Introduction to interferometry and VLBI

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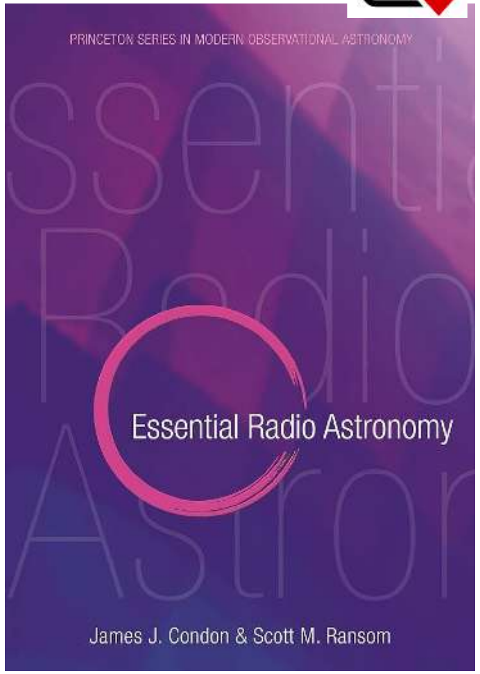
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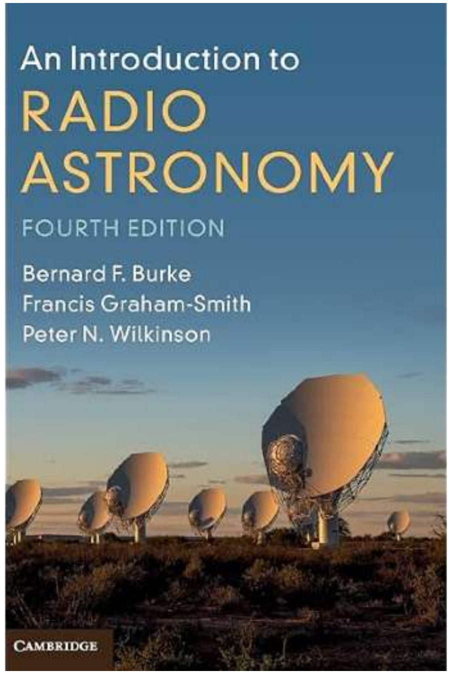
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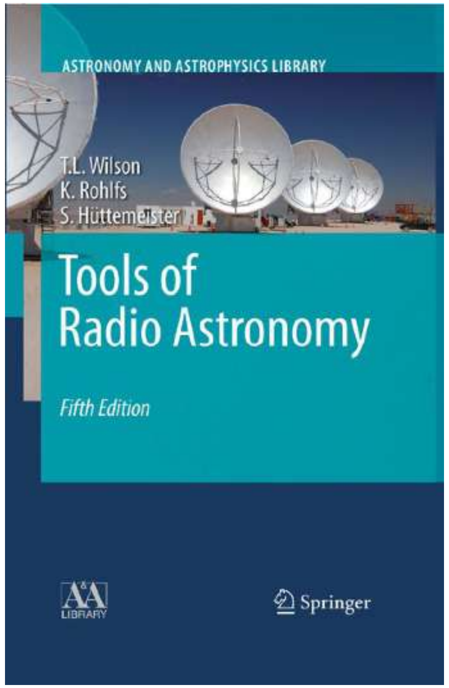
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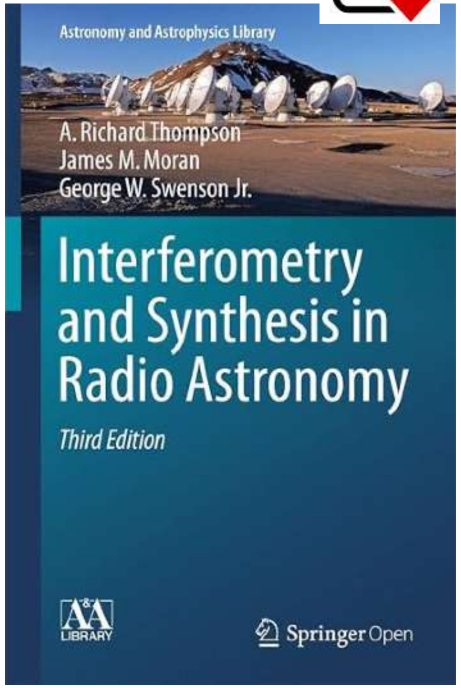
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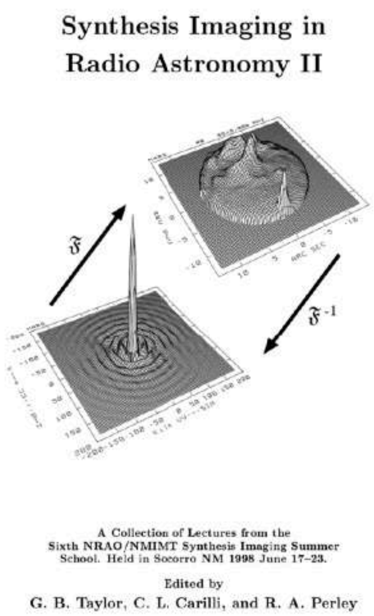
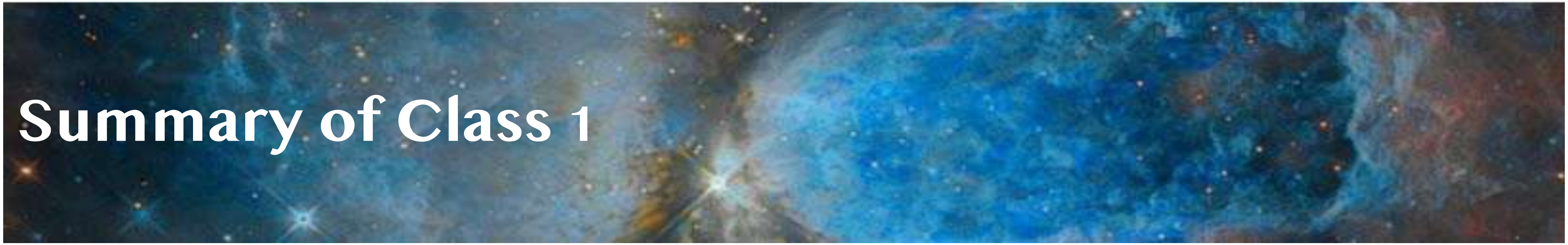
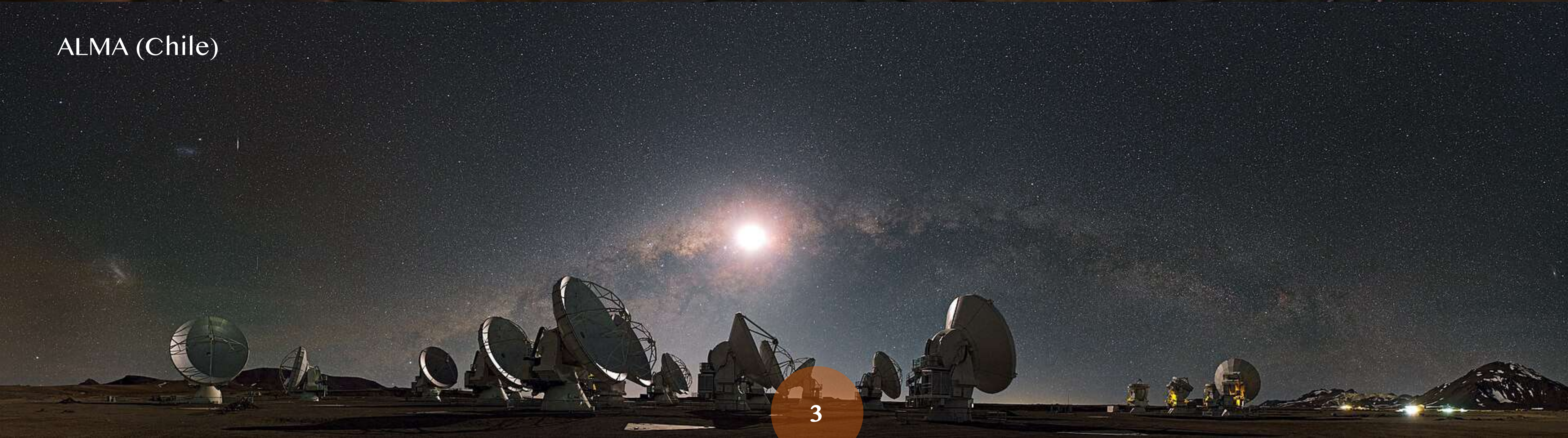

Synthesis Imaging in Radio Astronomy II
A Collection of Lectures from the Sixth NRAO/NMIMT Synthesis Imaging Summer School. Held in Socorro NM 1998 June 17–23.
Edited by
G. B. Taylor, C. L. Carilli, and R. A. Perley



Summary of Class 1

VLA (NM, USA)

ALMA (Chile)



Interferometers solve the problem of angular resolution in radio-astronomy

Biggest single-dish

Same resolution as human eye
(~ one arcminute)

"conventional" interferometers

Same resolution as large optical telescope
(~ 0.1 arcsecond)

VLBI arrays

Highest resolution in astronomy
(~ 1 milli-arcsecond down to 10 micro-arcsecond)

This is the highest angular resolution achievable in all of astronomy

Joseph Fourier (1768-1830)

Active during the French Revolution (1789)

Imprisoned during the “Terror”

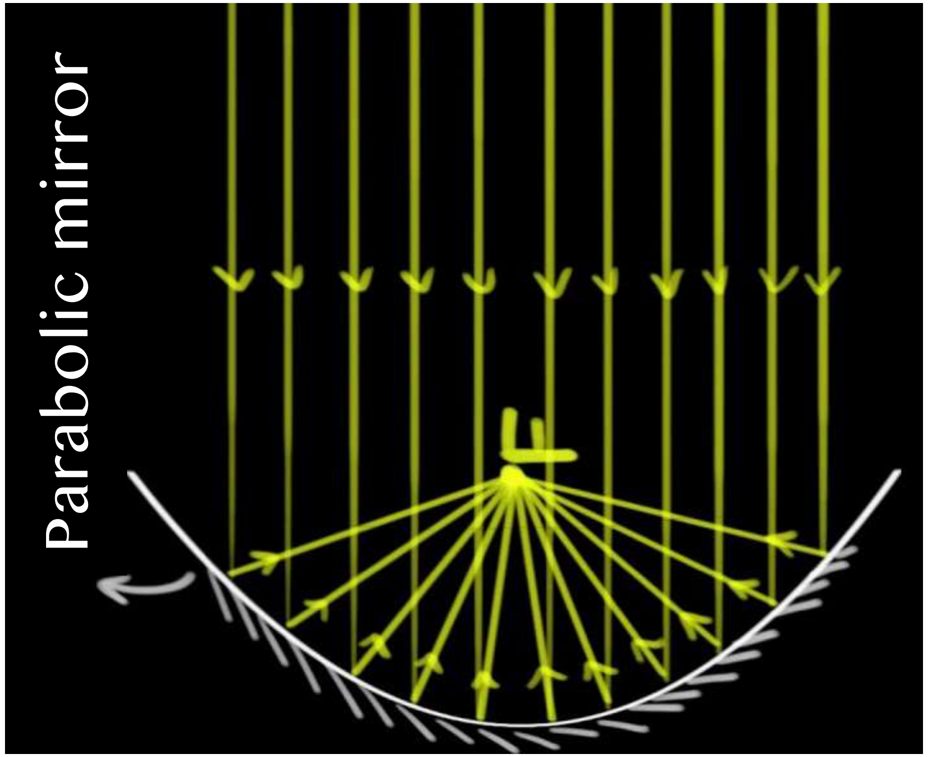
Appointment at École Normale, and then École Polytechnique (succeeded Joseph-Louis Lagrange)

Scientific advisor of Napoleon during “Egyptian Expedition” (i.e. war)

Perfect of Isère (a French department in the Alps)

Oh, and in his free time, he also invented some maths and discovered some physics….

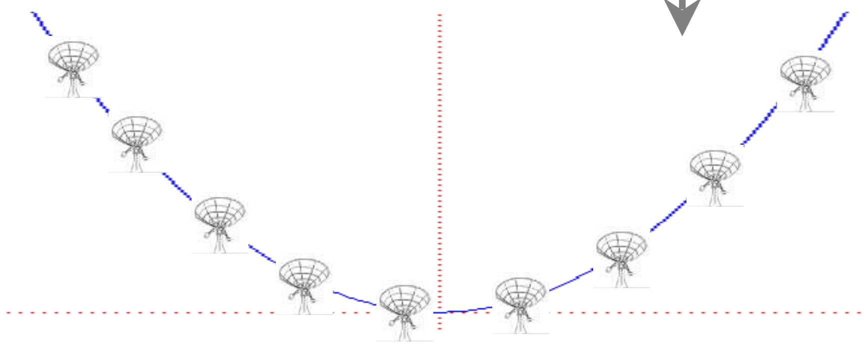
Pictorial principle of interferometry



(Cassegrain) reflector telescope

Replace with

Combination (to form the image at the focus) is done electronically



Fundamental result from Class 1

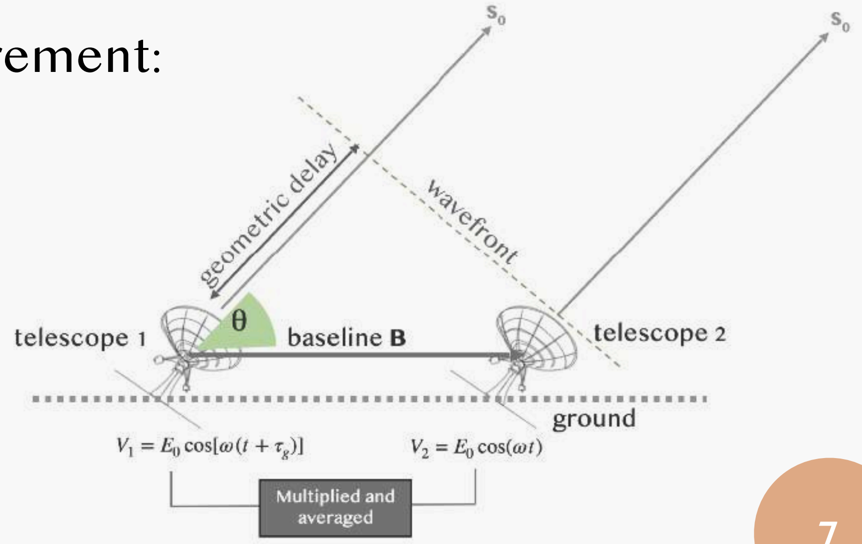
$$V(u,v) = \iint I(l,m) e^{-2\pi i(ul+vm)} \frac{dldm}{\sqrt{1-l^2-m^2}}$$

This is a **function** of (u, v)

$$I(l,m) = \sqrt{1-l^2-m^2} \iint V(u,v) e^{2\pi i(lu+mv)} dudv$$

This is a **function** of (l, m)

One measurement:



corresponds to one baseline : $\mathbf{B}$,

and therefore also to one value of (u_i, v_i)

(actually two points: also $(-u_i, -v_i)$
Including both baselines from telescope 1 to telescope 2 and from telescope 2 to telescope 1)

Part 3: The gory details of interferometry (dirty beam, CLEAN and all that…)

One simplification

Small field of view: $l \ll 1$ and $m \ll 1$

$\sqrt{1 - l^2 - m^2} \approx 1$

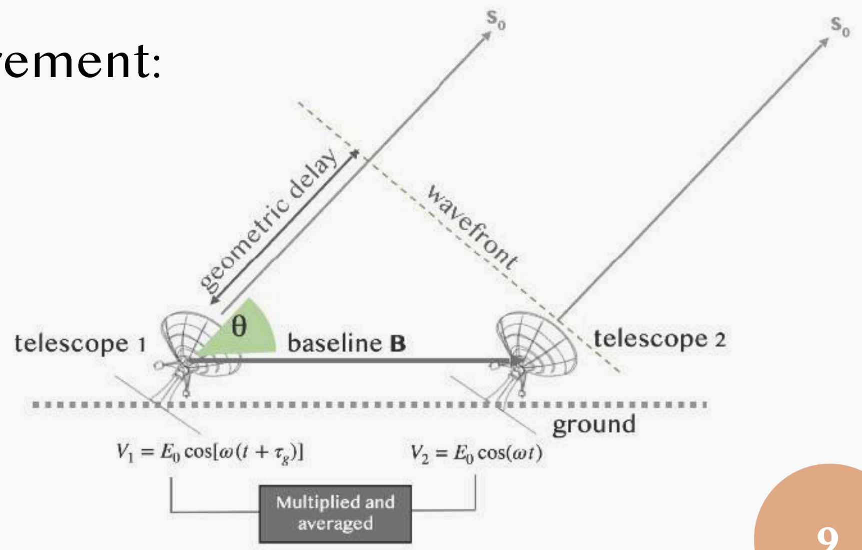
$$V(u, v) = \iint I(l, m) e^{-2\pi i(ul+vm)} \frac{dldm}{\sqrt{1 - l^2 - m^2}}$$

This is a **function** of (u, v)

$$I(l, m) = \sqrt{1 - l^2 - m^2} \iint V(u, v) e^{2\pi i(lu+mv)} dudv$$

This is a **function** of (l, m)

One measurement:



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(actually two points: also $(-u_i, -v_i)$
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One simplification

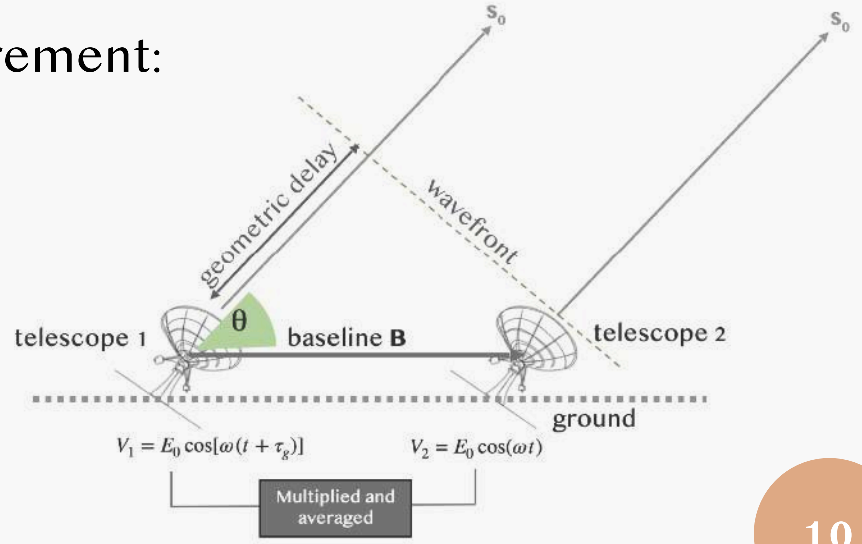
$$V(u,v) = \iint I(l,m)e^{-2\pi i(ul+vm)}dldm$$

This is a **function** of (u, v)

$$I(l,m) = \iint V(u,v)e^{2\pi i(lu+mv)}dudv$$

This is a **function** of (l, m)

One measurement:



corresponds to one baseline : $\mathbf{B}$,

and therefore also to one value of (u_i, v_i)

(actually two points: also $(-u_i, -v_i)$
Including both baselines from telescope 1 to telescope 2 and from telescope 2 to telescope 1)

Multiple measurements

Measurement 1 : baseline $\mathbf{B_1}$, values (u_1, v_1)

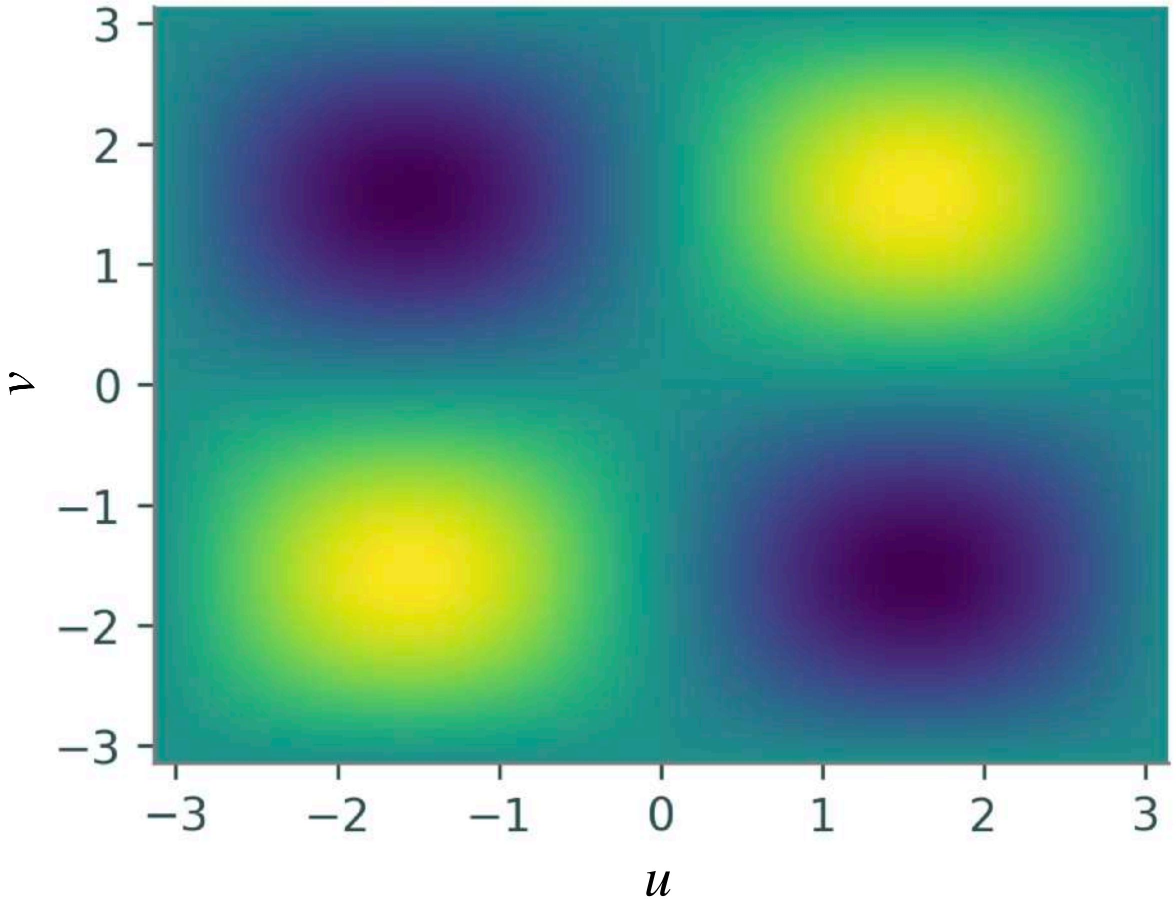
Measurement 2 : baseline $\mathbf{B_2}$, values (u_2, v_2)

…

Measurement k : baseline $\mathbf{B_k}$, values (u_k, v_k)

…

True function $V(u, v)$



Multiple measurements

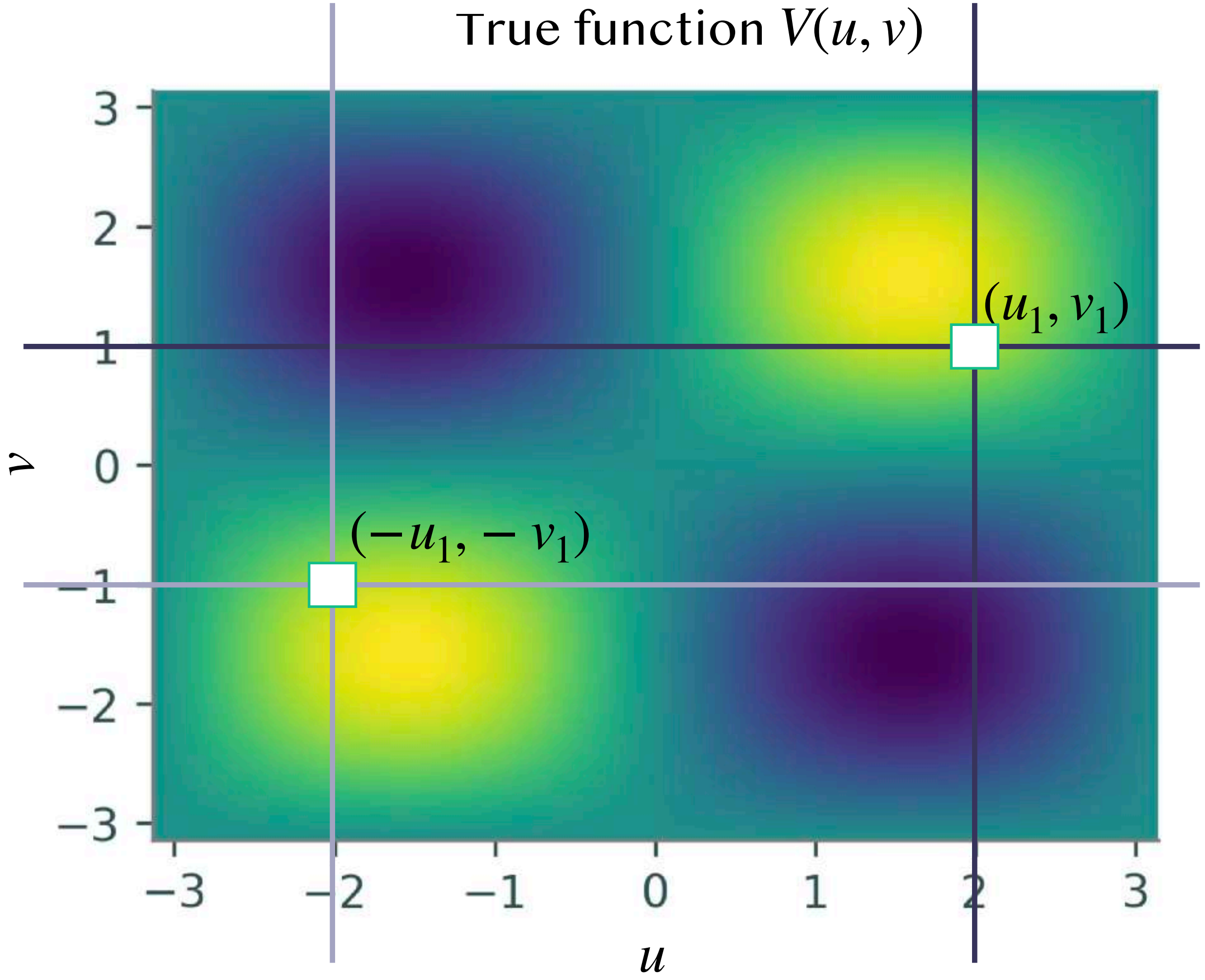
Measurement 1 : baseline $\mathbf{B_1}$, values (u_1, v_1)

Measurement 2 : baseline $\mathbf{B_2}$, values (u_2, v_2)

…

Measurement k : baseline $\mathbf{B_k}$, values (u_k, v_k)

…



Multiple measurements

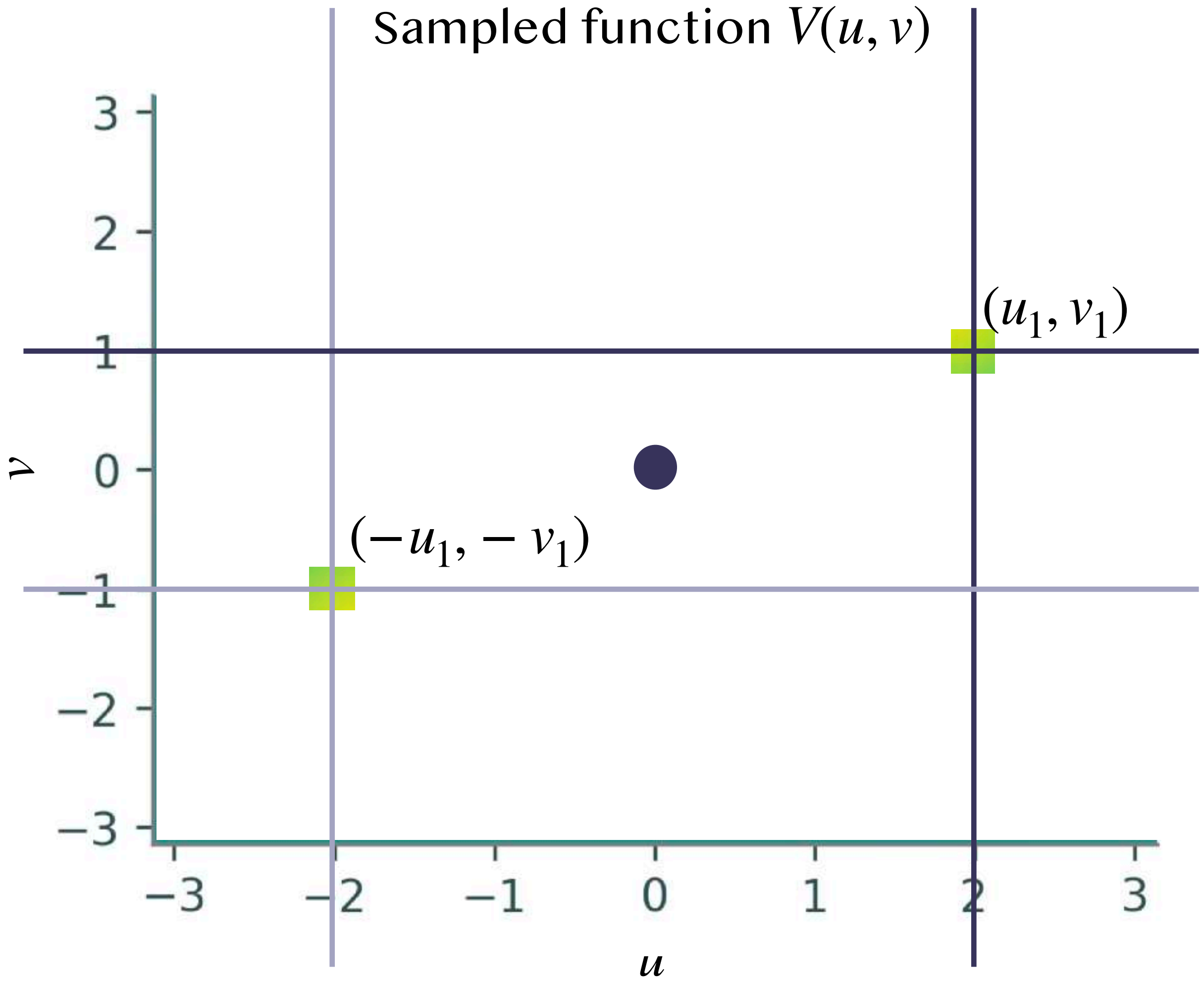
Measurement 1 : baseline $\mathbf{B_1}$, values (u_1, v_1)

Measurement 2 : baseline $\mathbf{B_2}$, values (u_2, v_2)

…

Measurement k : baseline $\mathbf{B_k}$, values (u_k, v_k)

…



Multiple measurements

Measurement 1 : baseline $\mathbf{B_1}$, values (u_1, v_1)

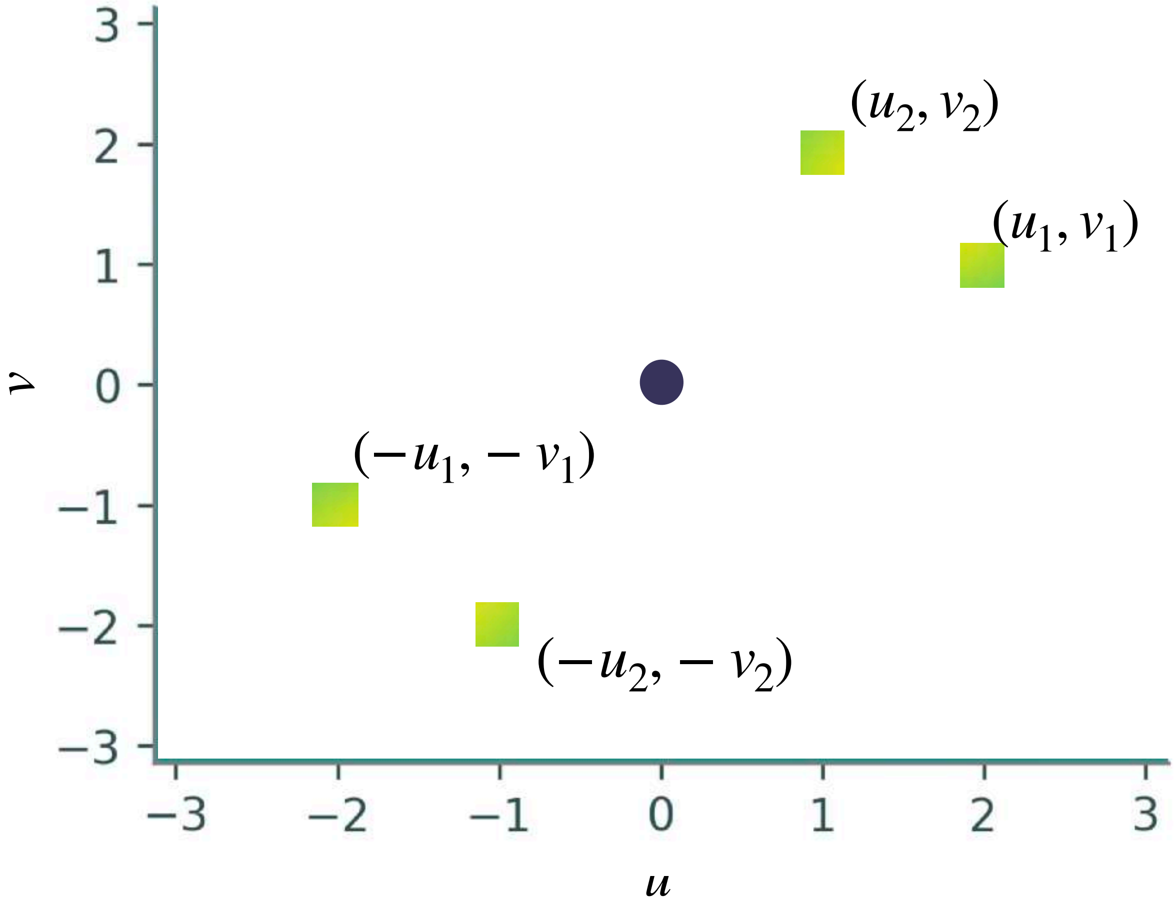
Measurement 2 : baseline $\mathbf{B_2}$, values (u_2, v_2)

…

Measurement k : baseline $\mathbf{B_k}$, values (u_k, v_k)

…

Sampled function $V(u, v)$



The sampling function

What the interferometer measures in **not** the complex visibility function $V(u, v)$

but the **sampled** complex visibility function $S(u, v)V(u, v)$ where $S(u, v)$ is the **sampling function**.

The sampling function is a sum of Dirac delta functions located at all sampled (u, v) points.

What you would **like** to get is: $$I(l, m) = \iint V(u, v)e^{2\pi i(lu+mv)}dudv$$

What you **actually** get is: $$I'(l, m) = \iint S(u, v)V(u, v)e^{2\pi i(lu+mv)}dudv$$

The "dirty beam"

$$I'(l,m) = \iint S(u,v)V(u,v)e^{2\pi i(lu+mv)}dudv$$

This is the Fourier transform of a product of two functions

The result if the convolution product of the individual Fourier transforms:

$$I'(l,m) = \iint S(u,v)e^{2\pi i(lu+mv)}dudv * \iint V(u,v)e^{2\pi i(lu+mv)}dudv$$

Dirty Beam = PSF = $B(l,m)$ True sky brightness $I(l,m)$

$$I'(l,m) = B(l,m) * I(l,m)$$

"Dirty image"

A simple example: single baseline

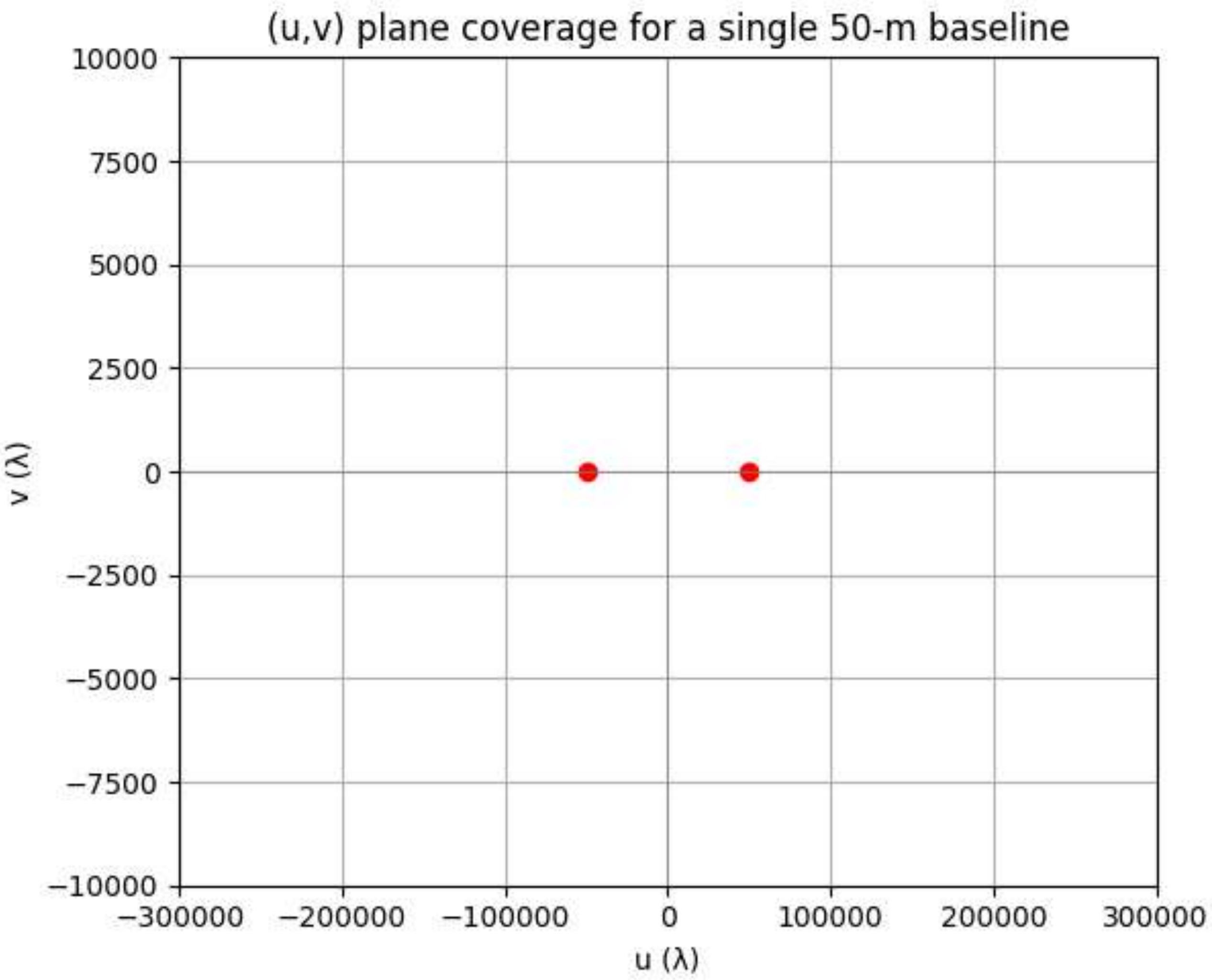
$$B(l,m) = \iint S(u,v) e^{2\pi i(lu+mv)} du dv$$

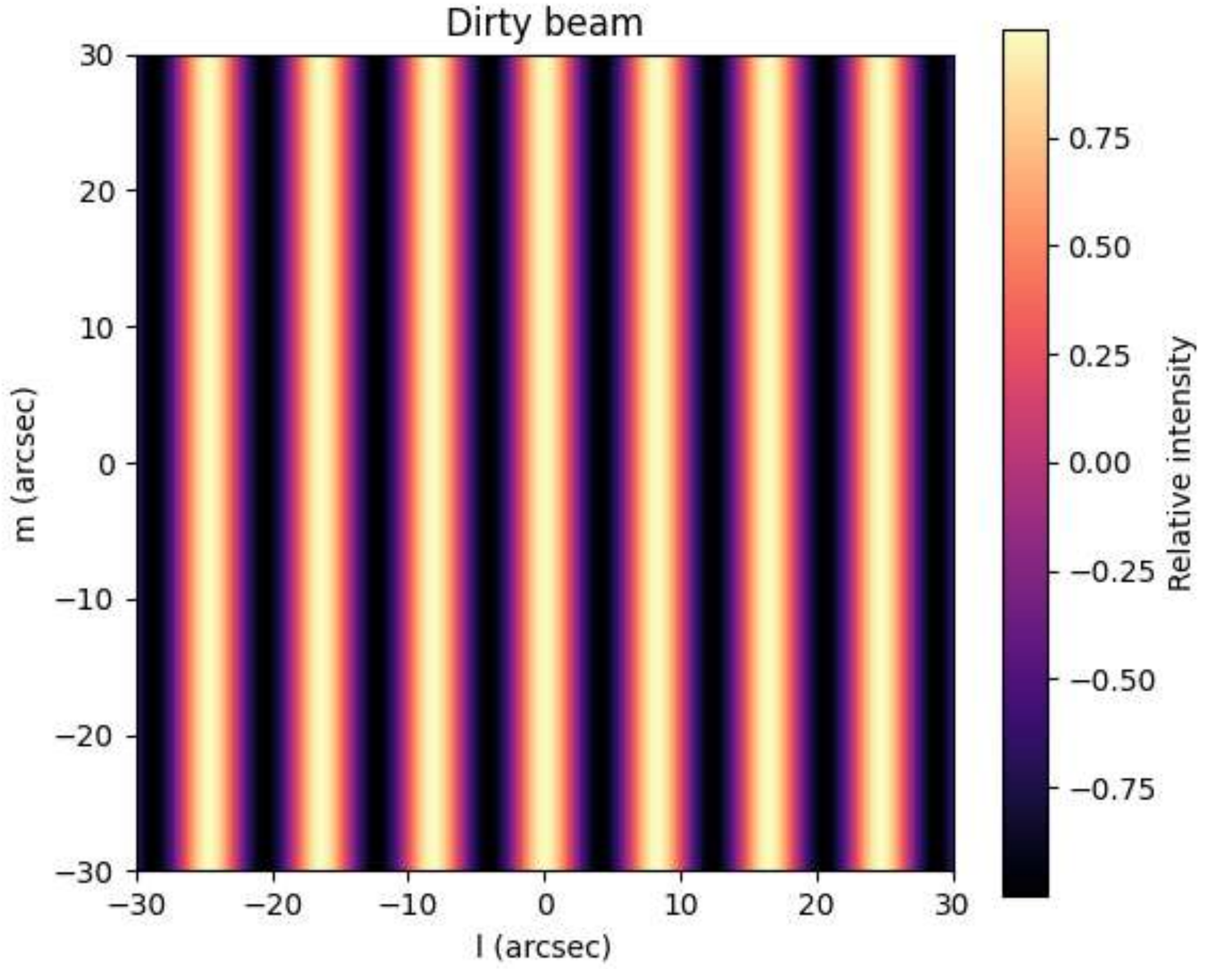
$$S_+(u,v) = \delta(u - u_0) \quad \Rightarrow \quad B_+(l,m) = e^{2i\pi u_0 l} = \cos(2\pi u_0 l) + i\sin(2\pi u_0 l)$$

$$S_-(u,v) = \delta(u + u_0) \quad \Rightarrow \quad B_-(l,m) = e^{-2i\pi u_0 l} = \cos(-2\pi u_0 l) + i\sin(-2\pi u_0 l)$$

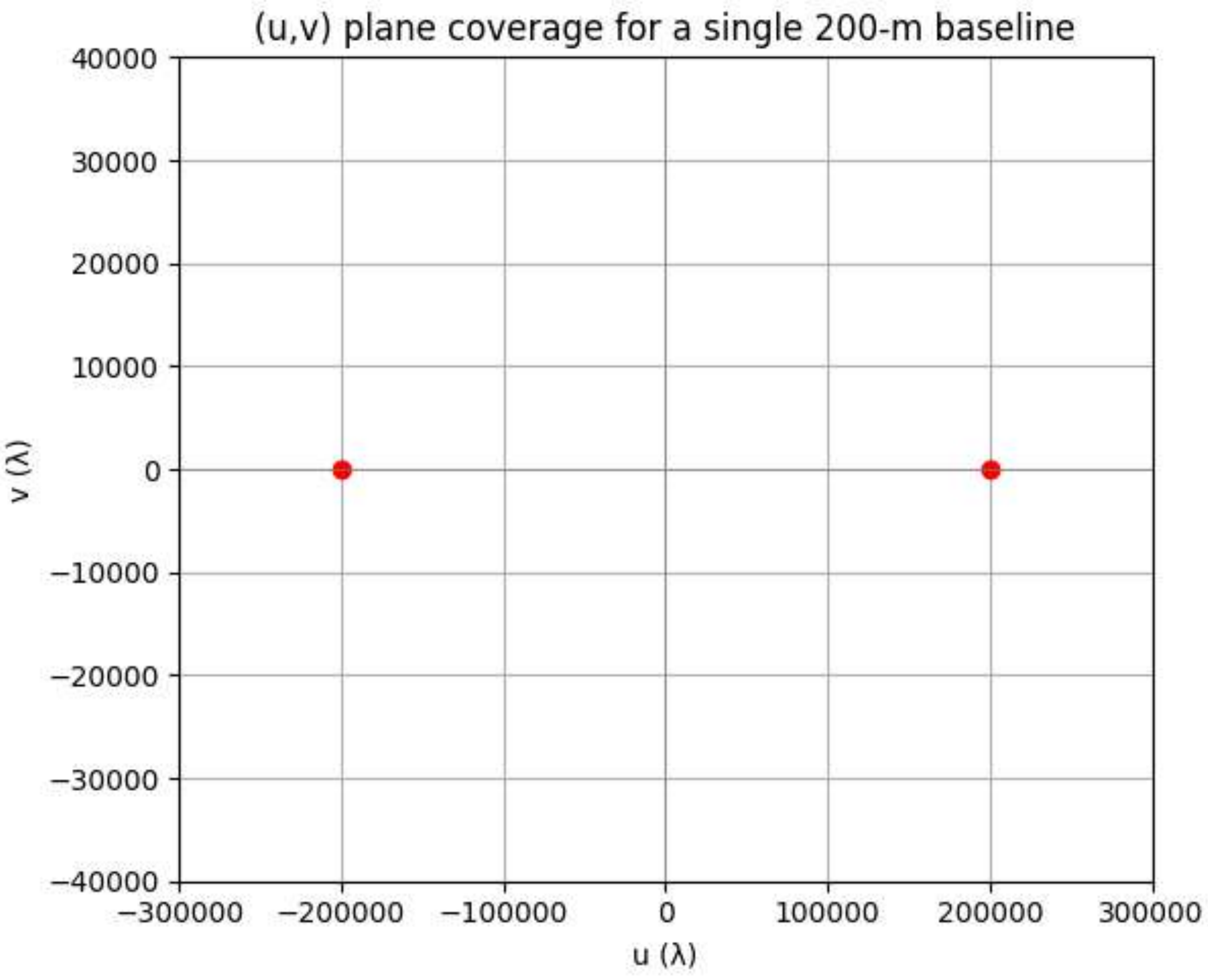
$$S(u,v) = \frac{1}{2}\left(\delta(u - u_0) + \delta(u + u_0)\right) \quad \Rightarrow \quad B(l,m) = \frac{1}{2}\left(B_+(l,m) + B_-(l,m)\right) = \cos(2\pi u_0 l)$$

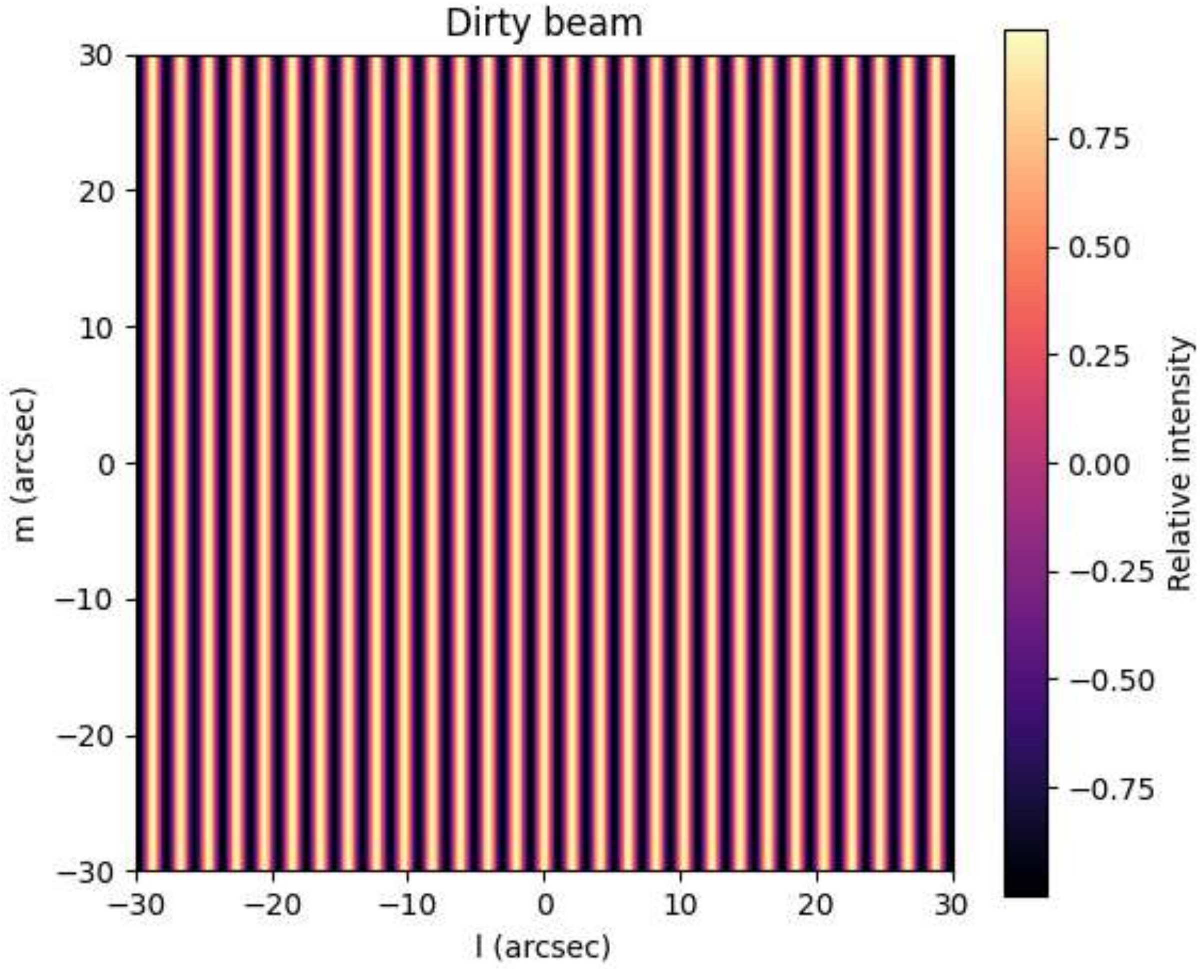
Response for single short baseline



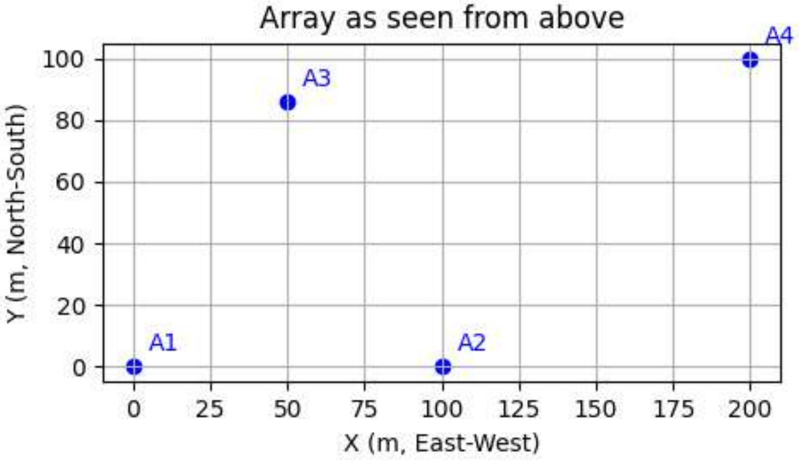


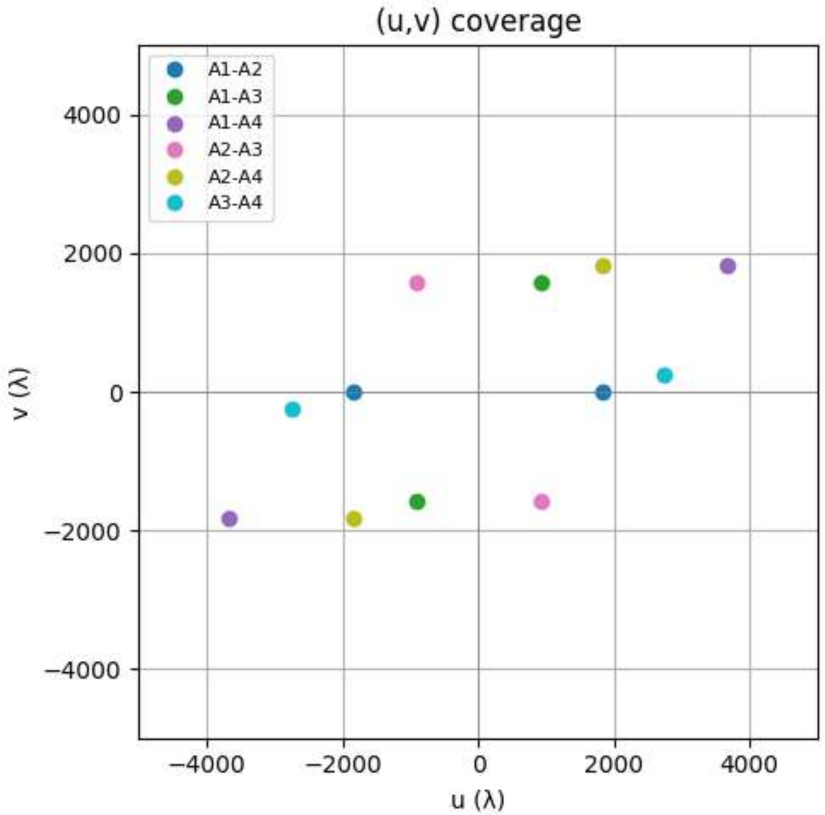
Response for single long baseline

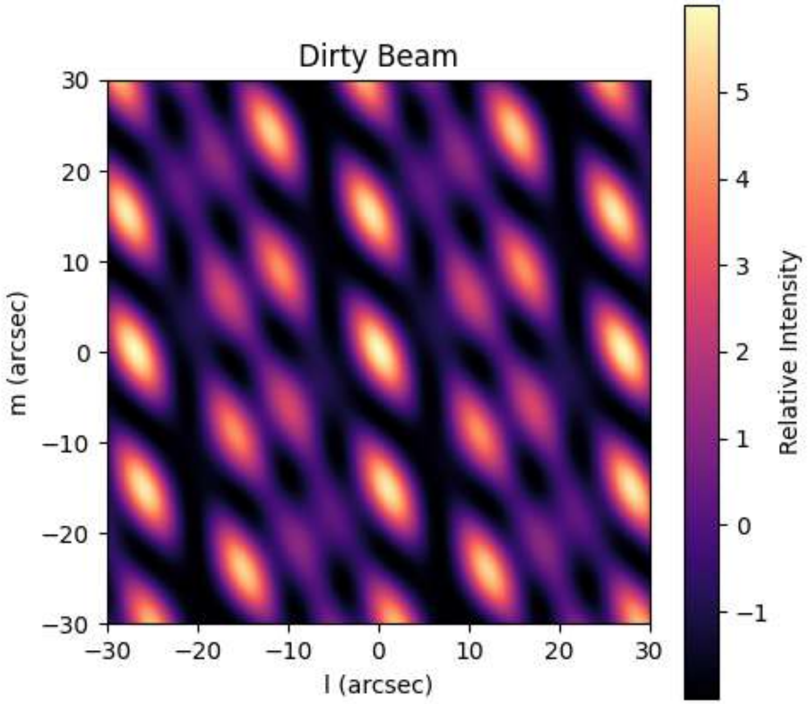




Response for four antennas

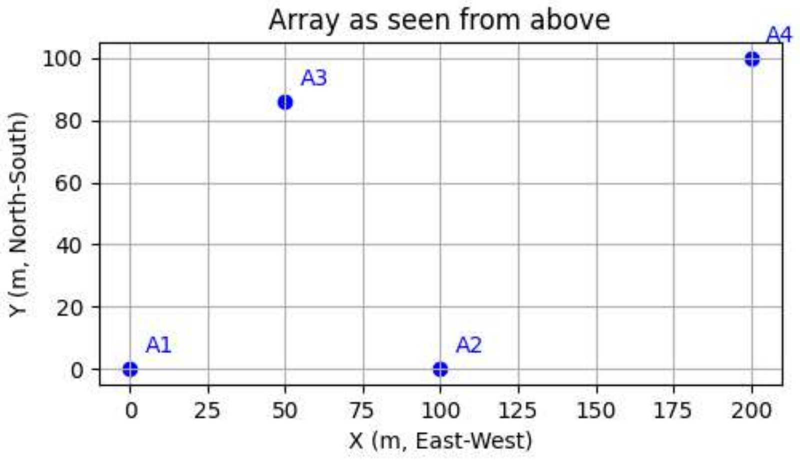


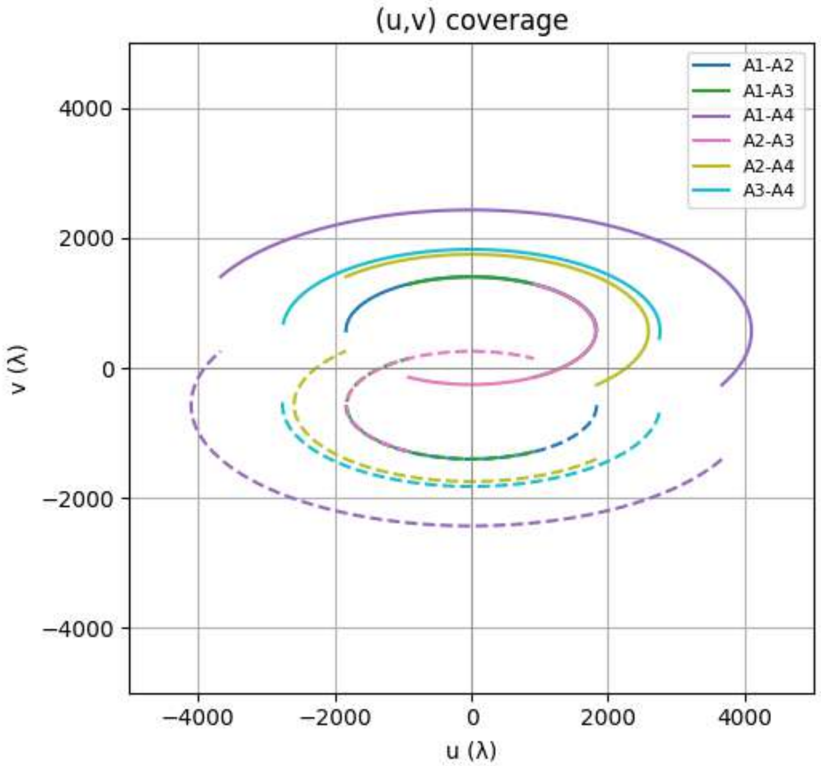


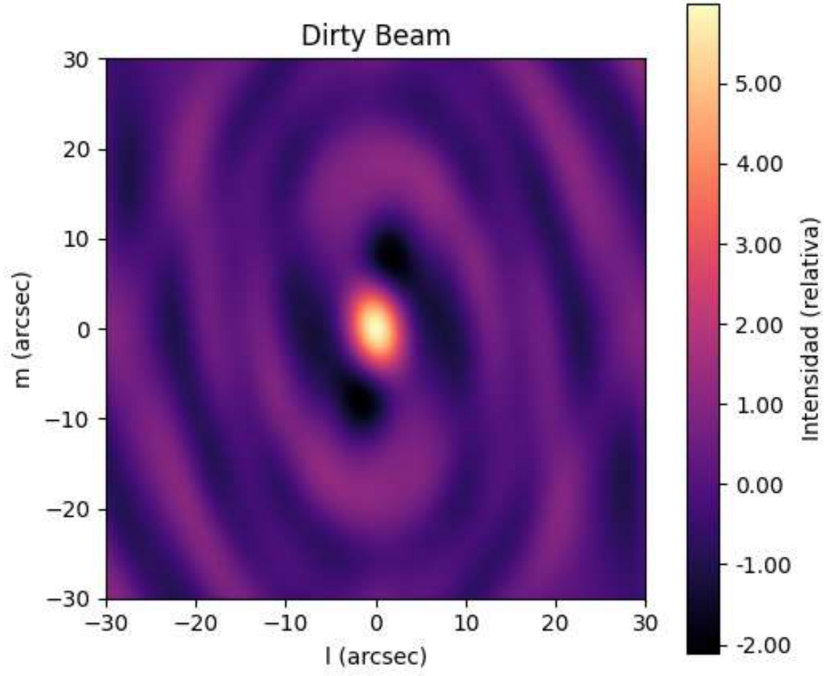


$$N_b = \frac{N_a(N_a - 1)}{2}$$

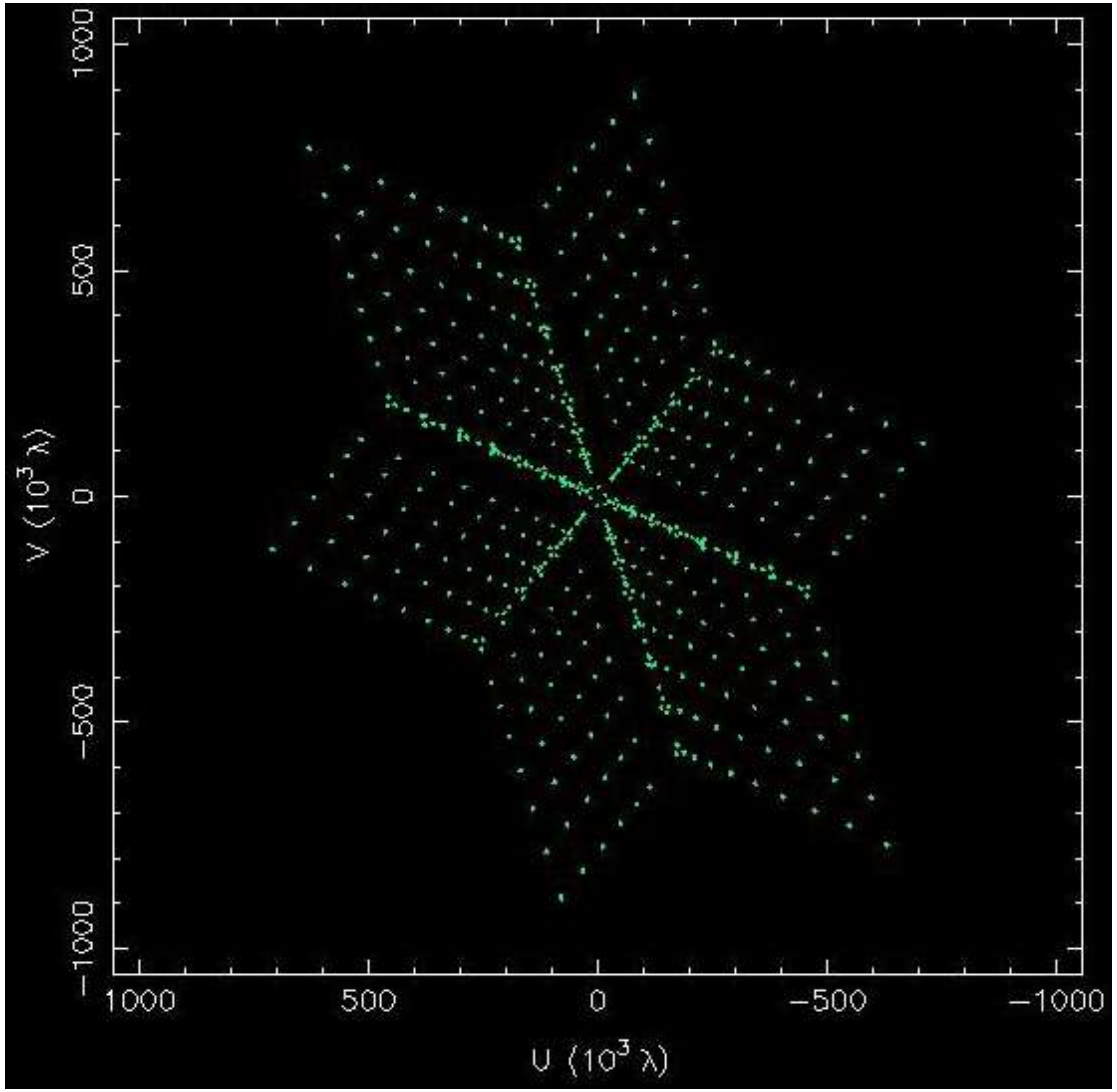
Response for four antennas with Earth rotation

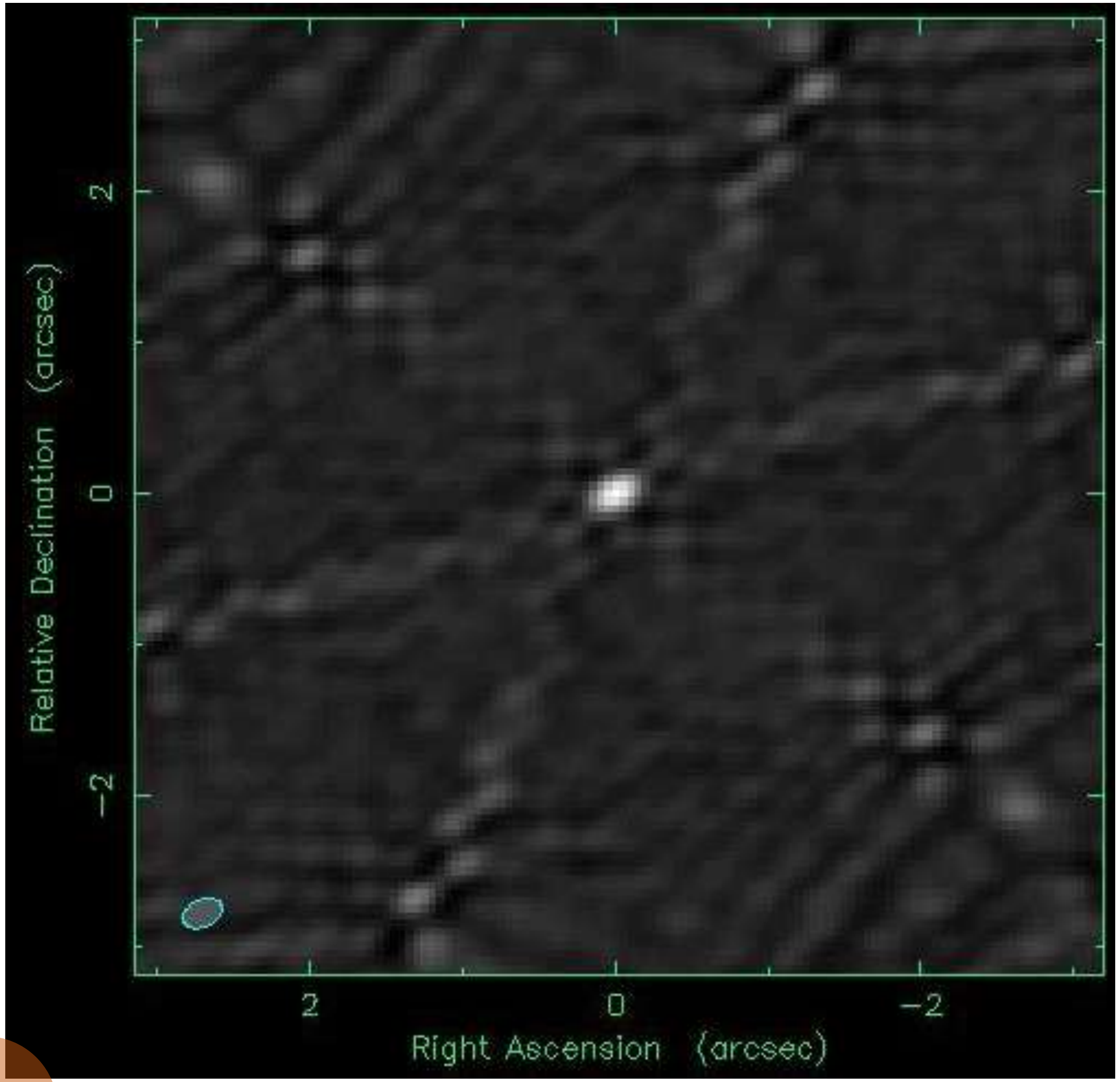




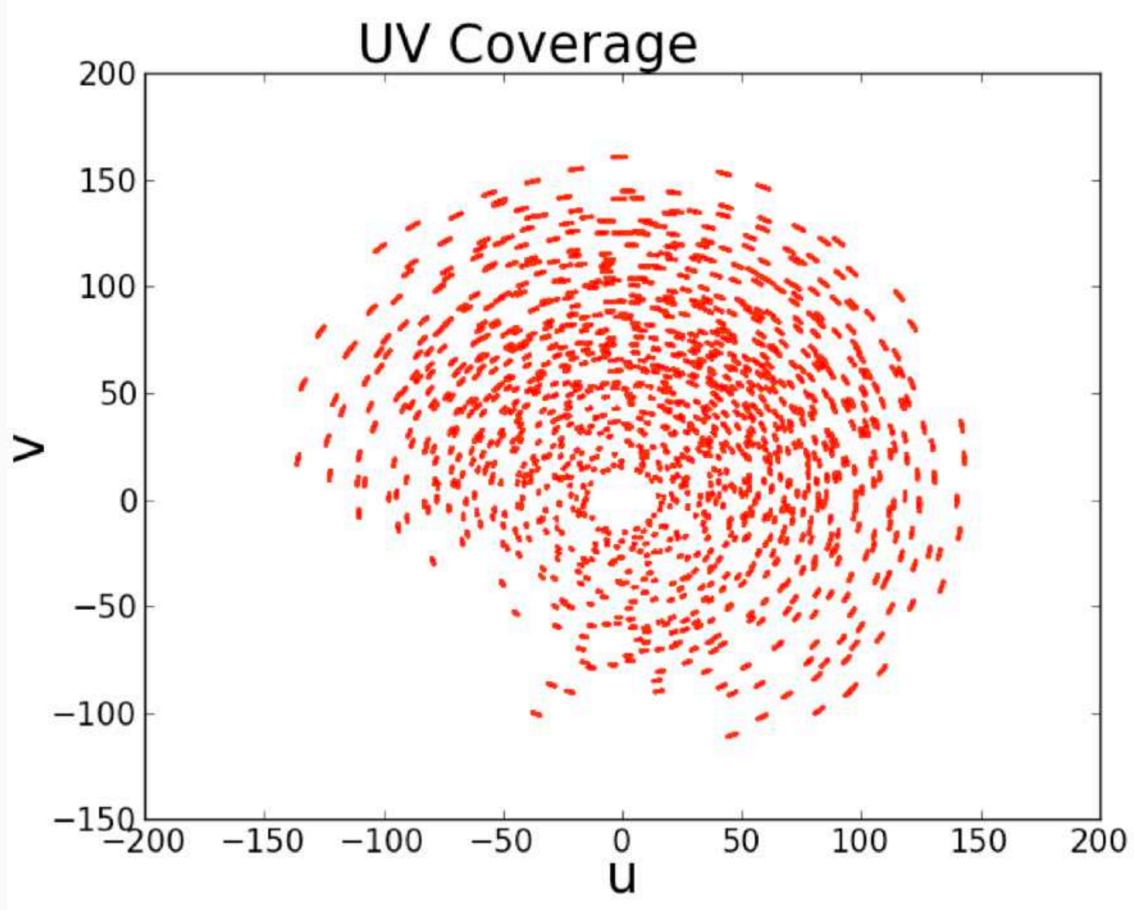


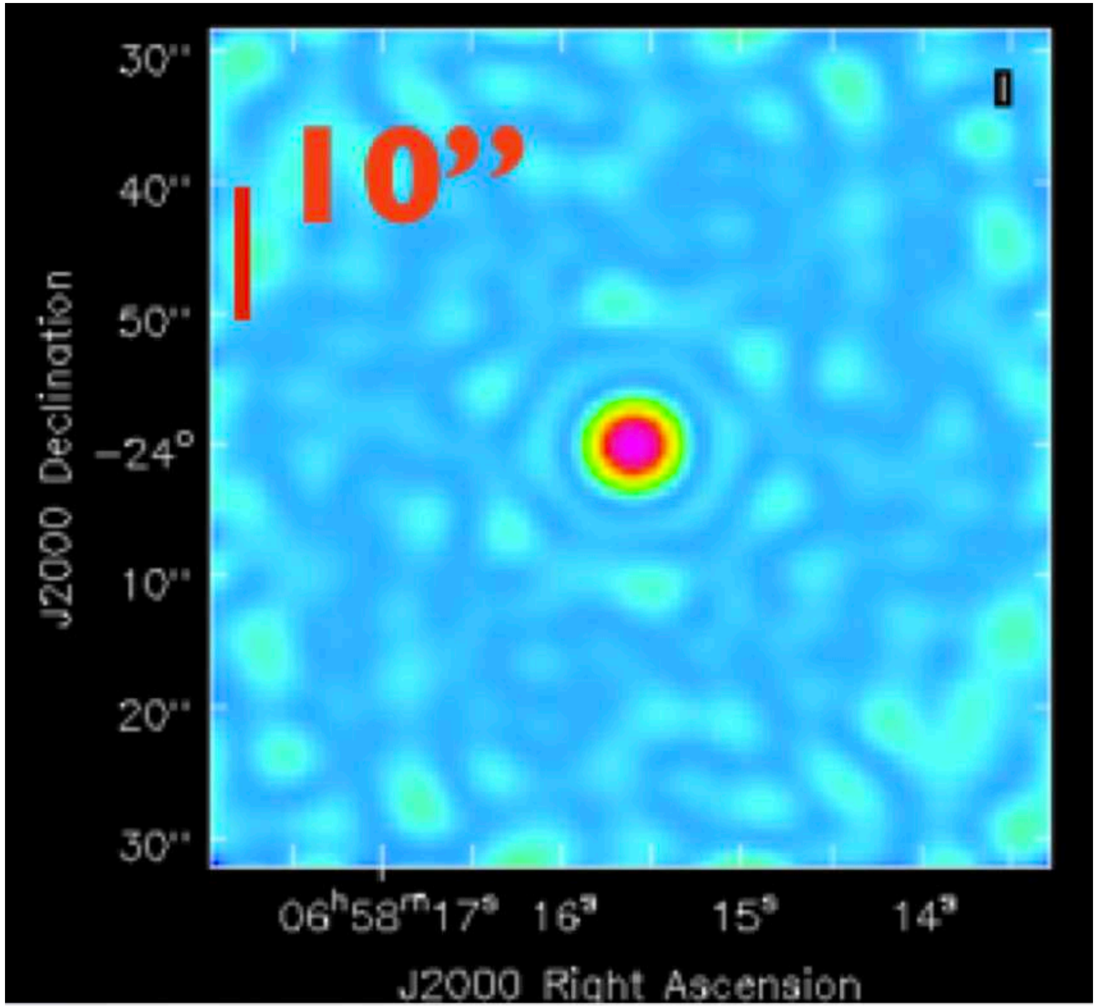
Responses from actual arrays (VLA)



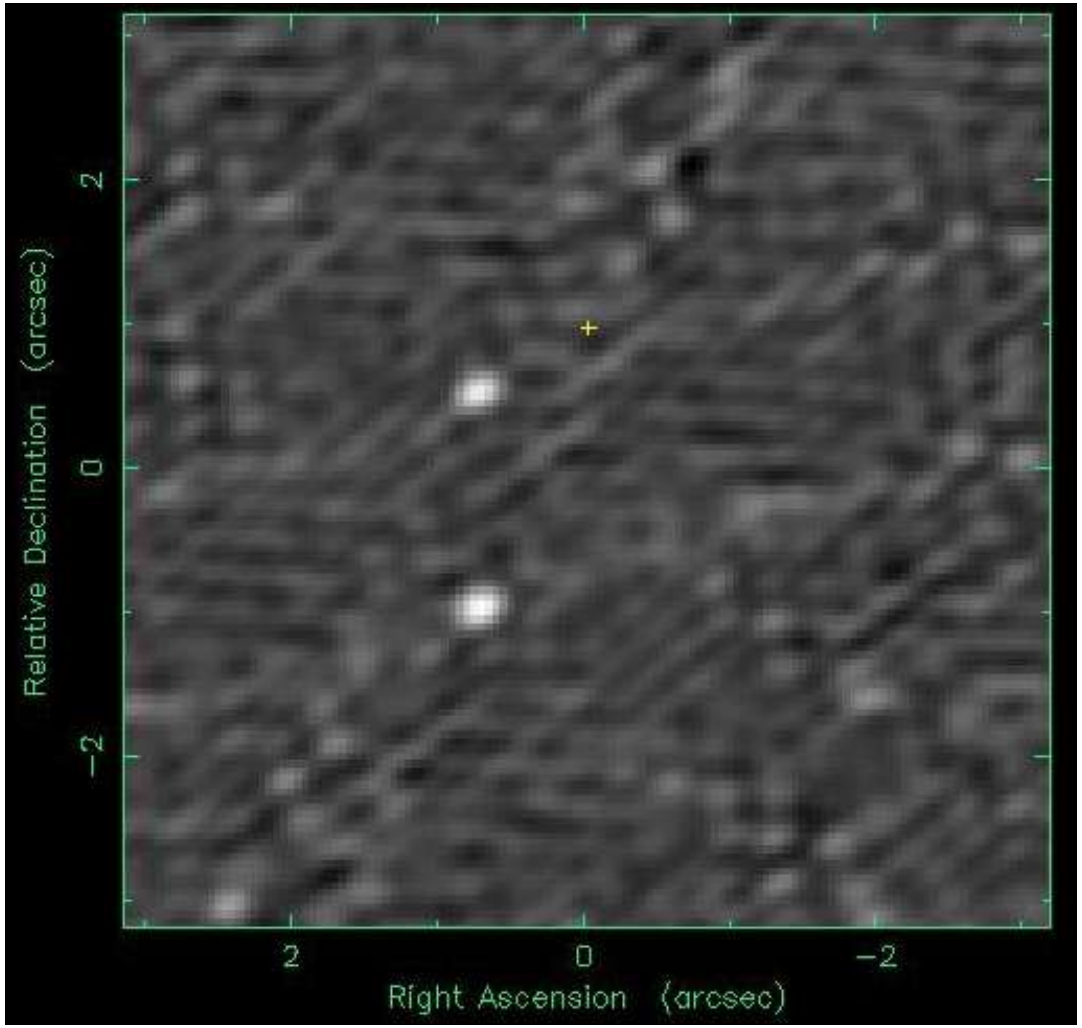


Responses from actual arrays (ALMA)

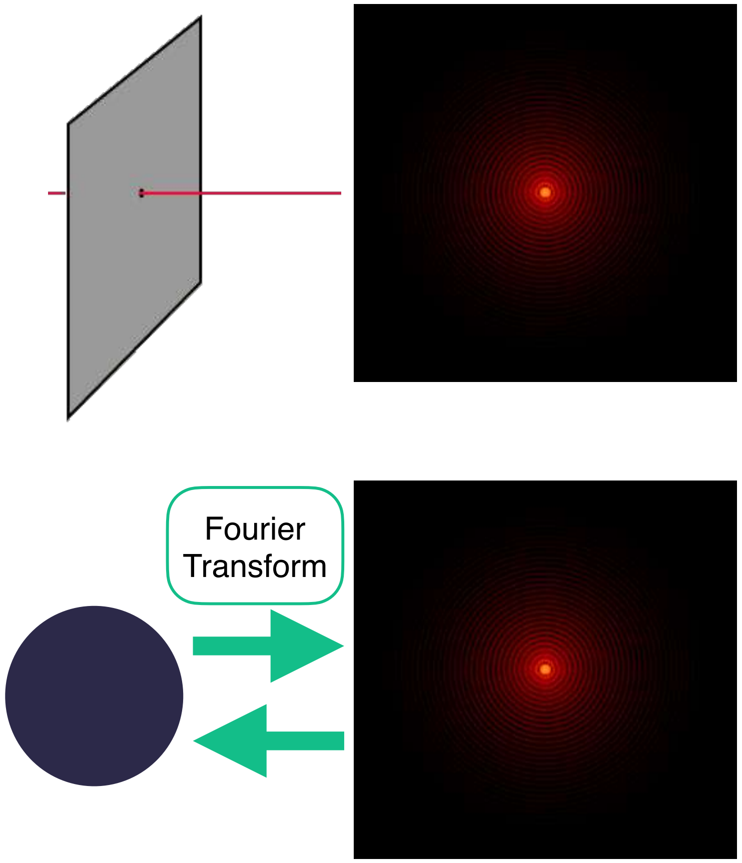




Consequences on image



Response to a circular aperture



Aperture Response

Actual observation of a binary system

The deconvolution idea

$$I'(l,m) = \iint S(u,v)V(u,v)e^{2\pi i(lu+mv)}dudv$$

$$I'(l,m) = \iint S(u,v)e^{2\pi i(lu+mv)}dudv * \iint V(u,v)e^{2\pi i(lu+mv)}dudv$$

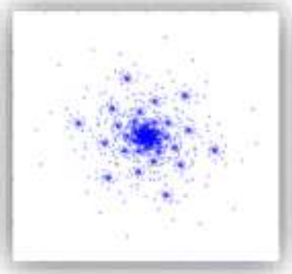
$$I'(l,m) = B(l,m) * I(l,m)$$

"Dirty image"	"Dirty Beam"	"True" Image

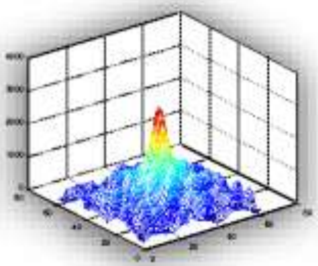
To recover the true image, we need to deconvolve the dirty image from the dirty beam
Fortunately, the dirty beam is ugly but very well known

The “CLEAN” algorithm

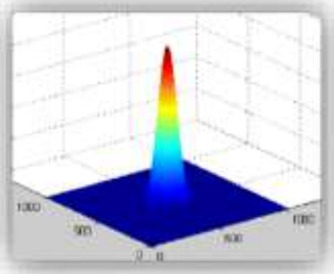
Step one



(u,v) coverage



dirty beam



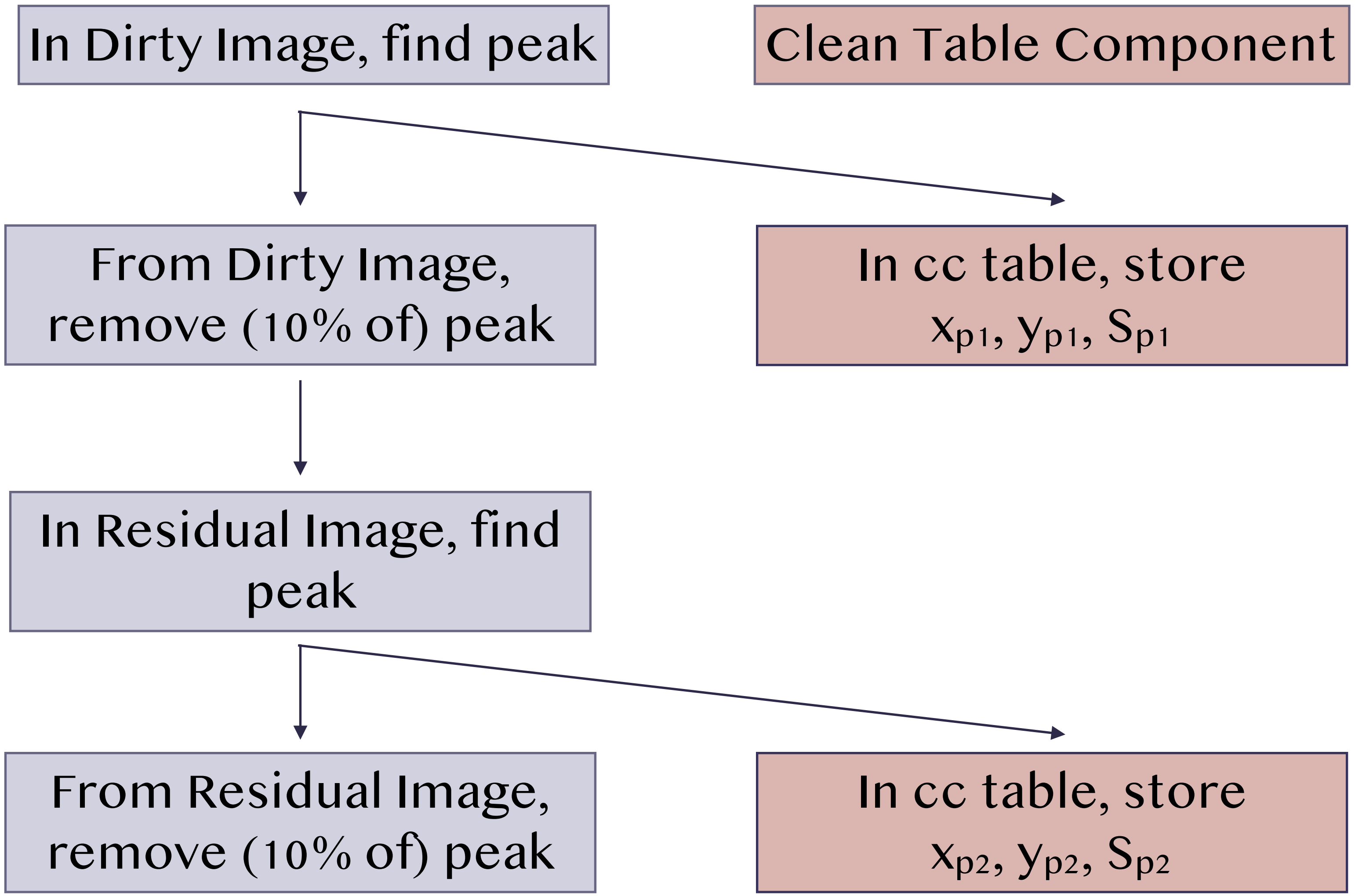
Gaussian fit
to central lobe

“CLEAN” beam

Step two

Visibilities → FT → Dirty Image + Dirty Beam → Iterative process

The "CLEAN" algorithm



continue until residual is put noise…

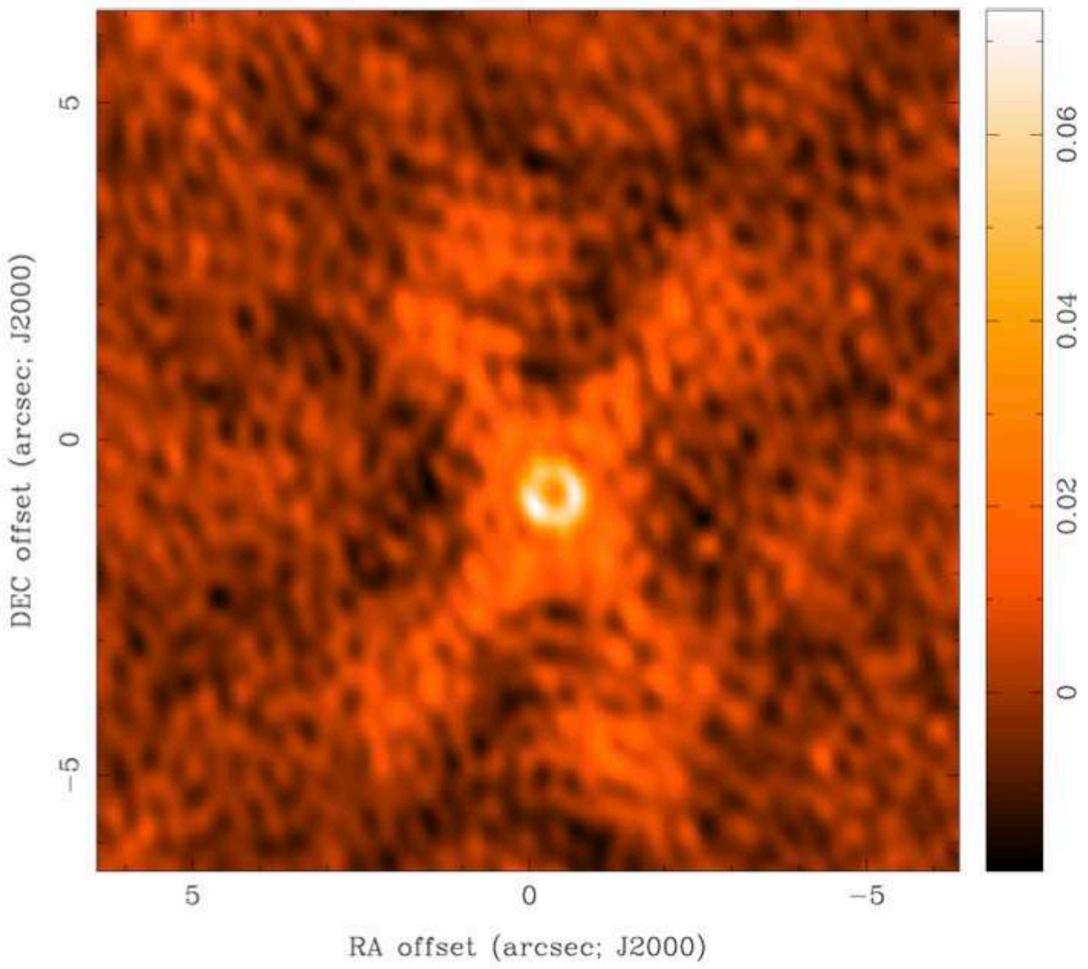
The "CLEAN" algorithm

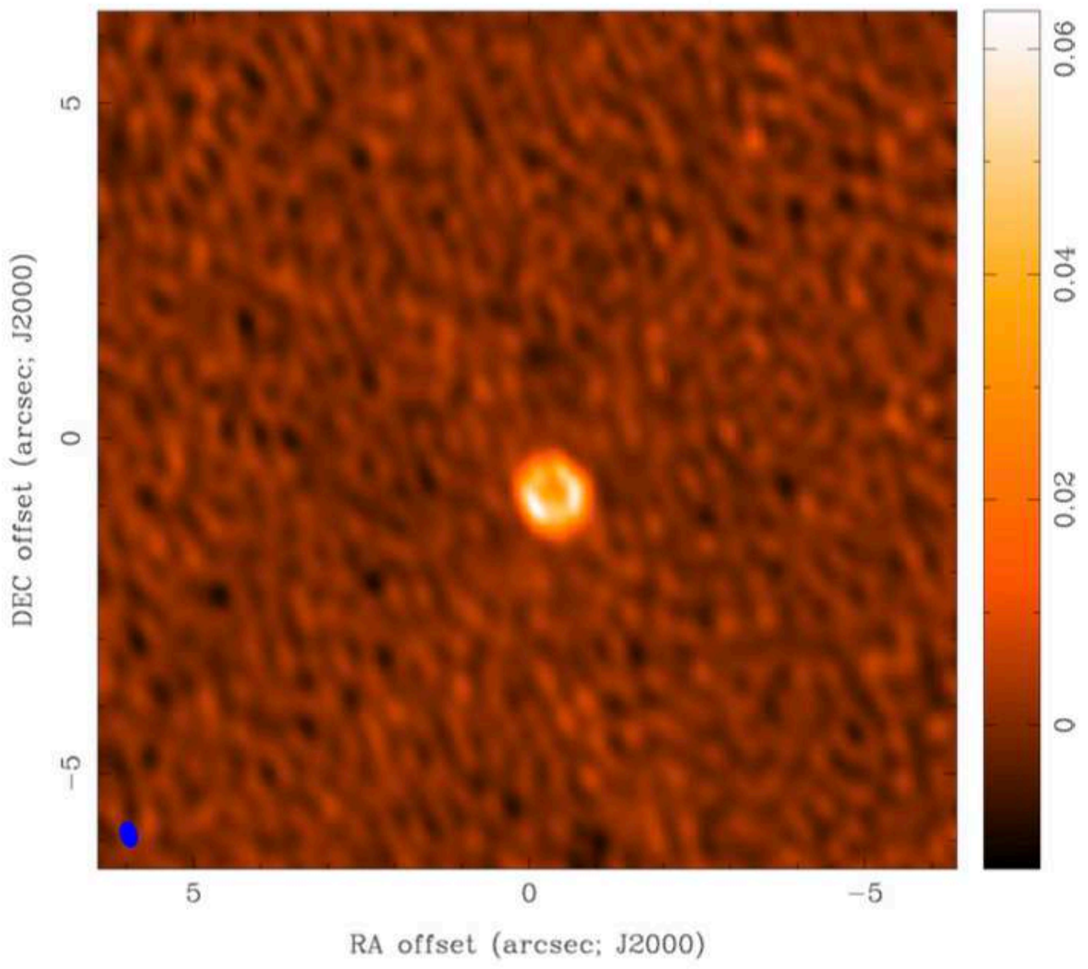
Step three

CLEAN image = Final Residual + $\sum$ Clean components x Clean Beam

See 1D example

CLEAN before/after





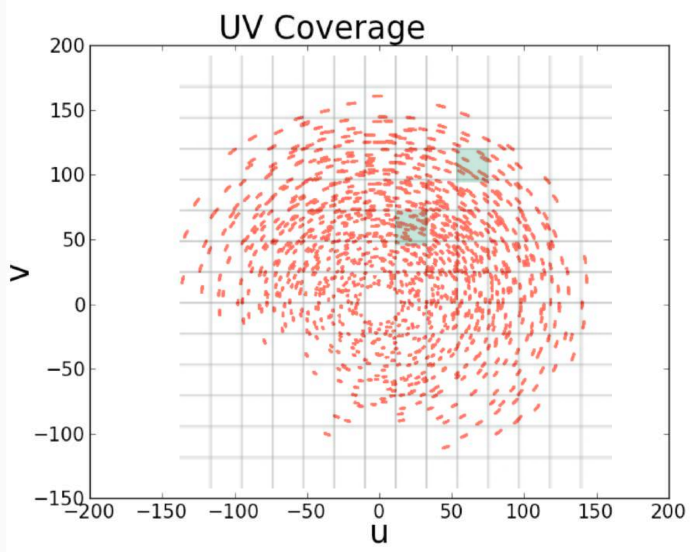
Two last comments

We wrote the dirty beam as:

$$B(l,m) = \iint S(u,v) e^{2\pi i(lu+mv)} du dv$$

We can modify this definition as:

$$B(l,m) = \iint S(u,v) w(u,v) e^{2\pi i(lu+mv)} du dv$$



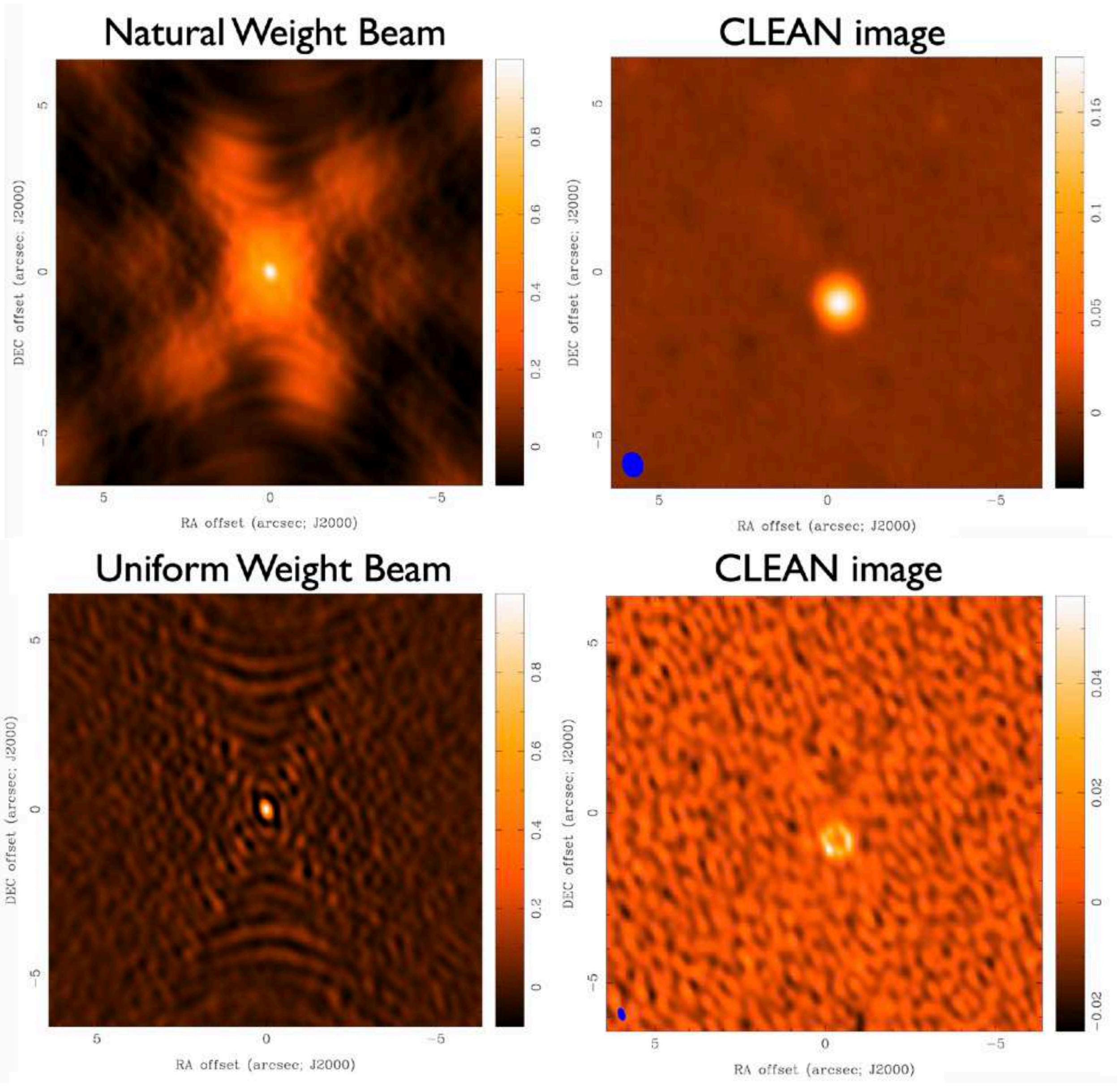
weight factor

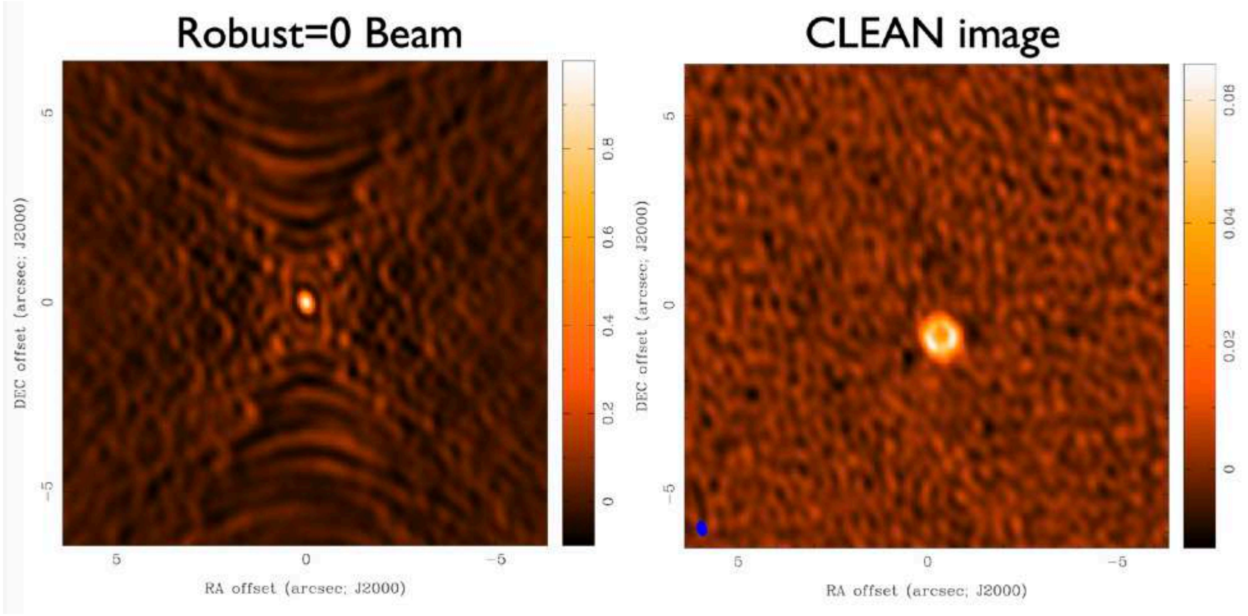
Natural
(Respect density of points)

Uniform
(Same everywhere)

Everything in-between

The effect of different weights

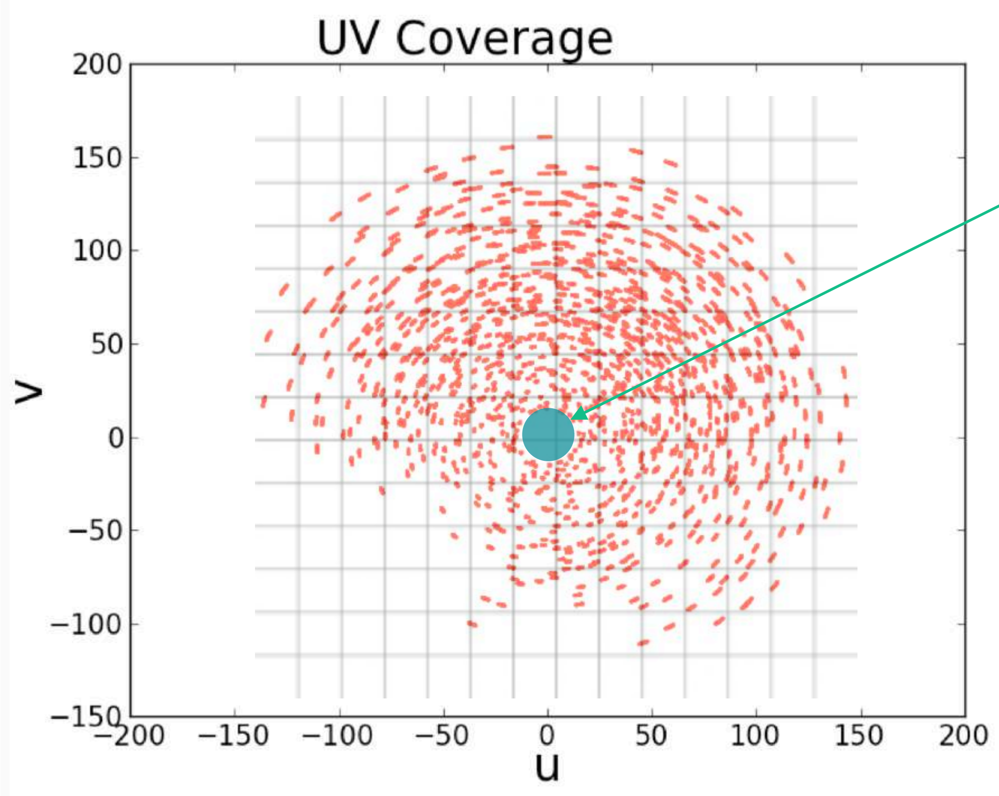




The choice is yours considering:

- Your science
- Your SNR
- Your target

Two last comments



No data near $(u, v) = 0$

Quiz: why is that?

Quiz 2: how do we fix this?

We merge interferometric and single-dish data

Interferometers+single dish