Exercise n. 1

Consider an infinite plane shock wave at z=0 as in the figure



Particles are accelerated at the shock and are free to leave the system from a region at $z=z_0$. Calculate the spectrum of accelerated particles and the flux of particles across the surface at $z=z_0$ in the test particle approximation. Discuss the physical relevance of the solution for cosmic ray physics

Hint to the solution: solve the transport equation

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x} \left[D \frac{\partial f}{\partial x} \right] - u \frac{\partial f}{\partial x} + \frac{1}{3} \frac{du}{dx} p \frac{\partial f}{\partial p} + Q(x, p, t)$$

separately at the shock and for z<0 (upstream) in the stationary situation, using the correct boundary condition at $z=z_0$ and assuming that the injection term is a delta function in both p and z (at z=0).

Exercise n. 2

Consider an infinite plane shock wave at z=0 as in the figure



Assume that there is no fresh injection of particles at the shock but the plasma that enters the shock from upstream carries with it a population of nonthermal particles with spectrum $f_{\infty}(p) \sim p^{-a}$. Discuss what happens to these particles at the shock and how the result changes depending on whether "a" is smaller or larger than 3r/(r-1) where $r=u_1/u_2$ is the compression factor. Discuss the physical relevance of the solution for cosmic ray physics. What happens to the total energy of particles? and to the total number?

Hint to the solution: solve the transport equation

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x} \left[D \frac{\partial f}{\partial x} \right] - u \frac{\partial f}{\partial x} + \frac{1}{3} \frac{du}{dx} p \frac{\partial f}{\partial p} + Q(x, p, t)$$

in the stationary case with the boundary condition that $f(z=-\infty,p)=f_{\infty}(p)$, recalling that the injection term is assumed to vanish.