NON-RADIAL OSCILLATIONS OF COLOR SUPERCONDUCTING SELF-BOUND QUARK STARS.

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4) Perturbed model (Cowling approximation).
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1) INTRODUCTION

**Compact stars** (without rotation) can oscillate in different forms:

**Radial oscillations:** They are useful to study the stability of compact stars when perturbed.

**Non-radial oscillations:** They are sources of gravitational radiation and their observation is a very important tool to study the internal composition of compact stars.
FAMILIES OF NON-RADIAL MODES

**f-modes**: frequency is proportional to the square root of the mean density of the star.

**p-modes**: pressure is the restoring force, frequencies are greater than those of the f-modes.

**g-modes**: buoyancy is the restoring force, frequencies are lower than those of the f-modes. They are present when there exist temperature gradients or density discontinuities.

**w-modes**: they are pure gravitational modes, fluid is not perturbed.
How we calculate the f and p modes?

1) Choose an EoS (hadronic/quarks).

2) Construct an equilibrium model by solving the TOV equations => pressure, energy density profiles, etc: \( p(r) \), \( \rho(r) \), ...

3) Solve the equations for the perturbed model (oscillation equations).
2) EQUATIONS OF STATE

Hadronic matter

We use the relativistic field model to describe hadronic matter. We adopt the following Lagrangian [Glendenning & Moszkowski 1991]

\[ \mathcal{L}_H = \sum_B \bar{\psi}_B [\gamma_\mu (i \partial^\mu - g_{\omega B} \omega^\mu - \frac{1}{2} g_{\rho B} \vec{\tau} \cdot \vec{\rho}^\mu) \]

\[ - (m_B - g_{\sigma B} \sigma) \psi_B + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) \]

\[ - \frac{1}{4} \omega_{\mu \nu} \omega^{\mu \nu} + \frac{1}{2} m_{\omega}^2 \omega_\mu \omega^\mu - \frac{1}{4} \vec{\rho}_{\mu \nu} \cdot \vec{\rho}^{\mu \nu} \]

\[ + \frac{1}{2} m_{\rho}^2 \vec{\rho}_{\mu} \cdot \vec{\rho}^\mu - \frac{1}{3} b m_{n} (g_{\sigma} \sigma)^3 - \frac{1}{4} c (g_{\sigma} \sigma)^4 \]

\[ + \sum_L \bar{\psi}_L [i \gamma_\mu \partial^\mu - m_L] \psi_L, \]

For matter composed by baryons, mesons, and leptons. For more details see [Lugones et al. 2010].
We use two different set of parameters shown in the following table [Glendenning & Moszkowski 1991, Lalazissis 1997]:

<table>
<thead>
<tr>
<th>Set</th>
<th>GM1</th>
<th>NL3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_\sigma$ (MeV)</td>
<td>512</td>
<td>508.194</td>
</tr>
<tr>
<td>$m_\omega$ (MeV)</td>
<td>783</td>
<td>782.501</td>
</tr>
<tr>
<td>$m_\rho$ (MeV)</td>
<td>770</td>
<td>763</td>
</tr>
<tr>
<td>$g_\sigma$</td>
<td>8.91</td>
<td>10.217</td>
</tr>
<tr>
<td>$g_\omega$</td>
<td>10.61</td>
<td>12.868</td>
</tr>
<tr>
<td>$g_\rho$</td>
<td>8.196</td>
<td>8.948</td>
</tr>
<tr>
<td>$b$</td>
<td>0.002947</td>
<td>0.002055</td>
</tr>
<tr>
<td>$c$</td>
<td>-0.001070</td>
<td>-0.002651</td>
</tr>
<tr>
<td>$M_{max}$</td>
<td>2.32</td>
<td>2.73</td>
</tr>
</tbody>
</table>
Strange Quark matter
We use Witten hypothesis.

Bag model and QCD corrections

We use the modified bag model [Alford et al. 2005, Weissenborn et al. 2011]

\[ \Omega_{QM} = \sum_{i=u,d,s,e} \Omega_i + \frac{3\mu^4}{4\pi^2} (1 - a_4) + B_{\text{eff}} \]

Where \( B_{\text{eff}} \) is the bag constant and the \( a_4 \) term is related to corrections from QCD.

\( a_4 = 1 \) corresponds to no QCD corrections.

Small values of \( a_4 < 1 \), corresponds to stronger corrections.
Color Flavor Locked phase (Color superconductivity)

There are Cooper pairs between quarks of different flavor and color.

We use the bag model [G. Lugones & J. Horvath 2002 ]

\[ \Omega_{CFL} = \Omega_{\text{free}} - \frac{3}{\pi^2} \Delta^2 \mu^2 + B \]

\( \Omega_{\text{free}} \) includes the potential for free quarks: up, down, and strange.

Where \( \Delta \) is the superconducting gap, and \( B \) is the bag constant.
How we set the parameters?

1) We require non-strange quark matter to have binding energy per baryon higher than that of the most stable atomic nucleus $^{56}$Fe (Farhi & Jaffe 1984).

$$E = \left( \frac{\varepsilon}{n_B} \right)_{u, d} \geq m_n = 939 \text{MeV}. $$

2) We also implement the strange matter hypothesis (Bodmer 1971, Witten 1984):

$$E = \left( \frac{\varepsilon}{n_B} \right)_{\text{SQM}} \leq m_n = 939 \text{MeV}. $$

We use set of parameters that allow masses greater than 2 solar masses in the light of recent observations. [Demorest et al. 2010]

By using the Lagrangian of hadronic matter and the potentials for quark matter we can obtain the equation of state: $P = P(\rho)$
3) EQUILIBRIUM MODEL

We consider the following background metric:

\[ ds^2 = -e^{\Phi(r)} dt^2 + e^{\Lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

and the stress-energy tensor of a perfect fluid:

\[ T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + pg_{\mu\nu} \]

We obtain the TOV equations from Einstein’s equations:

\[ \frac{dp}{dr} = - \frac{\rho m}{r^2} \left(1 + \frac{p}{\rho}\right) \left(1 + \frac{4\pi pr^3}{m}\right) \left(1 - \frac{2m}{r}\right)^{-1} \]

\[ \frac{d\Phi}{dr} = - \frac{2}{\rho} \frac{dp}{dr} \left(1 + \frac{p}{\rho}\right)^{-1} \]

\[ \frac{dm}{dr} = 4\pi r^2 \rho \]

\[ p = p(\rho) \]

From TOVs we can obtain \( m(r) \), \( p(r) \), and other necessary quantities to solve the perturbed model.
4) PERTURBED MODEL

We consider the perturbed metric \( g_{\mu\nu} = g^{(B)}_{\mu\nu} + h_{\mu\nu} \)

\[
h_{\mu\nu} = \begin{pmatrix}
  r^l \hat{H} e^{2\Phi} & i\omega r^{l+1} \hat{H}_1 & 0 & 0 \\
i\omega r^{l+1} \hat{H}_1 & r^l \hat{H} e^{2\Lambda} & 0 & 0 \\
0 & 0 & r^{l+2} \hat{K} & 0 \\
0 & 0 & 0 & r^{l+2} \hat{K} \sin^2 \theta
\end{pmatrix} Y^l_m e^{i\omega t}
\]

And obtain the perturbed Einstein’s tensor: \( G_{\mu\nu} = G^{(B)}_{\mu\nu} + \delta G_{\mu\nu} \)

We also consider perturbations in the fluid

\[
\xi_r = \frac{r^l}{r} e^\Lambda \hat{V} Y^l_m e^{i\omega t},
\]

\[
\xi_\theta = -\frac{r^l}{r^2} e^\Lambda \hat{V} \frac{\partial}{\partial \theta} Y^l_m e^{i\omega t},
\]

\[
\xi_\phi = -\frac{r^l}{r^2 \sin^2 \theta} e^\Lambda \hat{V} \frac{\partial}{\partial \phi} Y^l_m e^{i\omega t}
\]

And obtain the perturbed stress-energy tensor: \( T_{\mu\nu} = T^{(B)}_{\mu\nu} + \delta T_{\mu\nu} \)
We put the perturbed quantities into the Einstein’s equations and obtain the perturbed equations:

\[ G_{\mu\nu}^{(B)} = G_{\mu\nu} + \delta G_{\mu\nu} \quad \rightarrow \quad G_{\mu\nu} = T_{\mu\nu} \quad \rightarrow \quad \delta G_{\mu\nu} = \delta T_{\mu\nu} \]

We will consider the Cowling approximation: If the oscillations are present near the surface, the gravitational field is weakly perturbed, and we can set to zero the metric perturbations [MacDermott 1983].

Then, we can obtain the oscillation equations [Sotani et al. 2011]

\[
W' = \frac{d\rho}{dP} \left[ \omega^2 r^2 e^{\Lambda-2\Phi} V + \Phi' W \right] - \ell(\ell + 1)e^{\Lambda} V.
\]

\[
V' = 2\Phi' V - e^{\Lambda} \frac{W}{r^2}.
\]

The coefficients are determined from the TOV equations (equilibrium model)
Boundary conditions:

At the center we have the regularity conditions

\[ V(r) = -Cr^l / l + O(r^{l+2}) \]

\[ W(r) = Cr^{l+1} + O(r^{l+3}) \]

At the surface the lagrangian perturbation in the pressure is zero \( \Delta p = 0 \)

\[ \omega^2 r^2 e^{\Lambda - 2\Phi} V + \Phi' W = 0 \]
5) NUMERICAL METHOD

1) TOV equations are solved by a Runge Kutta method (integration until p is zero at the surface)

2) Oscillations equations are solved by a shooting method, which consists in:
   a) Choose a trial frequency, use boundary conditions at r = 0.
   b) Numerically integrate the Oscillations equations from the center to the surface.
   c) Check if the boundary conditions at the surface are satisfied, if not, take again a new trial frequency. If yes we have calculated the frequency.
6) RESULTS

f-mode (fundamental mode)

Blue lines for CFL stars,
Green lines for Bag model with QCD Corrections, black lines for hadronic stars.

1) Large maximum masses for small B and large Δ.
2) For CFL stars, \( f_f \) decreases as Δ is increased.
3) CFL stars have \( f_f \sim 2 - 3 \text{ KHz} \).

2) Profiles for hadron stars and quark stars are qualitatively very different.
3) For hadron stars \( f_f \) increases roughly linearly with the mass.
4) In contrast \( f_f \) doesn’t change considerably with the mass in the case of quark stars.
p1-mode:

1) Again there is a large difference between results for hadron and quark stars.

2) For a hadron stars the frequencies of the p1 modes are typically in the range 4 – 10 kHz.

3) For quark stars frequencies are in the same range for massive stars, but they increase significantly for low mass stars.
7) CONCLUSIONS

1) For quark stars the frequency of the fundamental mode has a small variation with the mass.

2) For CFL stars $f_f$ decreases as $\Delta$ is increased.

3) We have found $f_f \sim 2 - 3$ Khz for parameters of the EoS that result in stars with a maximum mass above 2 solar masses.

4) As in the case of $f_f$, the p1 modes also are very different to the corresponding modes of hadron stars.

5) Cowling approximations is a good tool to study qualitatively the effect of the EoS in the frequencies of the non-radial modes.

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