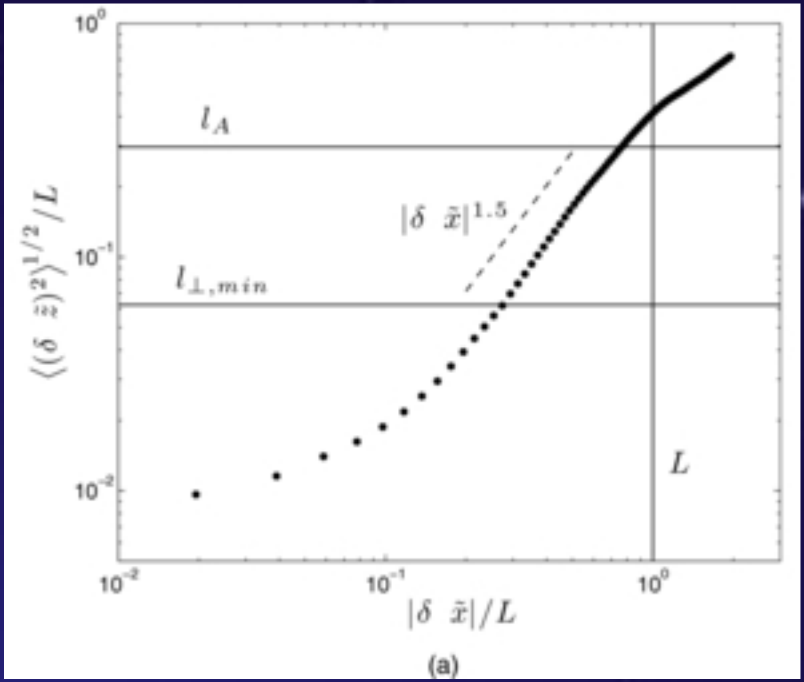
(differs from the M_A^2 dependence in literature)



Perpendicular diffusion depends on $M_A \equiv \delta B/B_0$.

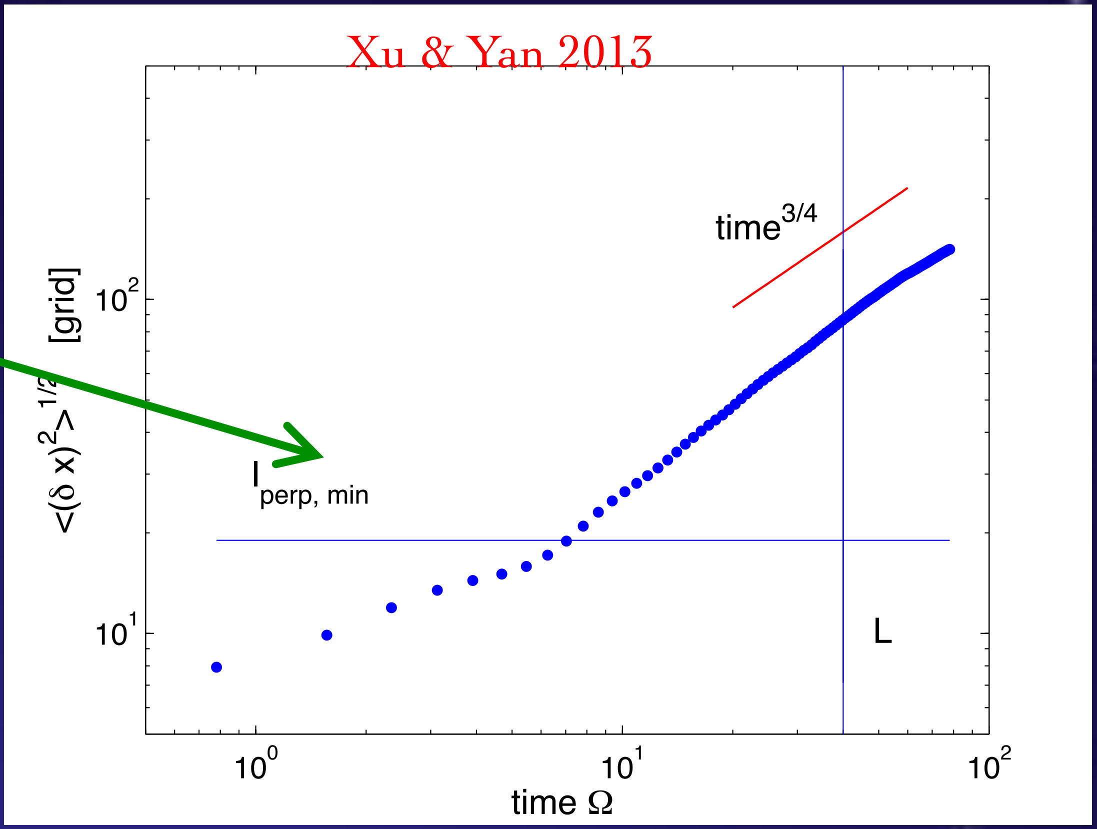
Field lines are superdiffusive on small scales

$$\langle |\mathbf{x}_1(t) - \mathbf{x}_2(t)|^2 \rangle \sim t^3.$$



Xu & Yan 2013

Superdiffusion (SD) of CRs is observed



Superdiffusion has M_A^4 dependence

- Theoretical prediction

  Lazarian & Vishniac 1999;
  Yan & Lazarian 2008

$$<(\delta z)^2> = \frac{|\delta x|^3}{3^3 L} M_A^4$$

- Numerical result

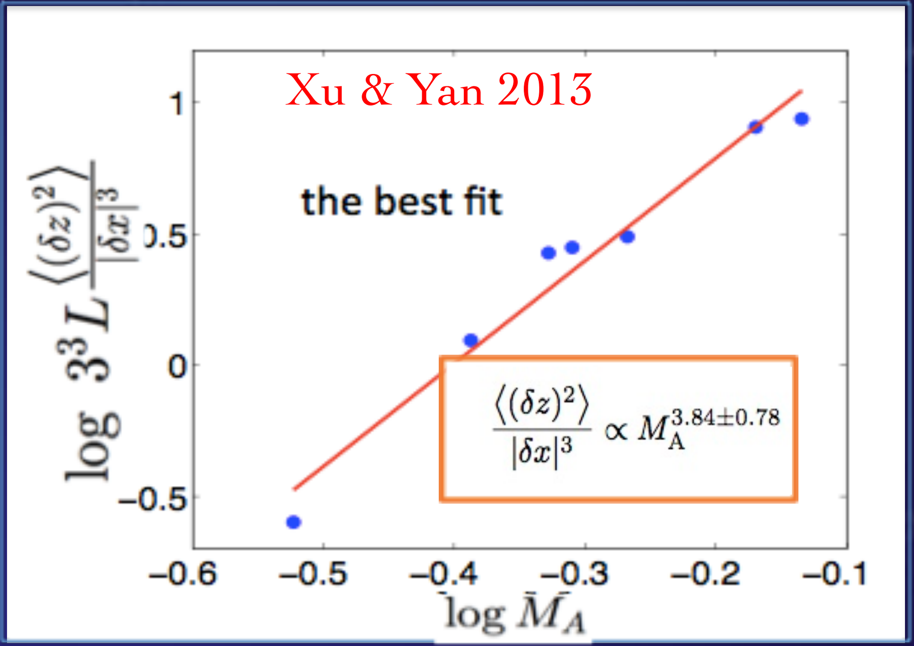


Table 1

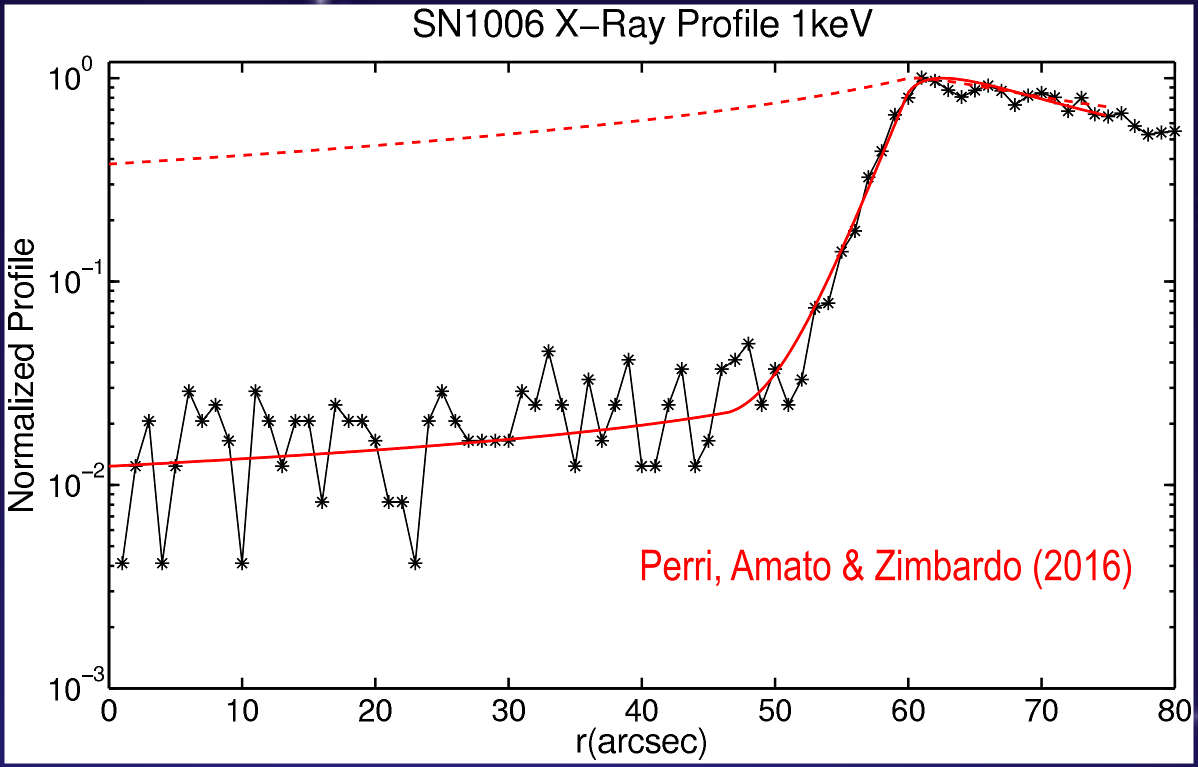
Regimes of MHD Turbulence and Magnetic Diffusion

Type of MHD turbulence	Injection velocity	Range of scales	Spectrum E(k)	Motion type	Ways of study	Magnetic diffusion	Squared separation of lines
Weak	$V_L < V_A$	$[l_{trans}, L]$	$k_\perp^{-2}$	wave-like	analytical	diffusion	$\sim sLM_A^2$
Strong subAlfvenic	$V_L < V_A$	$[l_{min}, l_{trans}]$	$k_\perp^{-5/3}$	anisotropic eddy-like	numerical	Richardson	$\sim \frac{s^3}{L} M_A^4$
Strong superAlfvenic	$V_L > V_A$	$[l_A, L]$	$k_\perp^{-5/3}$	isotropic eddy-like	numerical	diffusion	$\sim sl_A$
Strong superAlfvenic	$V_L > V_A$	$[l_{min}], l_A$	$k_\perp^{-5/3}$	anisotropic eddy-like	numerical	Richardson	$\sim \frac{s^3}{L} M_A^3$

Note. L and l_{min} are the injection and perpendicular dissipation scales, respectively. $M_A \equiv \delta B/B$, $l_{trans} = LM_A^2$ for $M_A < 1$ and $l_A = LM_A^{-3}$. for $M_A < 1$. For weak Alfvenic turbulence, $\ell_{\|}$ does not change. s is measured along magnetic field lines.

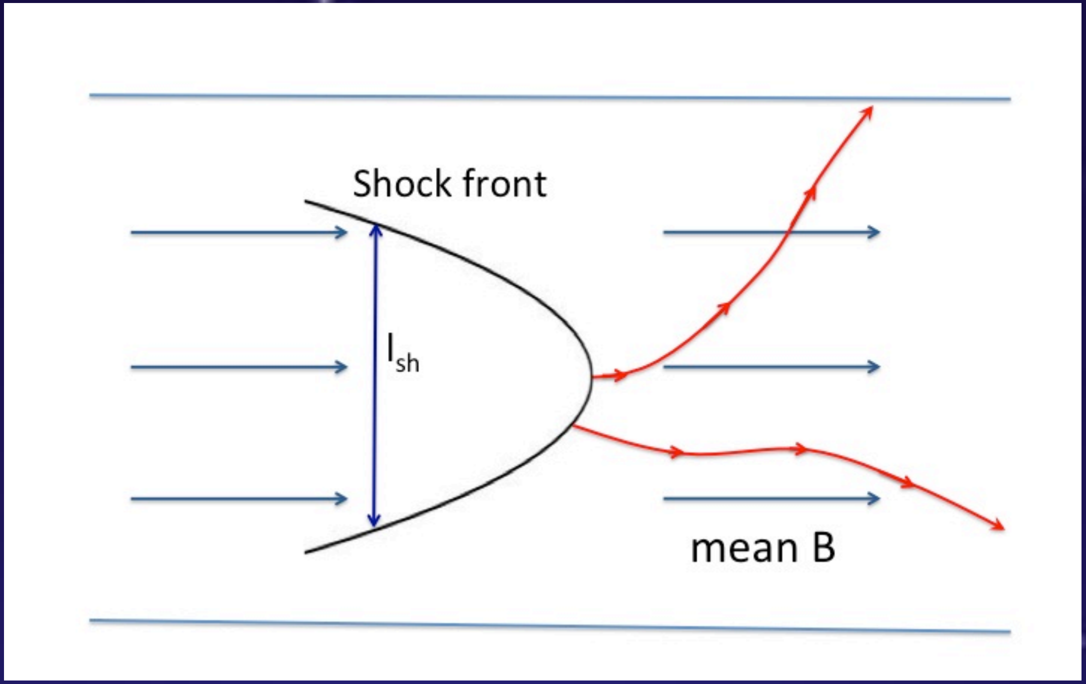
Lazarian & Yan (2014)

Observation of SNRs



Radial profile of the emission at about 1 keV for the SN1006 remnant. The thick red line corresponds to the model integrated along the line of sight for synchrotron-loss-dominated transport downstream, diffusive transport close upstream, and superdiffusive transport far upstream (in the flatter tail of the profile).

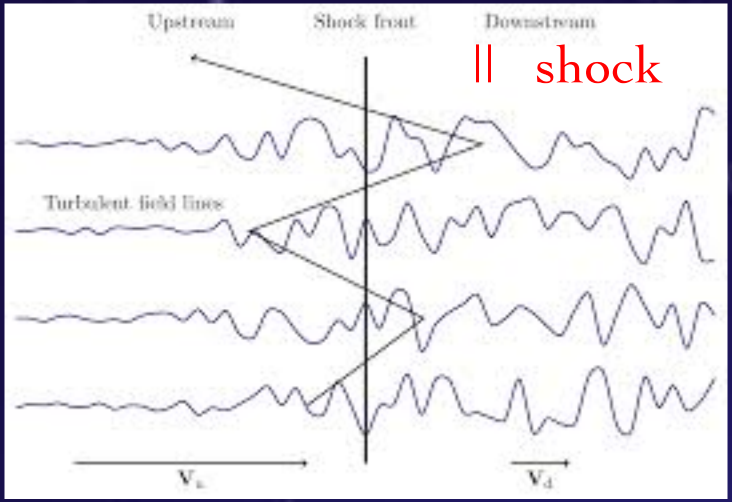
Implication I. Acceleration at shock w. finite size

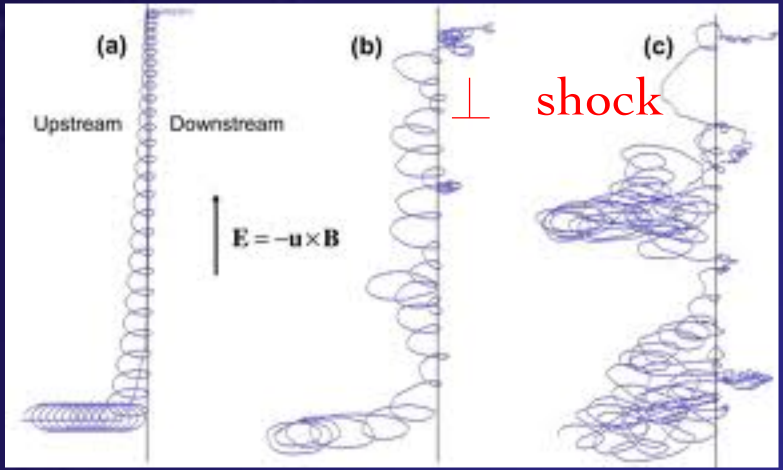


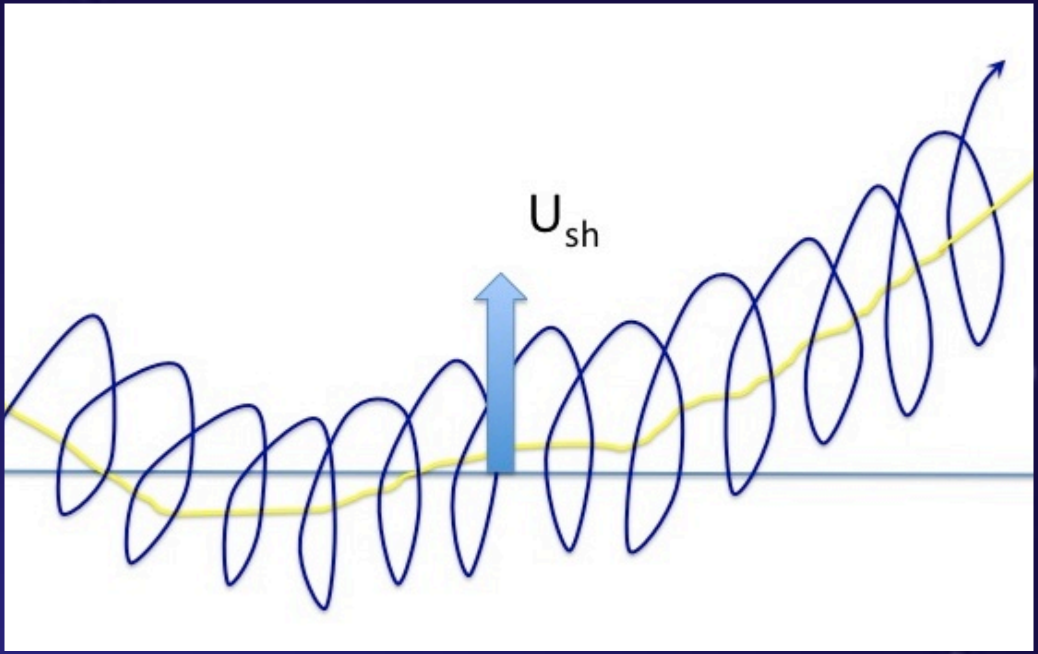
$$E_{max} = 32\left(\frac{U_1}{400\mathrm{km/s}}\right)^2\left(\frac{L}{100\mathrm{Au}}M_A^{\zeta}\right)^{\frac{4}{3}}\left(\frac{l_{sh}}{90\mathrm{Au}}\right)^{\frac{8}{3}}\left(\frac{10Au}{\lambda}\right)^4 \mathrm{MeV}\cdot\mathrm{nuc}^{-1}.$$

Lazarian & Yan (2014)

Implication II. Acceleration at ⊥ shock and || shock diminish w. SD

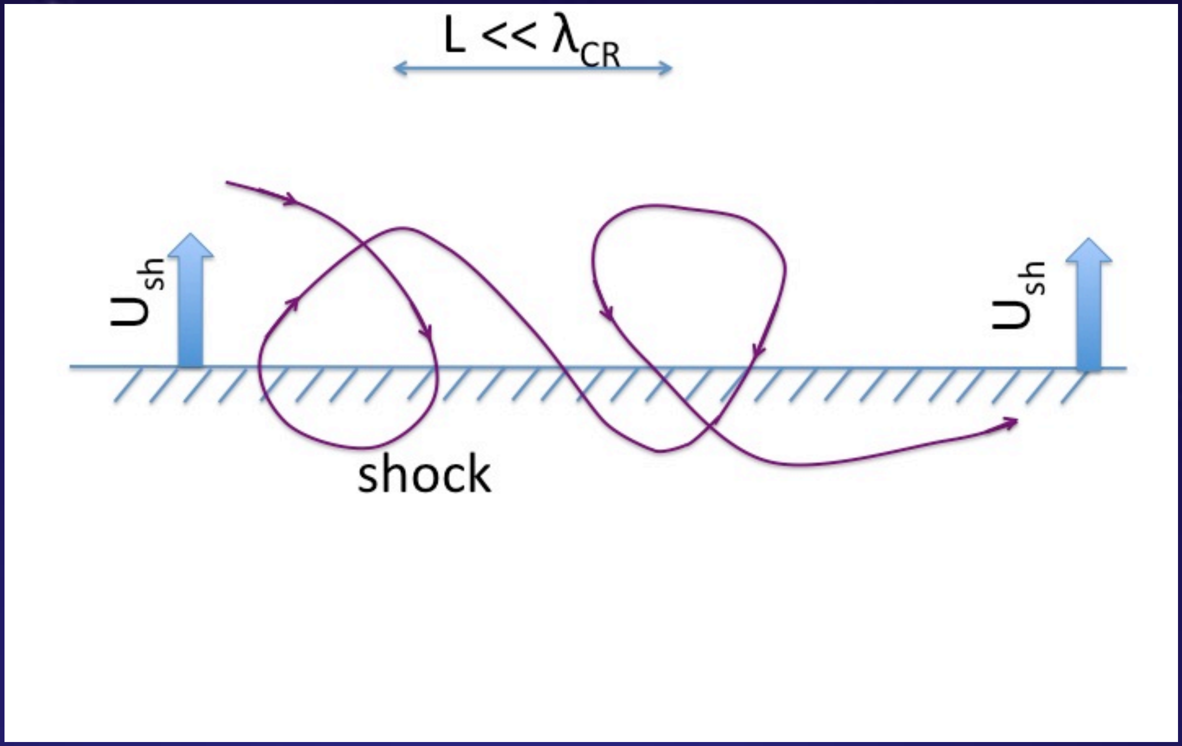






Lazarian & Yan (2014)

III. Fast acceleration w. local small scale turbulence



Lazarian & Yan (2014)

Outline

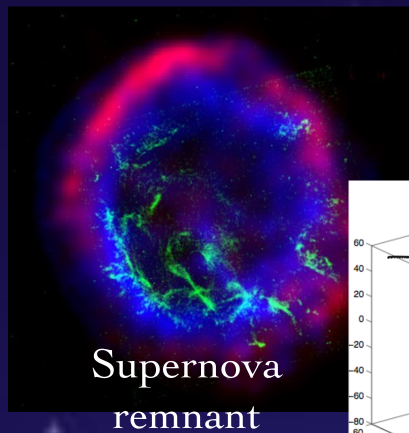
a. Particle Scattering in tested model of MHD turbulenc

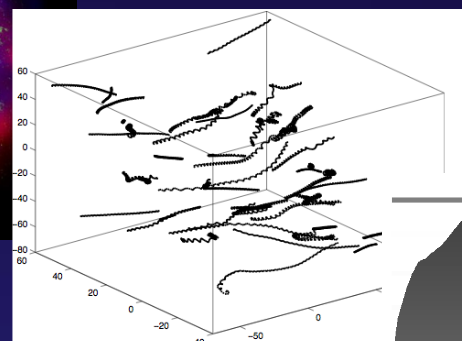
b. Cross field transport in MHD turbulence

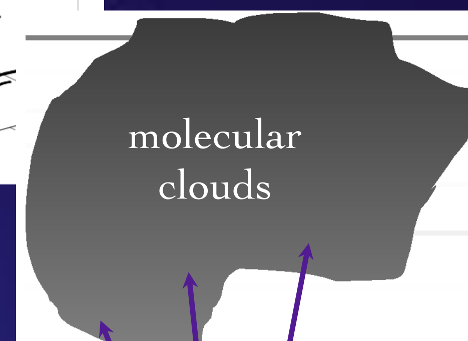
c. Instabilities and collisionless plasma

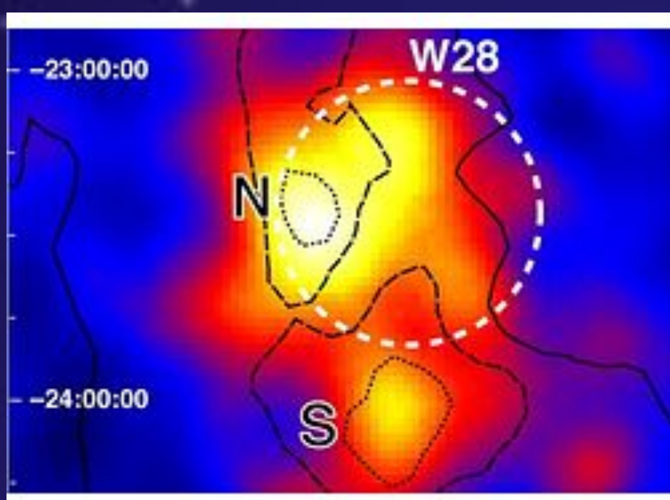
d. Turbulent reconnection model for γ ray burst (GRBs)

Streaming instability with enhanced CR flux

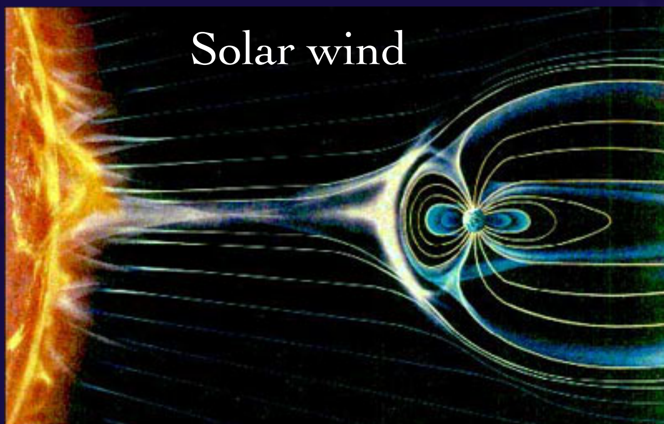




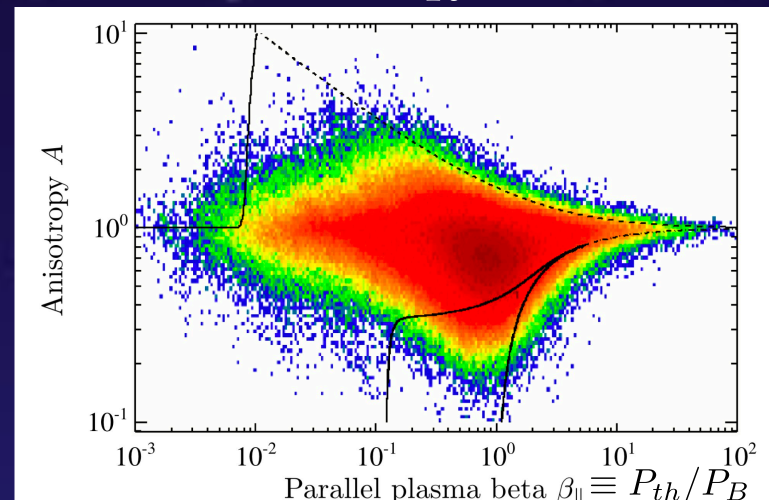




Turbulence in collisionless medium



Solar wind anisotropy (Schlickeiser 2011)



Highly collisionless:

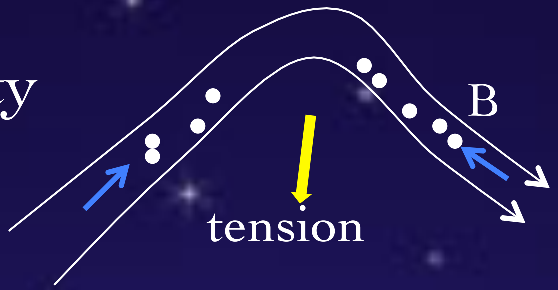
$n \sim 10^{-3}$-0.1

$T \sim 10^8$ K

$\lambda_{mfp} \sim 0.3$kpc $\approx 10^{21}$ cm

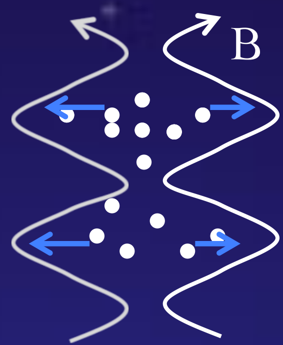
Anisotropy driven instabilities

- Firehose instability



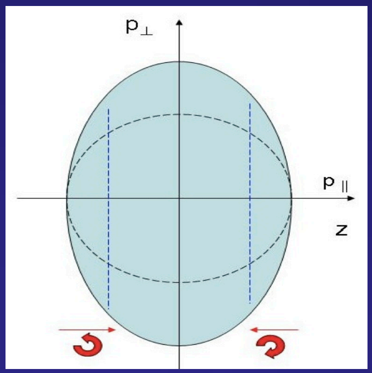
$$P_\parallel - P_\perp > \frac{B^2}{4\pi}$$

- Mirror instability



$$P_\perp - P_\parallel > \frac{B^2}{8\pi}$$

- Ion cyclotron (IC) instability



$$A \equiv \frac{P_\perp}{P_\parallel} - 1$$ $> 0.43\beta^{-0.42}$ thermal gas

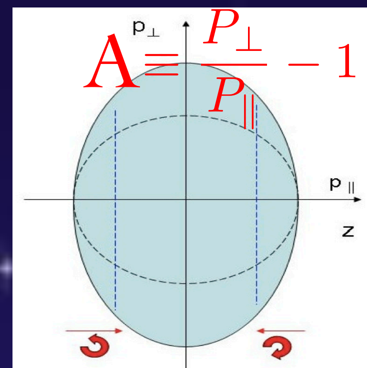
$> v_A/c$ CRs

$$\beta \equiv P_{gas}/P_B$$

B) Ion Cyclotron instability in turbulence

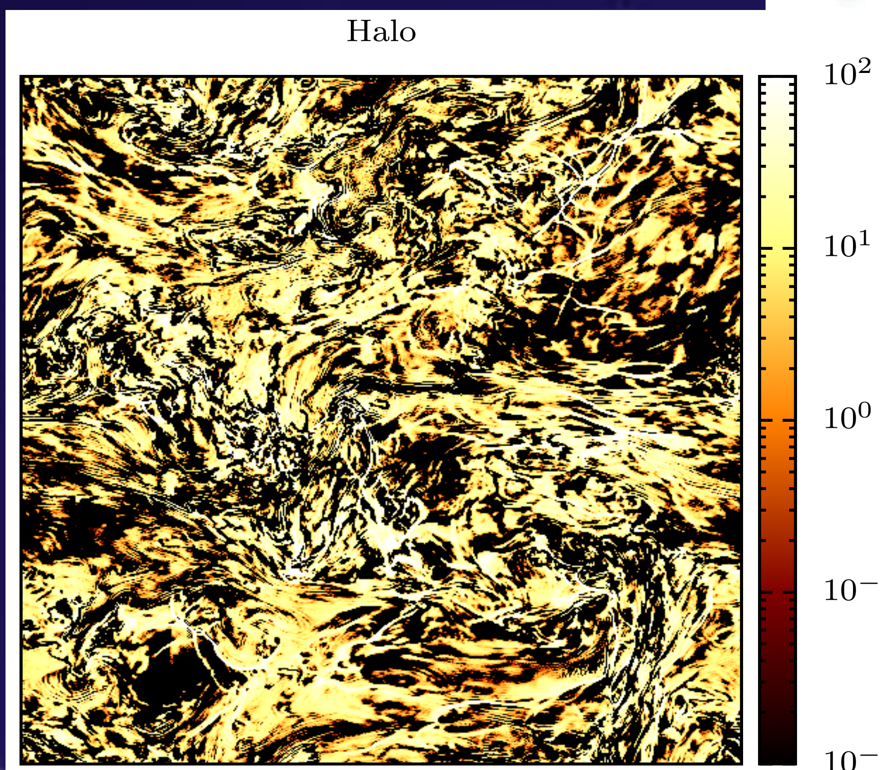
Turbulence stretching and compression

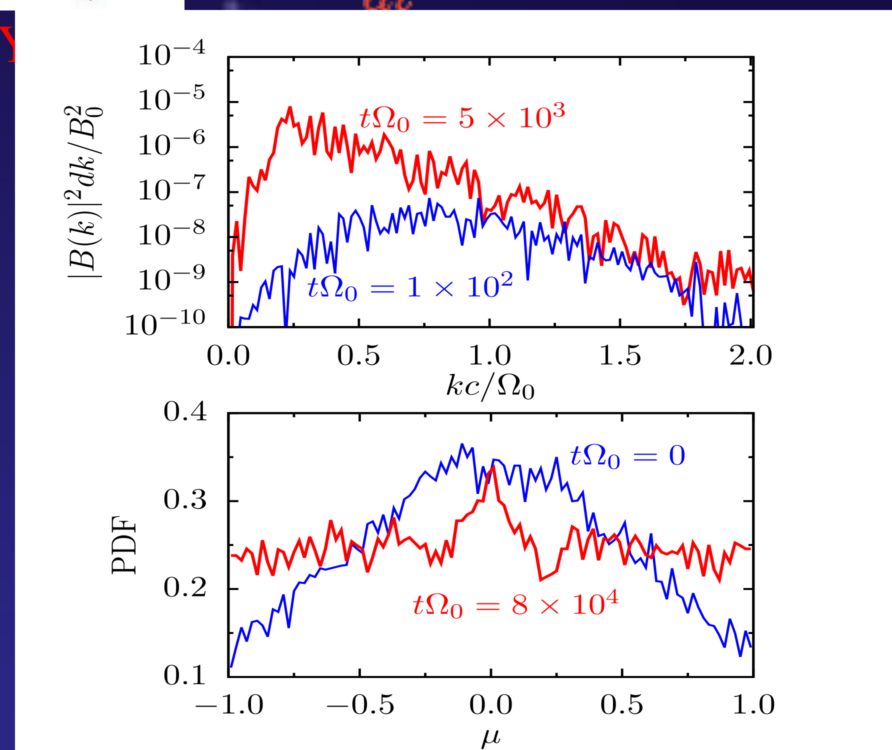
$$\frac{dA}{dt} > 0$$



Scattering by instability generated slab wave

$$\frac{dA}{dt} < 0$$





Santos-Lima+ 2017

Scattering by growing waves

Simple estimates:

$$\frac{dA}{dt} \sim -\nu A = -\frac{1}{W_{\|}}\left(\frac{dW_{\perp}}{dt} - \frac{W_{\perp}}{W_{\|}}\frac{dW_{\perp}}{dt}\right) \sim -\Gamma_{gr}\epsilon_N/\beta_{CR}$$

By balancing it with the rate of increase due to turbulence compression $\frac{1}{B}\frac{dB}{dt}$, we can get

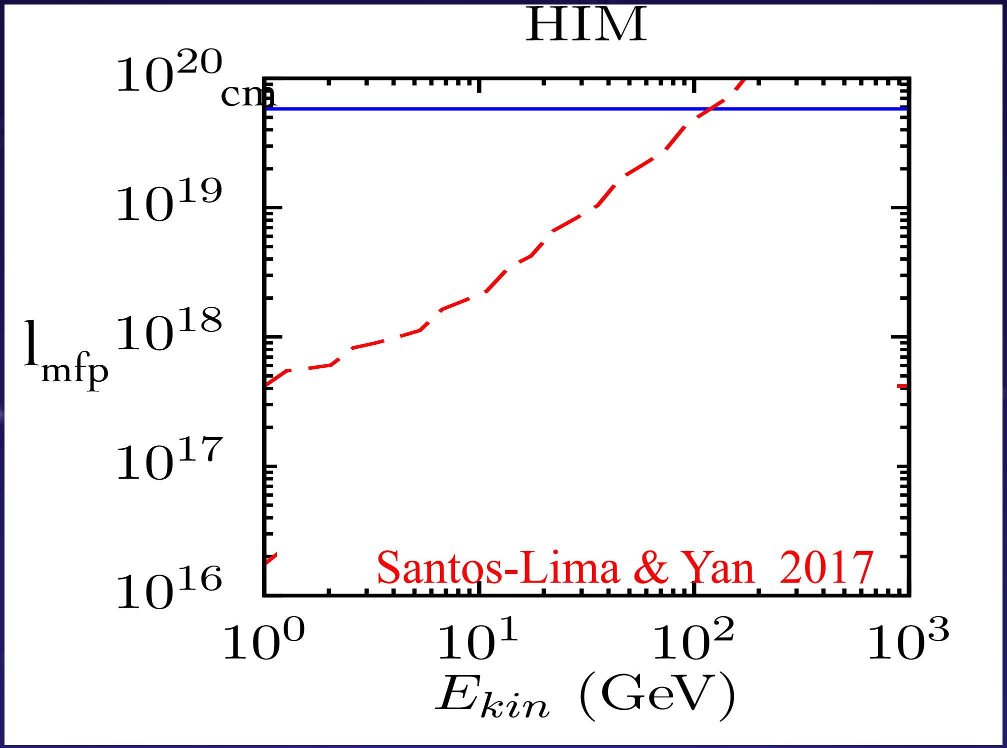
$$\epsilon_N \sim \frac{\beta_{CR}\omega\delta v}{\Gamma_{gr} v_A},\ \lambda \simeq r_p/\epsilon_N$$

Bottle-neck of growth due to energy constraint:

$$\epsilon_{N,max} \sim \frac{v_A}{L_{inj}\Gamma_{gr}}$$

Yan & Lazarian 2011

Scattering due to gyroresonance instability



Low energy CRs are scattered by instability generated small scale waves!

Outline

a. Particle Scattering in tested model of MHD turbulenc

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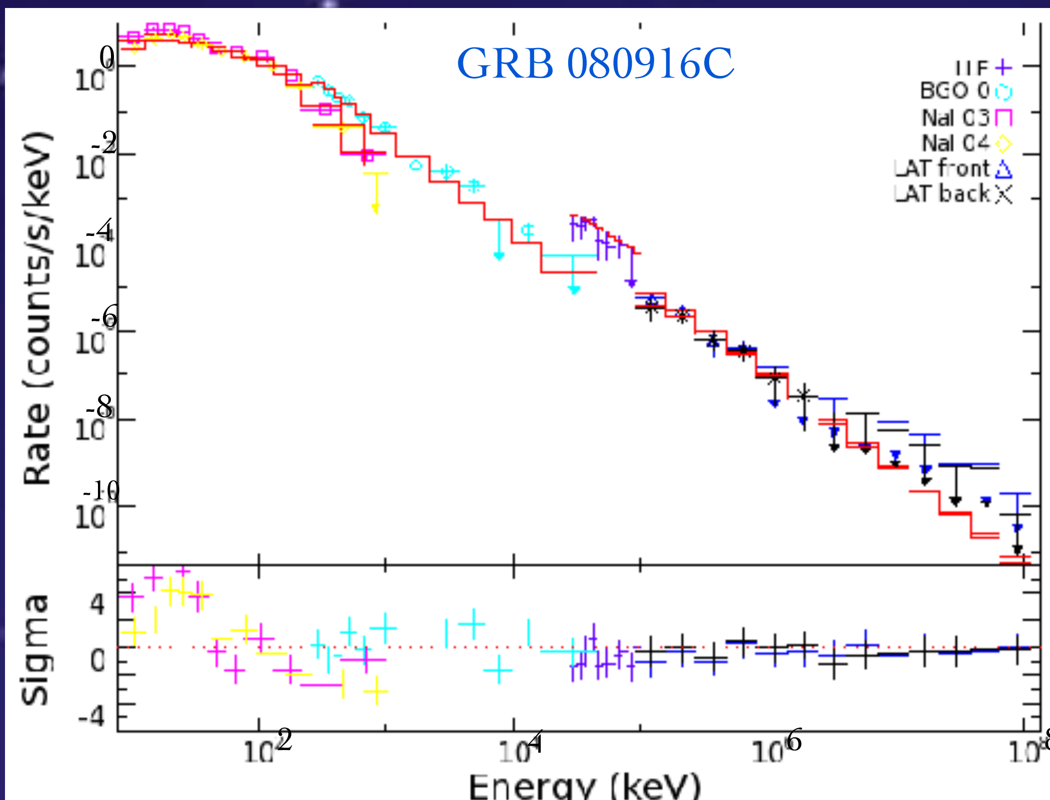
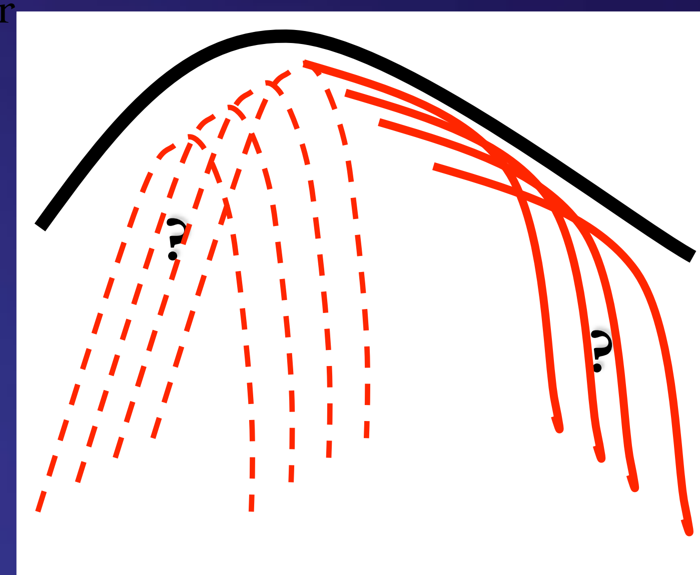
d. Turbulent reconnection model for ϒ ray burst (GRBs)

Is the band function emission from the photosphere?

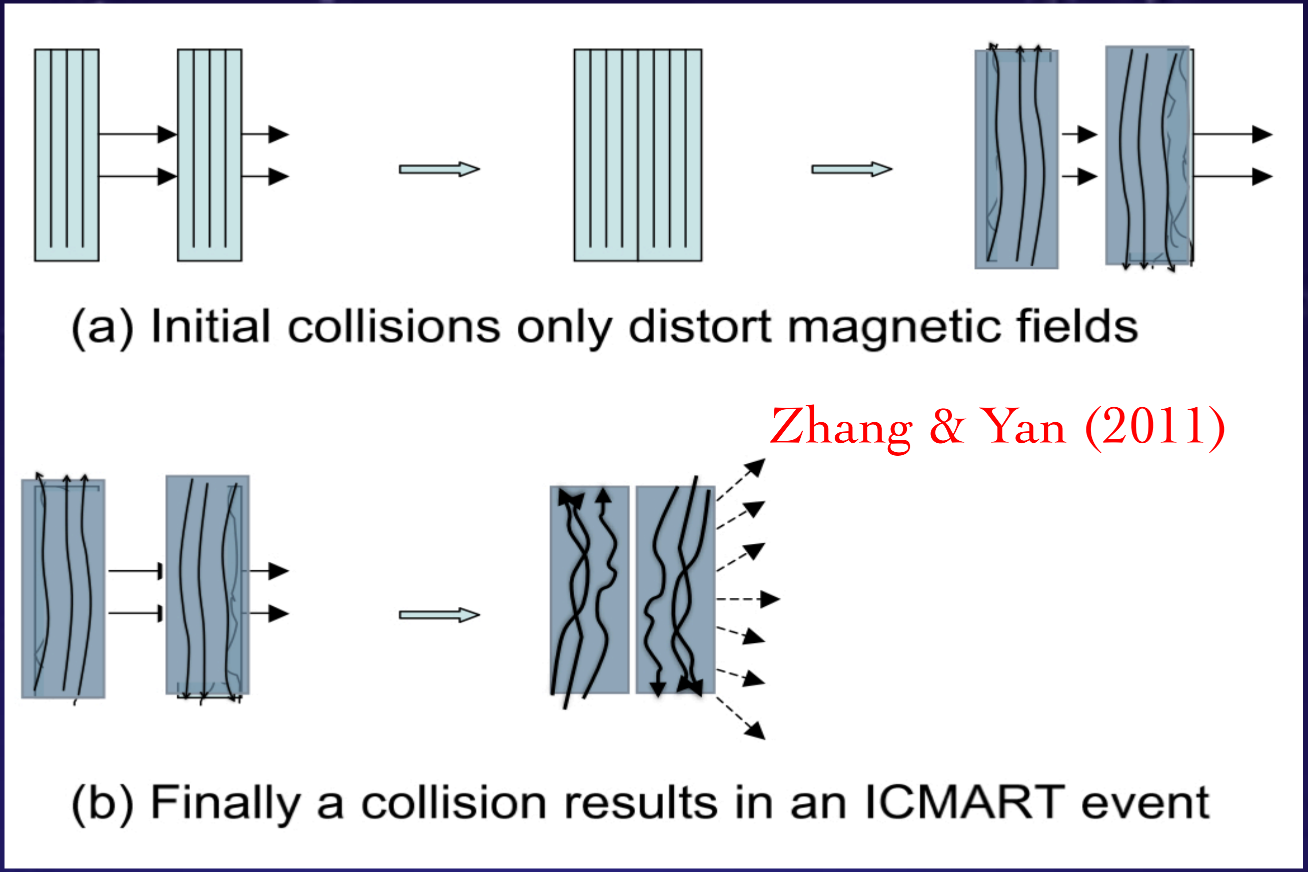


Abdo et al. (2009)

- Superposition from many shells (Toma et al. 2010; Li 2009)?
  - Contrived fine-tuning
  - Seems not supported by data with finer temporal resolution



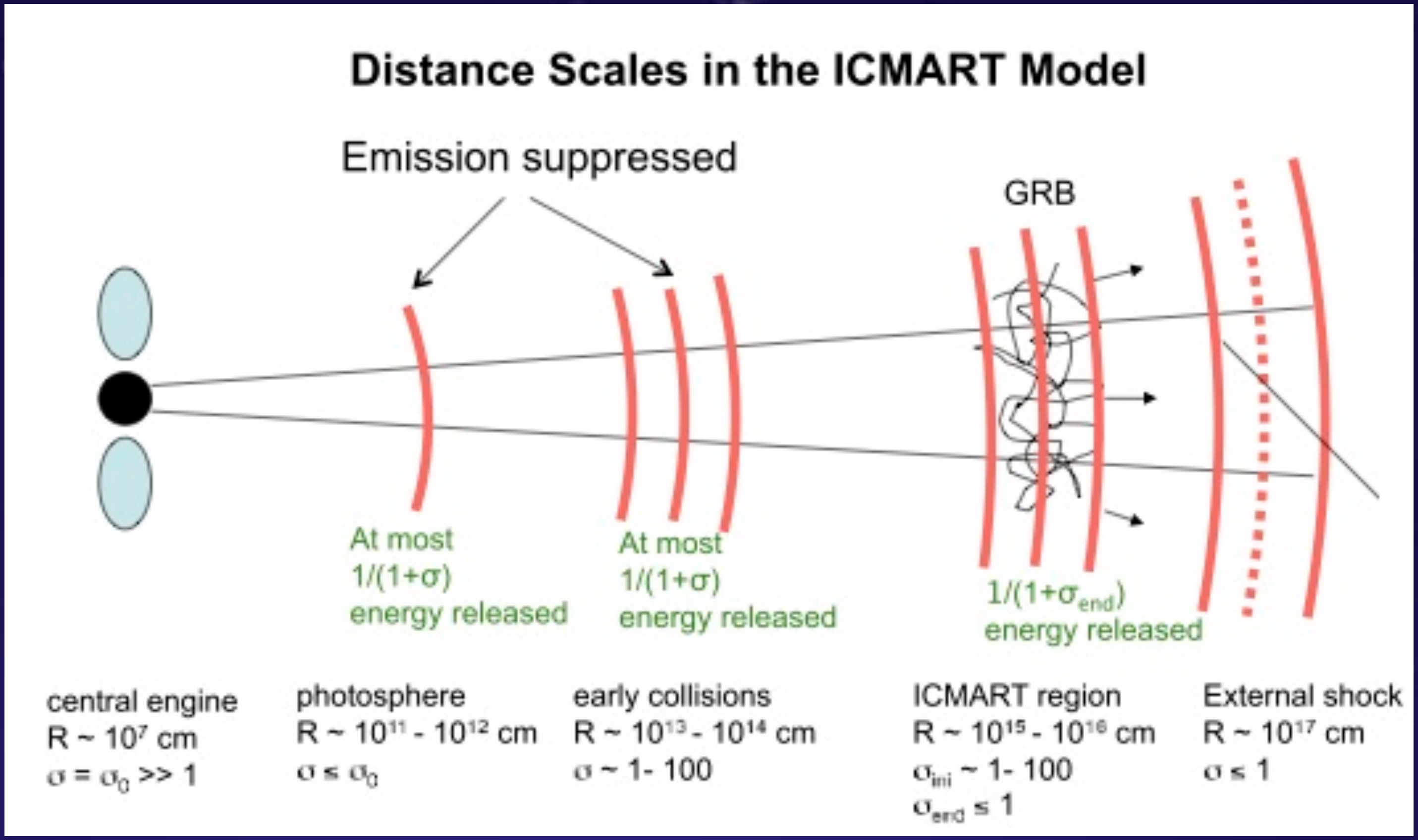
c) Turbulent reconnection model for GRBs



Zhang & Yan (2011)

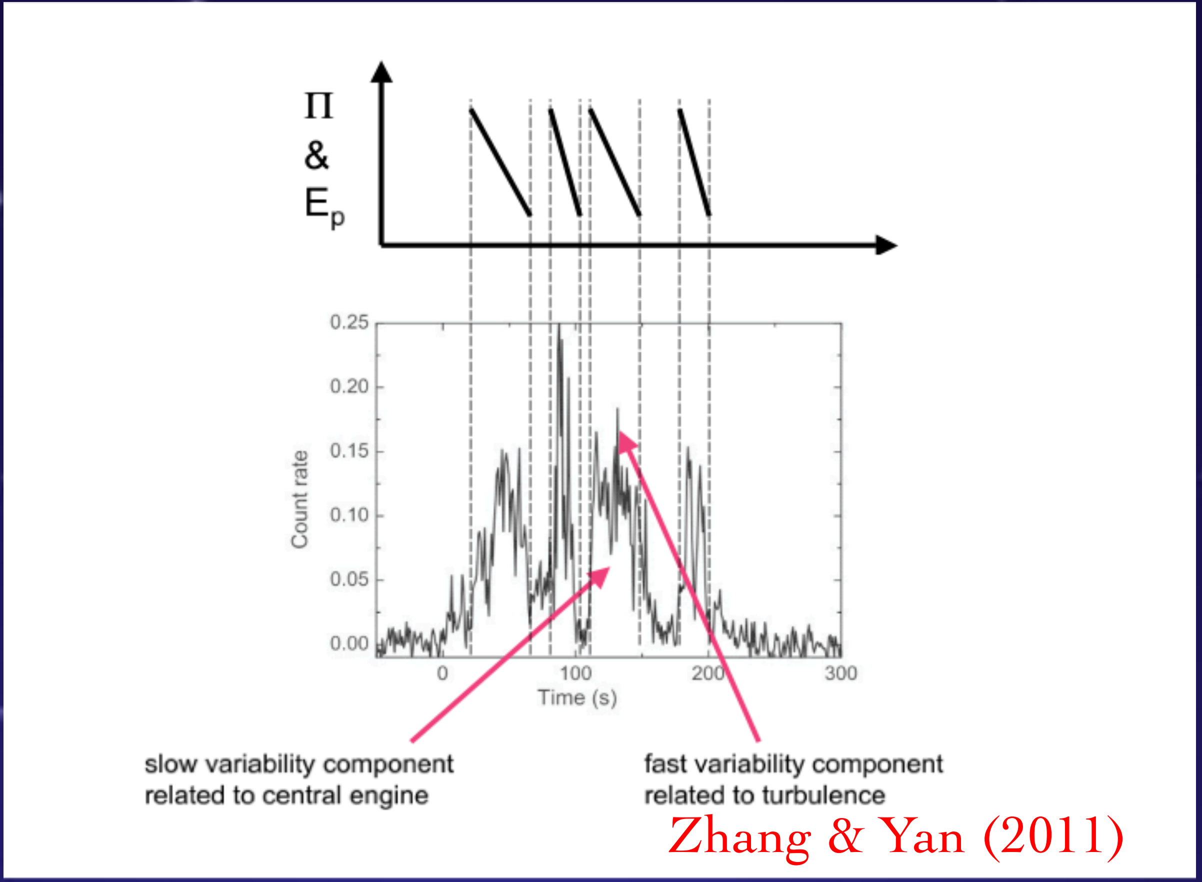
Bursty reconnection occurs as a nonlinear feedback of the increased stochasticity of B field.

Turbulent reconnection triggers a GRB at large R



Internal Collision triggered Magnetic Reconnection (ICMART) model provides a natural explanation for highly magnetized GRBs (Zhang & Yan 2011, >300 citations)

Variabilities of light curve are naturally explained!



Summary

- Changes in the MHD turbulence paradigm necessitates revision of CR theories. Anisotropy of turbulence should be accounted for.
- Compressible fast modes dominates CR transport through direct scattering. CR transport therefore varies from place to place.
- Near sources, GCRs <100GeV, and in collisionless plasma, instabilities are more important.
- CR perpendicular transport is diffusive in large scale turbulence and superdiffusive on small scales. As the result, the difference between perpendicular shock and parallel shock diminishes in particle acceleration.
- Existing codes (GalProp, Dragon, etc) should be modified to account for these new understandings.

Opportunities for Summer students, masters, PhDs… at DESY & UP!

Outline

- Basic formalism and interaction mechanism.
- Cosmic Ray (CR) scattering by numerically tested models of turbulence.
- Nonlinear theory and numerical testings
- Perpendicular transport
- Implications for various astrophysical problems
- Instabilities and Back-reaction of CRs (small scale)
- Turbulent reconnection model for γ ray burst (GRBs)