Introduction to interferometry and VLBI

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BLACK HOLE INITIATIVE

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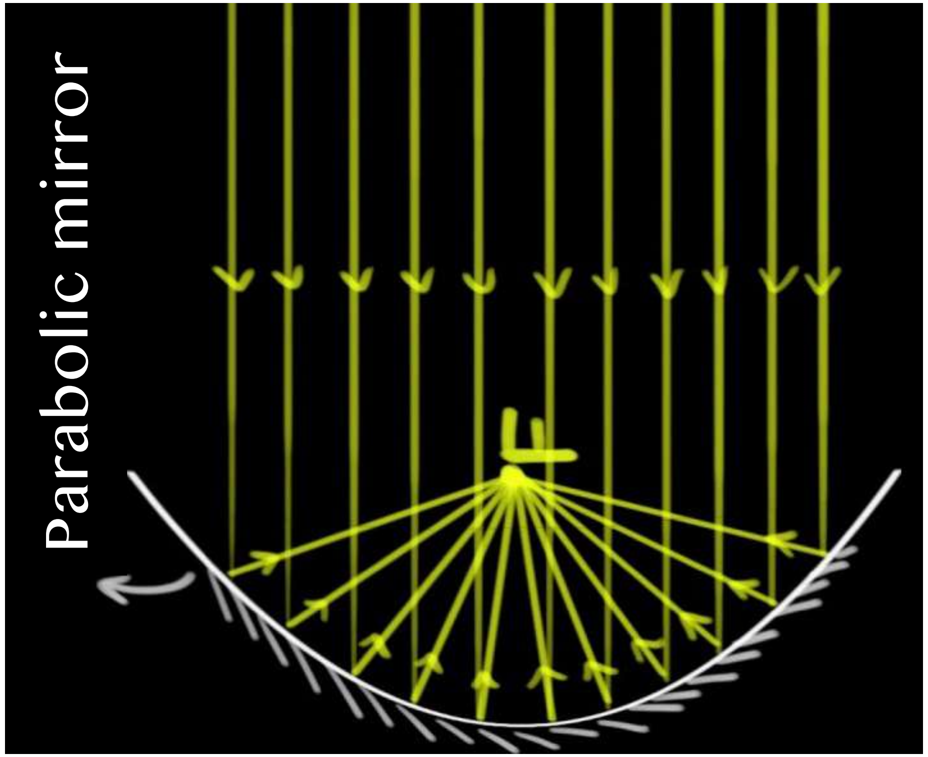
VLA (NM, USA)

ALMA (Chile)

Summary of Class 1 and 2

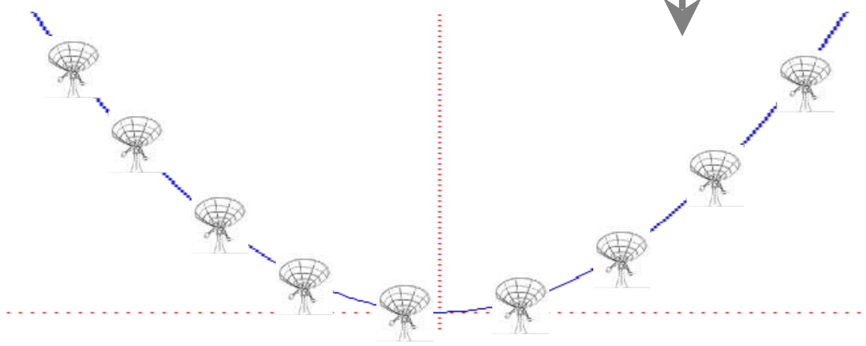
Pictorial principle of interferometry



(Cassegrain) reflector telescope

Replace with

Combination (to form the image at the focus) is done electronically



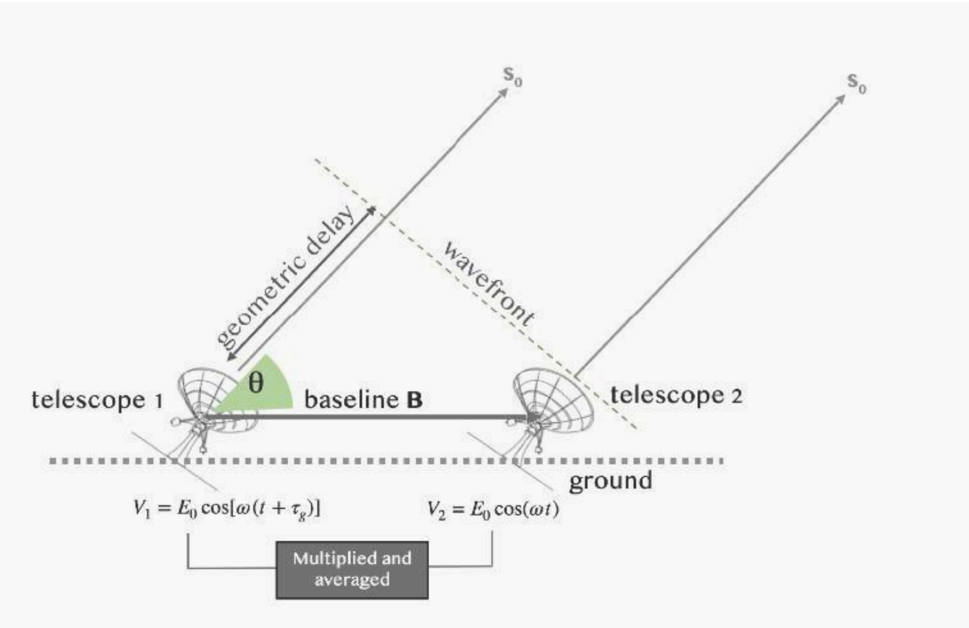
Fundamental results from Class 1 & 2

$$V(u,v) = \iint I(l,m) e^{-2\pi i(ul+vm)} dl dm$$

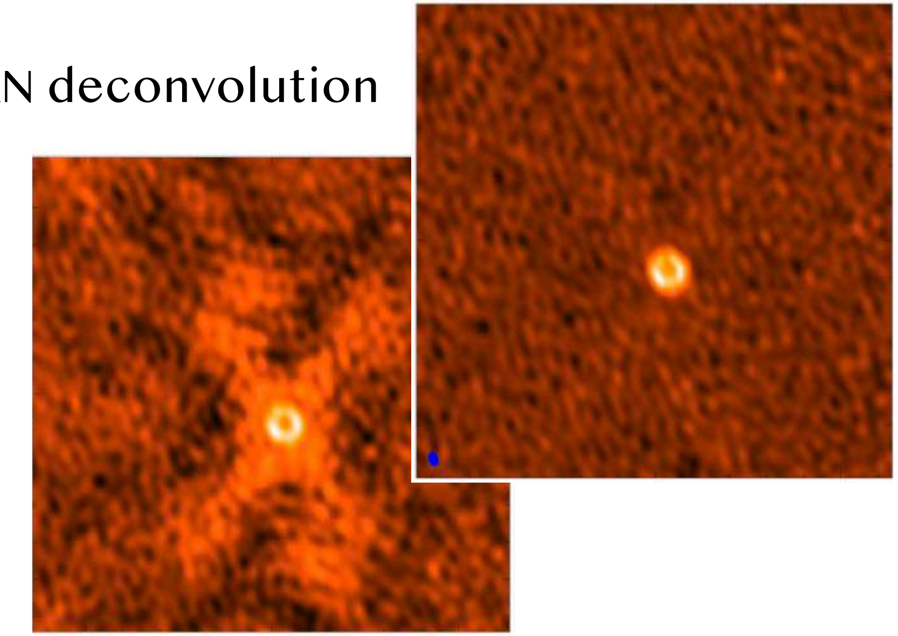
Complex visibility function

$$I(l,m) = \iint V(u,v) e^{2\pi i(lu+mv)} du dv$$

Sky brightness distribution

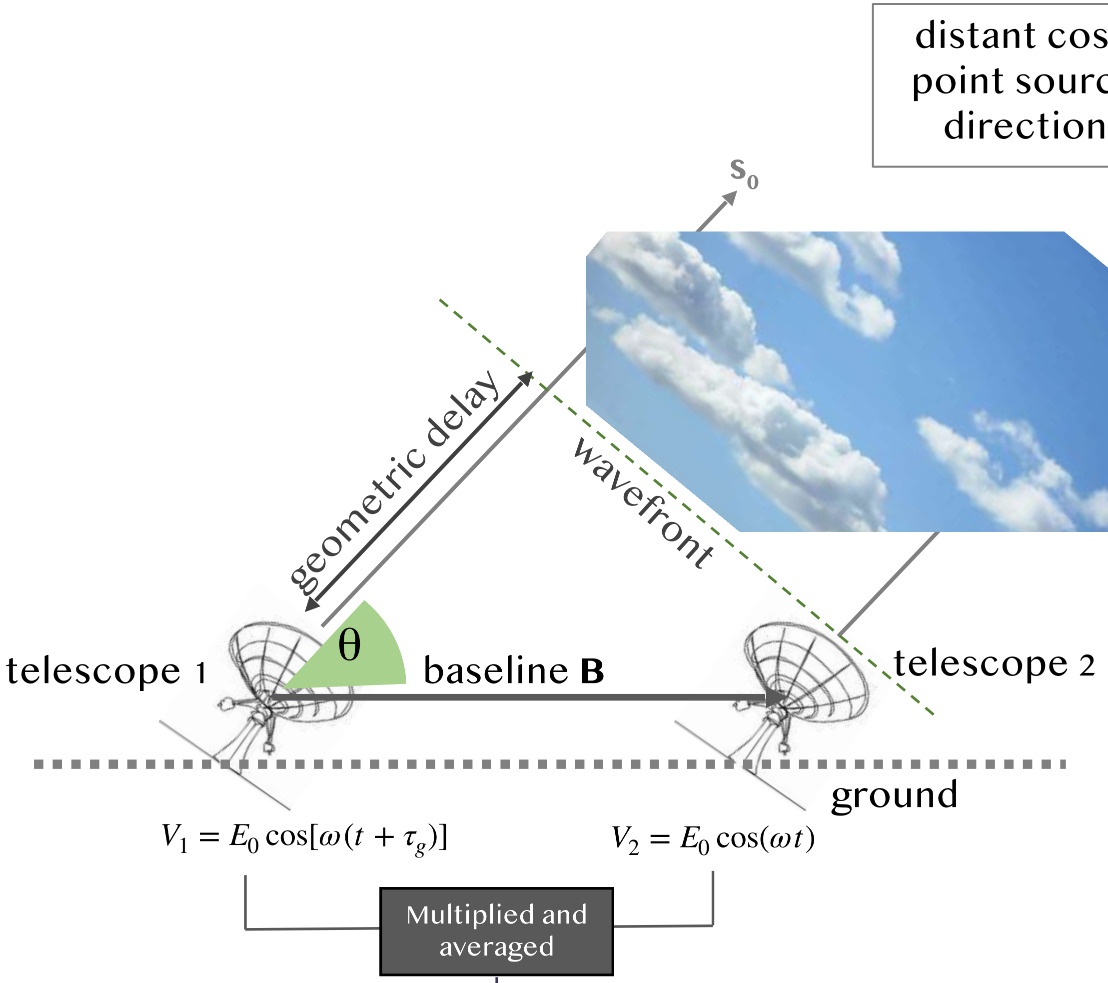


+ CLEAN deconvolution



Part 4: Calibration

Instrumental and atmospheric effects…



$$V(u,v) = \iint I(l,m) e^{-2\pi i(ul+vm)} \frac{dl dm}{\sqrt{1 - l^2 - m^2}}$$

$$V_{i,j}^{obs} = G_{i,j} V_{i,j}^{true}$$

Calibration term called the "gain".
Black box including all kinds of effects

$$G = B \,.\, G \,.\, D \,.\, E \,.\, X \,.\, P \,.\, T \,.\, F$$

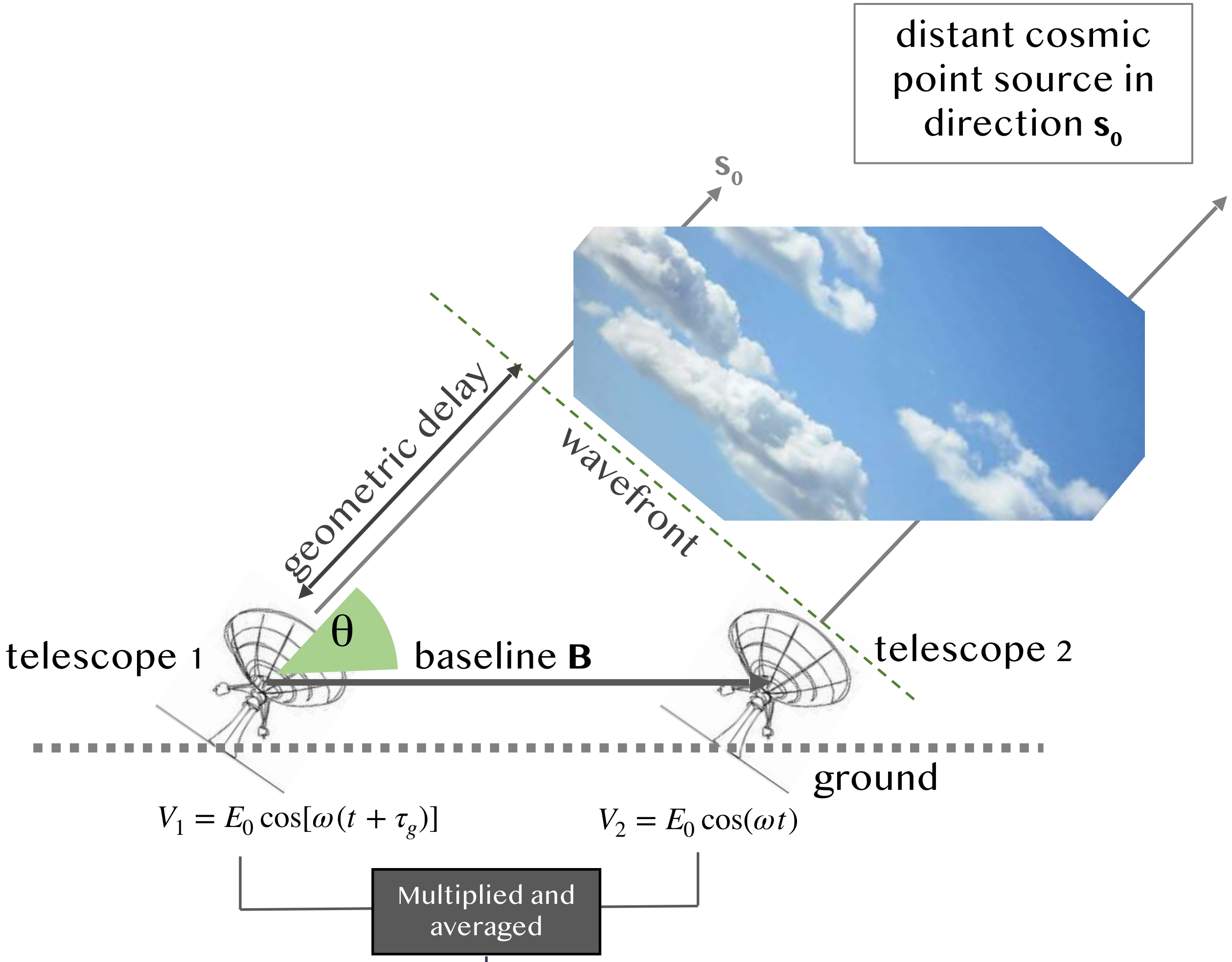
- B: Bandpass
- G: Electronic gains
- D: Polz leakages
- E: Gain curve
- X: Linear polz angle
- P: Parallactic angle
- T: Troposphere
- F: Ionosphere

Typical interferometric observation

- (Maybe) some initial setup strong source (anywhere in the sky)
- Bandpass calibrator (anywhere in the sky)
- Flux calibrator (anywhere in the sky)
- (Maybe) a polarization calibrator (anywhere in the sky)
- Gain calibrator (close to the target)
- Target
- Gain Calibrator (close to the target)
- Target

….

Example of atmospheric gain calibration



$$V(u,v) = \iint I(l,m) e^{-2\pi i(ul+vm)} \frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

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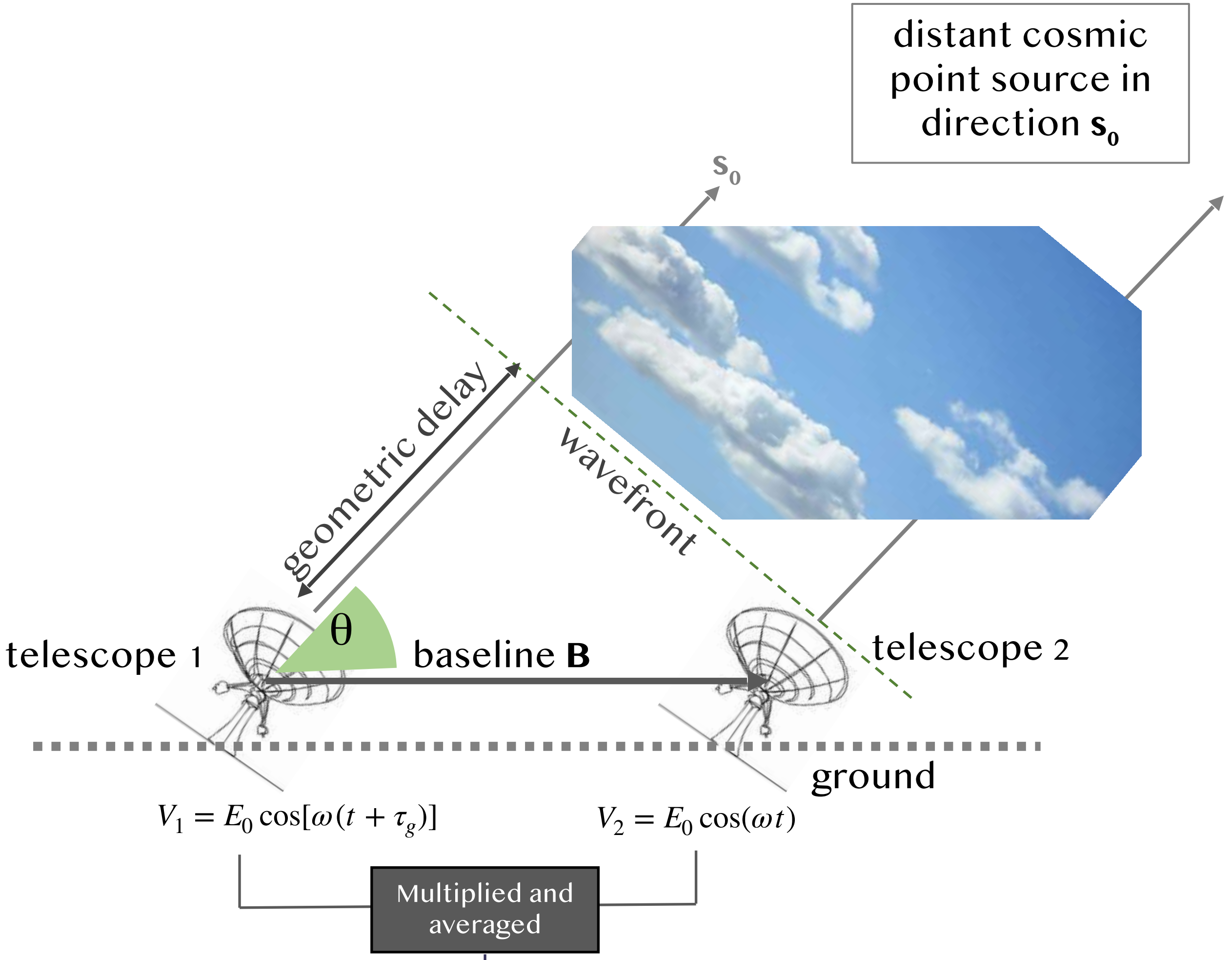

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$$V_{i,j}^{obs} = G_{i,j}V_{i,j}^{true}$$

$$G = B\,.\,G\,.\,D\,.\,E\,.\,X\,.\,P\,.\,T\,.\,F$$

Troposphere

Calibration is assumed to be antenna-based

- $V_{i,j}^{obs} = G_{i,j} V_{i,j}^{true} = G_i G_j^* V_{i,j}^{true}$ $\qquad G = B.G.D.E.X.P.T.F$

- Visibilities are complex quantities, so also are gains:

$$V_{i,j}^{obs} = a_{i,j}^{obs} \exp\left(i\delta_{i,j}^{obs}\right) \qquad V_{i,j}^{true} = a_{i,j}^{true} \exp\left(i\delta_{i,j}^{true}\right)$$

$$G_i = g_i \exp\left(i\phi_i\right) \qquad G_j^* = g_j \exp\left(-i\phi_j\right)$$

$$\Rightarrow \begin{cases} a_{i,j}^{obs} = g_i g_j a_{i,j}^{true} \\ \delta_{i,j}^{obs} = \phi_i - \phi_j + \delta_{i,j}^{true} \end{cases}$$

Calibration scan

- Calibrator source is a point source $V_{i,j}^{true} = \text{constant} \ (\in \mathbb{R}) = a \quad \forall i,j$

$$a_{i,j}^{obs} = g_i g_j a \quad \forall i,j \qquad \delta_{i,j}^{obs} = \phi_i - \phi_j + 0 \quad \forall i,j$$

- Here, a is assumed known (or taken to some arbitrary value) while $a_{i,j}^{obs}$ and $\delta_{i,j}^{obs}$ are observed (i.e. also known). On the other hand, g_i, g_j, ϕ_i and ϕ_j are unknowns.

- Assume we have 30 antennas. Then, we have 30 unknowns (the g_i's) for amplitude gains and 30 unknowns (the ϕ_i's) for the phase gains.

  But we have $\dfrac{30 \times 29}{2} = 435$ equations for each of the amplitude and phase gains.

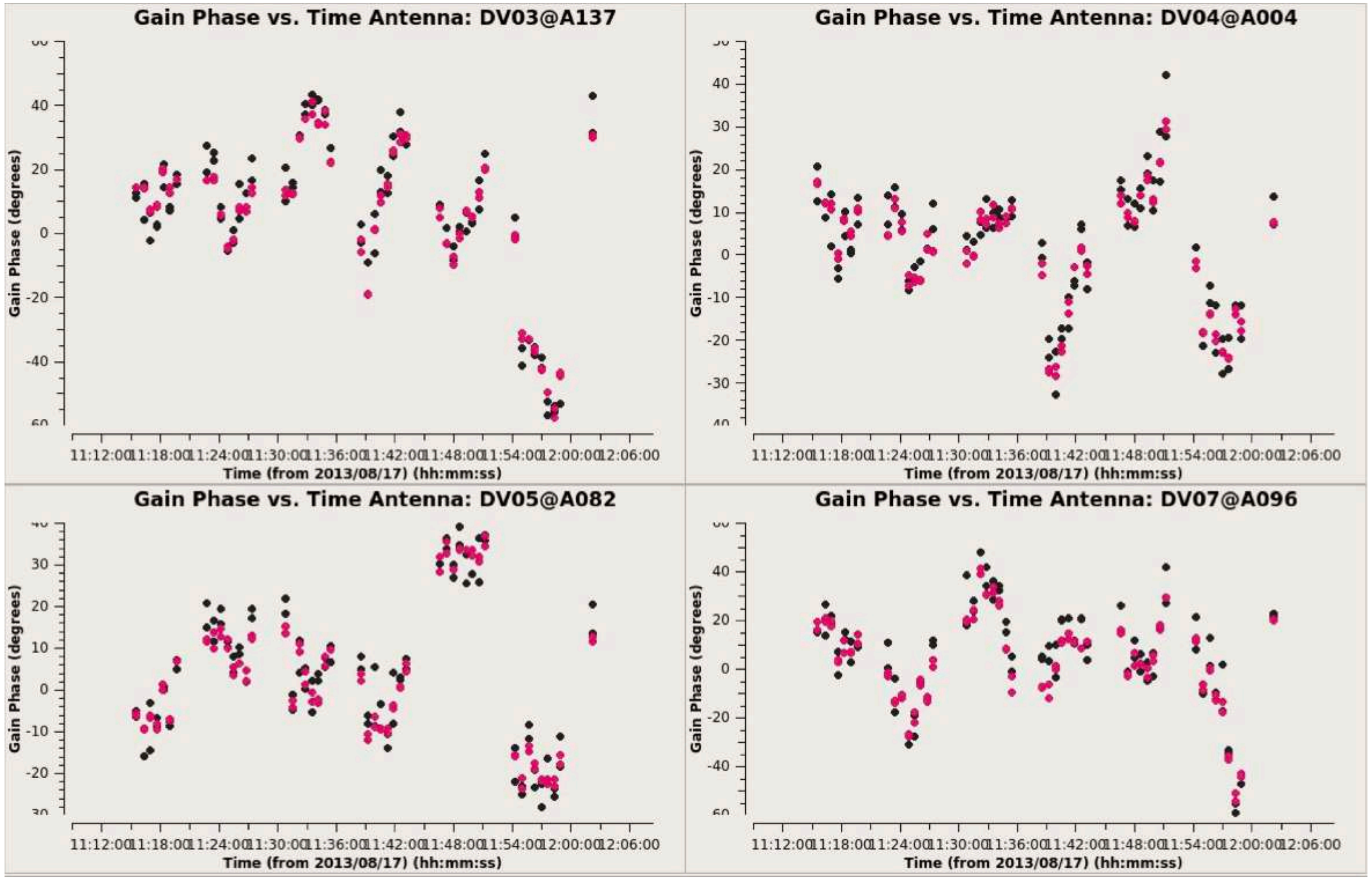
- By solving this 435 system of equation with 30 unknowns, we can measure the values of the g_i's and ϕ_i's.

Typical interferometric observation

- (Maybe) some initial setup strong source (anywhere in the sky)
- Bandpass calibrator (anywhere in the sky)
- Flux calibrator (anywhere in the sky)
- (Maybe) a polarization calibrator (anywhere in the sky)
- Gain calibrator (close to the target) → Gain measurements at the time t_1
- Target
- Gain Calibrator (close to the target) → Gain measurements at the time t_2
- Target

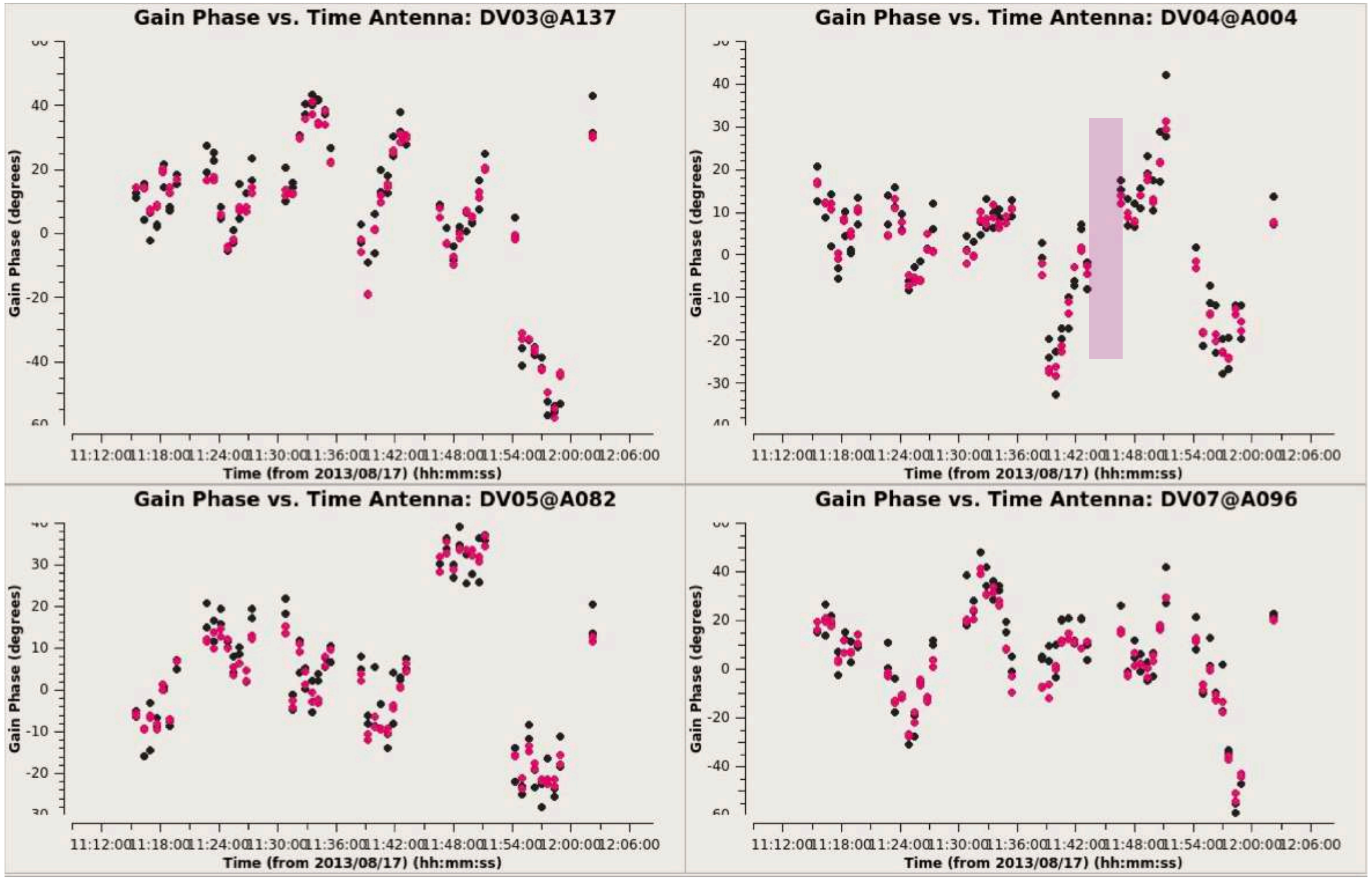
….

Typical phase gain curves

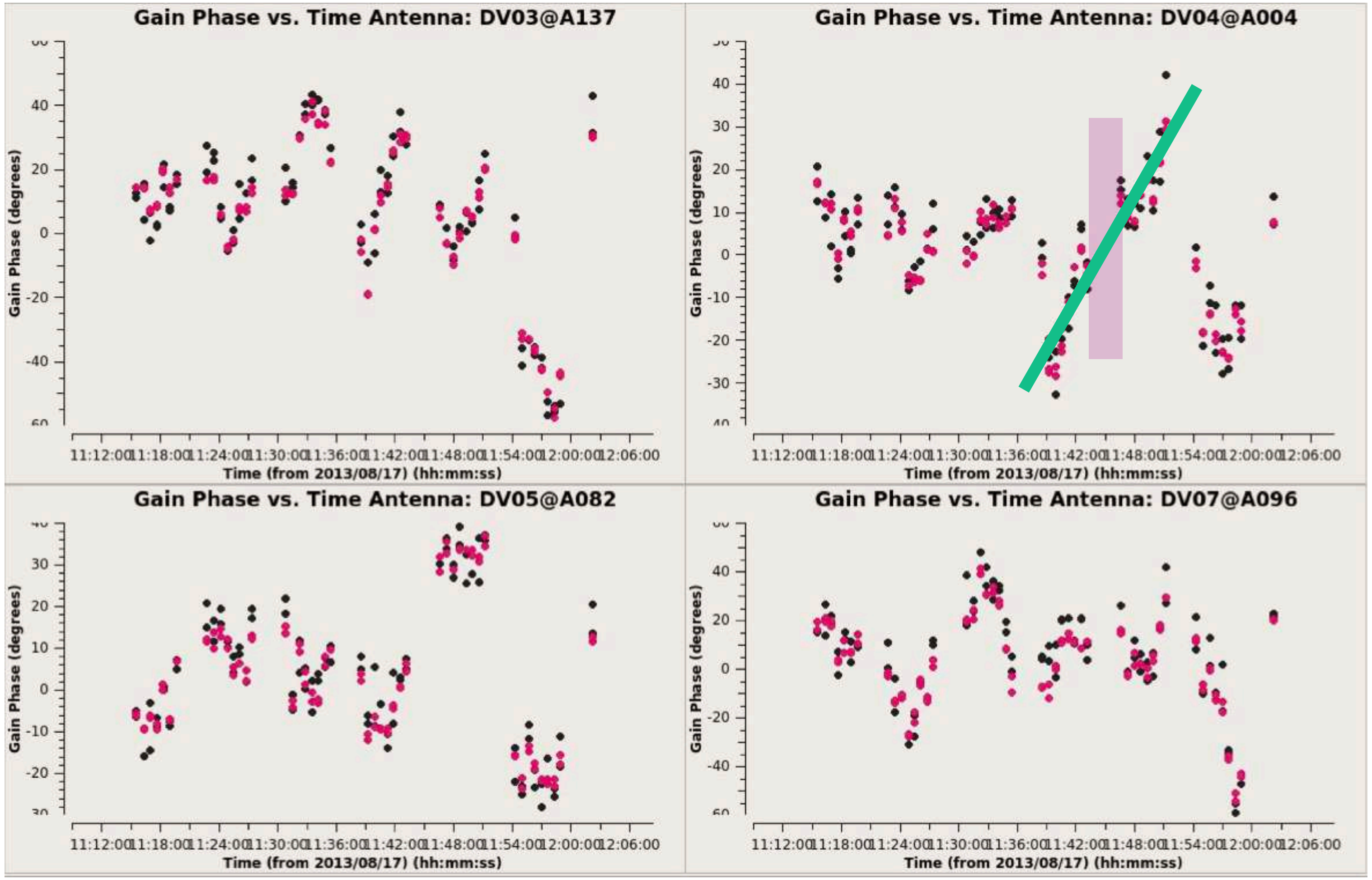


40 degrees

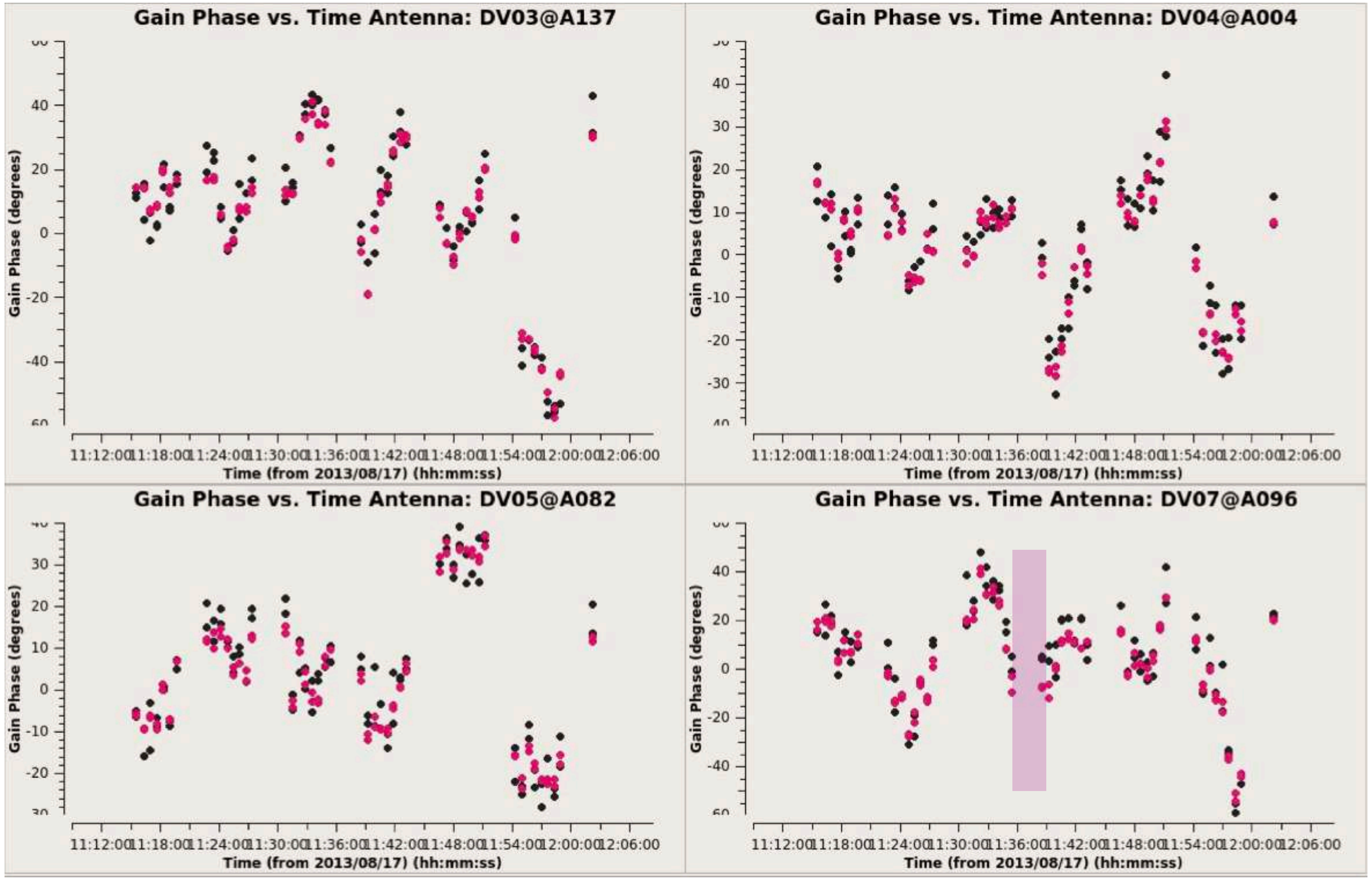
Interpolation



Interpolation



Interpolation



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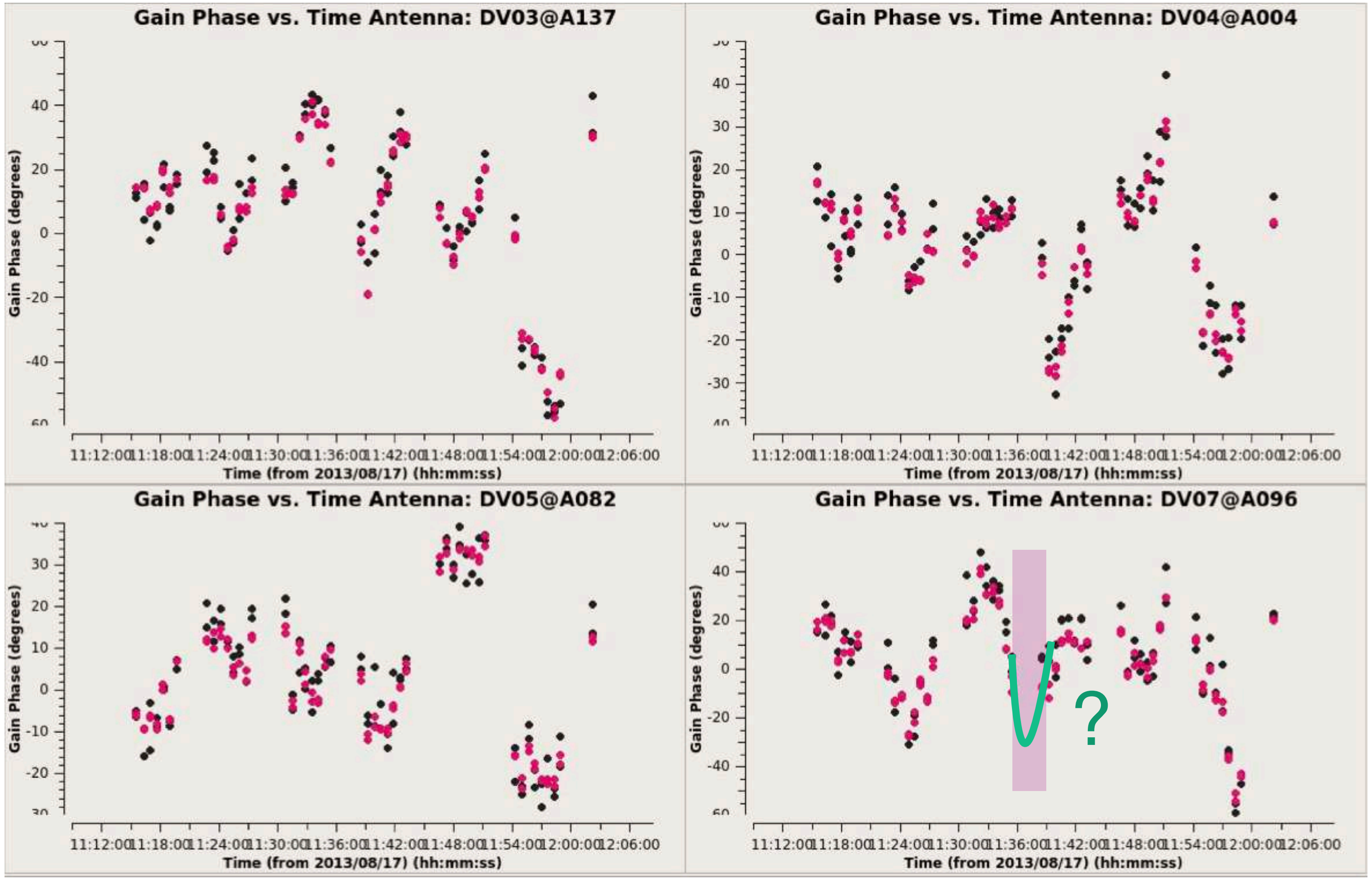

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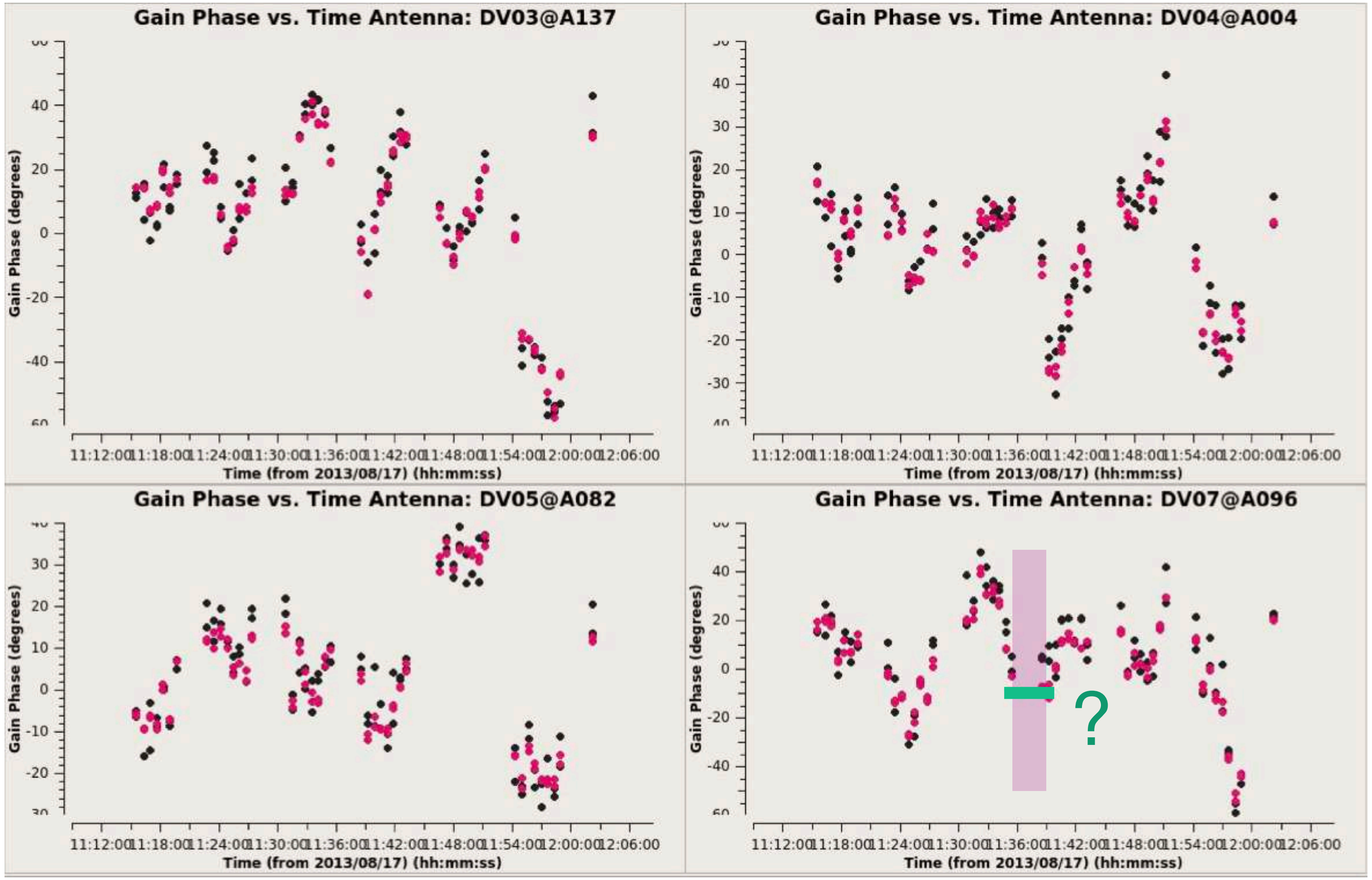
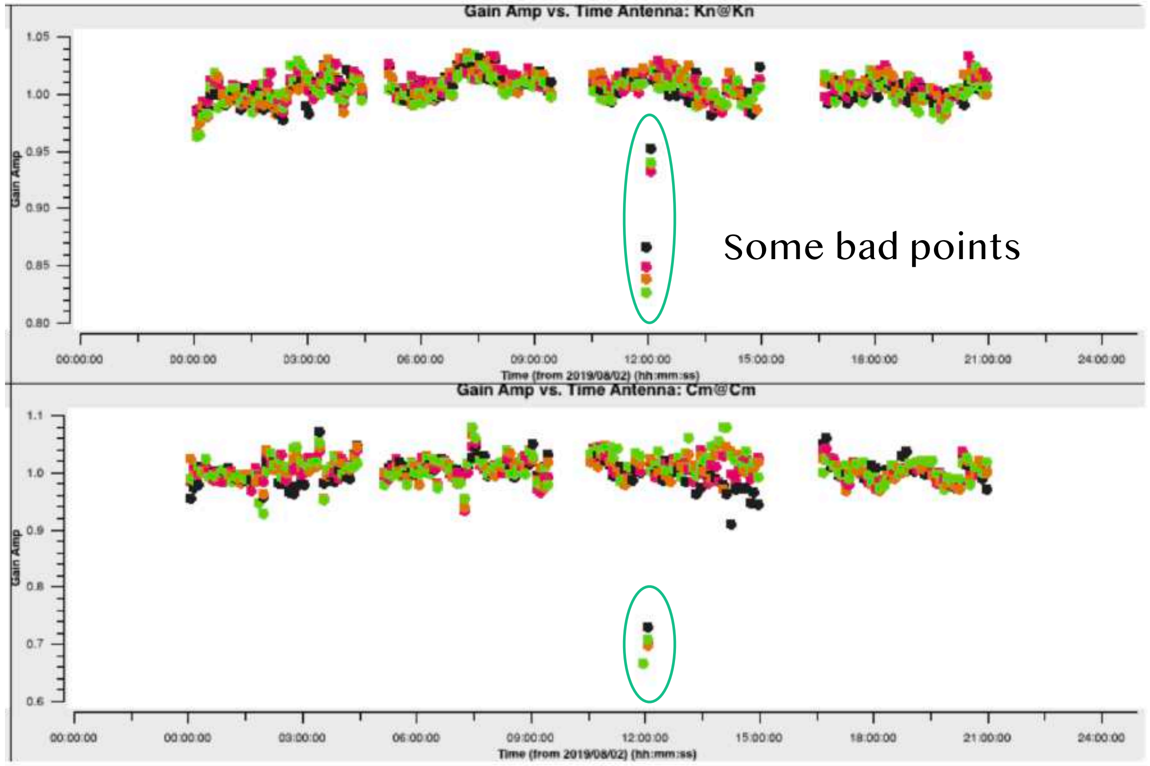
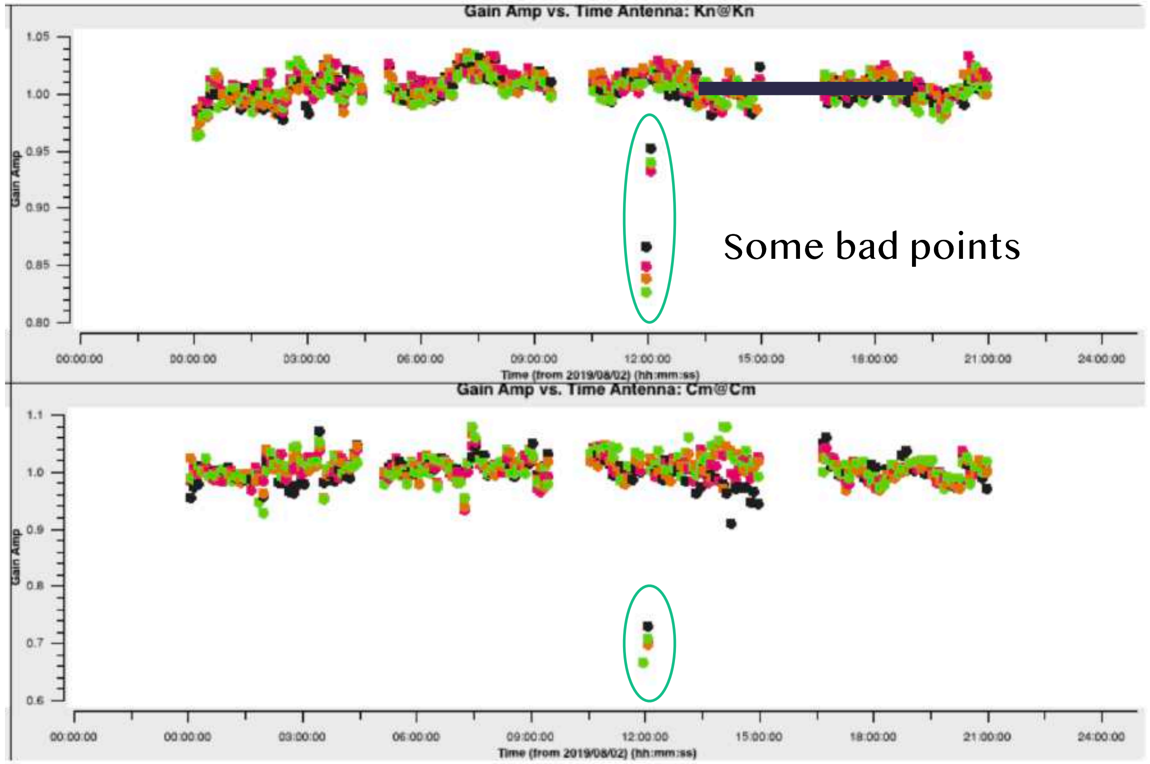


Typical amplitude gain curves

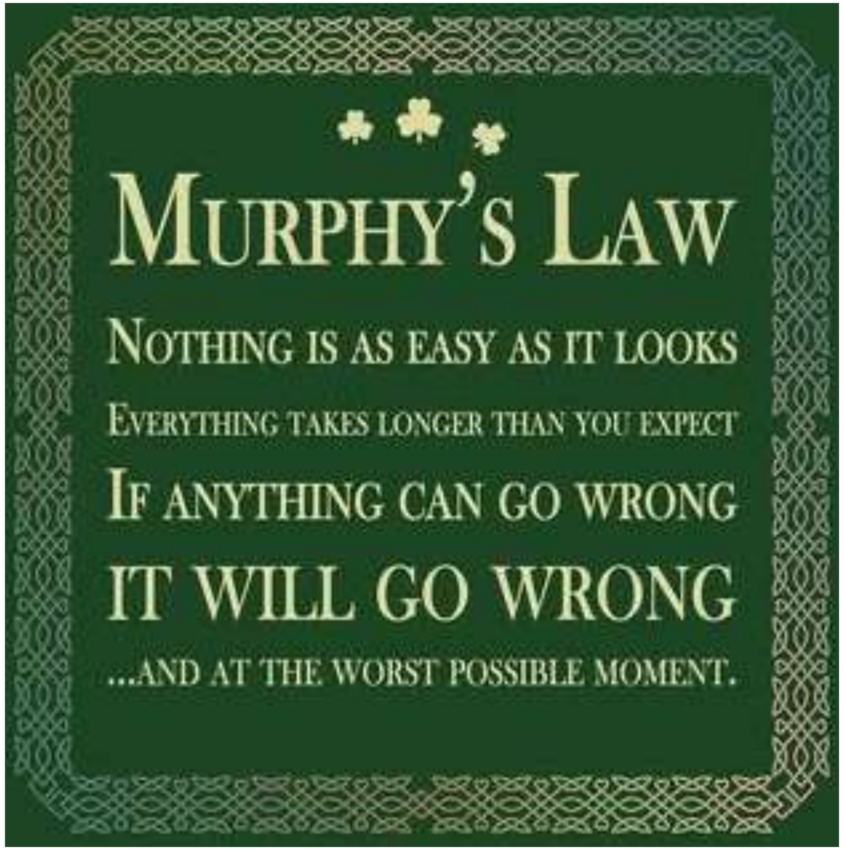


Typical amplitude gain curves



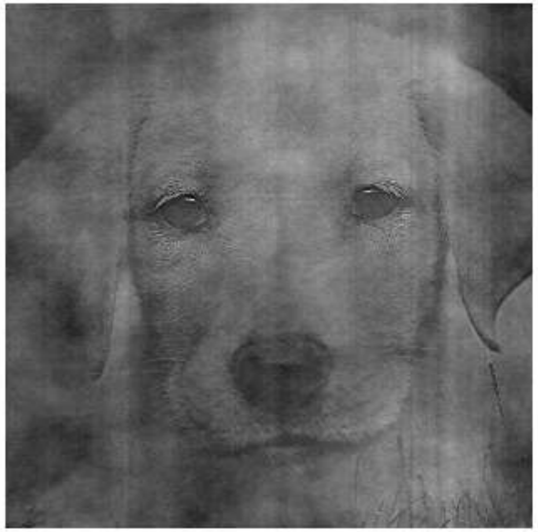
General rules:

Phase varies faster and more erratically than amplitude

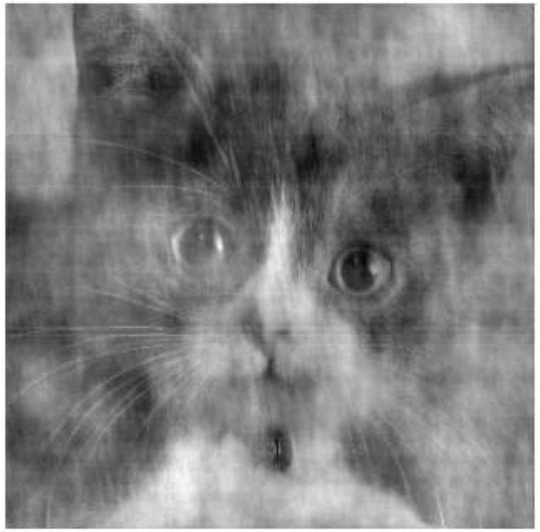


Effect of phase and amplitude errors

Phase of dog and amplitude of cat



Phase of cat and amplitude of dog



General rule:

For imaging, phases are more important than amplitudes

Part 5: A few special topics

Calibration errors

- (Maybe) some initial setup strong source (anywhere in the sky)
- Bandpass calibrator (anywhere in the sky)
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- Gain calibrator (close to the target)
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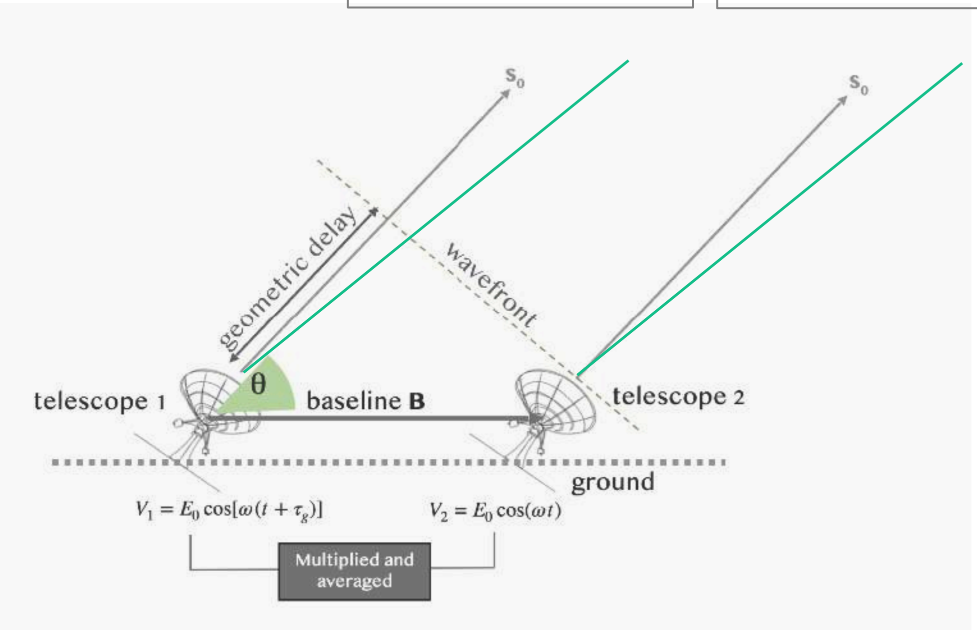
- ....

Target $\mathbf{s_0}$ | Calibrator $\mathbf{s_0'}$



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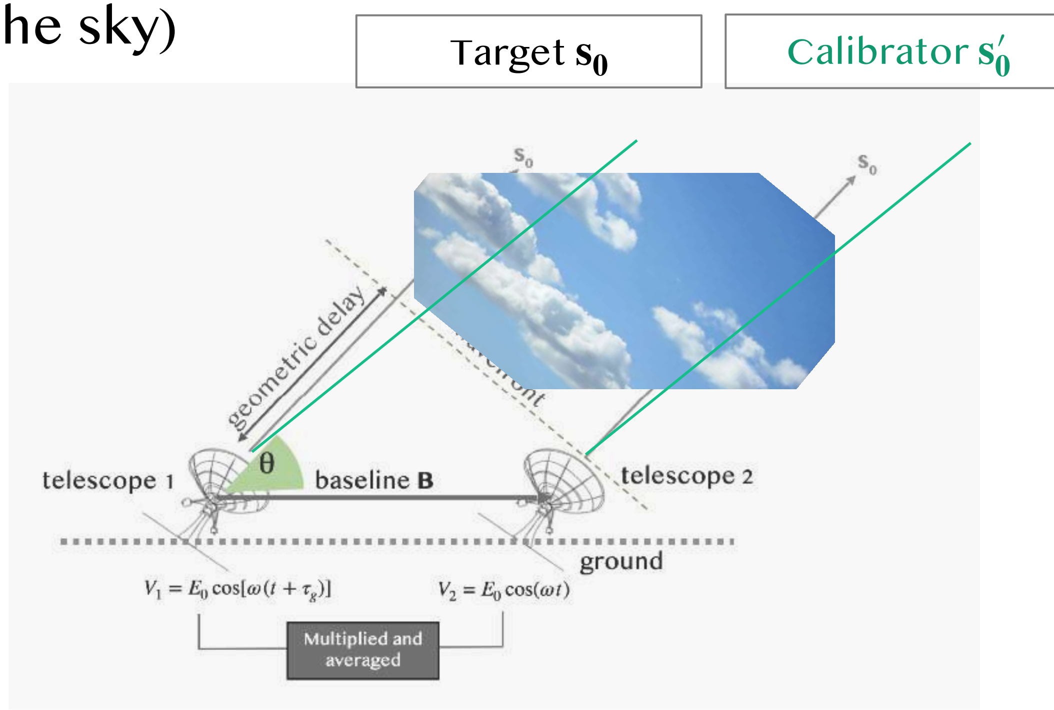
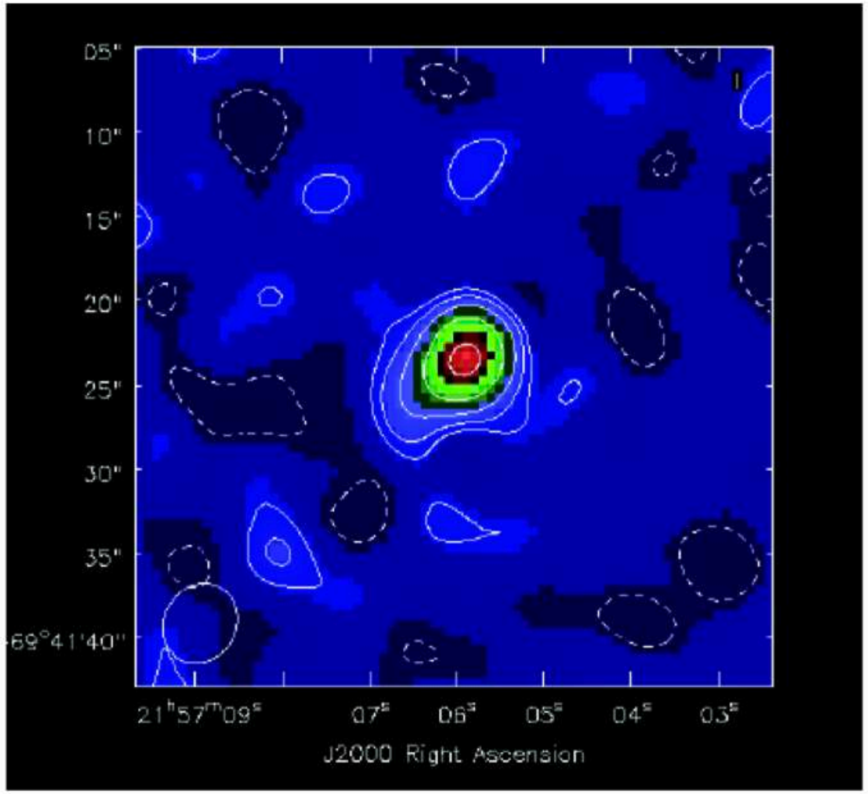


Calibration errors

There are **always** residual calibrator errors after initial calibration.

The effects may not be obvious.

Often seen a higher than expected noise in final image.



Self-calibration to the rescue

If you thought CLEAN was black magic, wait until you see this…

Calibration scan

- Calibrator source is a point source $V_{i,j}^{true} = \text{constant} \ (\in \mathbb{R}) = a \quad \forall i, j$

→ $a_{i,j}^{obs} = g_i g_j a \quad \forall i, j$ $\qquad \delta_{i,j}^{obs} = \phi_i - \phi_j + 0 \quad \forall i, j$

- Here, a is assumed known (or taken to some arbitrary value) while $a_{i,j}^{obs}$ and $\delta_{i,j}^{obs}$ are observed (i.e. also known). On the other hand, g_i, g_j, ϕ_i and ϕ_j are unknowns.

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Target scan

- Assume the first image is "the truth" $V_{i,j}^{true} = \text{TF(initial image)} \quad \forall i, j$

$$\Rightarrow \quad a_{i,j}^{obs} = g_i g_j a_{i,j}^{true} \quad \forall i, j \qquad \delta_{i,j}^{obs} = \phi_i - \phi_j + \delta_{i,j}^{true} \quad \forall i, j$$

- Here, a^{true} and δ^{true} are **known** while $a_{i,j}^{obs}$ and $\delta_{i,j}^{obs}$ are observed (i.e. also known). On the other hand, g_i, g_j, ϕ_i and ϕ_j are unknowns.

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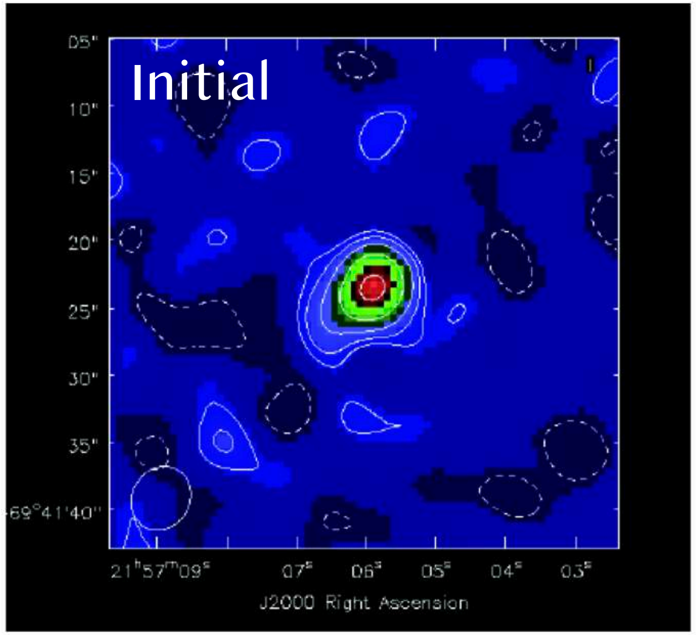
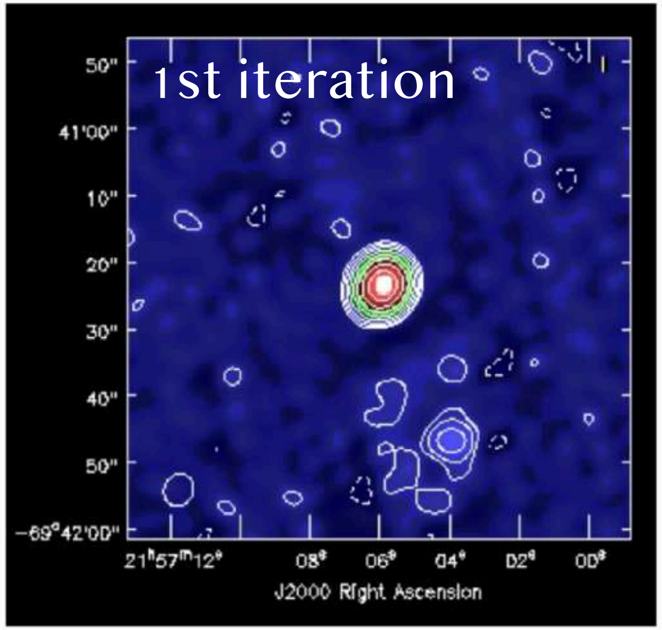
Self-calibration to the rescue

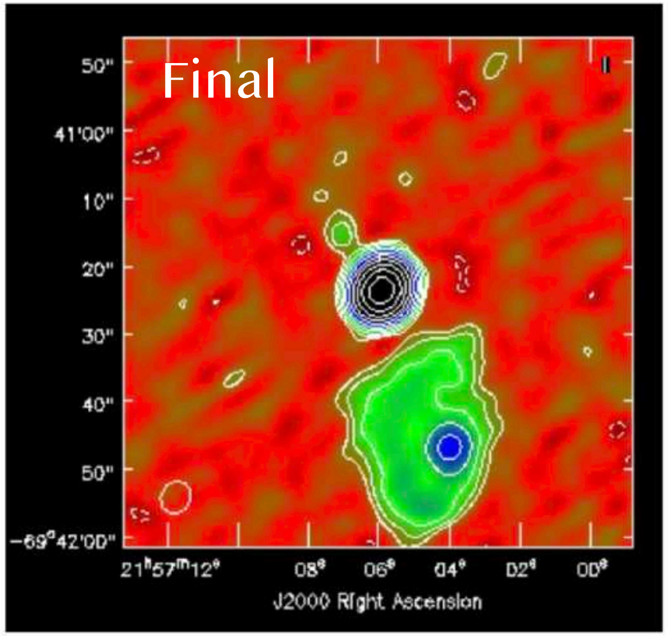
If you thought CLEAN was black magic, wait until you see this…

Only works if (i) you have enough redundancy (baselines) and (ii) sufficient SNR

Iterative process: in the second iteration, take image from first iteration as model (but **always** go back to initial visibilities…)

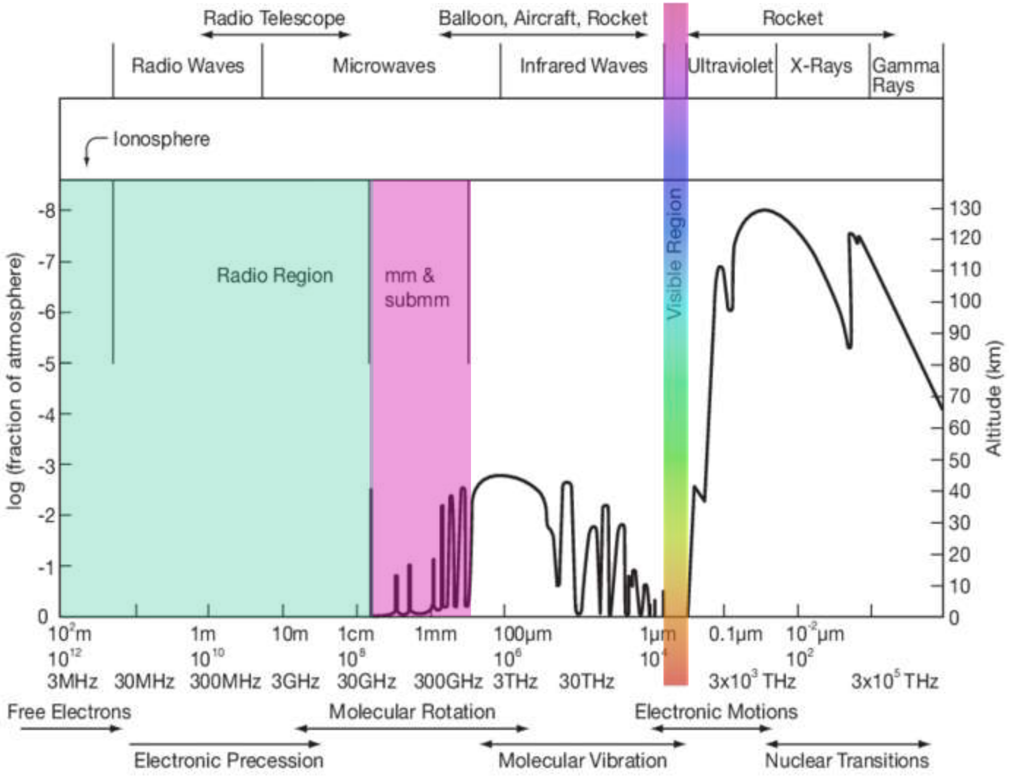






Dynamic range <100 ~500 > 2000

VLBI at mm wavelengths



Calibration becomes harder

Calibration becomes harder

mmVLBI is **really** hard

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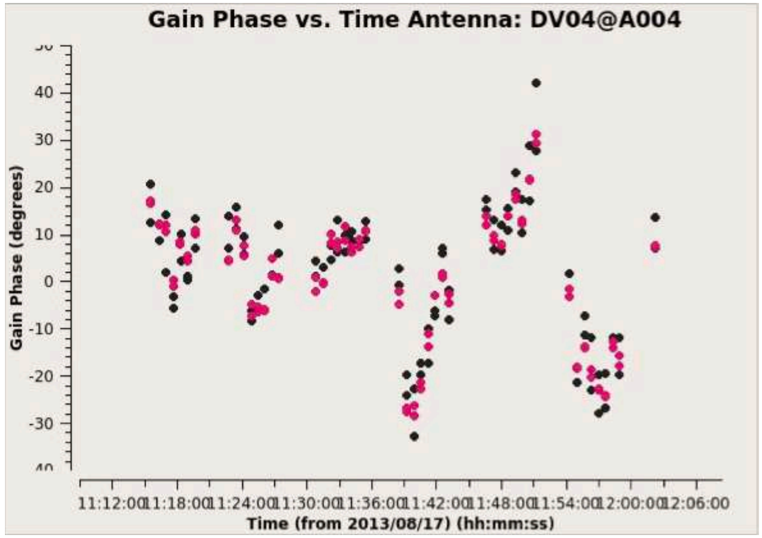


We can't transfer the phase gains

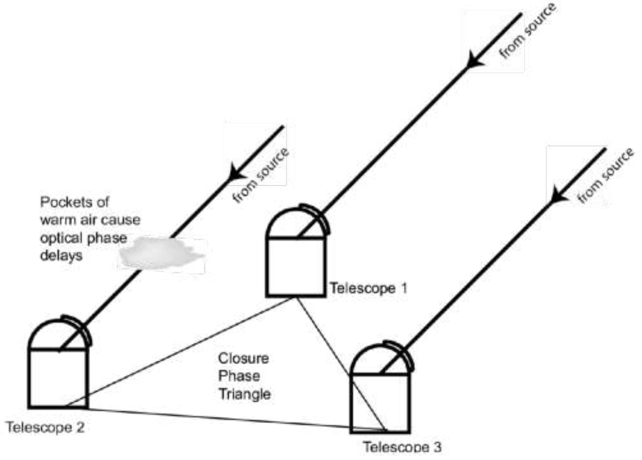
What do we do?

One option is self-calibration using an a-priori initial model (e.g. a small Gaussian)

Another option is to use **closure quantities** and **forward modeling**

And that is **not** black magic…

$$\delta_{i,j}^{obs} = \phi_i - \phi_j + \delta_{i,j}^{true} \quad \forall i, j$$



$$\delta_{1,2}^{obs} = \phi_1 - \phi_2 + \delta_{1,2}^{true}$$

$$\delta_{2,3}^{obs} = \phi_2 - \phi_3 + \delta_{2,3}^{true}$$

$$\delta_{3,1}^{obs} = \phi_3 - \phi_1 + \delta_{3,1}^{true}$$

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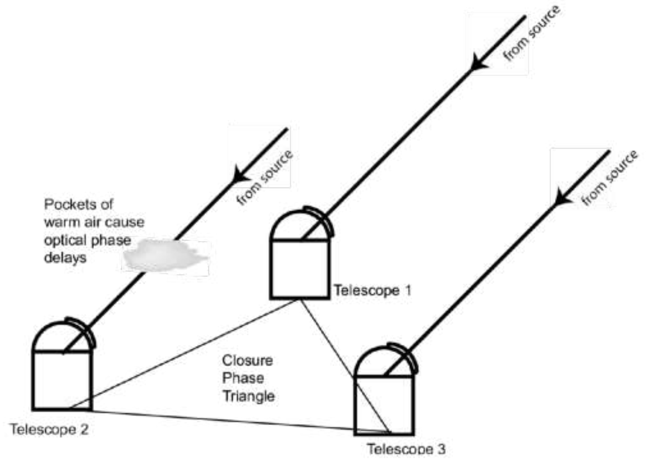


$$\delta_{1,2}^{obs} = \phi_1 - \phi_2 + \delta_{1,2}^{true}$$

+

$$\delta_{2,3}^{obs} = \phi_2 - \phi_3 + \delta_{2,3}^{true}$$

+

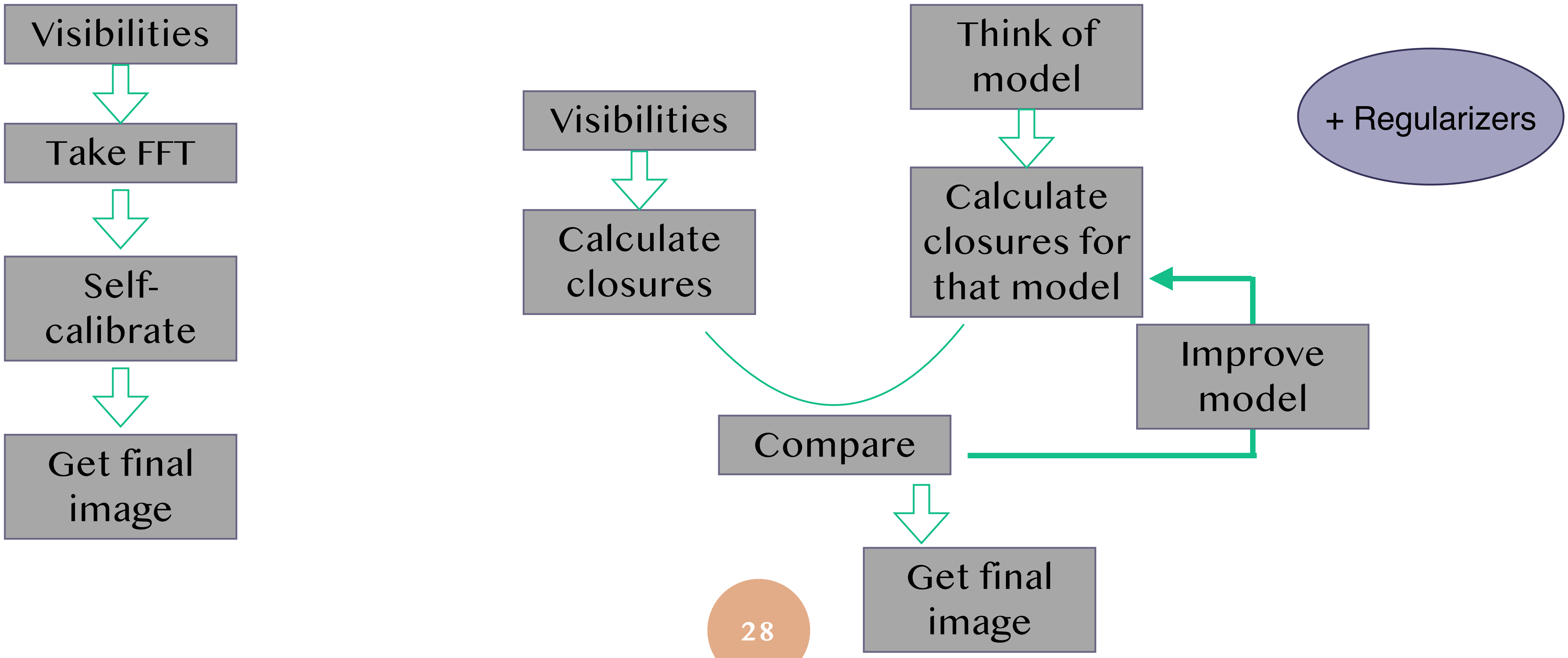
$$\delta_{3,1}^{obs} = \phi_3 - \phi_1 + \delta_{3,1}^{true}$$

$$\delta_{1,2}^{obs} + \delta_{2,3}^{obs} + \delta_{3,1}^{obs} = \delta_{1,2}^{true} + \delta_{2,3}^{true} + \delta_{3,1}^{true}$$

The sum of the phases on triangles of antennas are immune to atmospheric errors

Forward modeling

Problem is: you can't inverse Fourier transform a closure quantities



Forward modeling

Classical optimization problem

$$J(\boldsymbol{I}) = \sum_{\text{data terms}} \alpha_D \chi^2_D(\boldsymbol{I}, \boldsymbol{d}) - \sum_{\text{regularizers}} \beta_R S_R(\boldsymbol{I}).$$

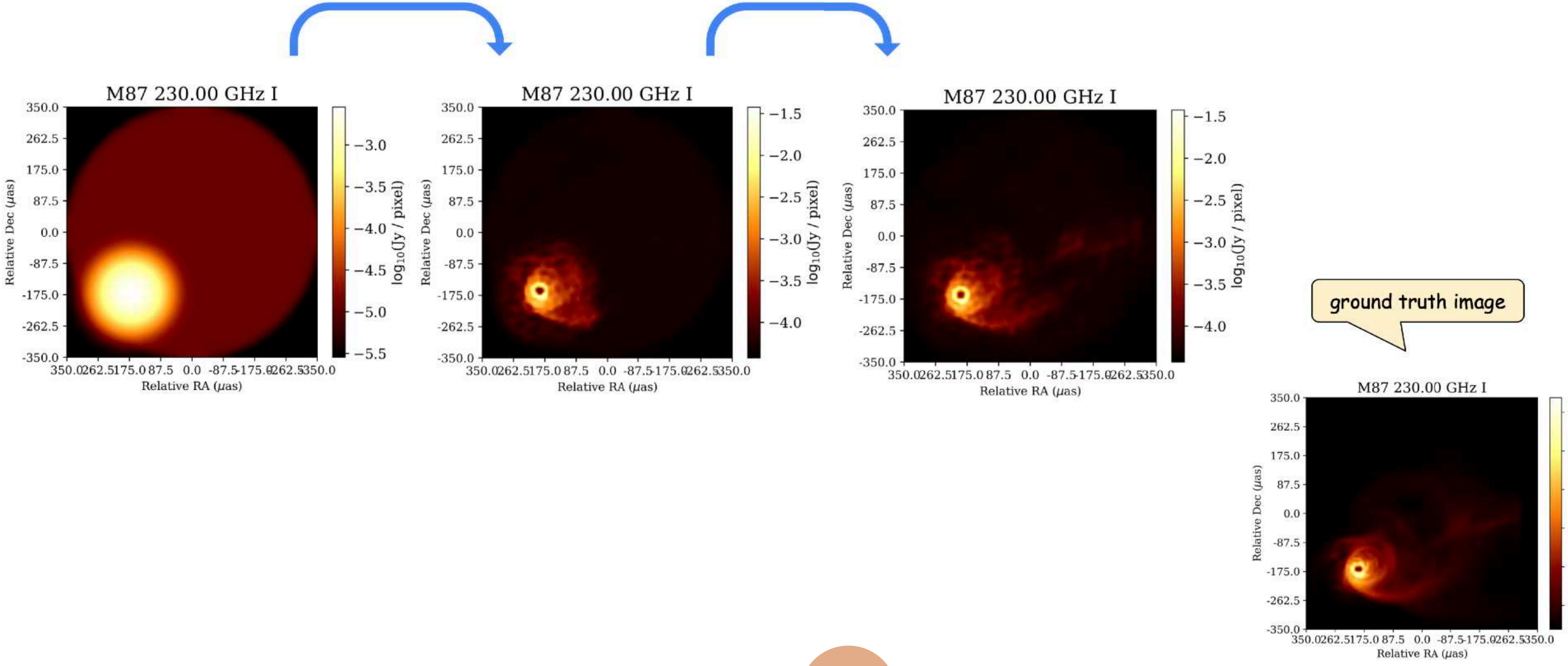
Objective function to be optimized

Comparison between data and model "Likelihood"

Regularizers enforcing some desired properties

Positiveness of emission
Continuity from pixel to pixel
Emission within the field
etc.

EHTim (Chael+2018)



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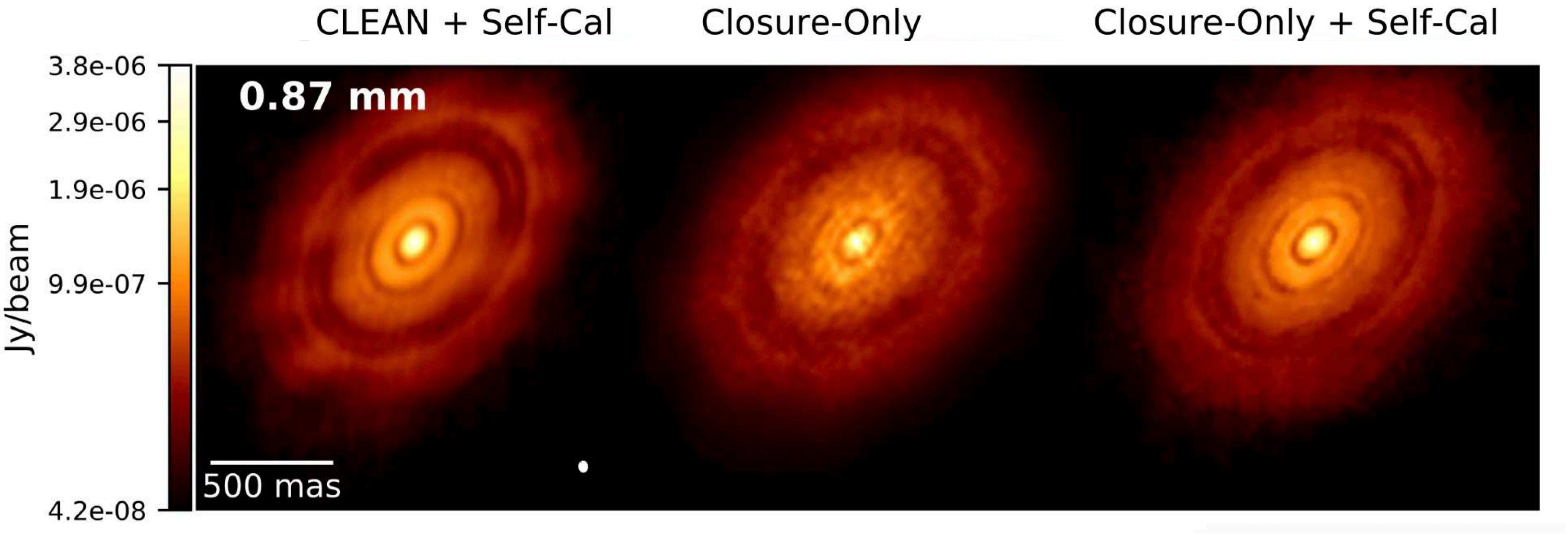

The Fourier transform in a nutshell…

Consider a function $f(z)$ of a complex variable z

We define its Fourier transform as

$$F(u) = \int_{-\infty}^{+\infty} f(z) e^{-2i\pi uz} dz$$

The variable u is the Fourier conjugate of z (z and u are conjugates of each other) and the product $u \,.\, z$ must be a-dimensional.

The inverse Fourier transform is

$$f(z) = \int_{-\infty}^{+\infty} F(u) e^{-2i\pi uz} du$$

Selected useful properties of the Fourier Transform

- Perhaps the most common use of the Fourier Transform (TF) is in signal processing. In this case, the function of interest is usually a function of time, $f(t)$, and the Fourier transform is a function of frequency $F(\omega)$.

- If $f(t)$ is real and even, $F(\omega)$ is real. If $f(t)$ is real and odd, $F(\omega)$ is imaginary.

- The TF of the $f(t) = \delta(t)$ (the delta Dirac function) is constant (and equal to 1).

- Translation property: $TF\left[f(t-t_0)\right] = e^{-i\omega t}TF\left[f(t)\right]$

f(delta) = 1
F(t-to) = e^-iwt F(t)
Consequence of the 0ther twoL
f((delta(t-to)) = cos(wt)
Transform of Gaussian air gaussian shuck tha