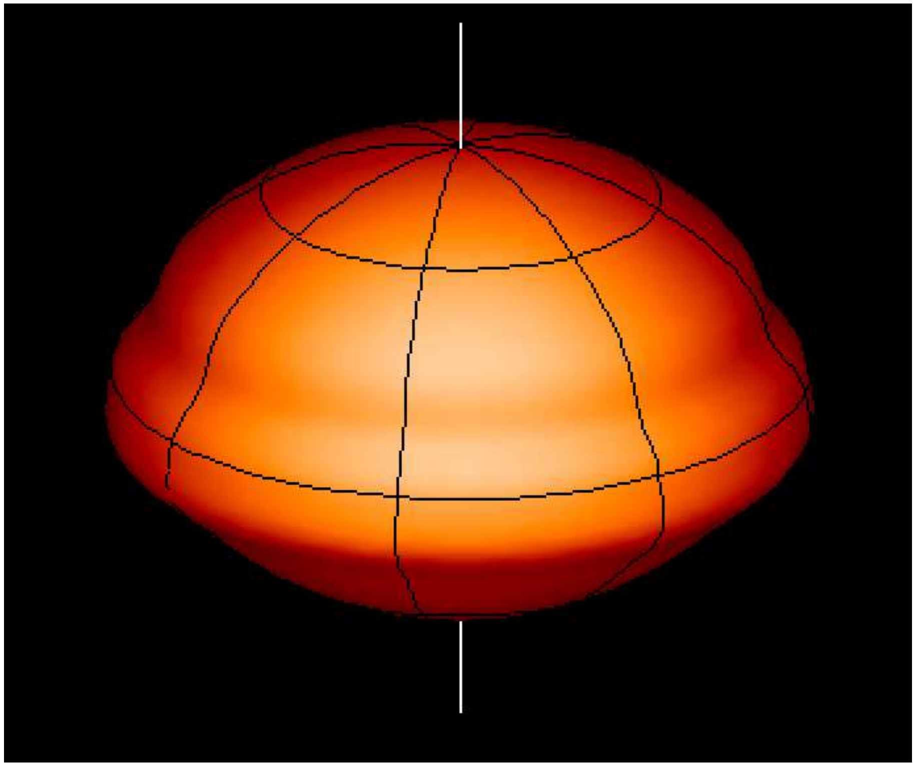
Seismology of rapidly rotating stars : A non-perturbative numerical approach



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Outline

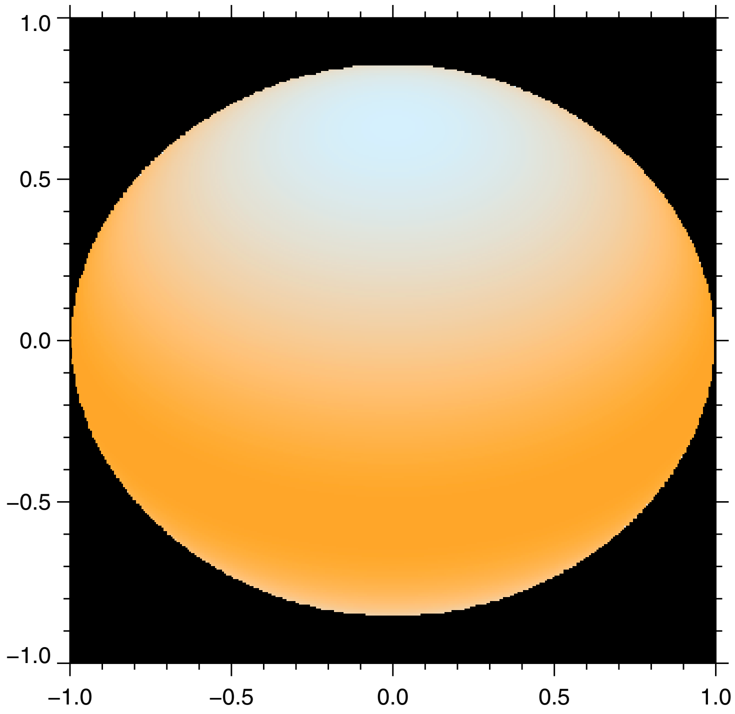
Introduction

Why study oscillations in rapidly rotating stars

✘ Physics of rotating stars not well understood

✘ High proportion of stars that rotate rapidly

- for example : δ Scuti and γ Doradus stars



Altair :

Interferometry :

Domiciano de Souza et al., 2005

Peterson et al., 2005

δ **Scuti pulsations :**

Buzasi et al., 2005

Models :

Suárez et al., 2005

Targets for Corot

Star HD	Spectral type	Type	Mass ($M_\odot$)	$v.\sin i$ ($km.s^{-1}$)
171834	F3		1.3	64[b]
177552	F1		1.4	41[b]
181555	A5	δ Scuti		170[a]
49434	F1 V	γ Dor		79[b]

[a] Poretti et al. 2003. A&A 406 :203-211.
[b] CorotWeek8, F. X. Schmider.

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1. Introduction
2. **Computational method**
3. The effects of rotation on pulsation frequencies
4. The effects of rotation on mode structure
5. Conclusion

Mathematical difficulties

- Two forces appear because of rotation
  - centrifugal force : stellar deformation, and $\vec{g}_{eff}$
  - Coriolis force
- Oscillation modes are no longer described by a single spherical harmonic
  - No longer $1D$, but $2D$

Comparison between perturbative and non-perturbative methods

Perturbative approach

✘ the rotation rate Ω is considered to be small

✘ equilibrium model and oscillation mode $=$ (a spherical solution) $+$ (a perturbation) $+ \mathcal{O}(\Omega^n)$

Non-perturbative approach

✘ the rotation rate Ω is not considered small

✘ equilibrium model and oscillation mode $=$ a solution to a 2D problem which fully includes the effects of rotation

Some references...

Perturbative methods

✘ 2^{nd} order methods :

- Saio (1981)
- Gough and Thompson (1990)
- Dziembowski and Goode (1992)

✘ 3^{rd} order methods :

- Soufi, Goupil and Dziembowski (1998)
- Karami et al. (2005)

Numerical methods

✘ Clement (1981-1998)

✘ Ipser and Lindblom (1990)

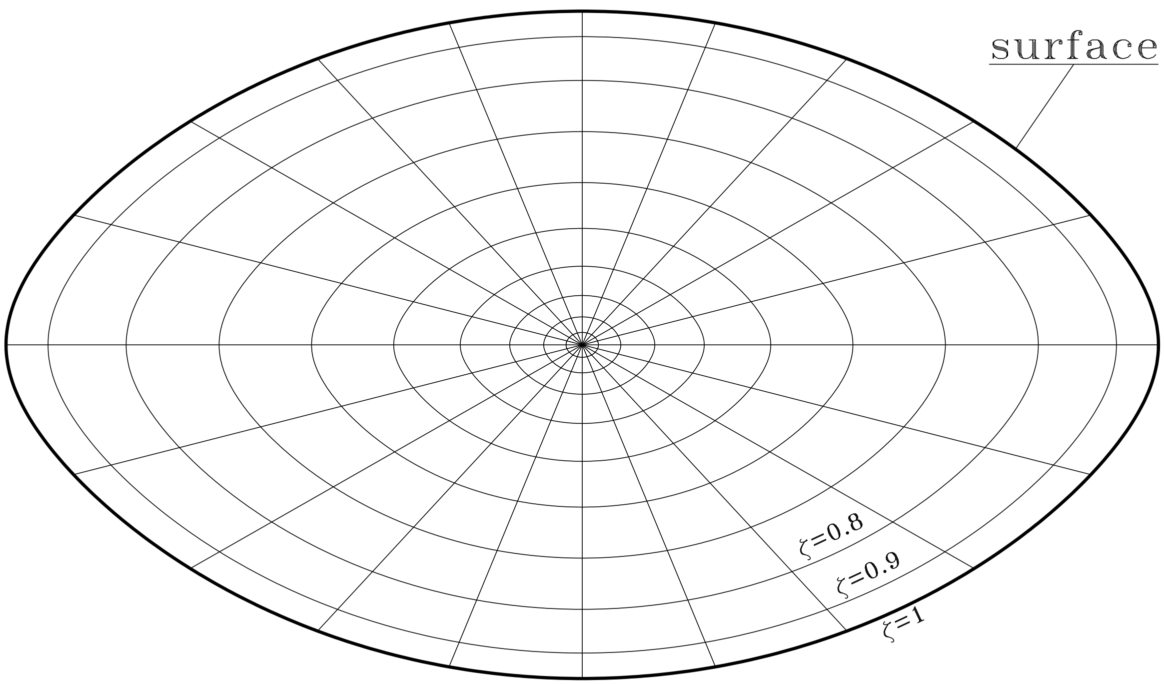
✘ Yoshida and Eriguchi (2001)

✘ Espinosa et al. (2004)

Our work

Objective : to accurately take into account the effects rotation on stellar pulsations

- ✘ Non-perturbative numerical method
- ✘ Use of spectral methods in both directions
  - 120 points with spectral method $\Longleftrightarrow$ 5000 points with finite differences
- ✘ Use of surface-fitting coordinate system (ζ, θ, ϕ) (cf. Bonazzola et al., 1998)



What we calculate

Equilibrium model

- ✘ polytropic model ($N = 1.5$ or 3)
- ✘ uniform rotation

Oscillations : adiabatic, acoustic modes

Typical numerical resolution :

- ✘ $\ell_{max} = 80$, $Nr = 60$, for $N = 3$
- ✘ $\ell_{max} = 70$, $Nr = 80$, for $N = 1.5$

Method

1. Write explicit equations in spheroidal coordinates

Continuity equation
Euler's equation
Adiabatic energy equation
Poisson's equation

Method

1. Write explicit equations in spheroidal coordinates

2. Express unknowns using spherical harmonics

$$\textit{For example}: \Phi(\zeta,\theta,\varphi) = \sum_{\ell=|m|}^{\ell_{max}} \Phi_m^\ell(\zeta) Y_\ell^m(\theta,\varphi)$$

Method

1. Write explicit equations in spheroidal coordinates

3. Project equations onto spherical harmonic base

2. Express unknowns using spherical harmonics

For example : $$\iint_{4\pi} \{Y_\ell^m\}^* \{\mathrm{Continuity\ \ equation}\}\, d\Omega$$

Method

1. Write explicit equations in spheroidal coordinates

3. Project equations onto spherical harmonic base

2. Express unknowns using spherical harmonics

4. Discretise system onto Chebyshev collocation grid

$$\textit{Generalised eigenvalue problem}: Ax = \lambda Bx$$

Tests and Accuracy of the method

Tests

- ✘ Comparison with Christensen-Dalsgaard and Mullan (1994) in the non-rotating case : $\Delta\omega/\omega \sim 10^{-7}$
- ✘ Comparison with Lignières (2003, CW5) : $\Delta\omega/\omega \sim 10^{-6}$
- ✘ Comparison with Saio for small rotation rates
- ✘ Variational test : $\Delta\omega/\omega \sim 10^{-7}$ for $N = 3$ and $\Delta\omega/\omega \sim 10^{-5}$ for $N = 1.5$

Accuracy

- ✘ High numerical accuracy of frequencies, up to $\Delta\omega/\omega \sim 10^{-10}$

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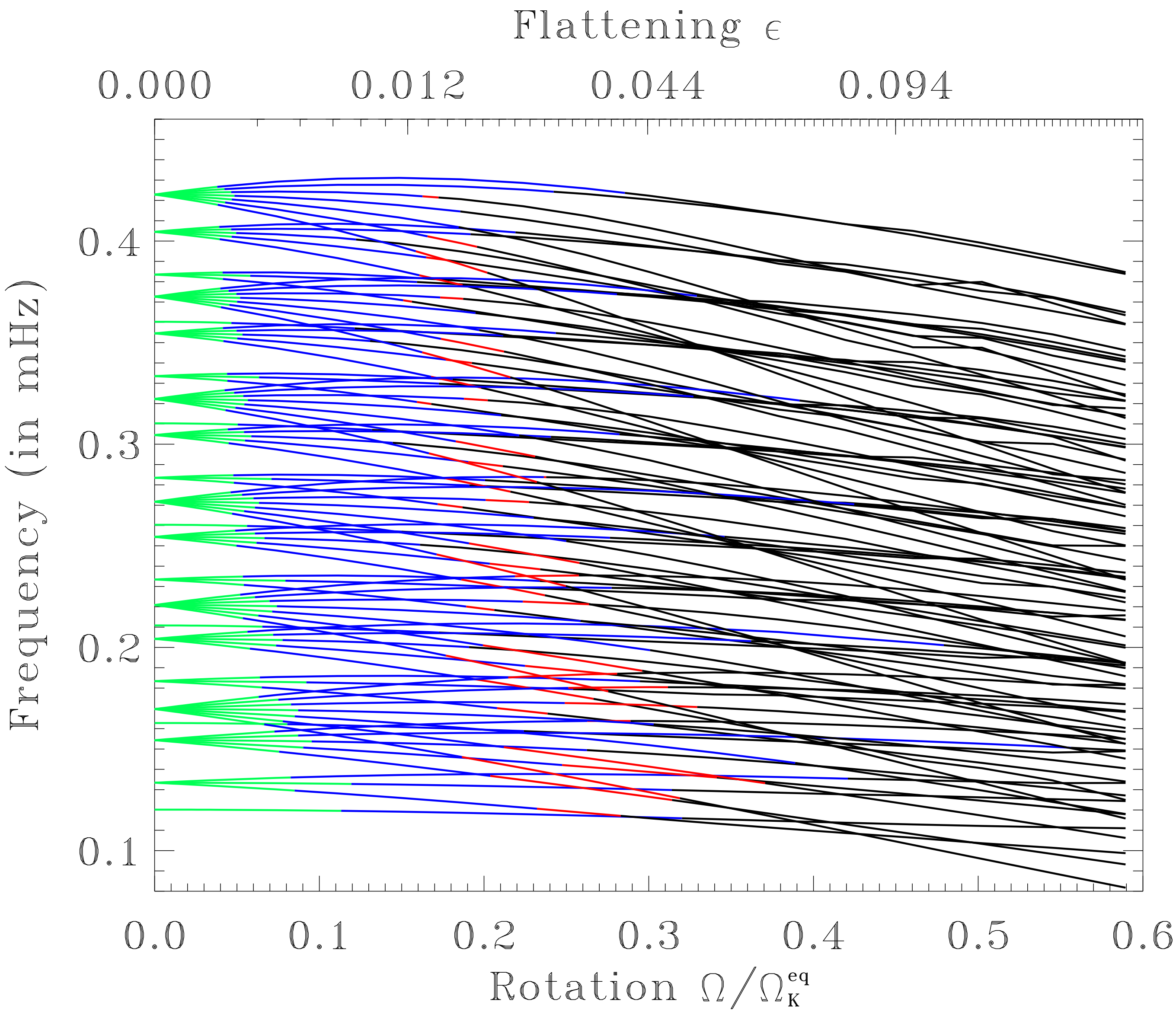
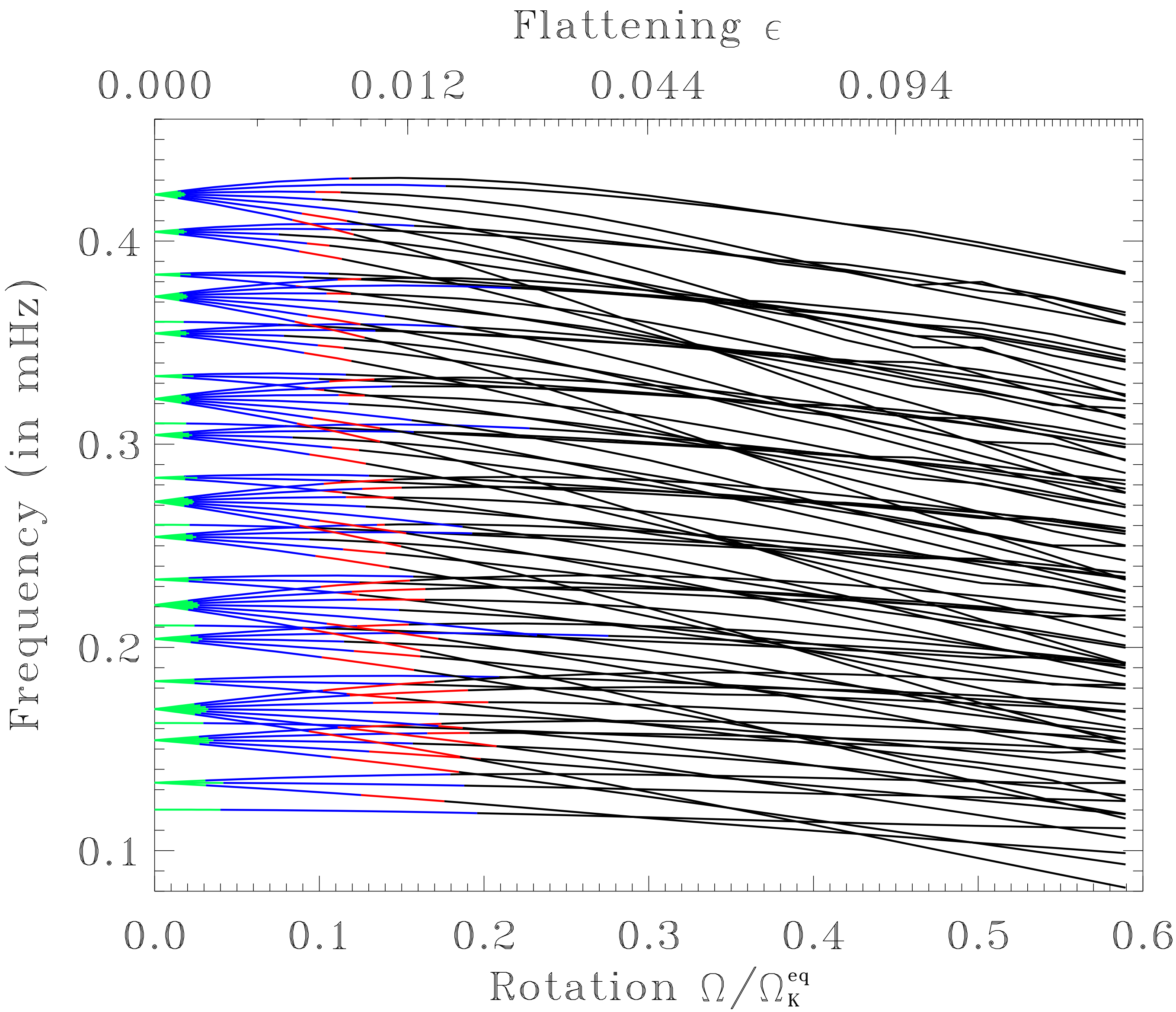
A few frequencies...



Modes :

$$n = 1 \text{ to } 6$$
$$\ell = 0 \text{ to } 3$$
$$m = -\ell \text{ to } \ell$$

A few frequencies...



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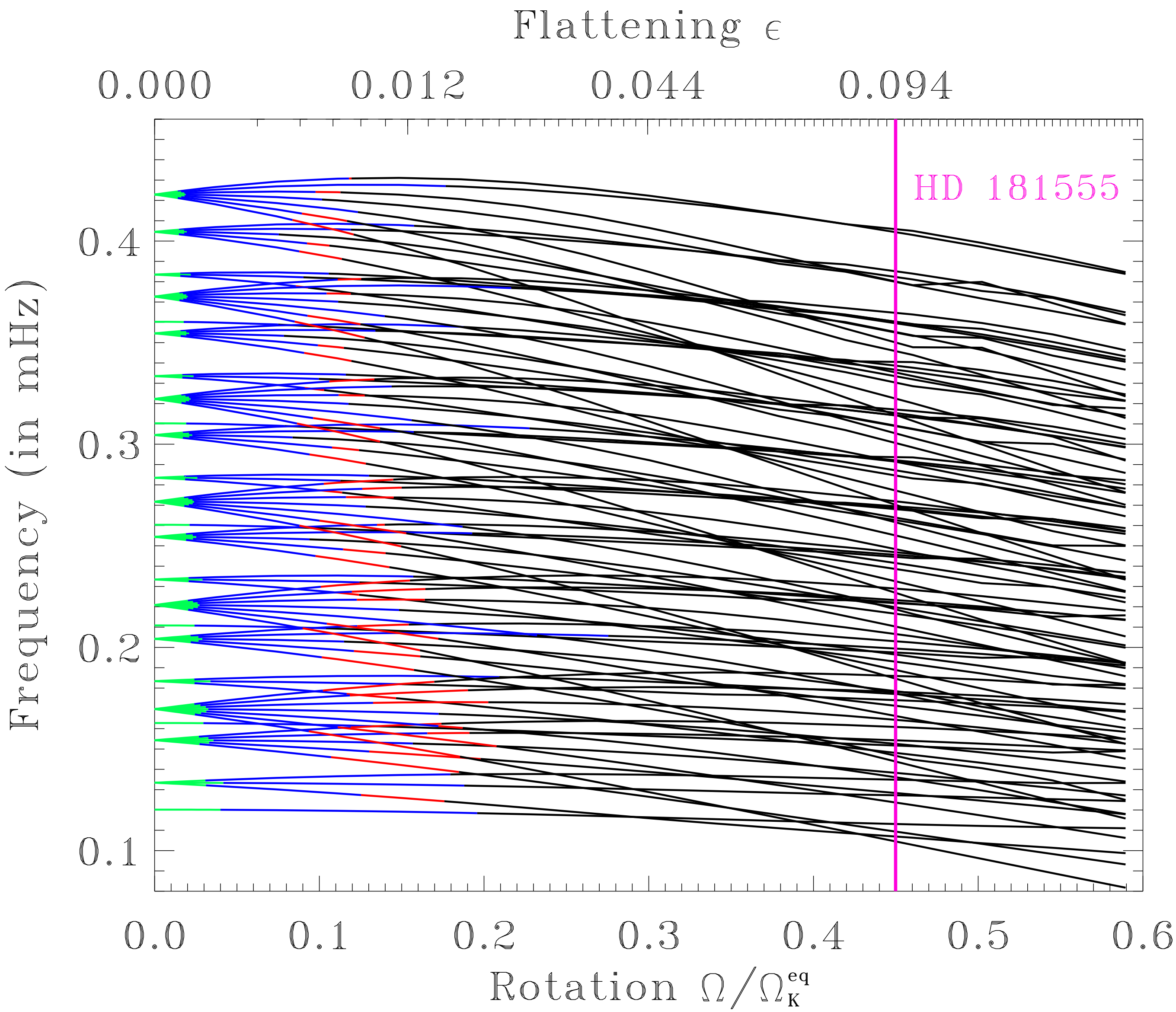
A few frequencies...



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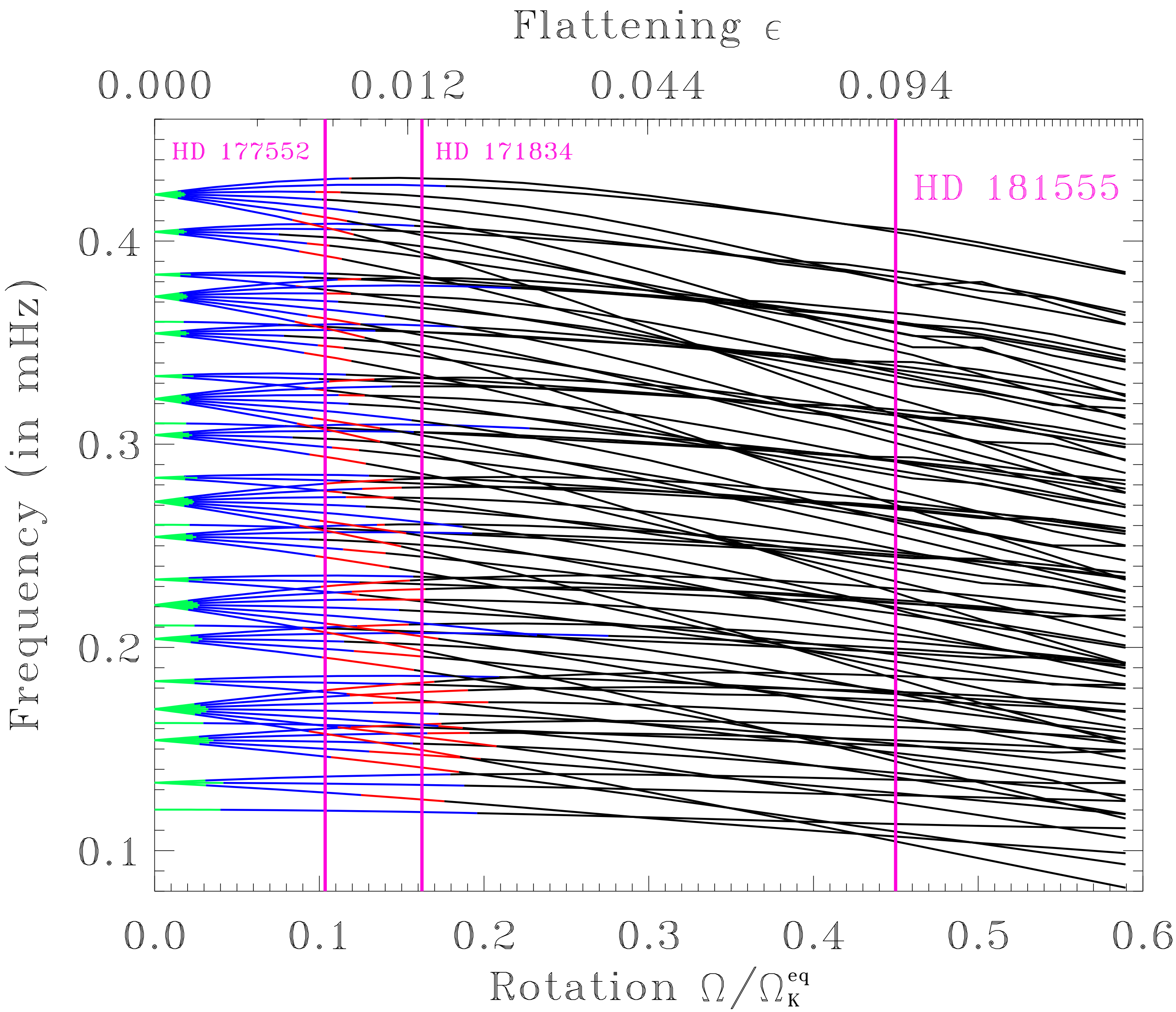
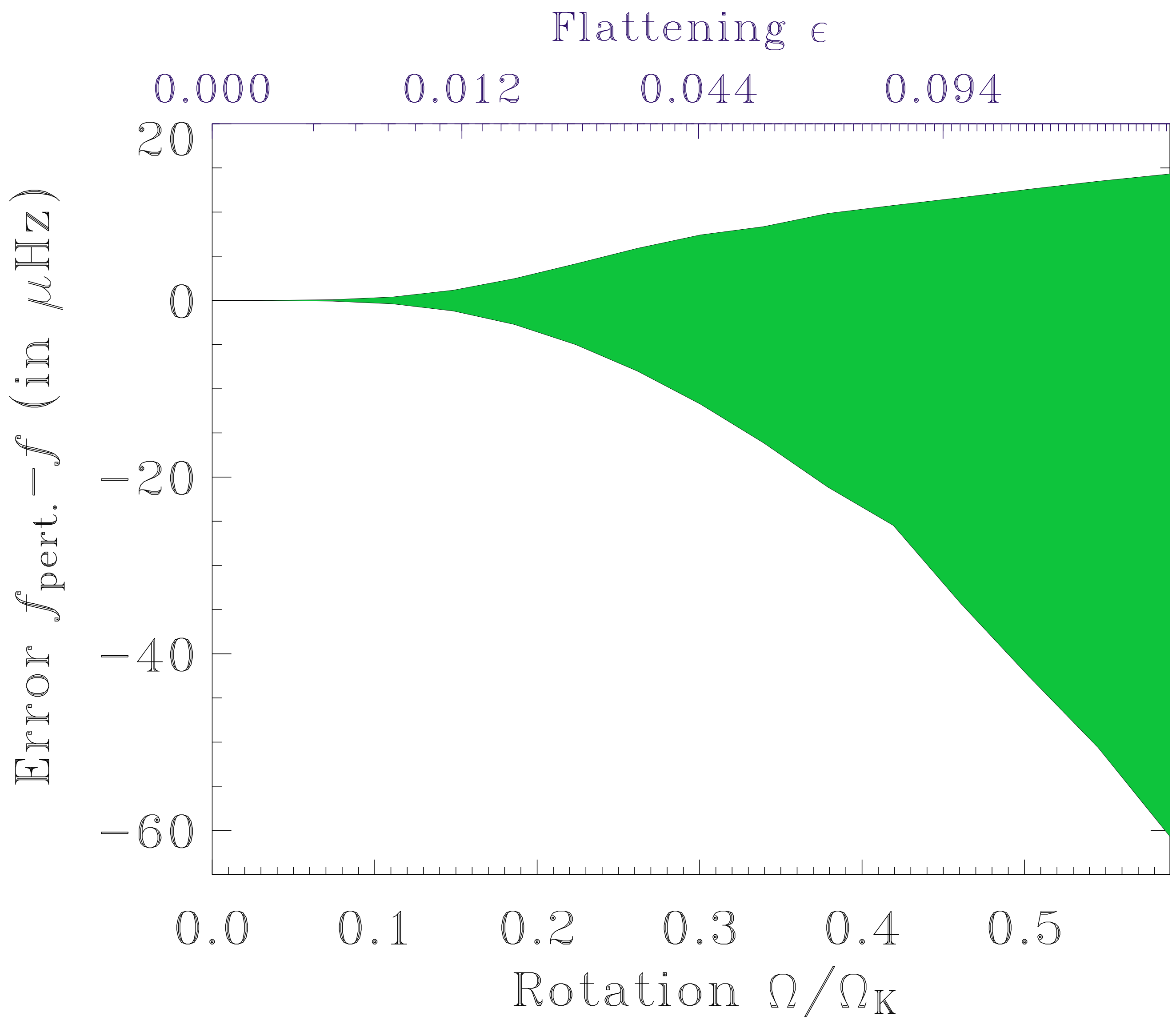
A few frequencies...



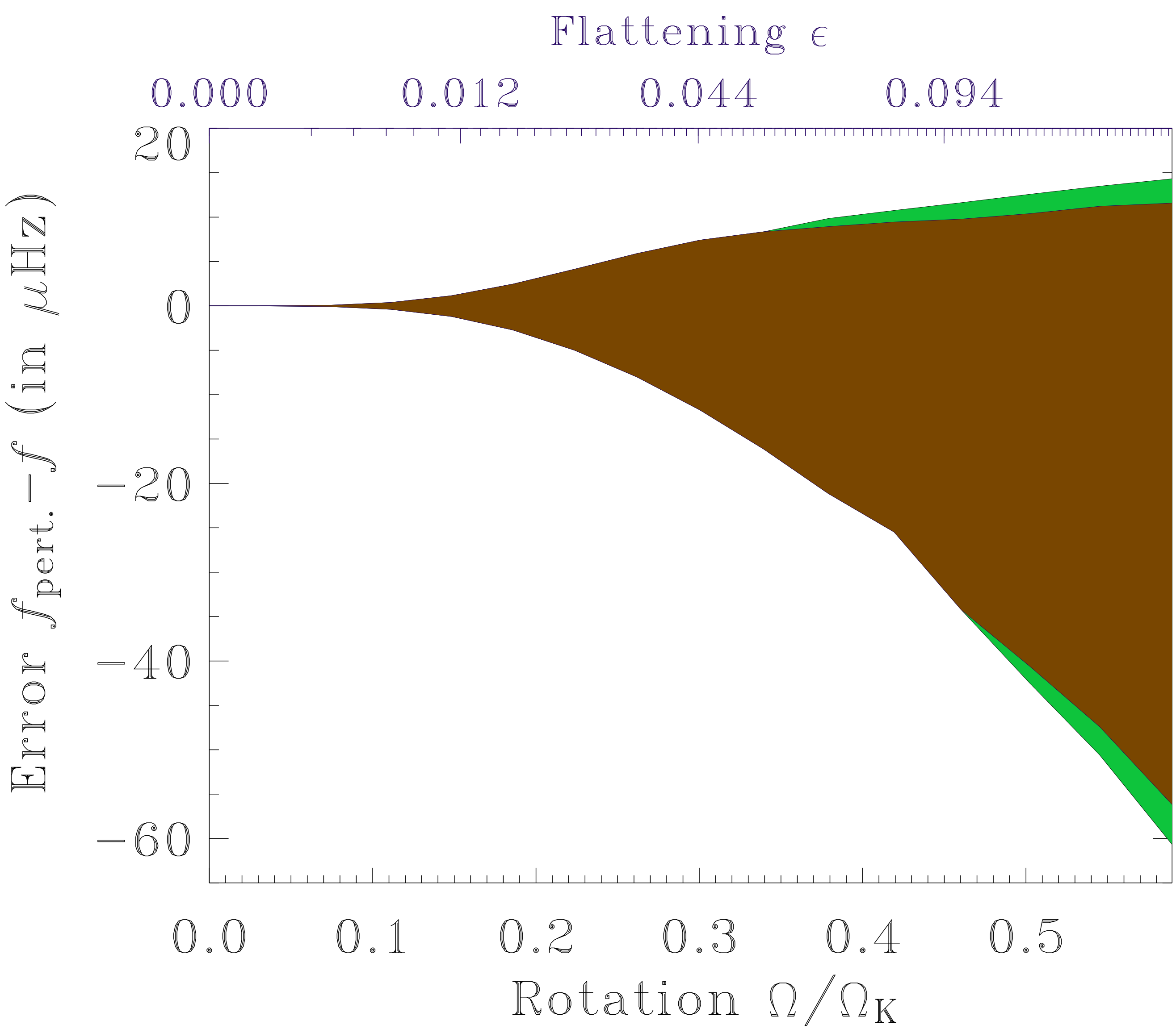
Modes :

n = 1 to 6
ℓ = 0 to 3
m = $-\ell$ to ℓ

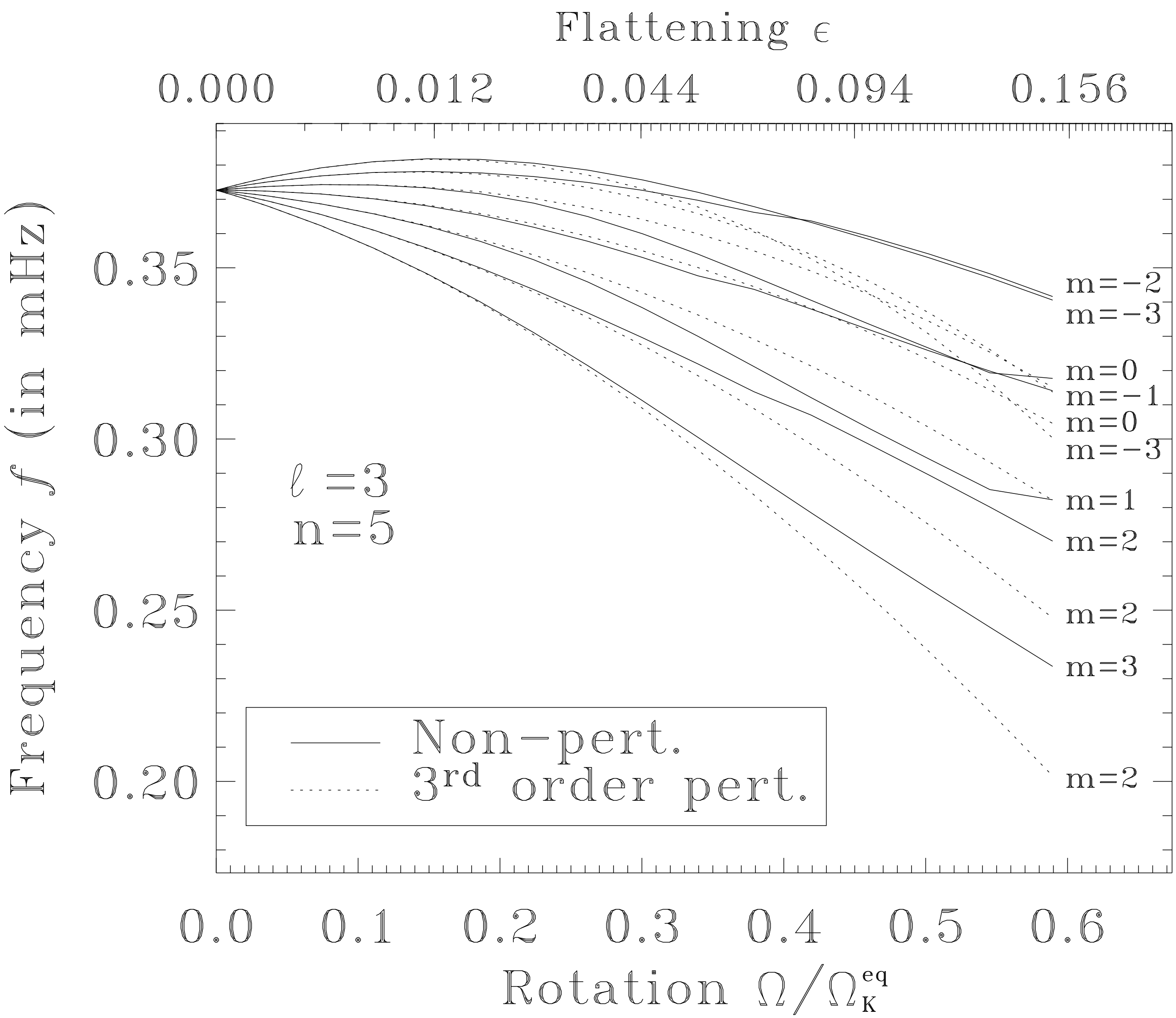
Error envelope



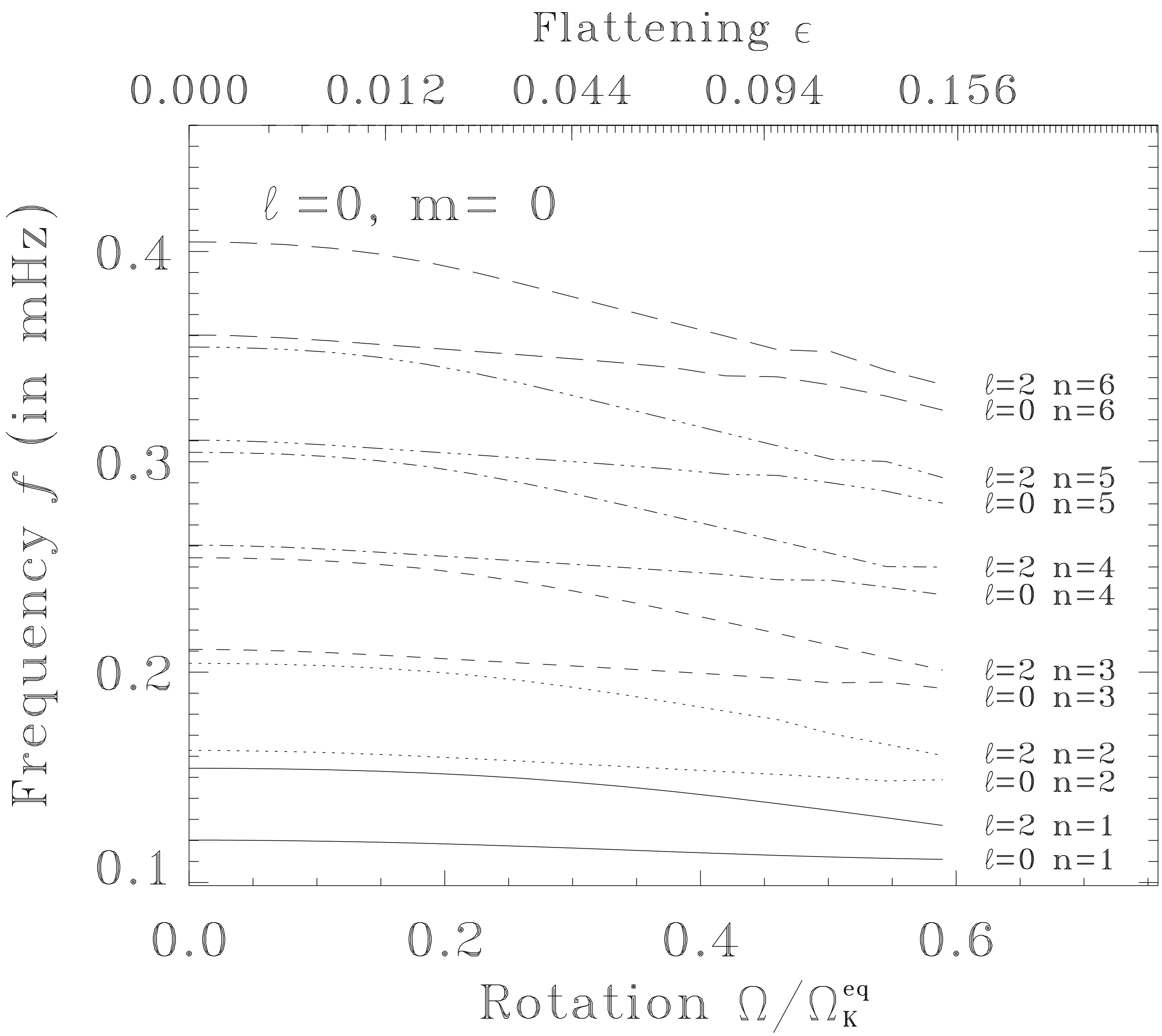
Error envelope



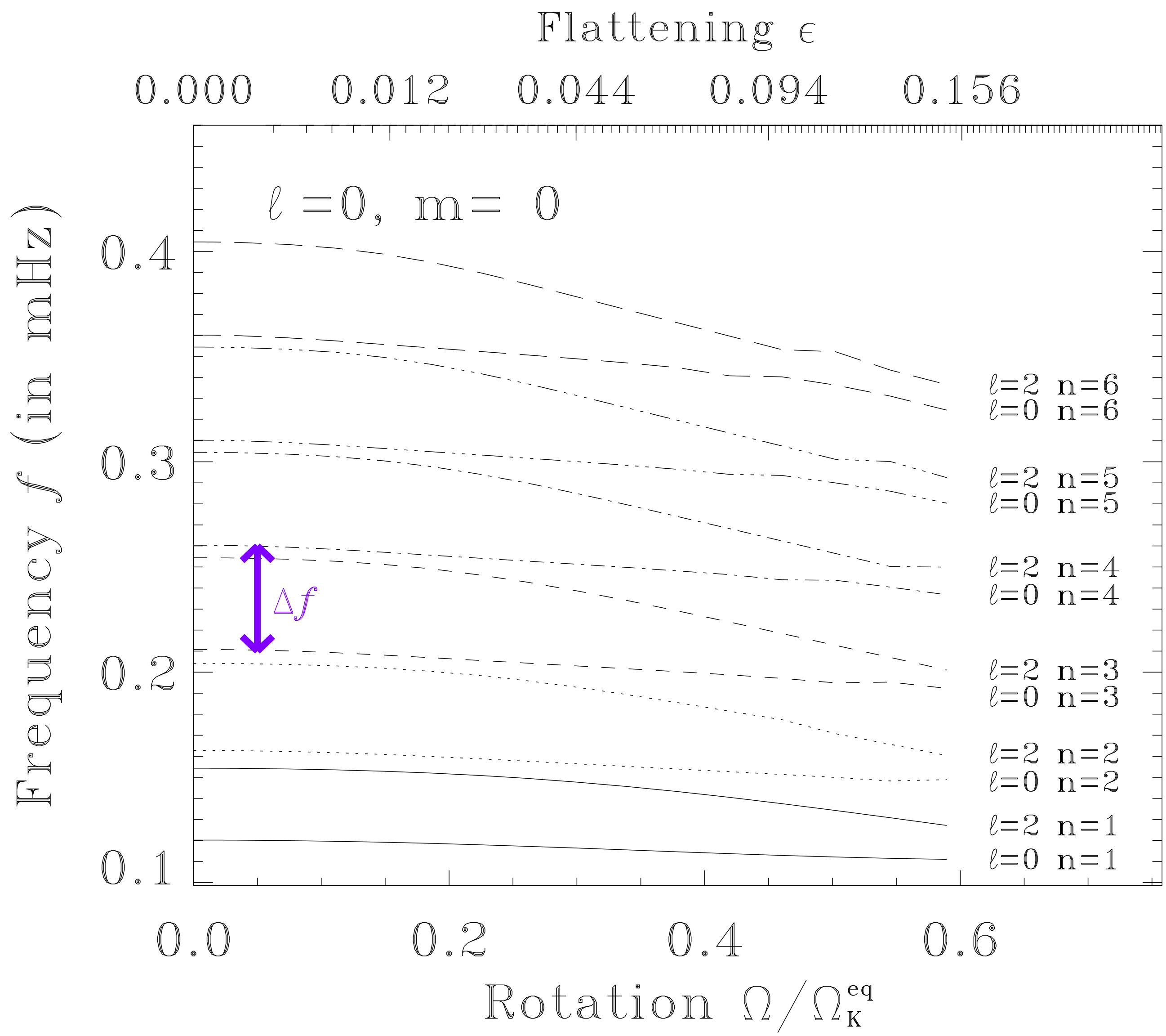
A multiplet


Flattening ϵ
0.000
0.012
0.044
0.094
0.156
Frequency f (in mHz)
0.35
0.30
0.25
0.20
m=−2
m=−3
m=0
m=−1
m=0
m=−3
m=1
m=2
m=2
m=3
m=2
ℓ =3
n=5
Non−pert.
3rd order pert.
0.0
0.1
0.2
0.3
0.4
0.5
0.6
Rotation Ω/Ω_K^{eq}

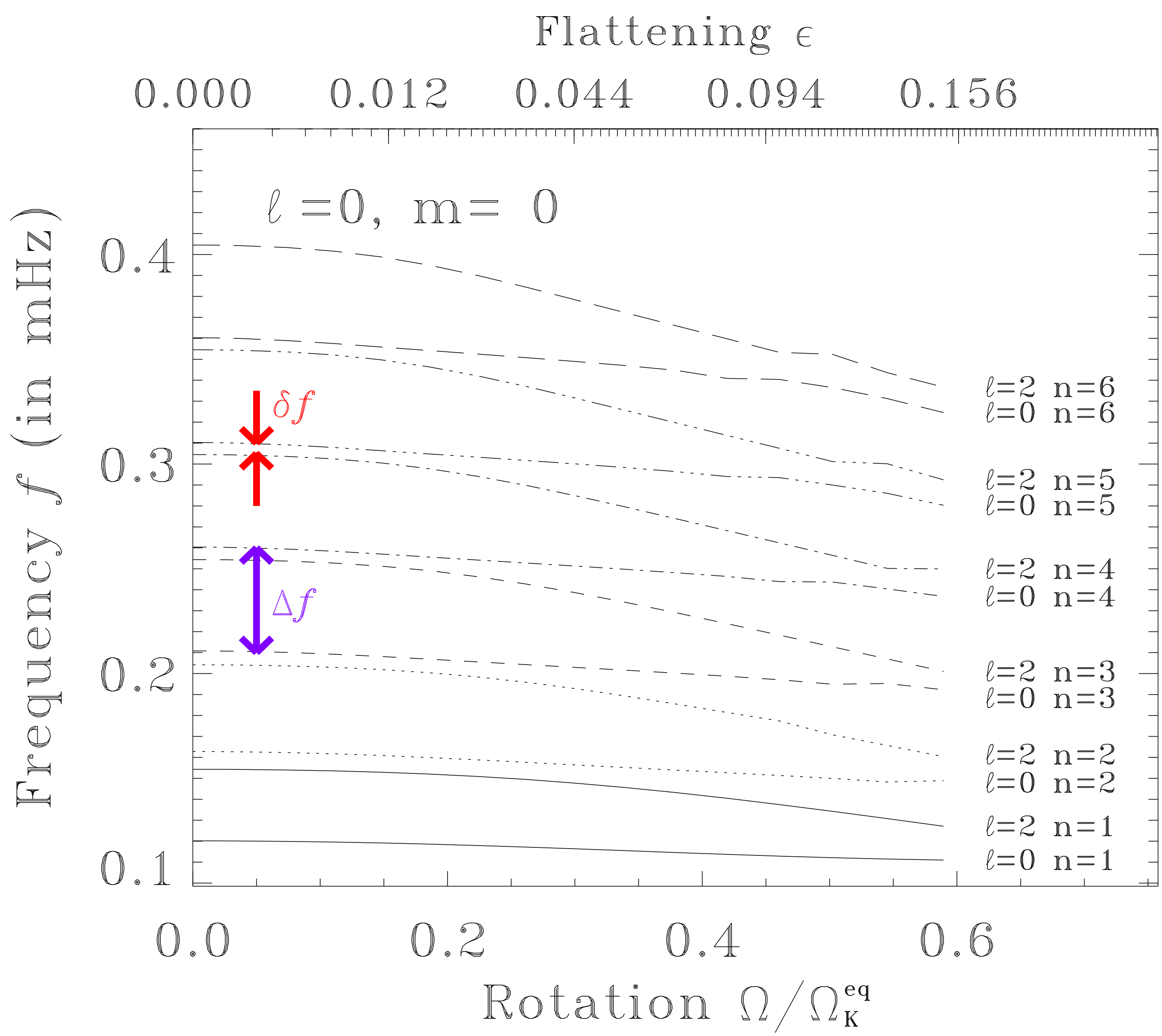
Large and small frequency separations



Large and small frequency separations

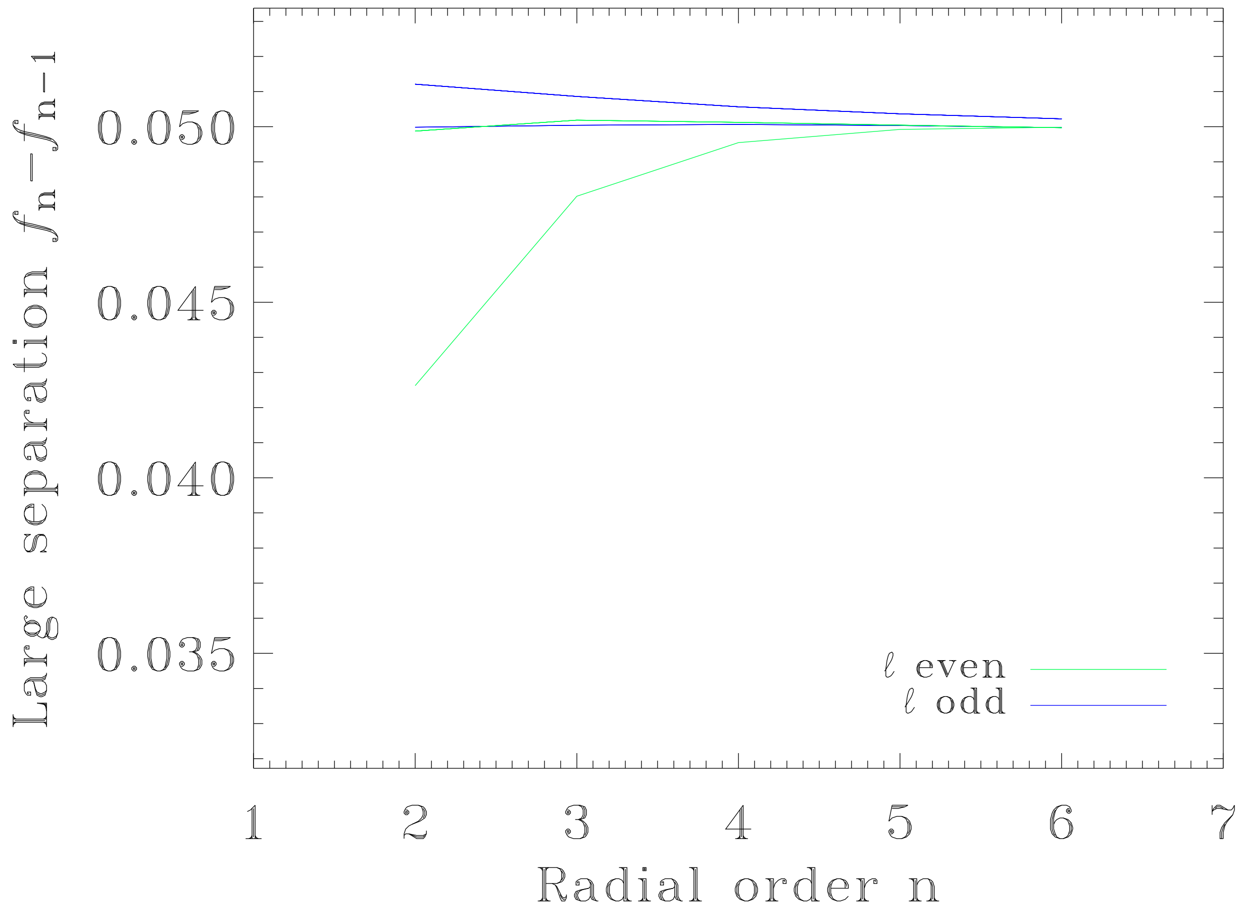


Large and small frequency separations



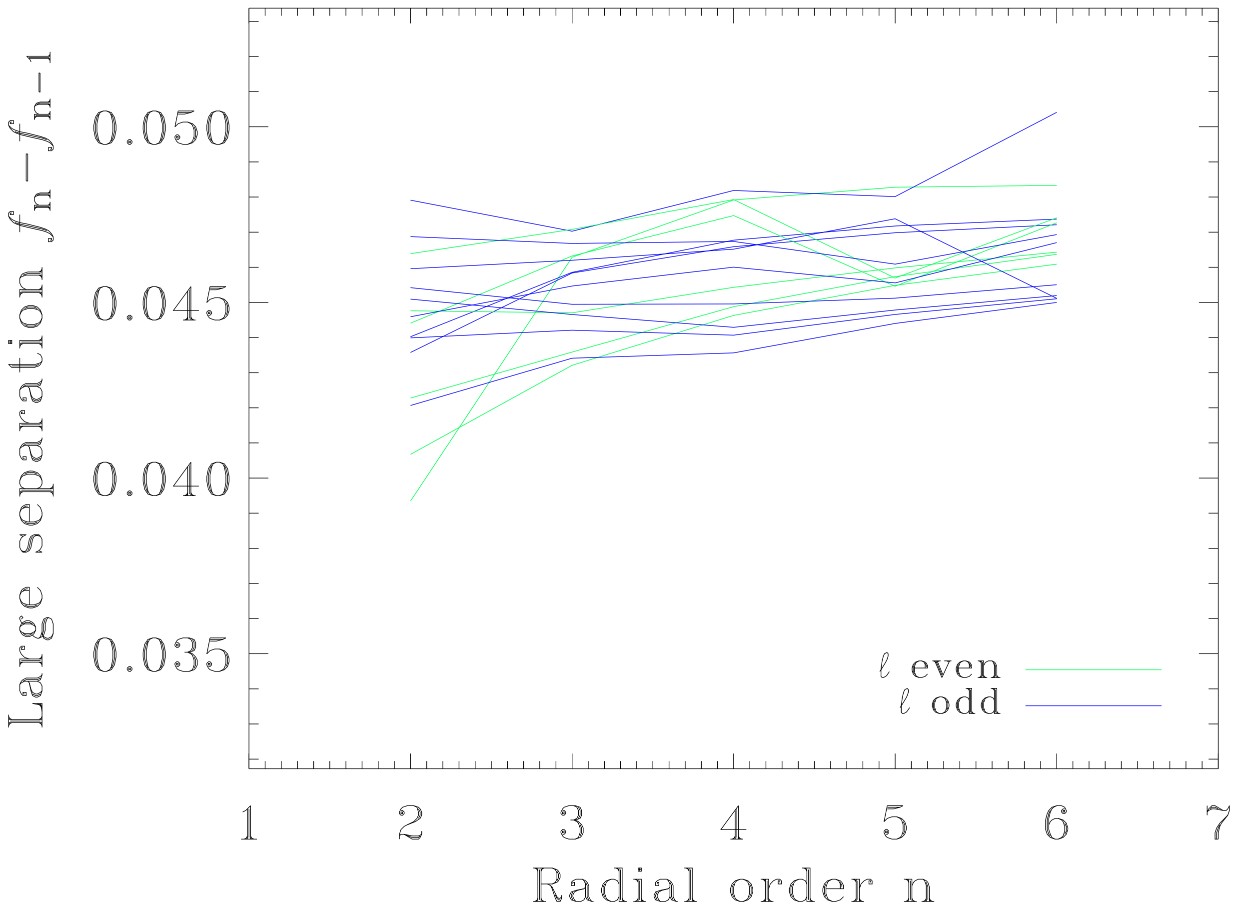
Large frequency separation

$$\Omega = 0.00\,\Omega_K$$



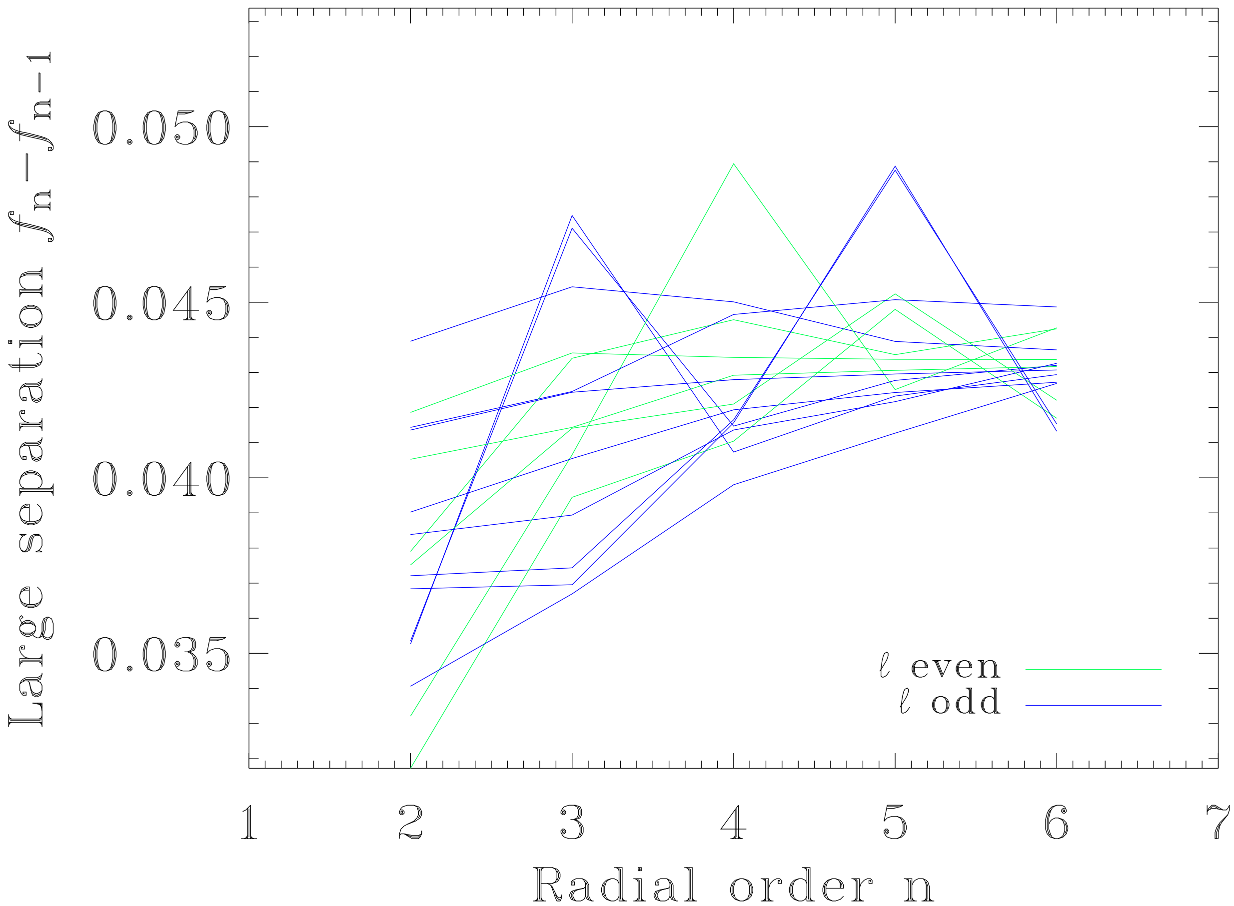
Large frequency separation

$$\Omega = 0.38\,\Omega_K$$

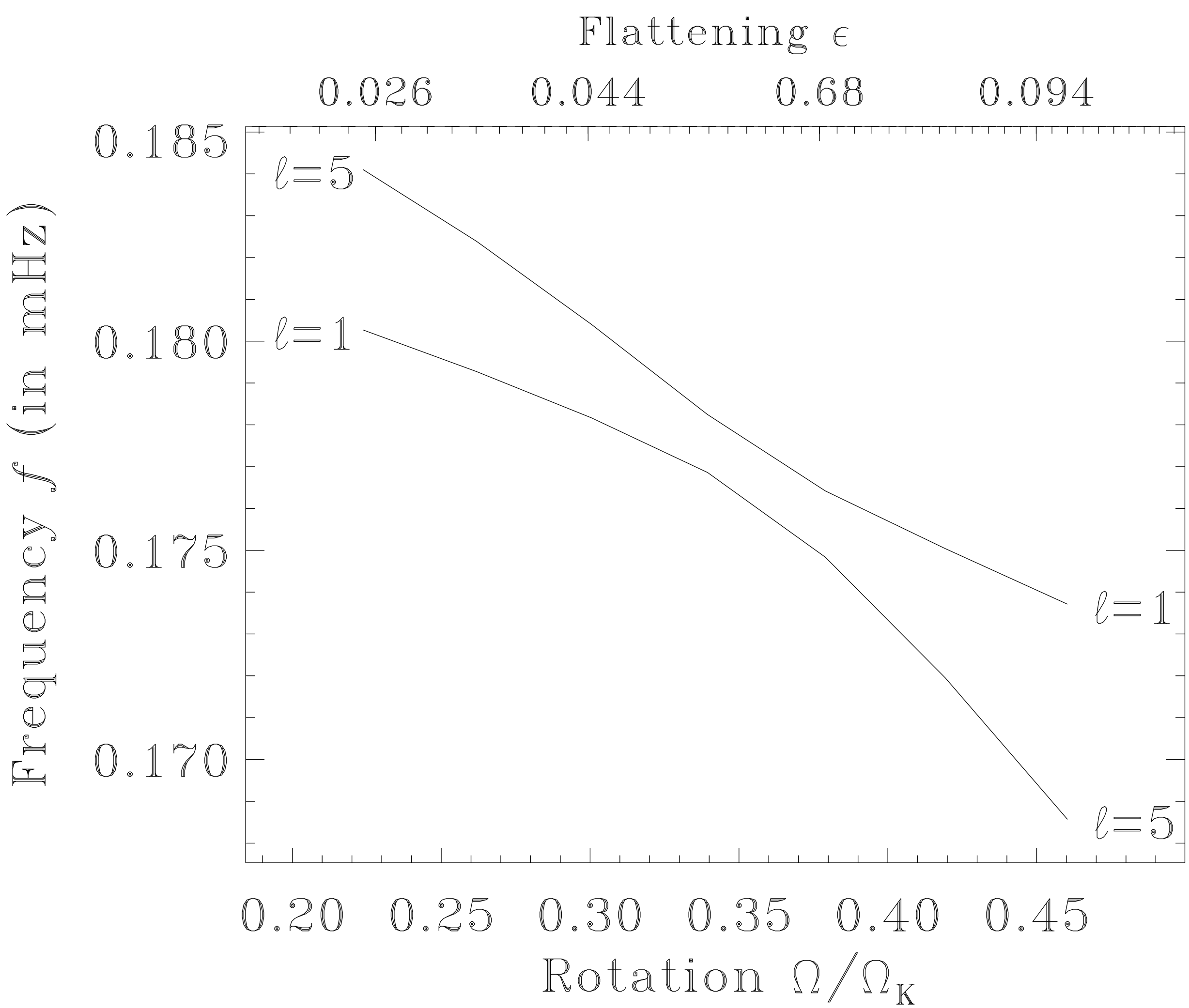


Large frequency separation

$$\Omega = 0.59\,\Omega_K$$



Avoided crossings



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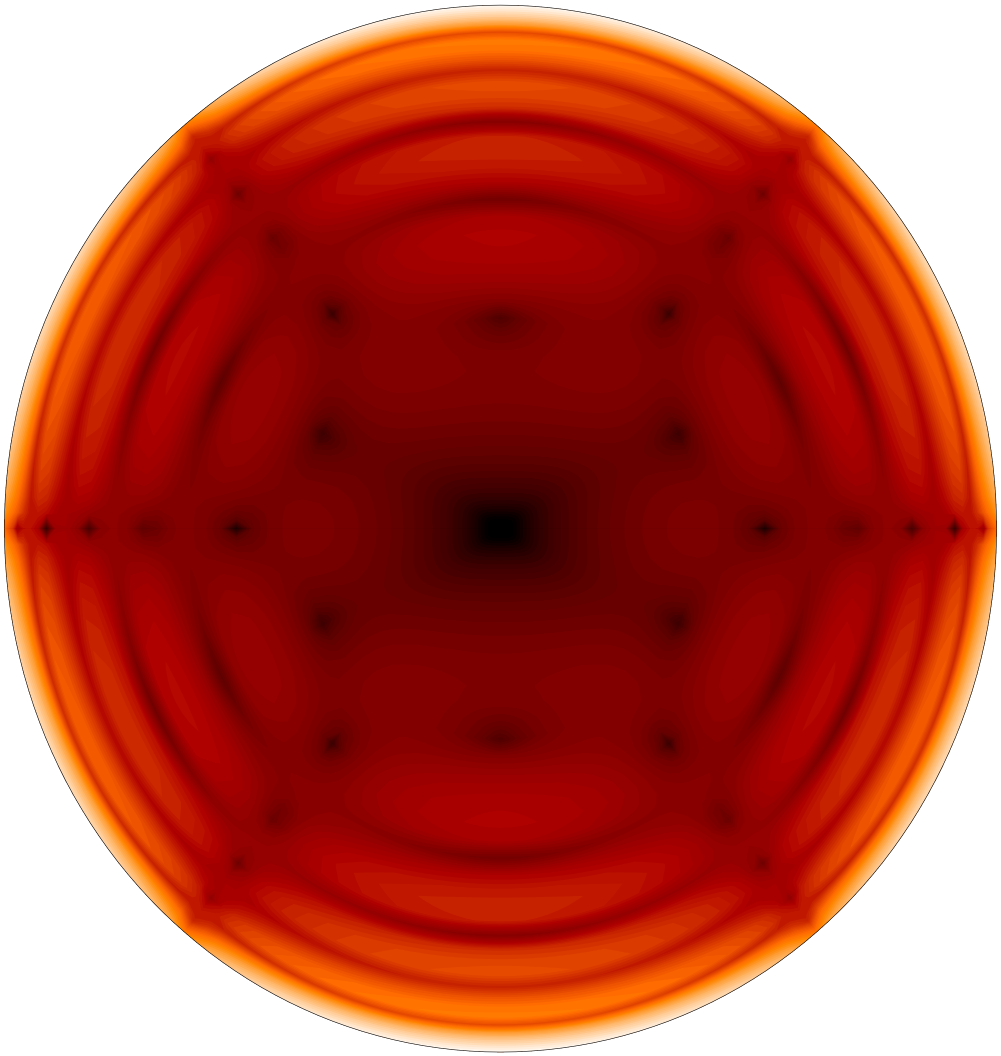
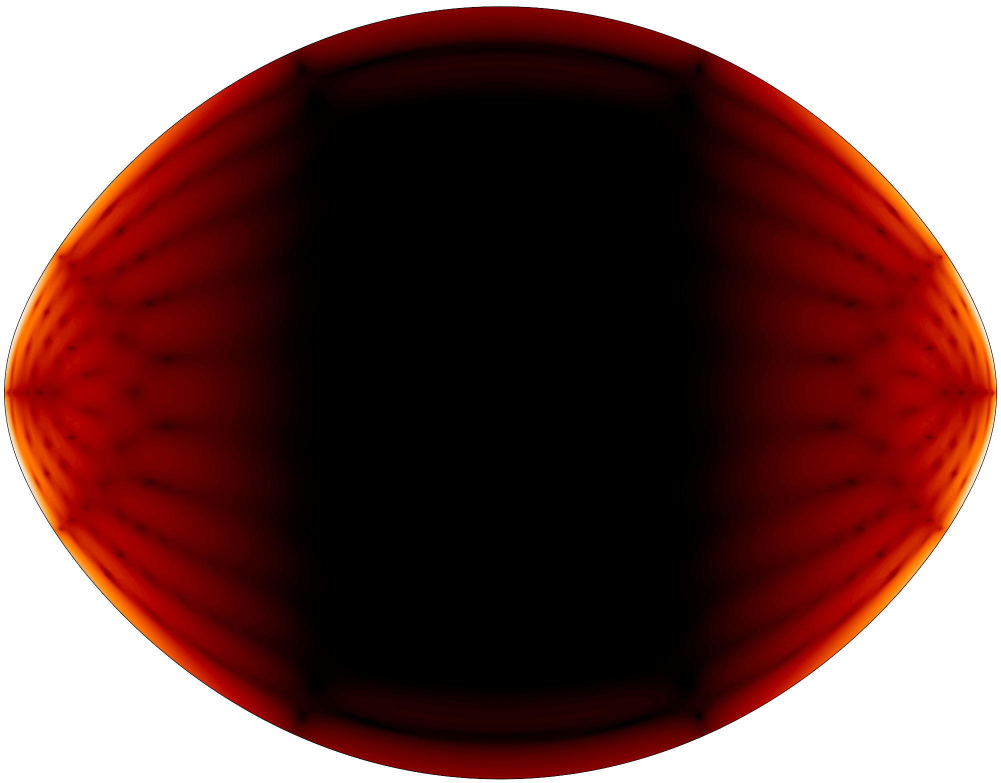
Rotation and mode structure



Mode :

$$n = 5$$
$$\ell = 3$$
$$m = 0$$
$$\Omega = 0$$
$$f = 0.373\ mHz$$

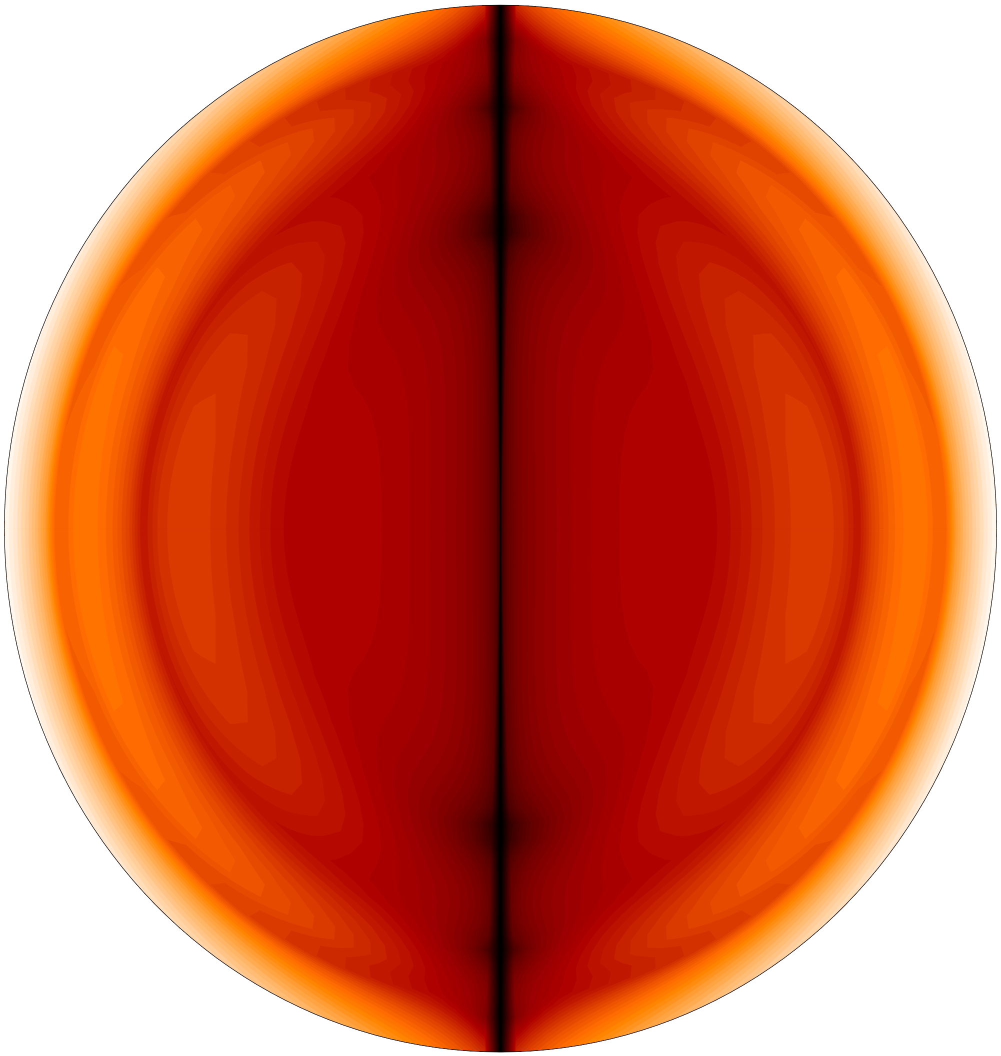
Rotation and mode structure



Mode :

$$n = 5$$
$$\ell = 3$$
$$m = 0$$
$$\Omega = 0.84\,\Omega_K$$
$$f = 0.260\ mHz$$

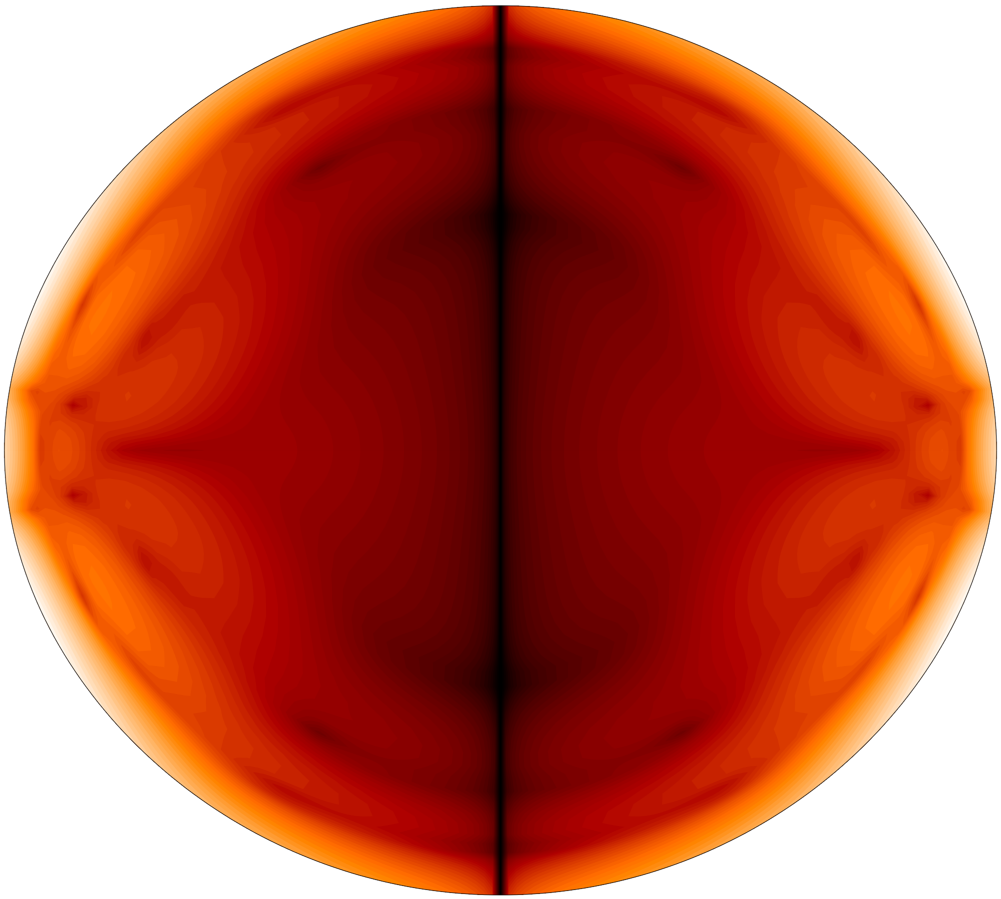
Rotation and mode structure



Mode :

$$n = 3$$
$$\ell = 2$$
$$m = 2$$
$$\Omega = 0$$
$$f = 0.254\ mHz$$

Rotation and mode structure

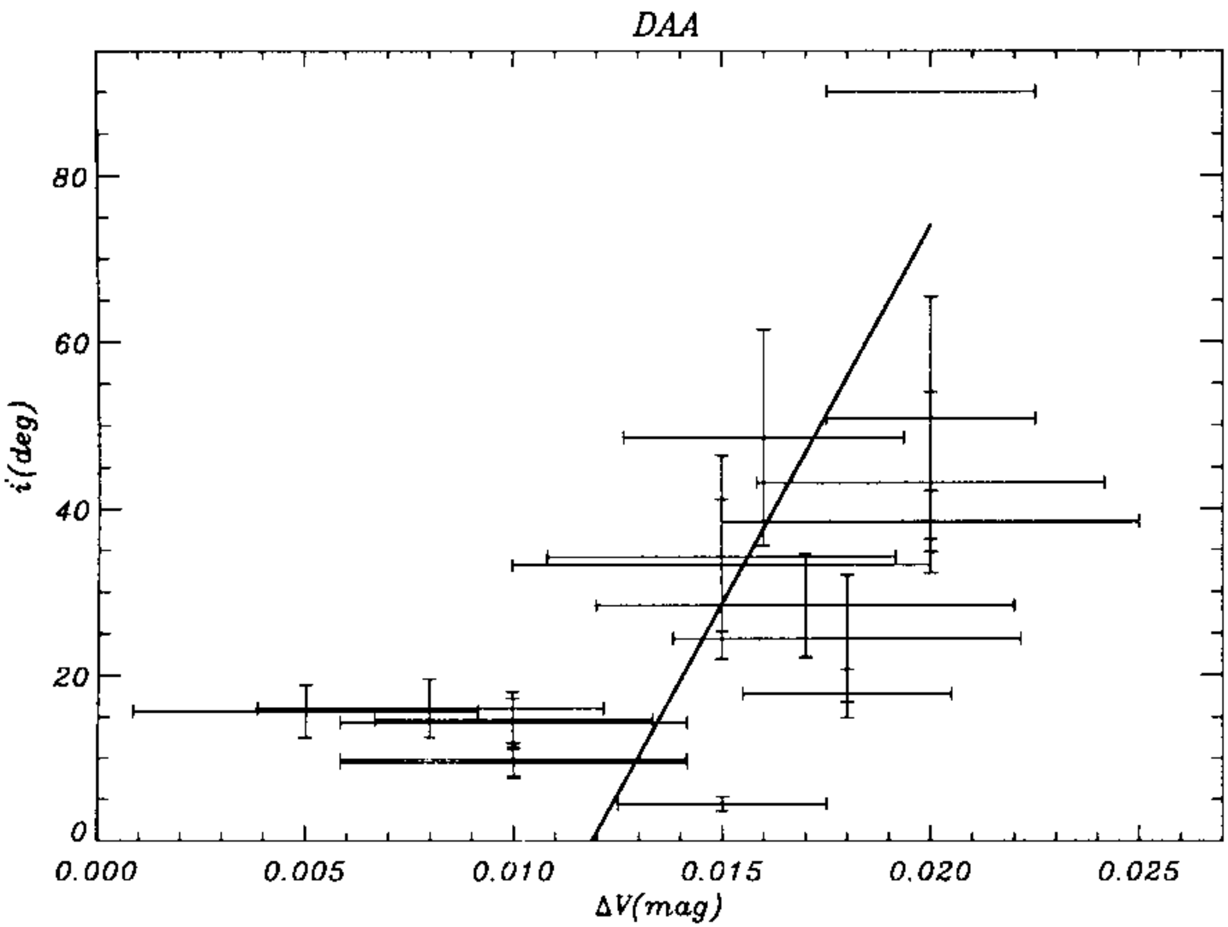


Mode :

$$n = 3$$
$$\ell = 2$$
$$m = 2$$
$$\Omega = 0.59\,\Omega_K$$
$$f = 0.164\ mHz$$

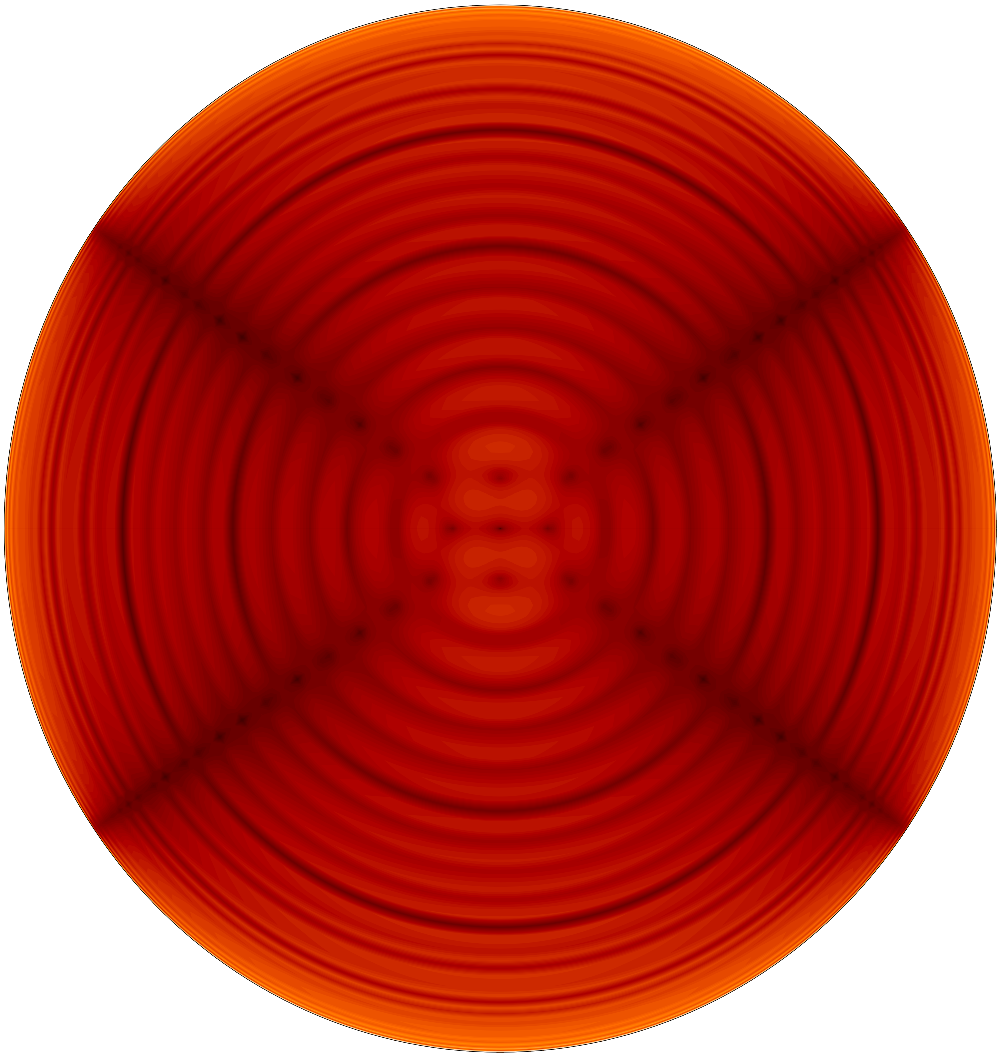
Rotation and mode structure

What do observations say ?



[Suárez et al., 2002]

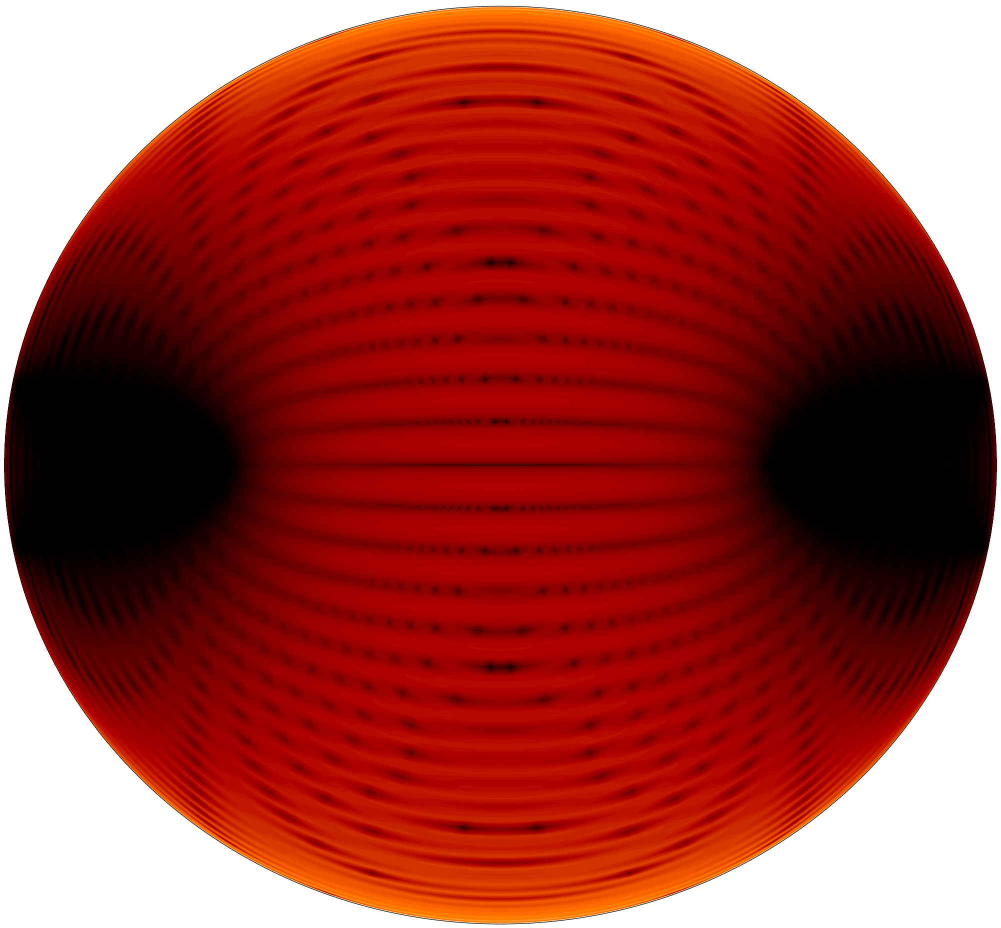
Rotation and mode structure



Mode :

$$n = 20$$
$$\ell = 2$$
$$m = 0$$
$$\Omega = 0$$
$$f = 31.4\,\Omega_K$$

Rotation and mode structure



Mode :

$$n = 20$$
$$\ell = 2$$
$$m = 0$$
$$\Omega = 0.46\,\Omega_K$$
$$f = 36.2\,\Omega_K$$

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Conclusion

Summary

- ✘ rotation greatly complicates the oscillation spectrum
- ✘ the large frequency separation seems to be preserved
- ✘ the geometry and visibility of modes are greatly altered

Future work includes :

- ✘ quantitative study of mode visibilities
- ✘ pulsations for solar-like stars
- ✘ study of g-modes

Estimating perturbative coefficients

The pulsation frequencies are a function of the rotation :

$$\begin{aligned}
\omega_{n\ell m} &= \omega_0 + \omega_1\Omega + \omega_2\Omega^2 + \omega_3\Omega^3 + \mathcal{O}\left(\Omega^4\right) \\
\omega_{n\ell\ -m} &= \omega_0 - \omega_1\Omega + \omega_2\Omega^2 - \omega_3\Omega^3 + \mathcal{O}\left(\Omega^4\right)
\end{aligned}$$

We calculate ω for small values of Ω

Least squares fit of $\frac{\omega_{n\ell m}+\omega_{n\ell\ -m}}{2}$ and $\frac{\omega_{n\ell m}-\omega_{n\ell\ -m}}{2}$

Obtain in this way ω_0, ω_1, ω_2 and ω_3.

Logarithmic graph of error envelope