Exercise n. 1

Consider an infinite plane shock wave at $z=0$ as in the figure

Particles are accelerated at the shock and are free to leave the system from a region at $z=z_0$. Calculate the spectrum of accelerated particles and the flux of particles across the surface at $z=z_0$ in the test particle approximation. Discuss the physical relevance of the solution for cosmic ray physics.

**Hint to the solution:** solve the transport equation

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial x} \left[ D \frac{\partial f}{\partial x} \right] - u \frac{\partial f}{\partial x} + \frac{1}{3} \frac{du}{dx} p \frac{\partial f}{\partial p} + Q(x, p, t)
\]

separately at the shock and for $z<0$ (upstream) in the stationary situation, using the correct boundary condition at $z=z_0$ and assuming that the injection term is a delta function in both $p$ and $z$ (at $z=0$).
Exercise n. 2

Consider an infinite plane shock wave at \( z=0 \) as in the figure

Assume that there is no fresh injection of particles at the shock but the plasma that enters the shock from upstream carries with it a population of non-thermal particles with spectrum \( f_{\infty}(p) \sim p^{-a} \). Discuss what happens to these particles at the shock and how the result changes depending on whether “\( a \)” is smaller or larger than \( 3r/(r-1) \) where \( r = u_1/u_2 \) is the compression factor. Discuss the physical relevance of the solution for cosmic ray physics. What happens to the total energy of particles? and to the total number?

**Hint to the solution:** solve the transport equation

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial x} \left[ D \frac{\partial f}{\partial x} \right] - u \frac{\partial f}{\partial x} + \frac{1}{3} \frac{du}{dx} p \frac{\partial f}{\partial p} + Q(x,p,t)
\]

in the stationary case with the boundary condition that \( f(z=-\infty,p) = f_{\infty}(p) \), recalling that the injection term is assumed to vanish.