

# Introduction to interferometry and VLBI

Laurent Loinard

BHI and DRCLAS, Harvard University

Instituto de Radioastronomía y Astrofísica, UNAM

(ng-)Event Horizon Telescope Collaboration



BLACK HOLE  
INITIATIVE



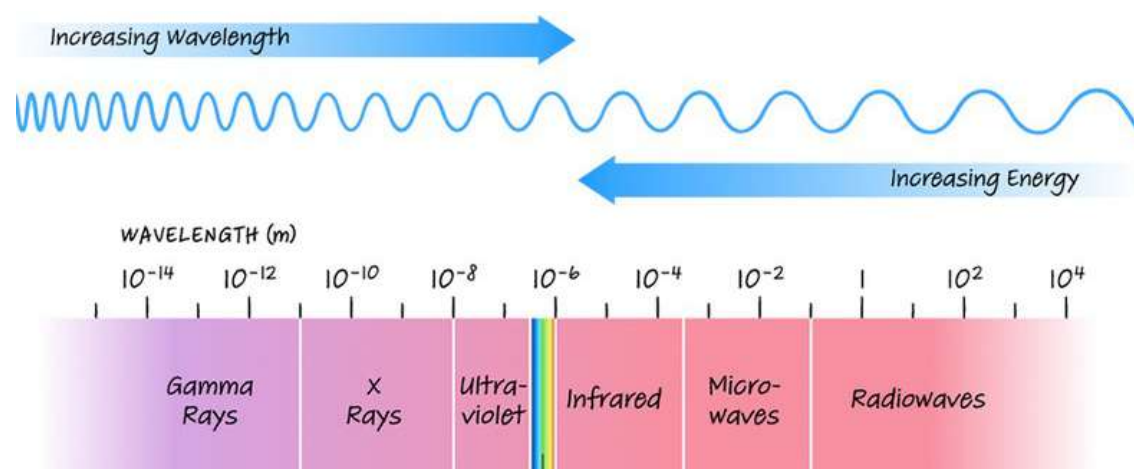
DAVID ROCKEFELLER CENTER  
FOR LATIN AMERICAN STUDIES  
HARVARD UNIVERSITY



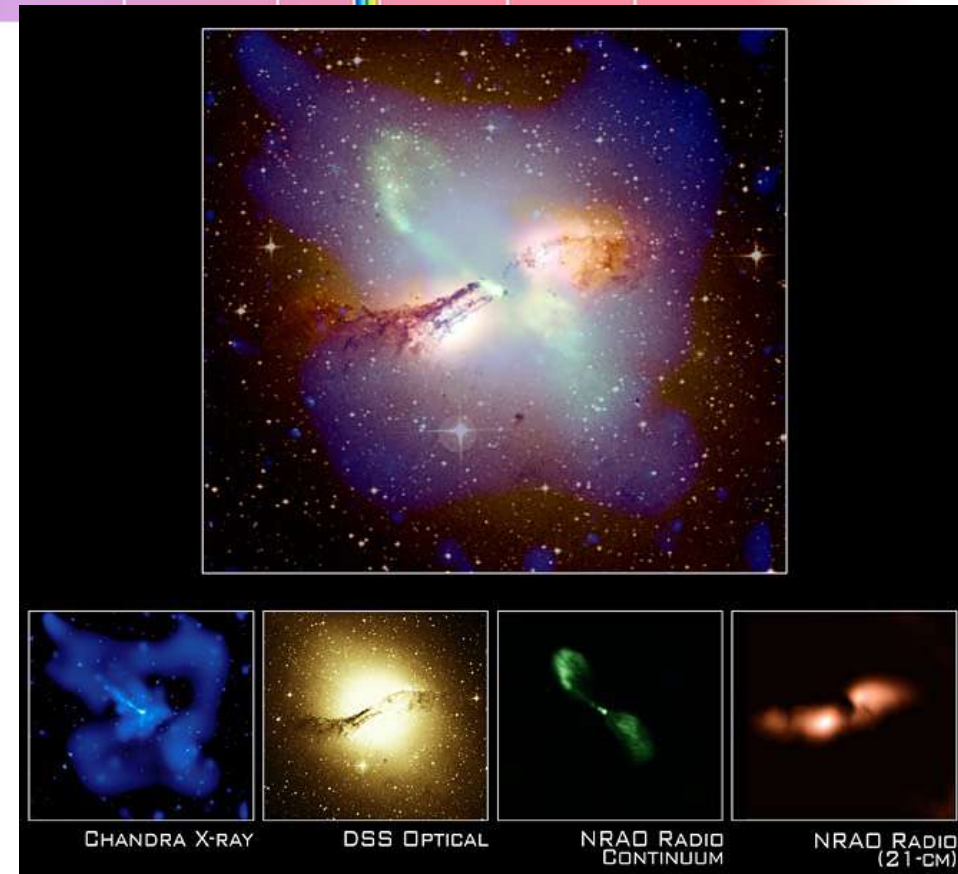
# Part 1: Radioastronomy and Interferometry in a Nutshell



# Multi-wavelength Astronomy

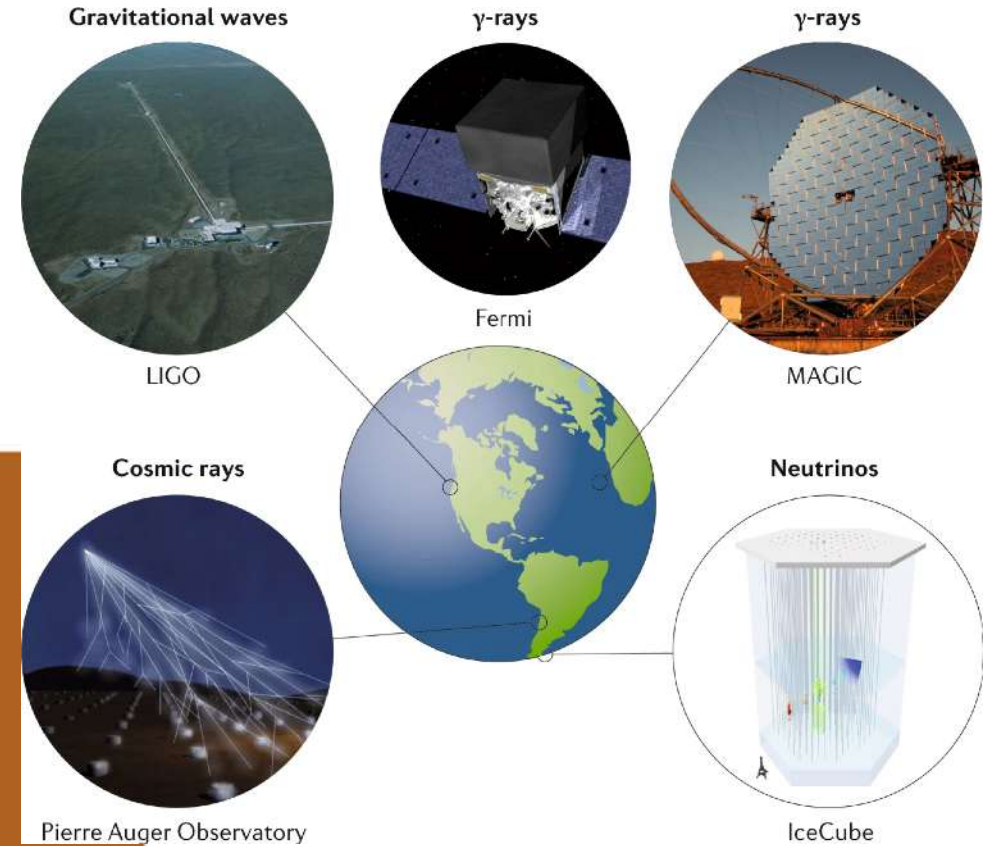
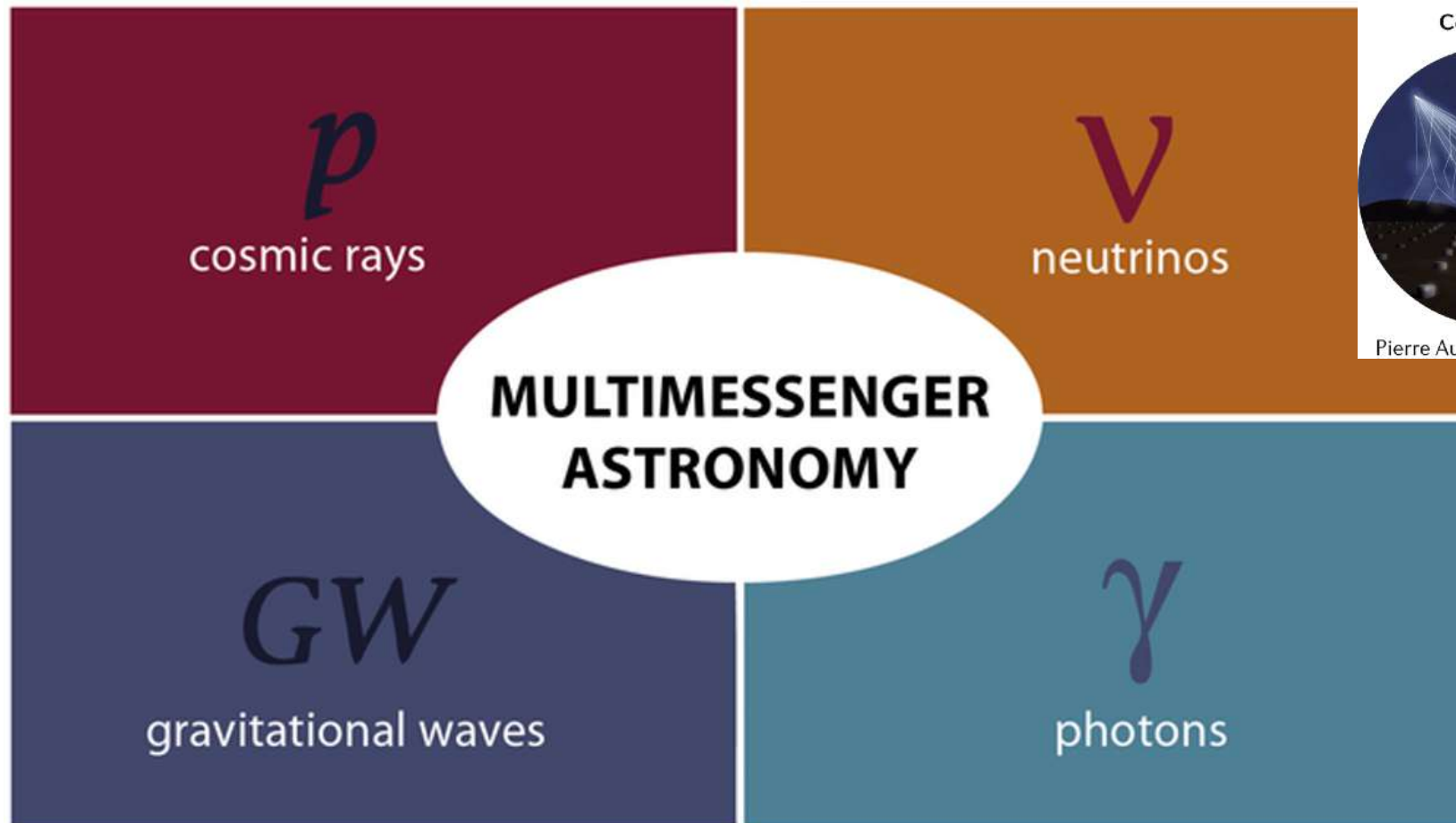


Spiral Galaxy M51



Active galaxy Centaurus A

# Multi-messenger Astronomy





# Astronomy Nobel Prizes



1974: M. Ryle & A. Hewish (J. Bell)  
Imaging synthesis; pulsar



1978: A. Penzias & R. Wilson  
Cosmic Microwave Background



1993: R. Hulse & J. Taylor  
Double pulsar



2002: R. Giacconi, M. Koshiba & R. Davis Jr.  
Cosmic X-ray and neutrino sources



2006: J. Mather & G. Smoot  
Cosmic Microwave Background



2011: S. Perlmutter, B. Schmidt & A. Riess  
Dark energy (Cosmic expansion)



2017: R. Weiss, B. Barrish & K. Thorne  
Gravitational wave detection



2019: M. Mayor & D. Queloz  
Extrasolar planets

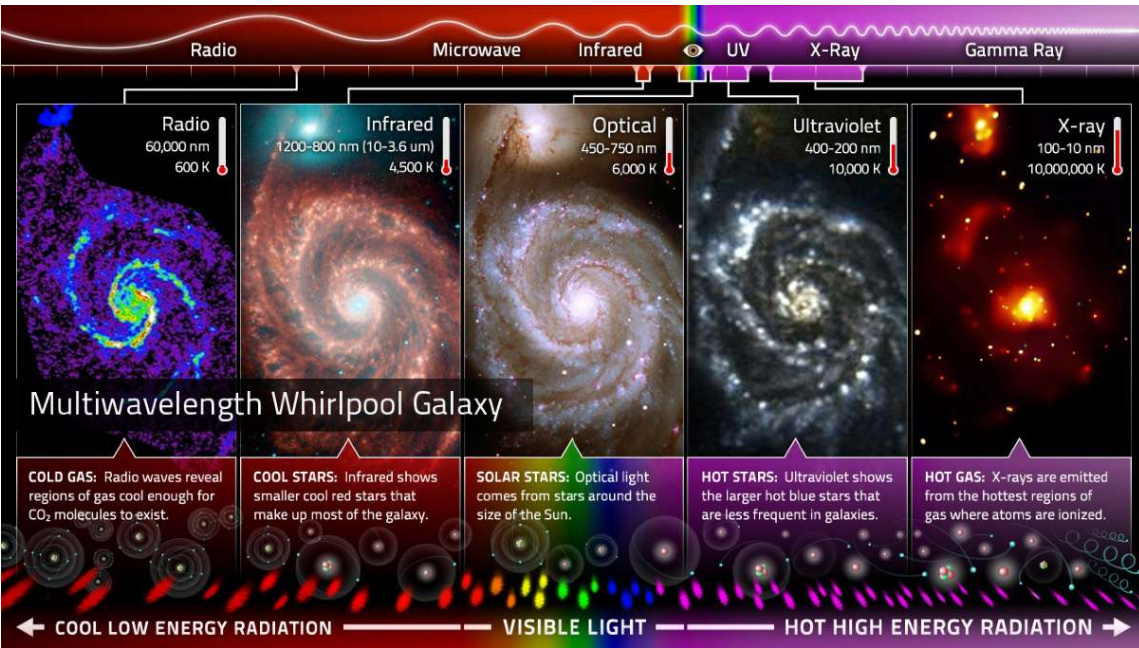


2020: R. Genzel & A. Ghez  
Black hole @ Galactic center

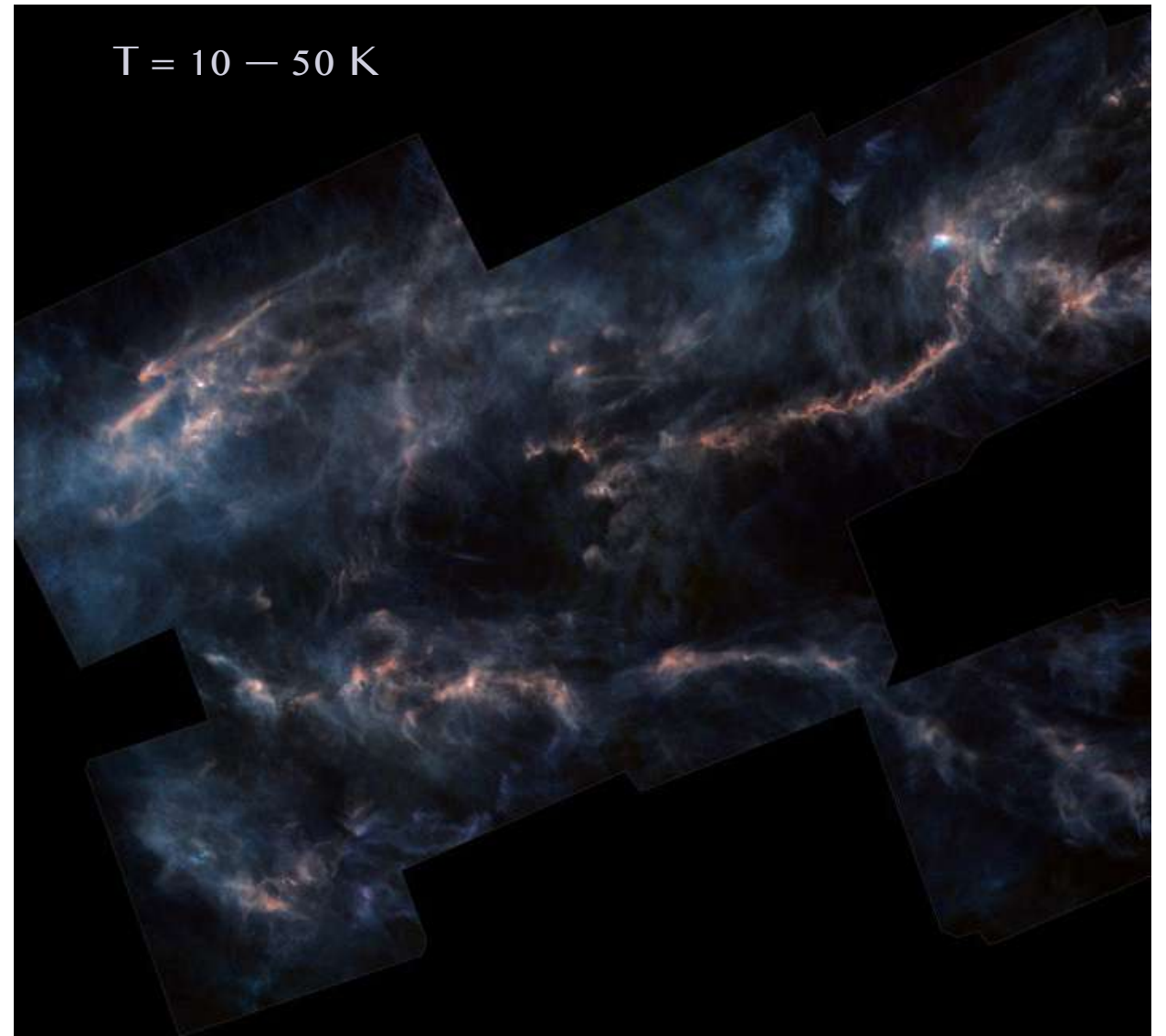
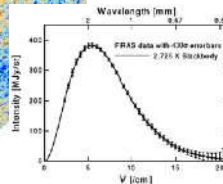
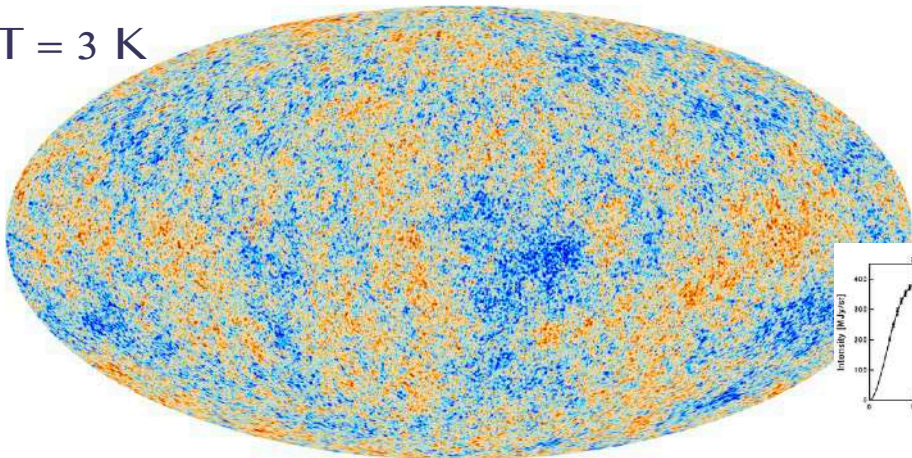
# Astronomy Nobel Prizes



# Emission processes: thermal (blackbody)



$T = 3 \text{ K}$

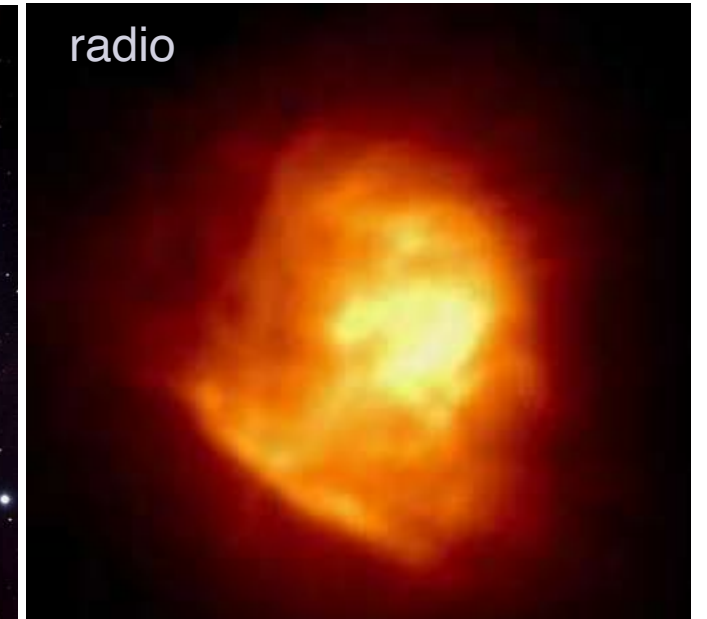
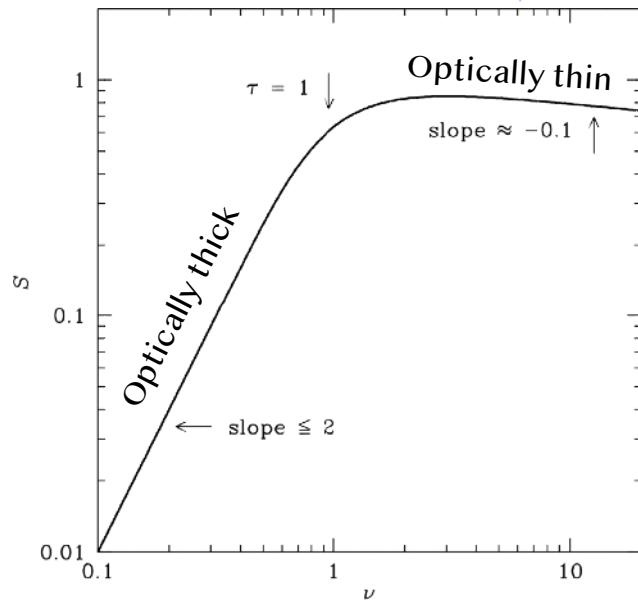
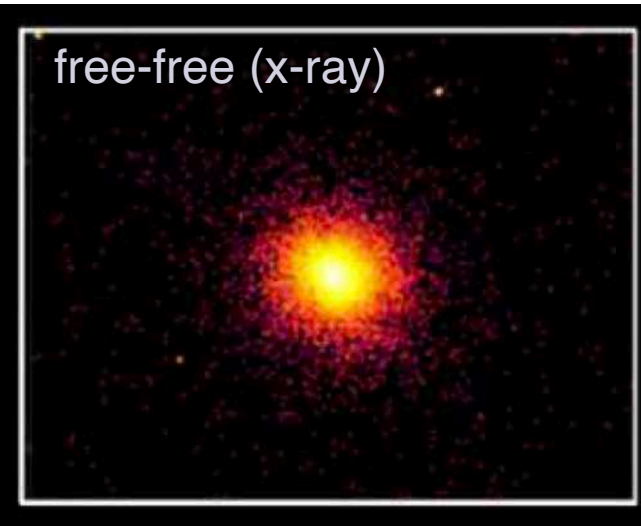
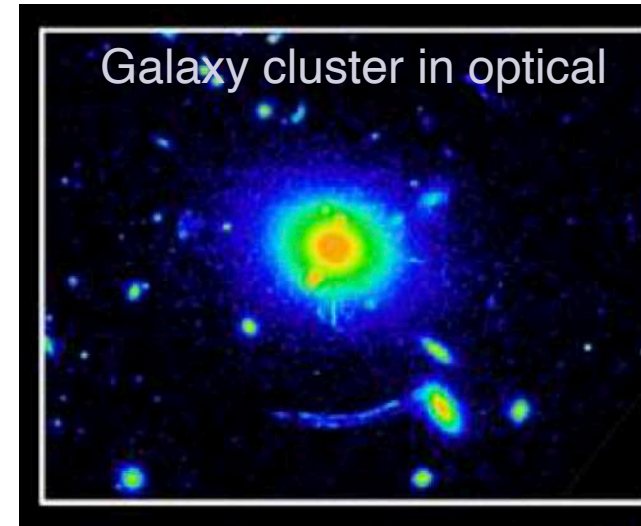
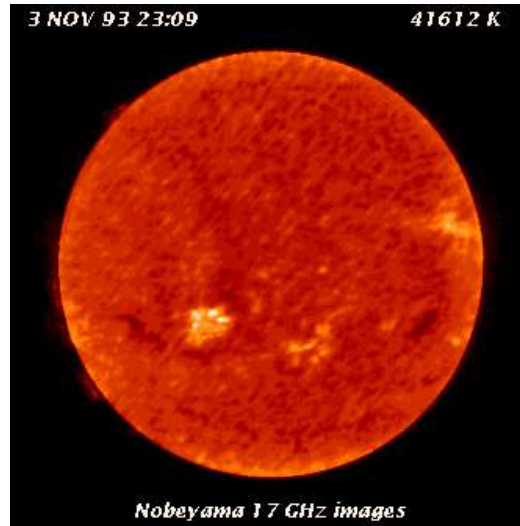
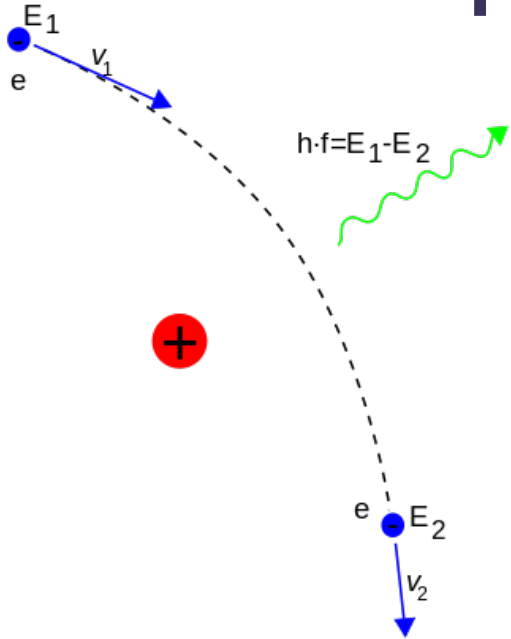


The cosmological microwave background

Taurus clouds with the Herschel satellite (150, 250, 350, 500  $\mu\text{m}$ )



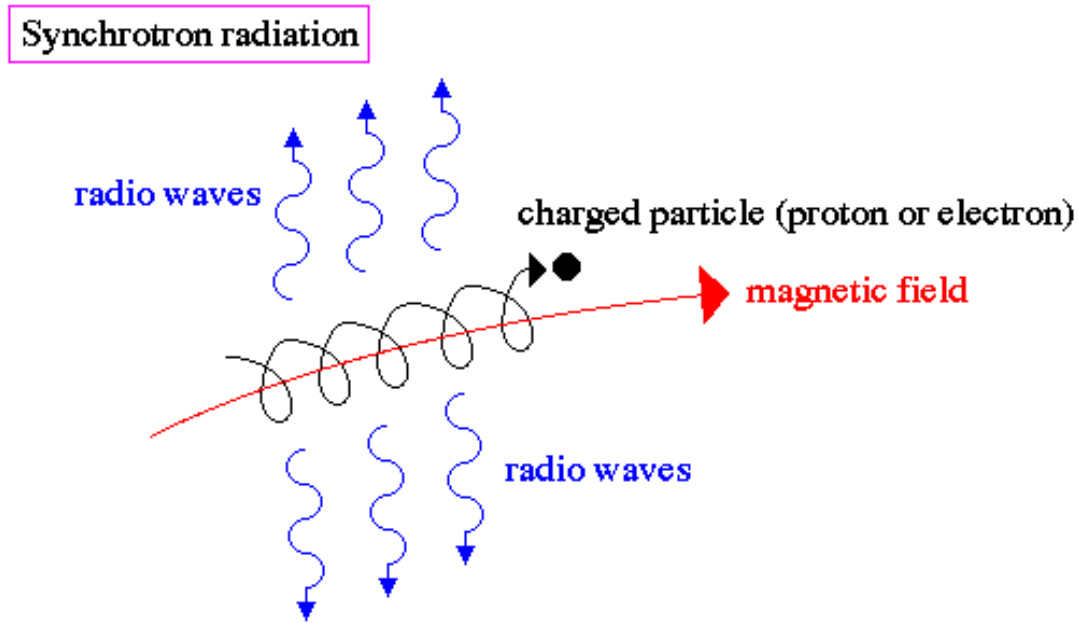
# Emission processes: free-free (thermal)



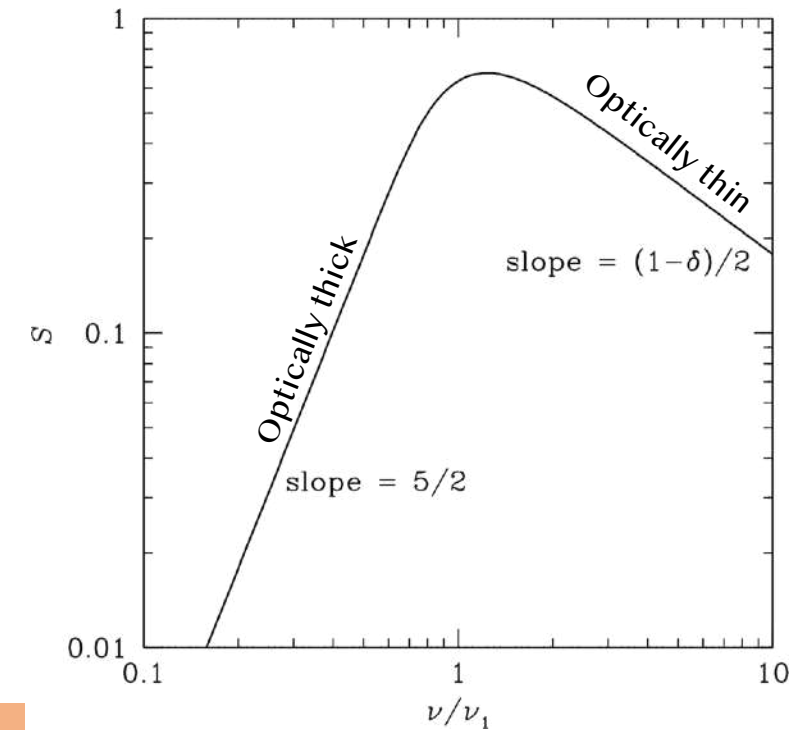


# Emission processes: synchrotron (non-thermal)

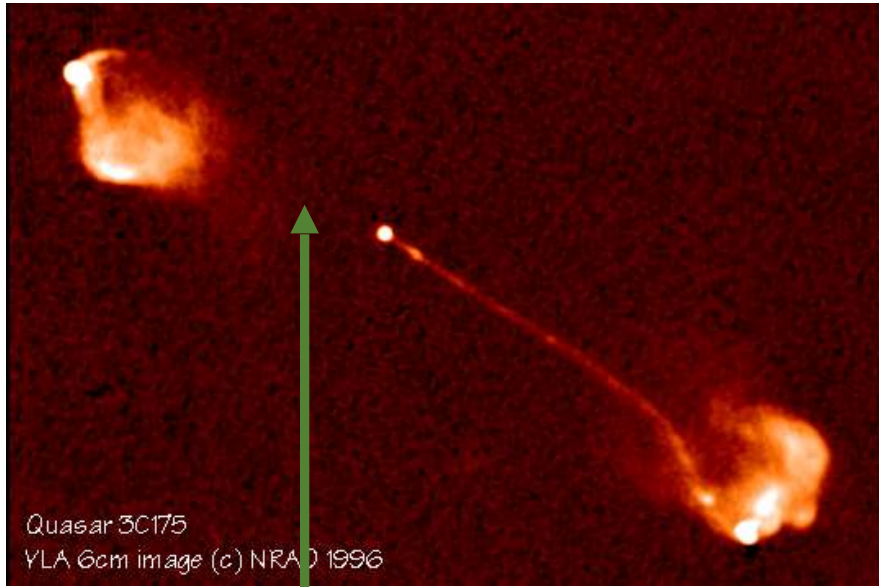
Radioastronomy (and, as we will see, particularly VLBI) is the realm of non-thermal processes



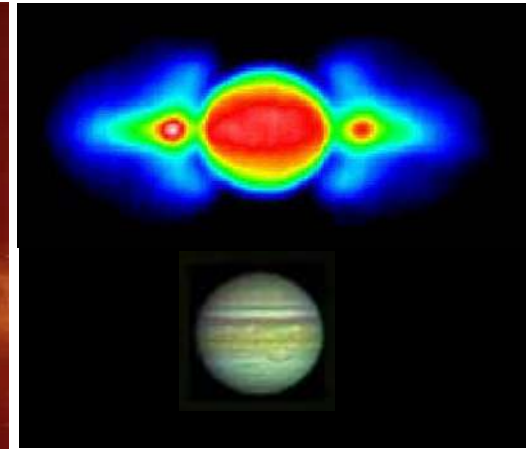
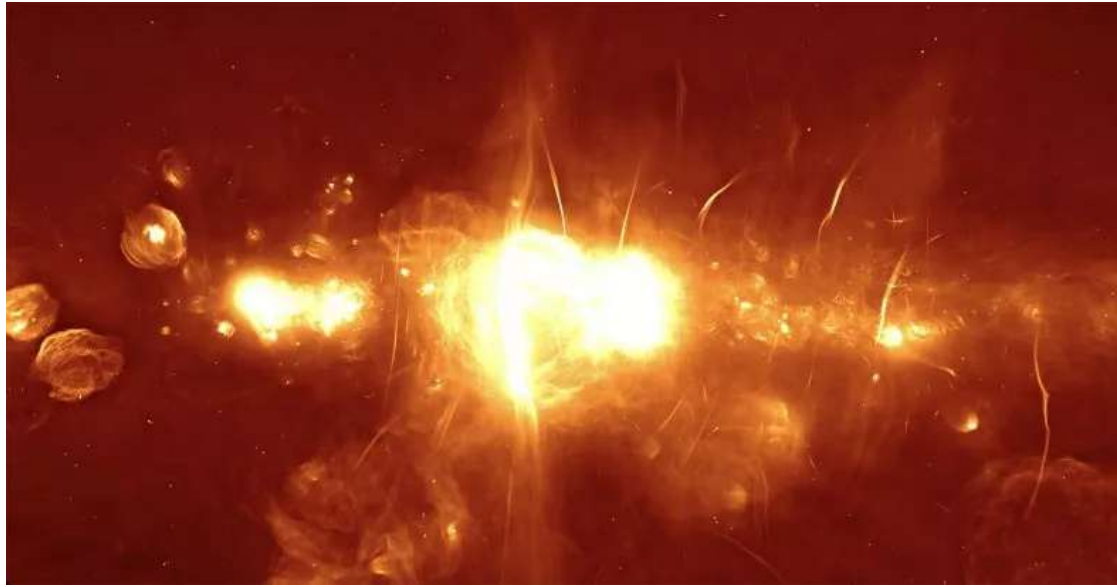
Relativistic electrons needed  
(acceleration mechanism)



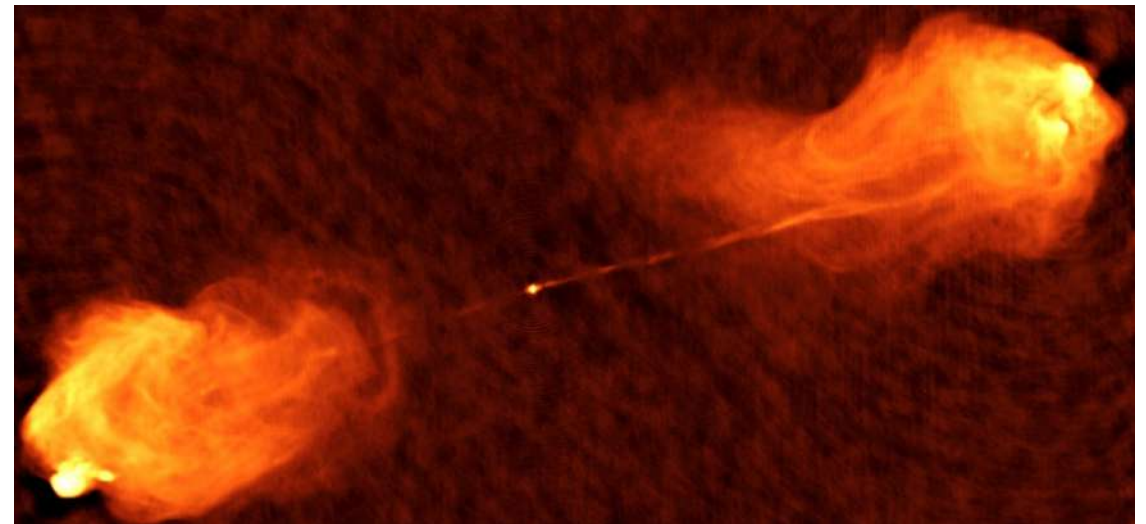
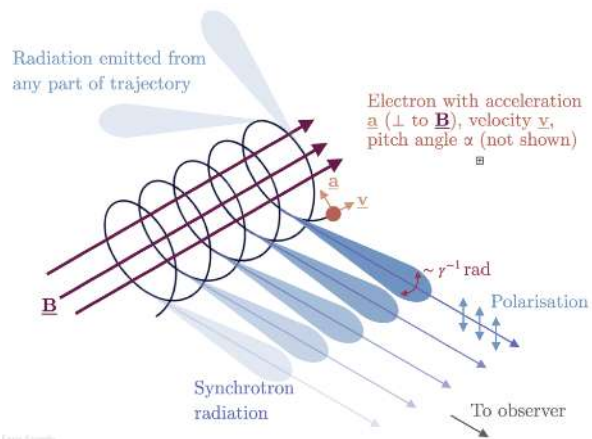
# Emission processes: synchrotron (non-thermal)



Quiz: why isn't there a counterjet?



The Galactic center in radio waves

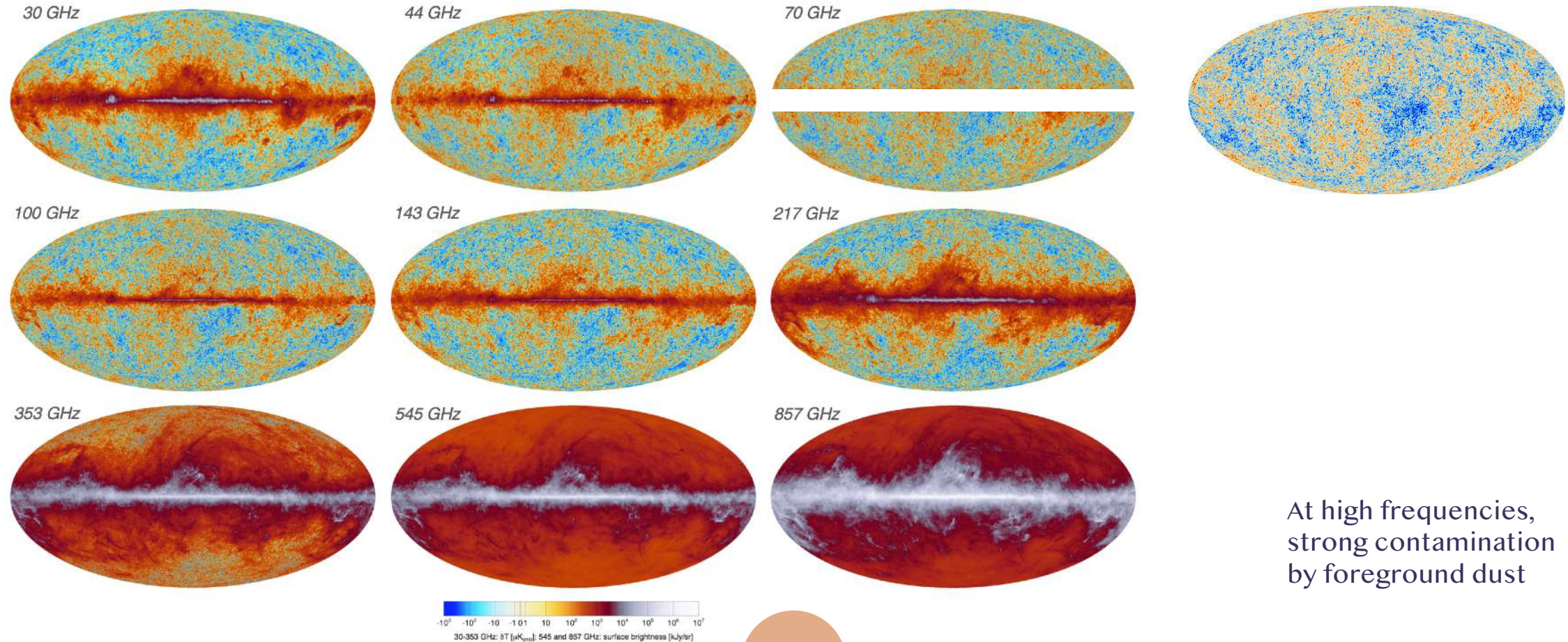




# All-sky Planck microwave maps

At low frequencies,  
strong contamination  
by foreground free-  
free and synchrotron

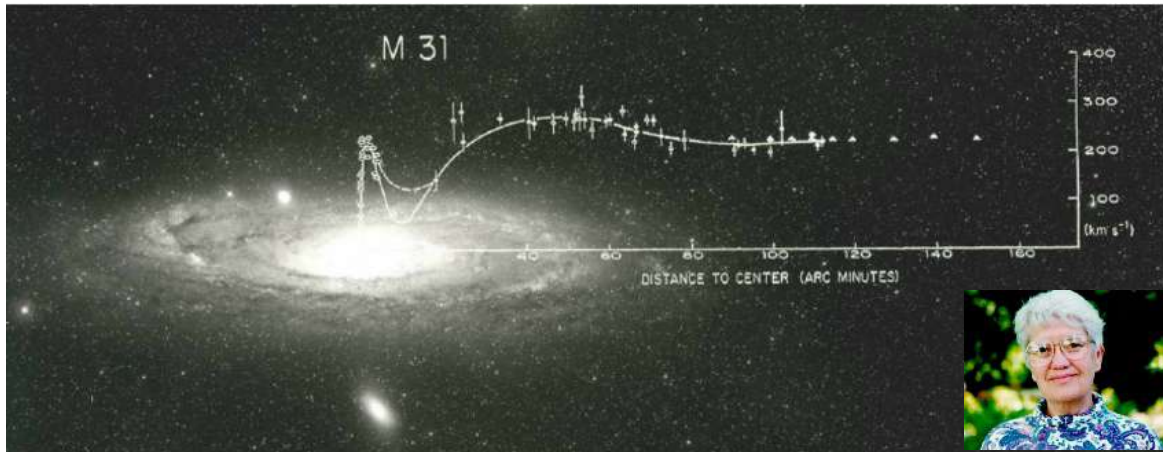
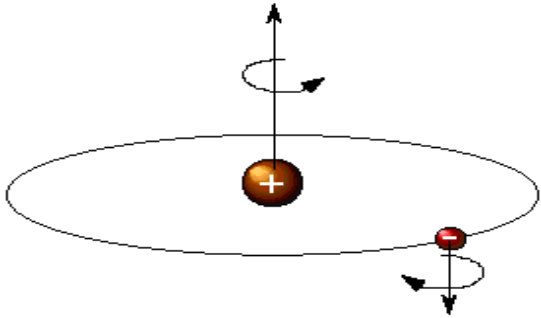
The sweet spot for  
cosmological  
microwave background



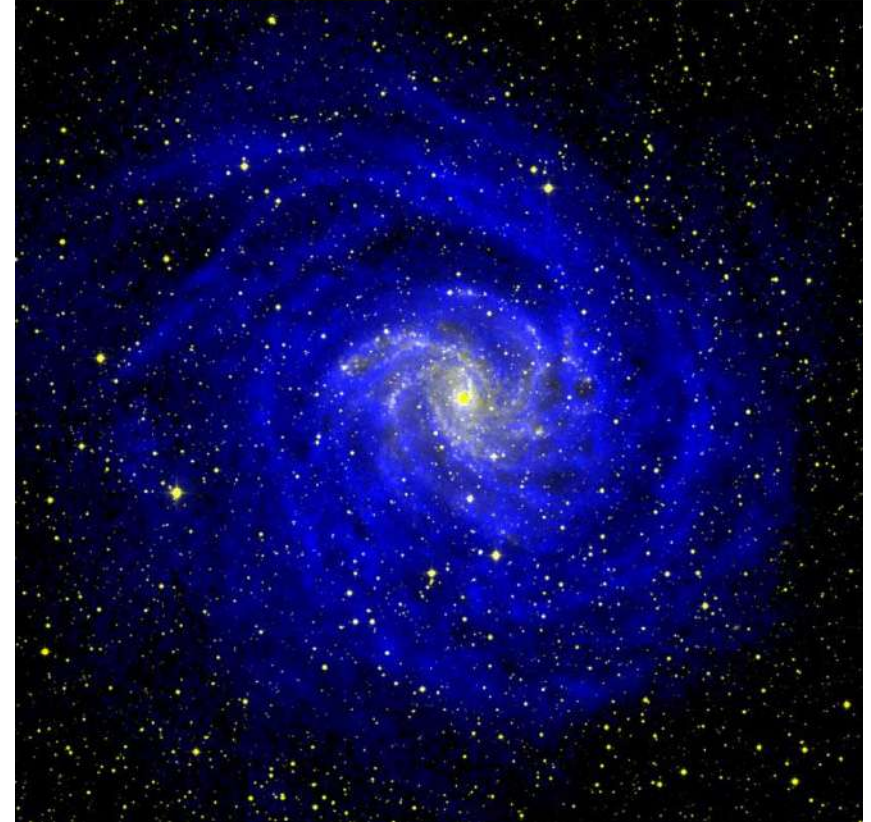
At high frequencies,  
strong contamination  
by foreground dust



# Line emission: 21-cm hydrogen line



Vera Rubin

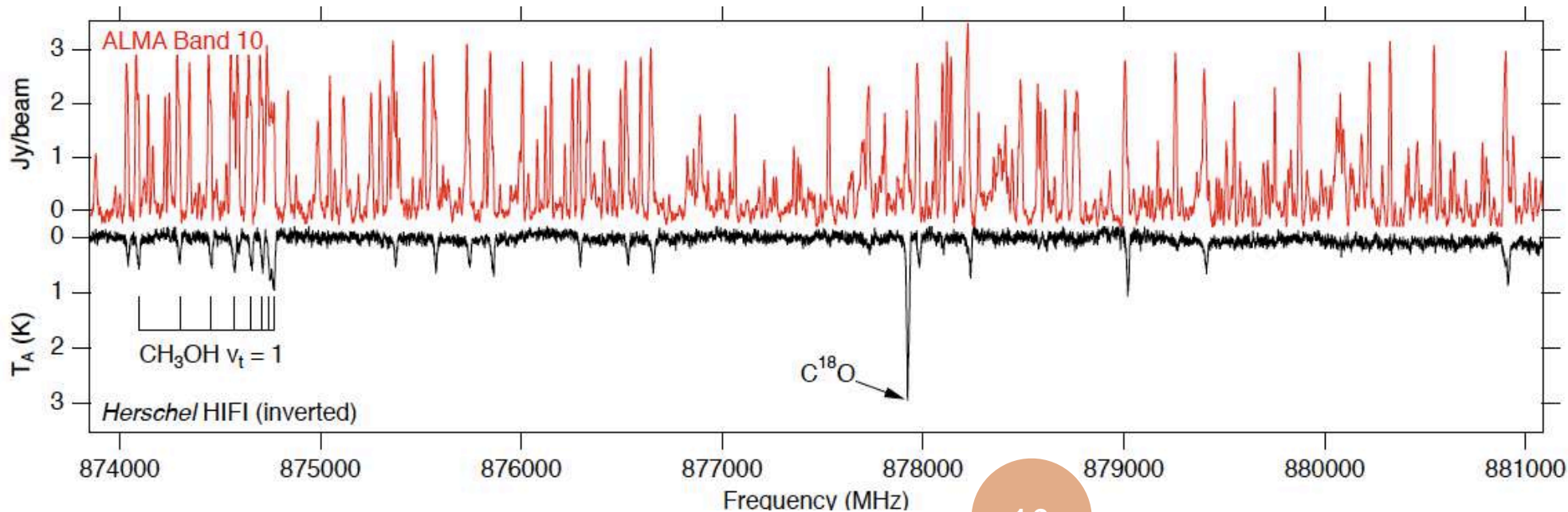
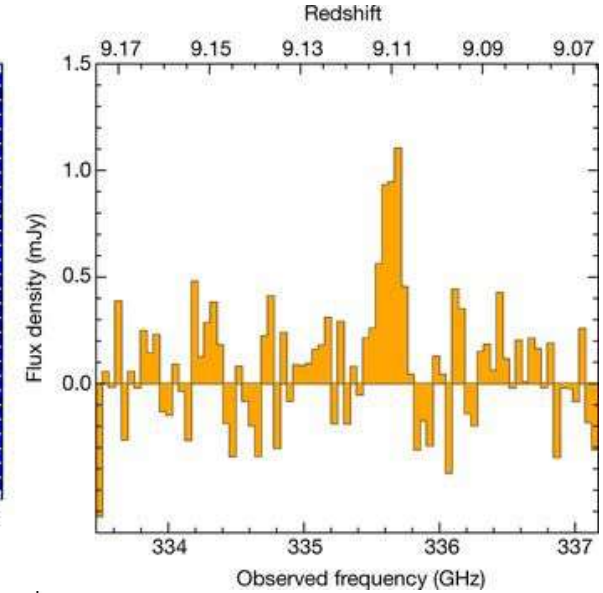
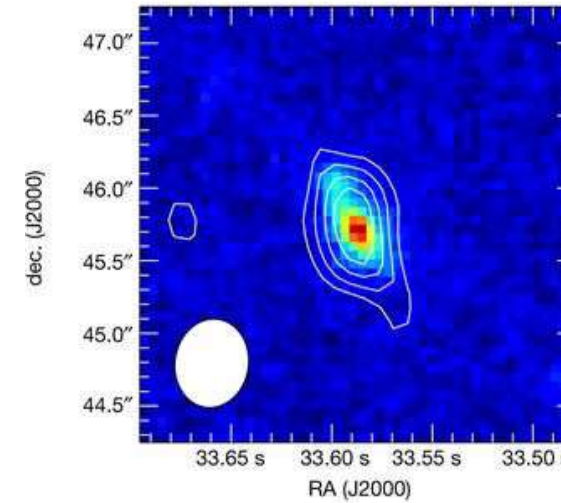
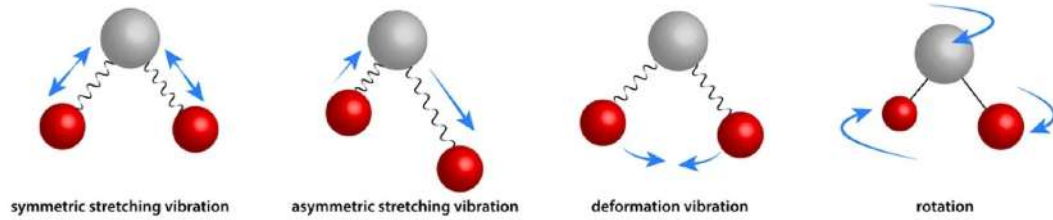


Optical image

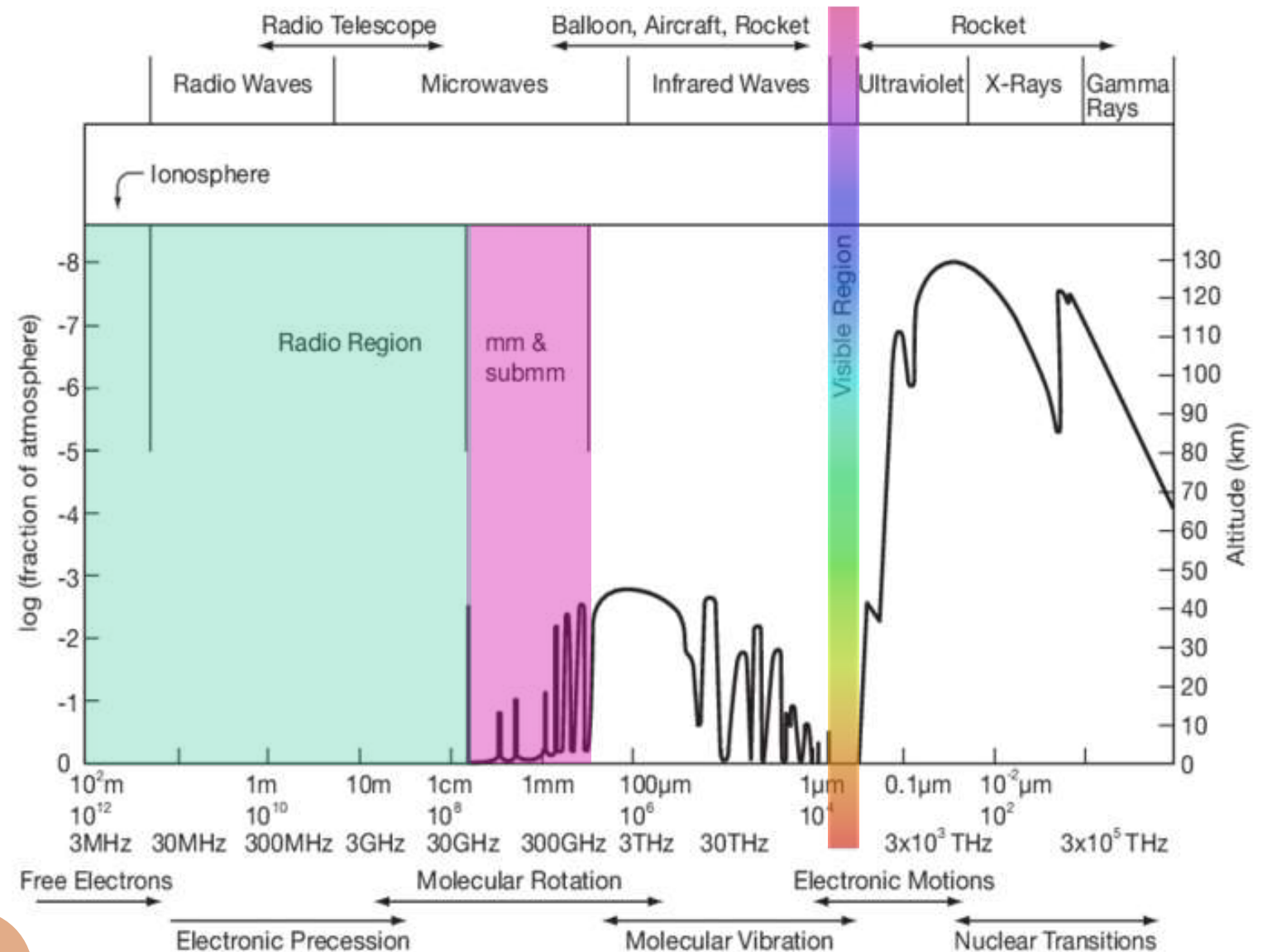
21-cm map

# Molecular emission lines

Oxygen at  $z = 9$  with ALMA



# Atmospheric transparency/opacity





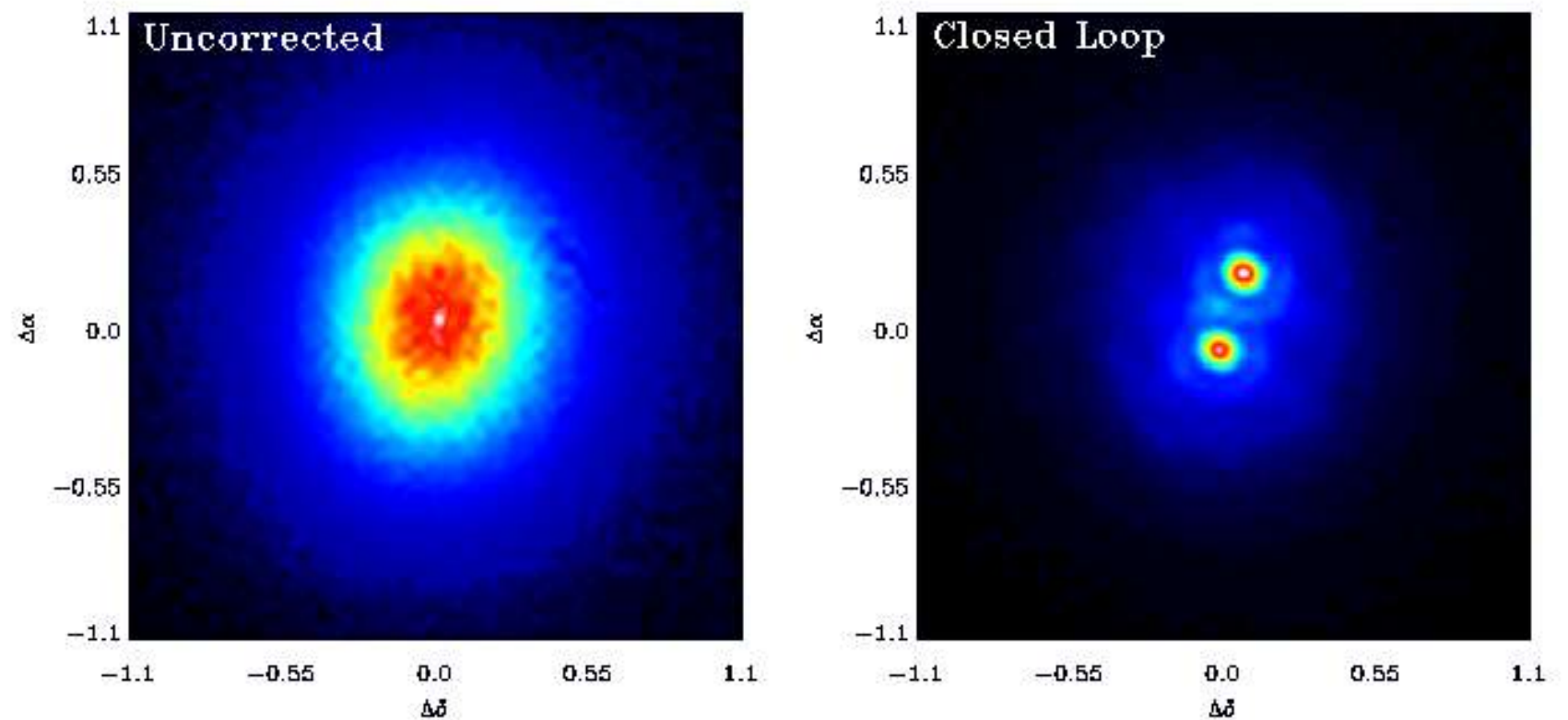
# Angular resolution (image sharpness)



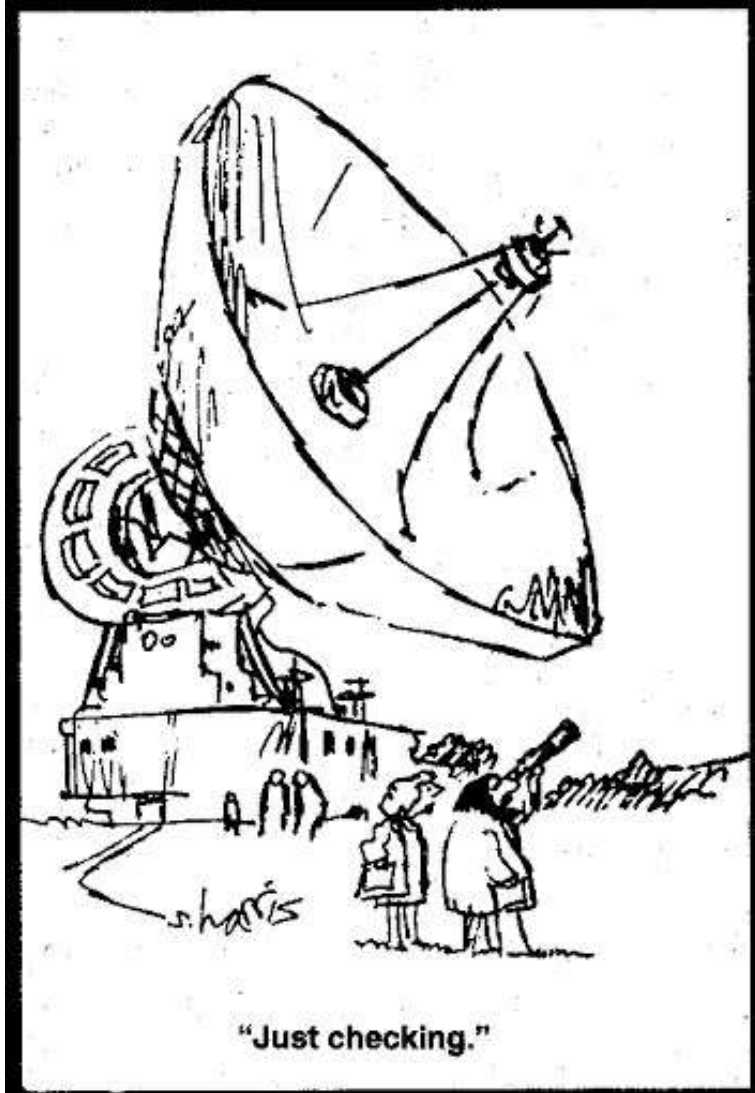
# Angular resolution definition

The angular resolution is the minimum separation that an optical instrument can resolve:

$$\theta \approx \frac{\lambda}{D}$$



# The angular resolution **problem** in radioastronomy



$$\theta \approx \frac{\lambda}{D}$$

$$\lambda = 5 \text{ cm} \text{ \& } D = 160 \text{ m} \longrightarrow \theta = 1'$$

Radio-astronomers need to build **BIG** telescopes

$$\lambda = 0.5 \text{ }\mu\text{m} \text{ \& } D = 1.6 \text{ mm} \longrightarrow \theta = 1'$$



# That's hard and expensive...



Five Hundred Meter Aperture Spherical Telescope (China)



Green Bank Telescope (100m, USA)



# ... and potentially risky

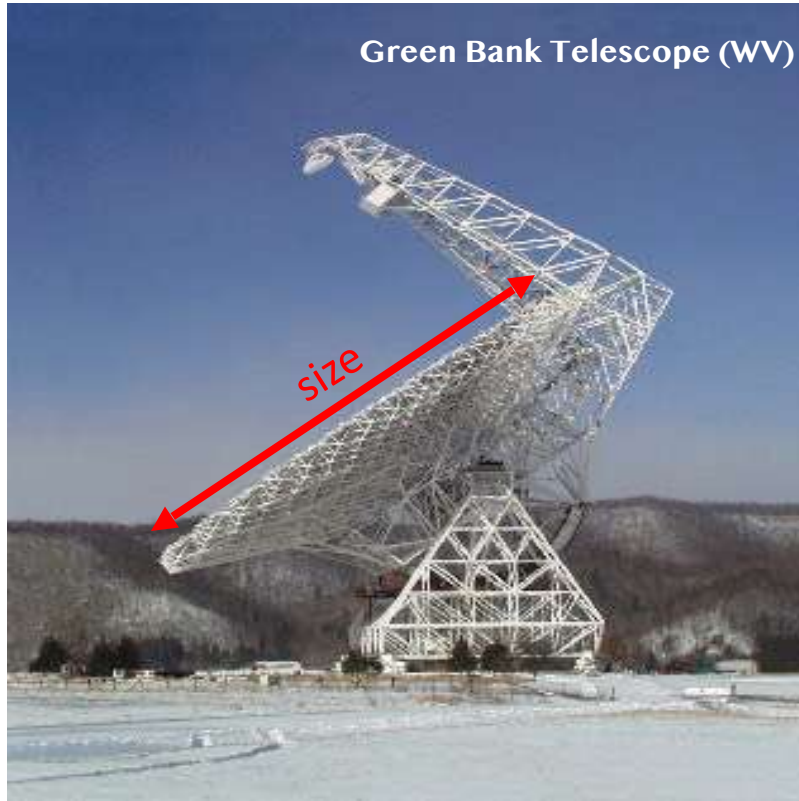


300 foot telescope in Green Bank (November 15, 1988)



300 foot telescope in Green Bank (November 16, 1988)

# Two kinds of radio telescopes



Single-dish

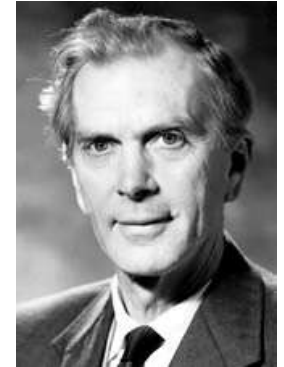
$$\theta \approx \frac{\lambda}{D}$$



Interferometers

$$\theta \approx \frac{\lambda}{B_{max}}$$

Maximum "baseline"



Sir Martin Ryle (Nobel 1974)

Imaging synthesis

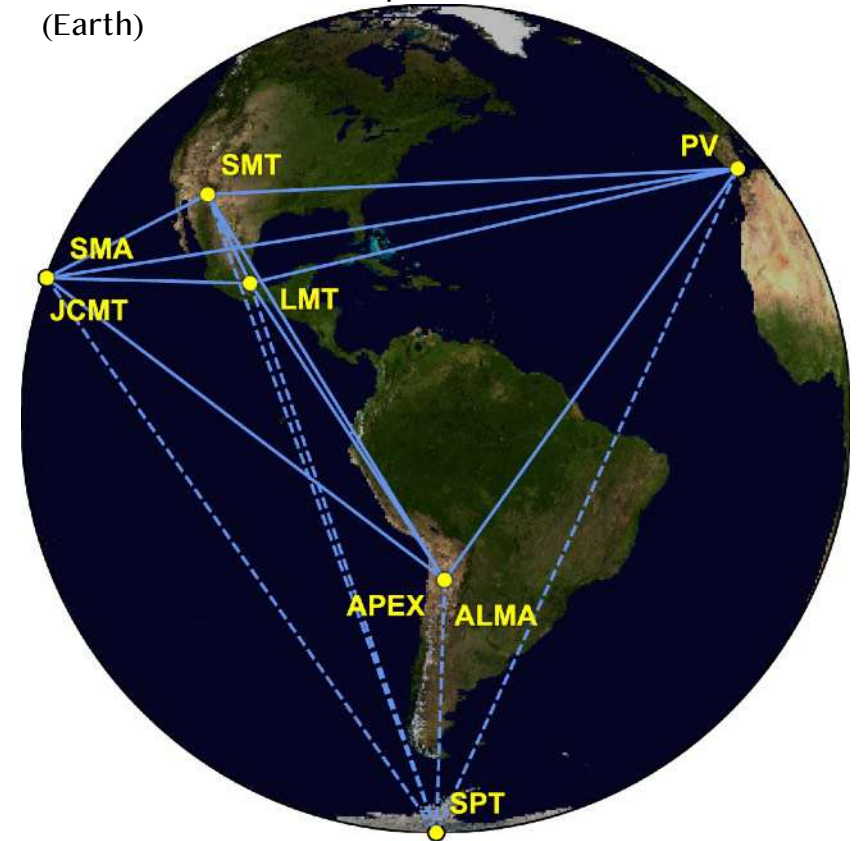


# Very Long Baseline Interferometry (VLBI)



1 milli-arcsecond resolution

Event Horizon Telescope  
(Earth)



25 microarcsecond resolution

# The angular resolution **solution** in radioastronomy



Biggest single-dish

Same resolution as human eye  
(~ one arcminute)



“conventional” interferometers

Same resolution as large optical telescope  
(~ 0.1 arcsecond)



VLBI arrays

Highest resolution in astronomy  
(~ 1 milli-arcsecond down to 10 micro-arcsecond)

**This is the highest angular resolution achievable in all of astronomy**



VLA (NM, USA)



ALMA (Chile)





ASKAP (Australia)



Lower (Europe)



Keercat (South-Africa)

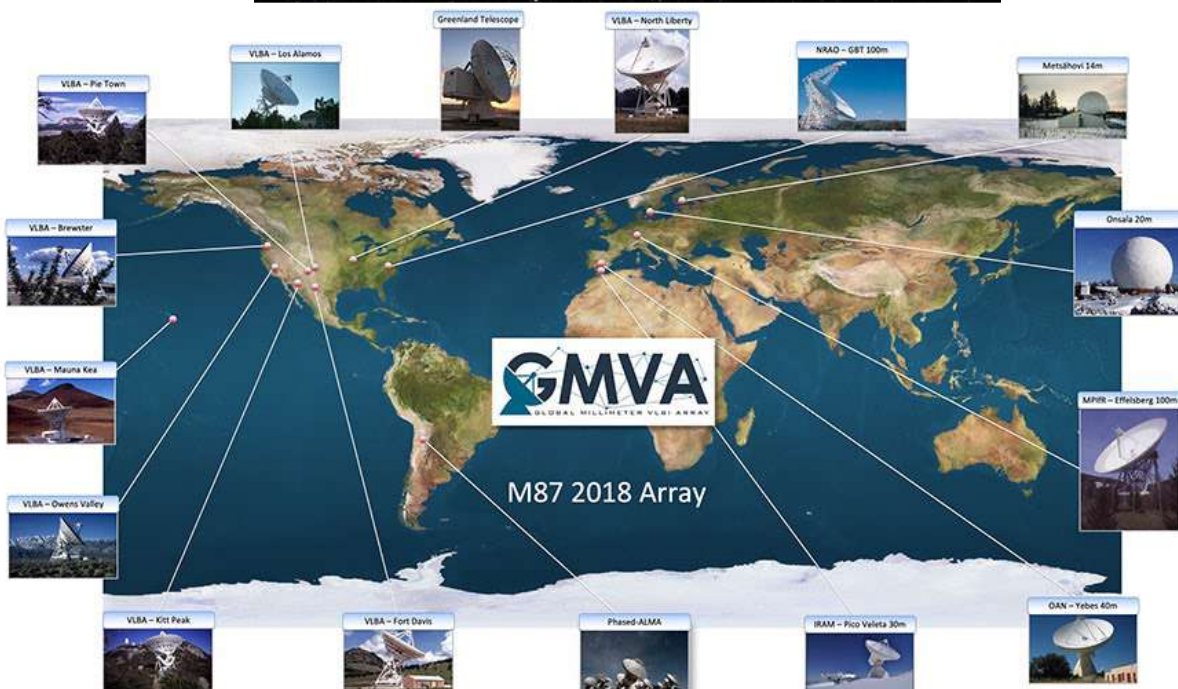
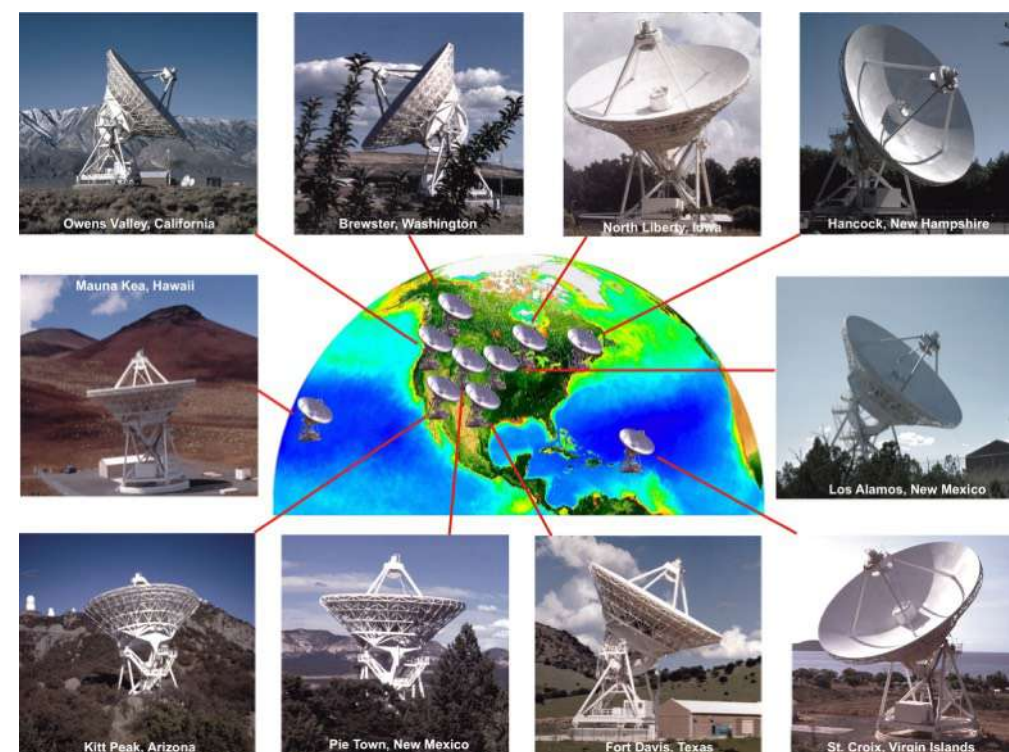




EHT

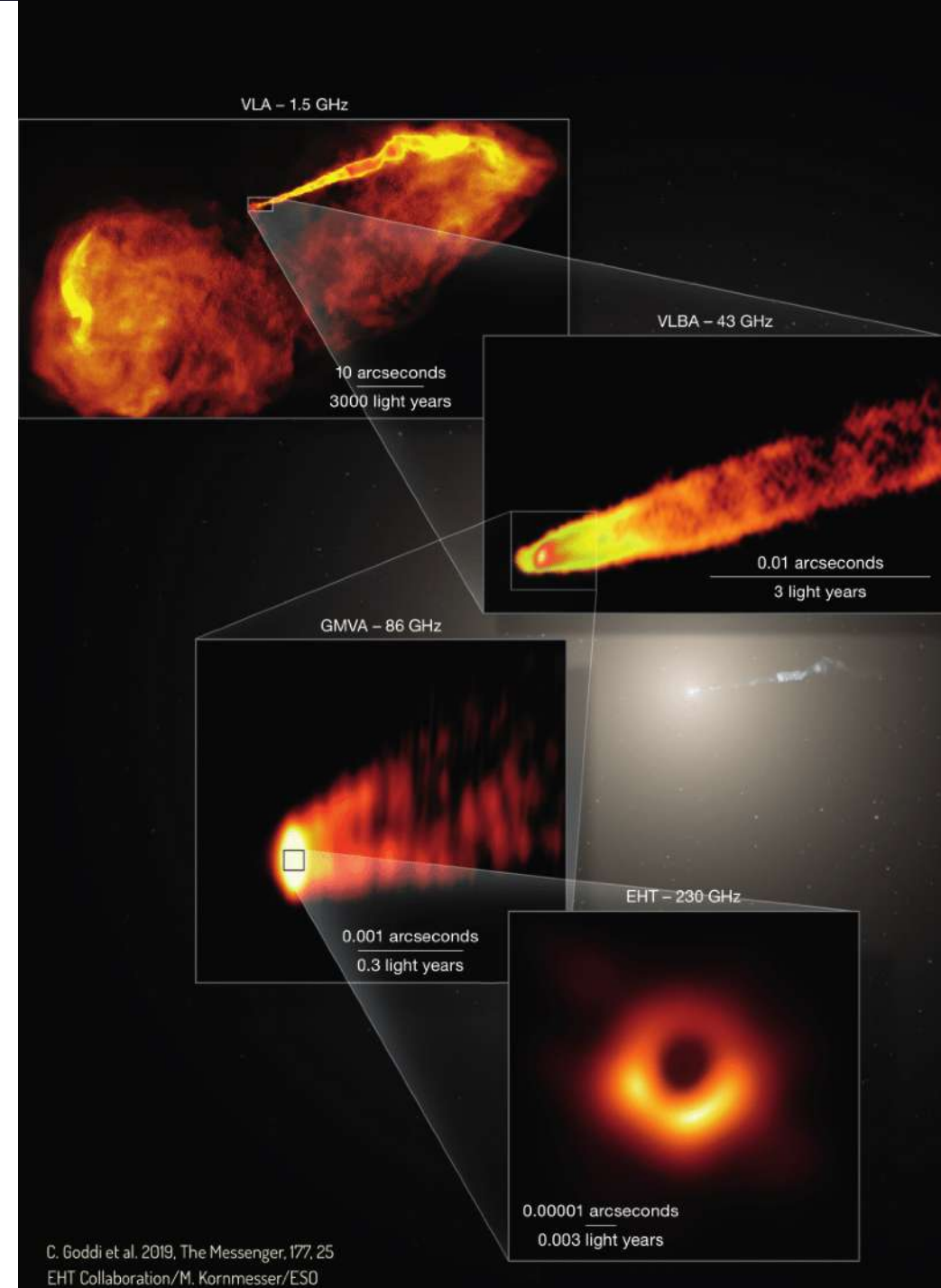
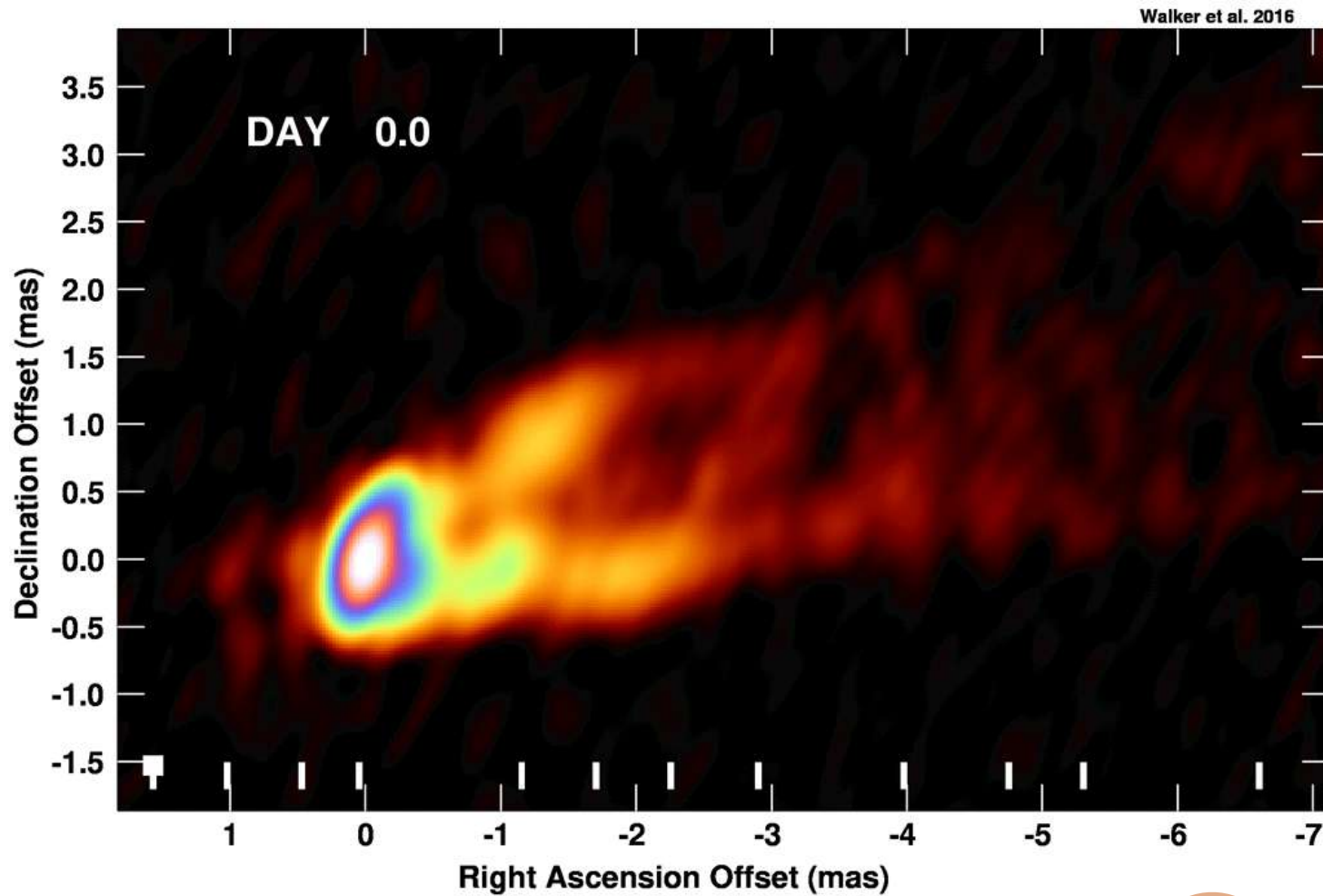


VLBA





# VLBI highlights





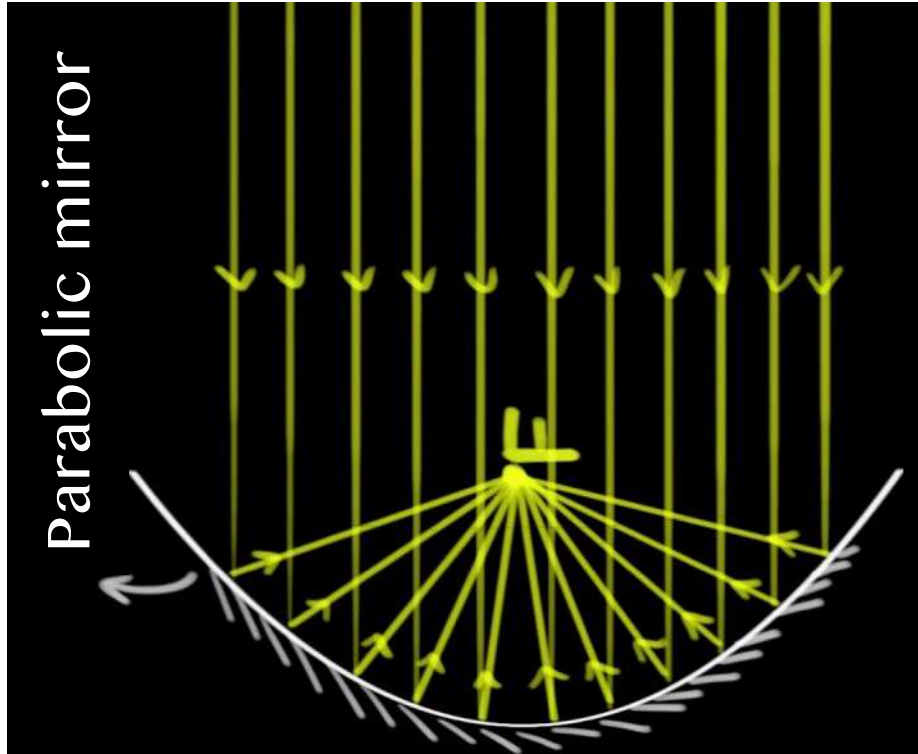


## Part 2: Fundamentals of Interferometry

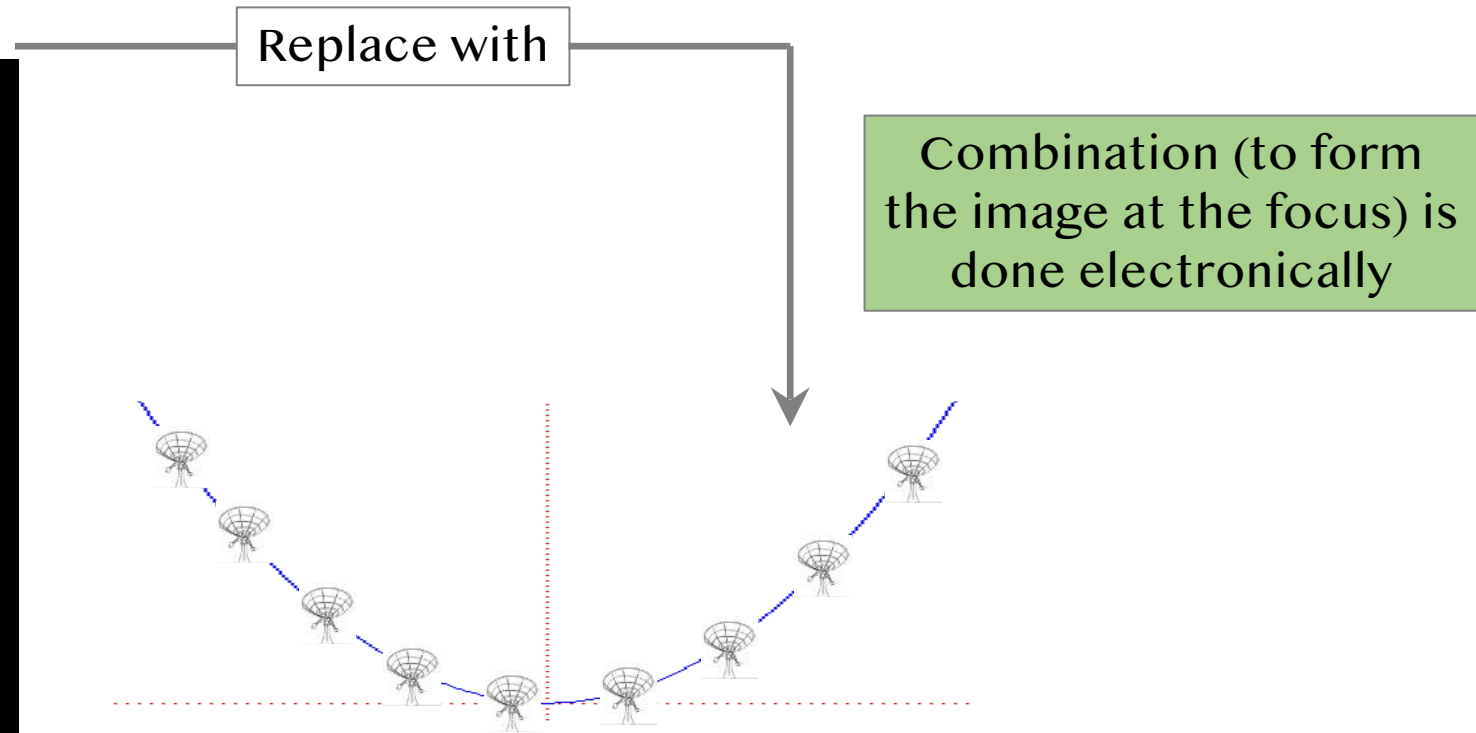
# Fair warning: interferometry is more work than single-dish...

- Observing with  $N \gg 1$  antennas, is  $N$  times more work than observing with one antenna (or maybe even  $N^2$  times more work...)
- As we will see in details later, interferometers do not provide directly an image. Rather they deliver complex quantities ( $\in \mathbb{C}$ ) that need to be manipulated mathematically to computationally reconstruct an image
- The resulting images contain artifacts caused by the geometry of the array
- Sensitivity is limited
- Calibration is more work than for single-dish telescopes

# Pictorial principle of interferometry

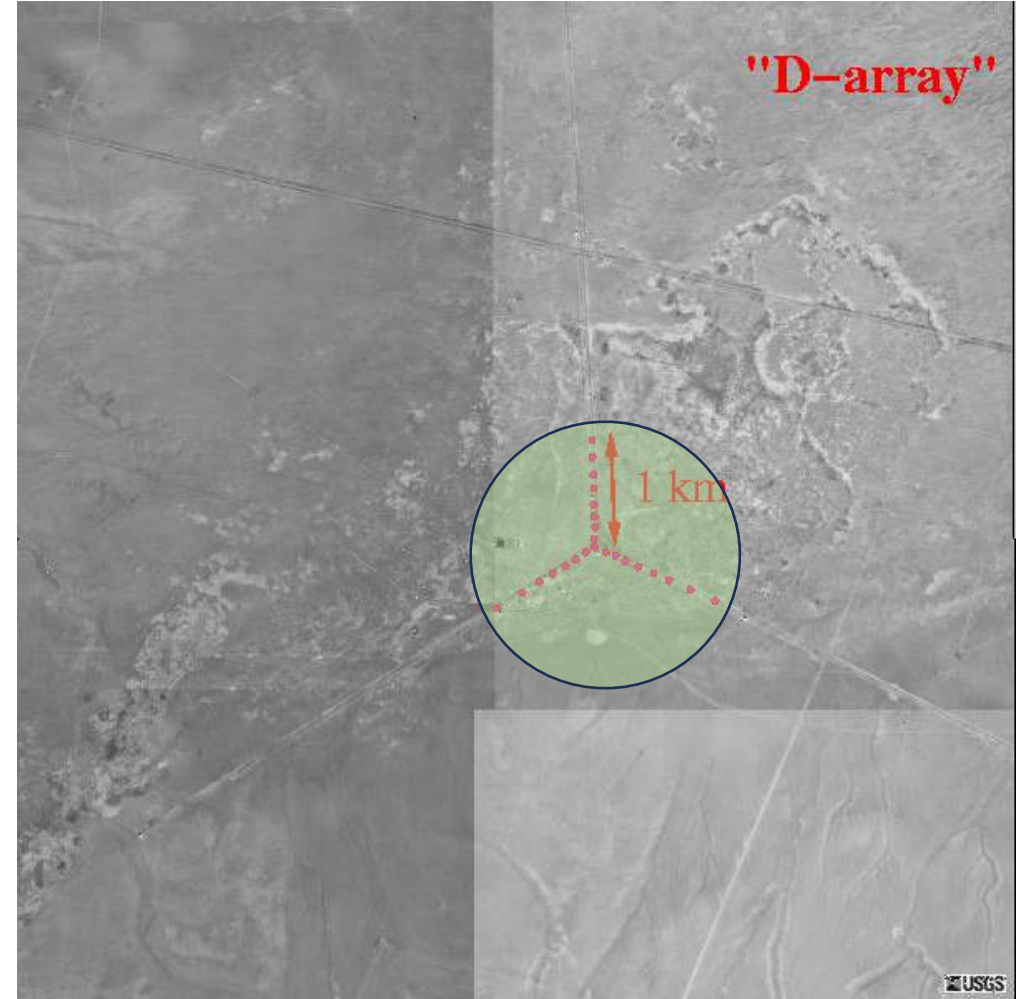


Reflector telescope

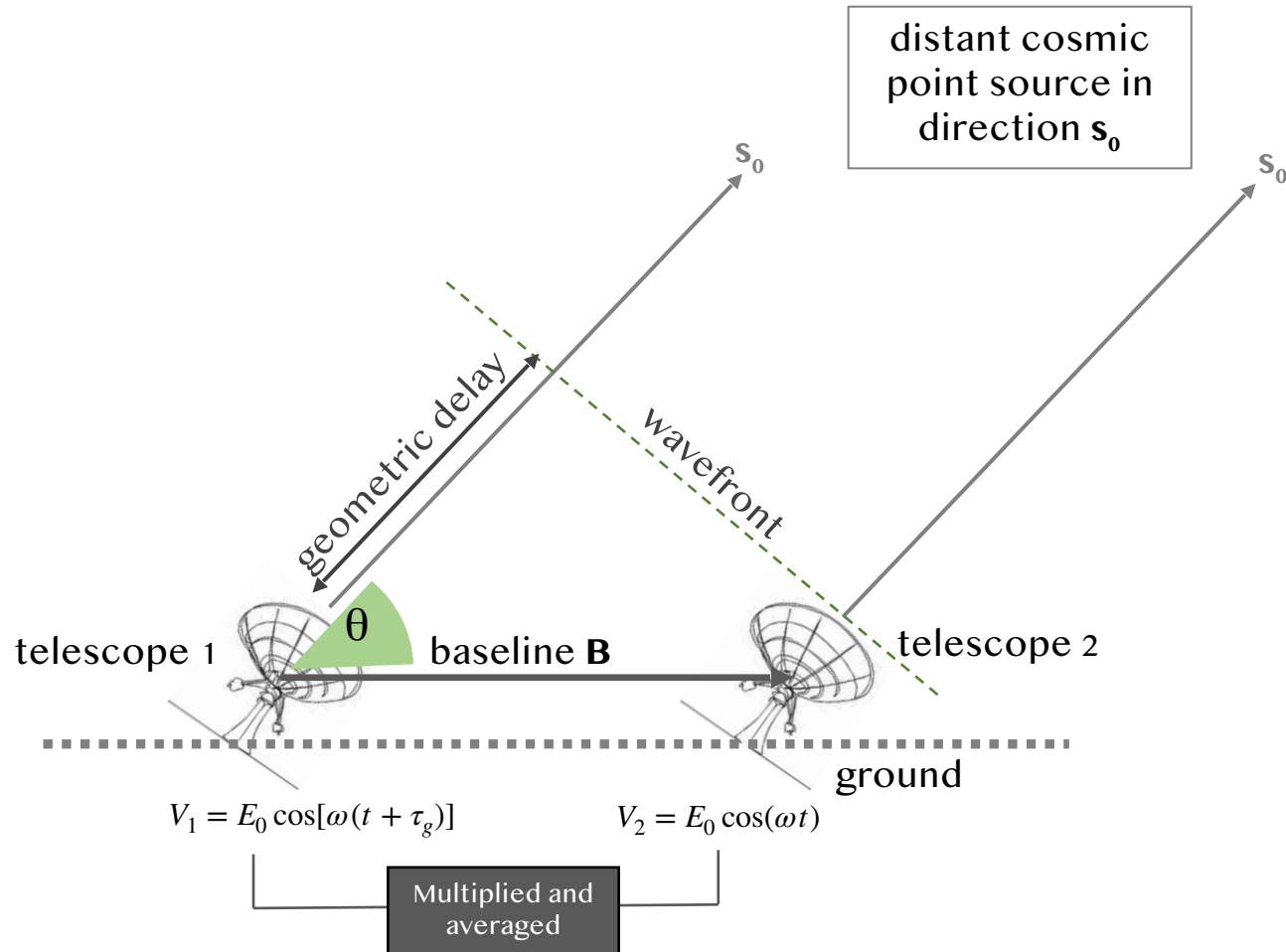




# Interferometers as fragmented mirrors



# How it really works...



- ❑ Each telescope of the array acts as a **coherent sensor** of the incoming electric field  $\mathbf{E}$
- ❑ The **geometric delay**,  $\tau_g$ , is:

$$\tau_g = \frac{\mathbf{B} \cdot \hat{\mathbf{s}}_0}{c}$$

- ❑ The output of the interferometer is:

$$R_c = P_0 \cos(\omega \tau_g)$$

$P_0 = E_0^2/2$  is the received power in the EM wave



# How it really works...

- ❑ If the source is not point-like,

$$R_c = \iint I(\hat{\mathbf{s}}) \cos(\omega \tau_g) d\Omega$$

$I(\hat{\mathbf{s}})$  is the sky brightness

- ❑ By adding a  $\pi/2$  phase delay to the output of telescope 2 before multiplying, we can get a complementary interferometer response:

$$R_s = \iint I(\hat{\mathbf{s}}) \sin(\omega \tau_g) d\Omega$$

- ❑ The specialized hardware that produces  $R_c$  and  $R_s$  from the output of the telescopes is called a “correlator”

# How it really works...

- We now **define** a new **complex** function  $R$ , by combining  $R_c$  and  $R_s$ :

$$R = R_c - iR_s$$

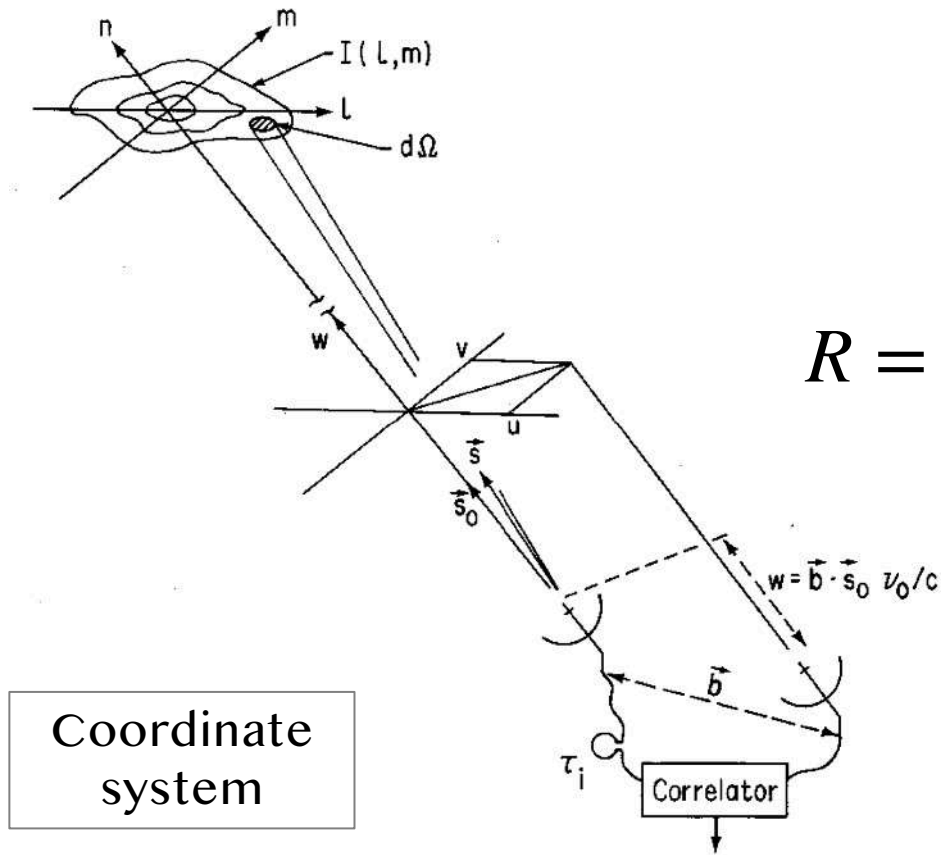
- From the formulae of  $R_c$  and  $R_s$ , we see that:

$$R = \iint I(\hat{\mathbf{s}}) e^{-i\omega\tau_g} d\Omega = \iint I(\hat{\mathbf{s}}) e^{-2\pi i \mathbf{B} \cdot \hat{\mathbf{s}} / \lambda} d\Omega$$

- Because  $R$  is a complex number, the specialized hardware that produces  $R$  (i.e.  $R_c$  and  $R_s$ ) from the output of the telescopes is called a “complex correlator”



# How it really works...



$$e^{-2\pi i \mathbf{B} \cdot \hat{\mathbf{s}} / \lambda} = e^{-2\pi i \mathbf{B} \cdot \hat{\mathbf{s}}_0 / \lambda} \cdot e^{-2\pi i (ul + vm)}$$

$$R = e^{-2\pi i \mathbf{B} \cdot \hat{\mathbf{s}}_0 / \lambda} \iint I(l, m) e^{-2\pi i (ul + vm)} \frac{dl dm}{\sqrt{1 - l^2 - m^2}}$$

Complex visibility function

# Fundamental result

$$R = e^{-2\pi i \mathbf{B} \cdot \hat{\mathbf{s}}_0 / \lambda} \iint I(l, m) e^{-2\pi i (ul + vm)} \frac{dl dm}{\sqrt{1 - l^2 - m^2}}$$

Response of the  
interferometer

$$V(u, v) = \iint I(l, m) e^{-2\pi i (ul + vm)} \frac{dl dm}{\sqrt{1 - l^2 - m^2}}$$

Complex visibility  
function

Van Cittert-Zernike Theorem

Fourier Transform

$$I(l, m) = \sqrt{1 - l^2 - m^2} \iint V(u, v) e^{2\pi i (lu + mv)} du dv$$

Sky brightness



# But, careful...

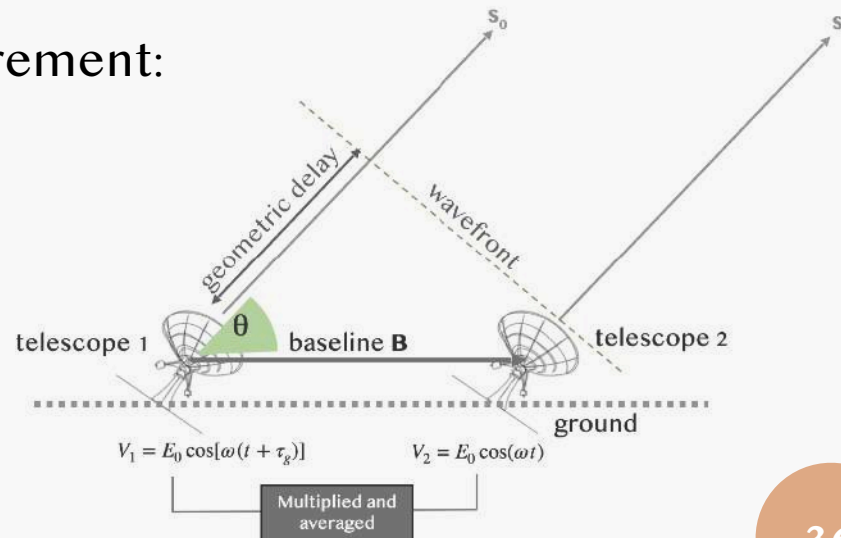
$$V(u, v) = \iint I(l, m) e^{-2\pi i(ul+vm)} \frac{dl dm}{\sqrt{1-l^2-m^2}}$$

This is a **function** of  
 $(u, v)$

$$I(l, m) = \sqrt{1-l^2-m^2} \iint V(u, v) e^{2\pi i(lu+mv)} du dv$$

This is a **function** of  
 $(l, m)$

One measurement:



corresponds to one baseline : **B**,

and therefore also to one value of  $(u_i, v_i)$

(actually two points: also  $(-u_i, -v_i)$   
Including both baselines from telescope 1  
to telescope 2 and from telescope 2 to  
telescope 1)

# Summary

- The complex visibility function,  $V(u, v)$ , and the sky brightness,  $I(l, m)$ , are Fourier conjugates (van Cittert-Zernike Theorem).
- Measuring the visibility function and taking the inverse Fourier transform enables us to obtain the sky brightness.
- This is the (electronic and computational) process by which radio interferometers re-construct an image “in the focal plane” of the telescope they simulate.
- For each observation with two antennas, one only gets one the values of the complex visibility at two points  $(u_i, v_i)$  and  $(-u_i, -v_i)$ .