Looking for Monochromatic GW Signals in ALLEGRO's Data

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Outline

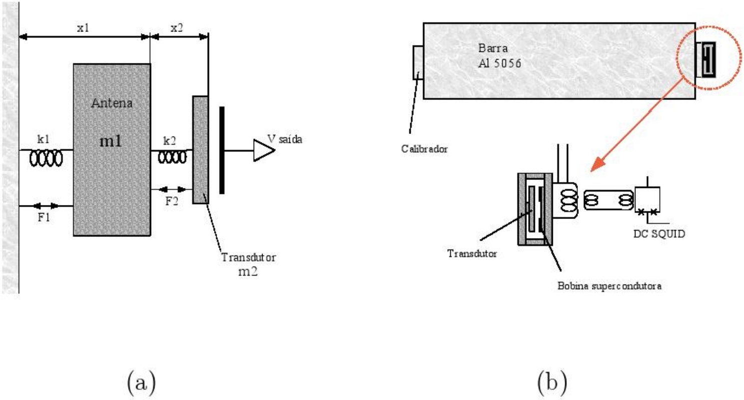
The Gravitational Wave Detector ALLEGRO

Antenna caracteristics
Louisiana State University

Material	Aluminum alloy 5056
Physical mass	2296 kg
Diameter	60 cm
Length	3 m
Therm. temperature	4.2 K
First long. mode	913 Hz
Minus mode	895.4146 Hz
Lockin reference frequency	907.5370 Hz
Mode Plus	919.6594 Hz
Longitude	91.178627^o W
Latitude	30.41258^o N

Outline

ALLEGROS's Model



$$m_1\ddot{x}_1 + h_1\dot{x}_1 + k_1x_1 - h_2\dot{x}_2 - k_2x_2 = F_1 - F_2 + F_T + \frac{1}{2}m_1\ell\ddot{h}_{xx}$$

$$m_2(\ddot{x}_2 + \ddot{x}_1) + h_2\dot{x}_2 + k_2x_2 = F_2 - F_T$$

Outline

ALLEGRO’s data aquisition system

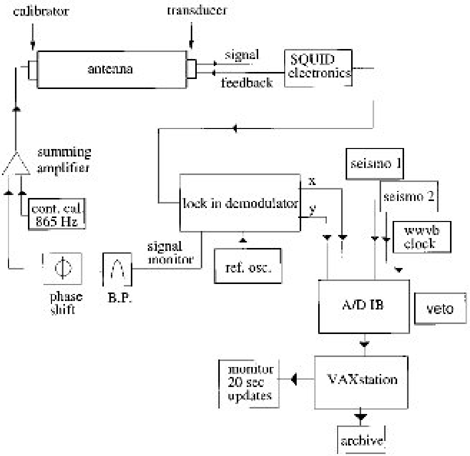


FIG. 5. Schematic of the Allegro data acquisition system.

Data Aquisition

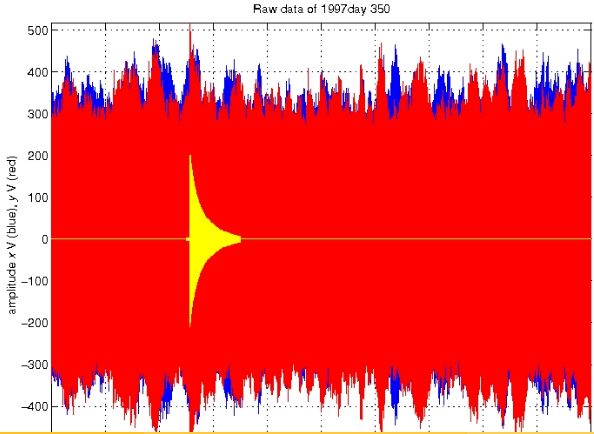
- The signal s from the SQUID is sent to the lockin
- The lockin demodulates and low pass the signal:

$$z = \mathrm{e}^{-i2\pi(f-f_{\mathrm{ref}})}\mathcal{H}[s]$$

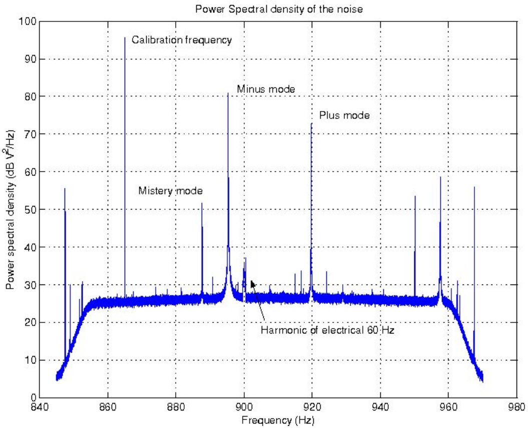
- The reference frequency of the lockin f_{ref} is set halfway betwen the frequency of the plus and minus mode
- The output of the lockin is the in phase $x = \Re[z]$ and quadrature $y = \Im[z]$
- The signal is sent to a A/D converter with sampling frequency of 125 Hz
- The resulting data is save in file for digital processing

Raw Data x and y

- Blue: in phase x
- Red: minus mode
- Yellow: signal put by hand



Power Spectral Density of the Noise



Outline

Mached Filter

- Given a signal $m(t)$ embedded in noise (a stochastic process)

$$f(t) = m(t) + n(t)$$

- and the filtered version with the filter $k(t)$

$$\phi(t) = \mu(t) + \nu(t),$$

- we have a matched filter if it produces the maximum signal to noise ratio

$$\rho = \frac{\mu(t_0)^2}{\mathrm{var}[\nu(t)]}$$

- In frequency domain this filter is

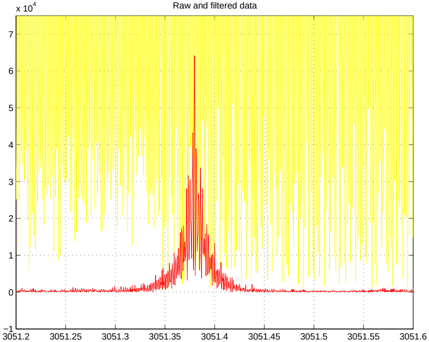
$$K(w) = ce^{-iwt_0}\frac{M^*(w)}{S_n(w)}$$

- and

$$\rho = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{|M(w)|^2}{S_n(w)}dw$$

Filtered Signal

- Red: filtered signal with matched filter ($\rho \sim 10$)
- Yellow: raw data



- If the signal is slightly different from the template the signal to noise ratio is severely degraded
- If the signal have unknown parameters, we need many templates to span the entire space of parameters

Outline

Monochromatic GW Signals (MGW)

- Let us suppose that we are receiving a MGW of the form

$$h_{+} = A\cos(\nu_s t + \alpha)$$

- Because of the translational and rotational movement of the Earth the signal received is Doppler shifted

$$\nu = \nu_s \gamma \left(1 + \frac{\mathbf{n}\cdot\mathbf{v}}{c}\right)$$

- If the GW signal comes from a star in the direction **n**, the Doppler shift is

$$\begin{aligned} \nu = \nu_s + \frac{\nu_s}{c}\mathbf{n}\cdot\mathbf{v} \quad &= \quad \nu_s + \frac{\nu_s}{c}[(-wr\cos\theta\sin wt - \Omega R\sin\Omega t)\, n_x \\ &+ \quad (wr\cos\theta\cos\epsilon\cos wt + \Omega R\cos\Omega t)\, n_y \\ &- \quad wr\cos\theta\sin\epsilon\cos wt\, n_z] \end{aligned}$$

The Expression for Doppler Shift ν

- For a source with 887.5 Hz (mystery mode) the Doppler shift is

$$\begin{aligned}\nu \;&=\; 887.5\times 10^3 - (1.186\sin wt + 88.17\sin\Omega t)n_x \\ &\quad +(1.088\cos wt + 88.17\cos\Omega t)n_y \\ &\quad -0.4722\cos wt\, n_z \qquad [\mathrm{mHz}]\end{aligned}$$

- The minimum diurnal Doppler shift variation

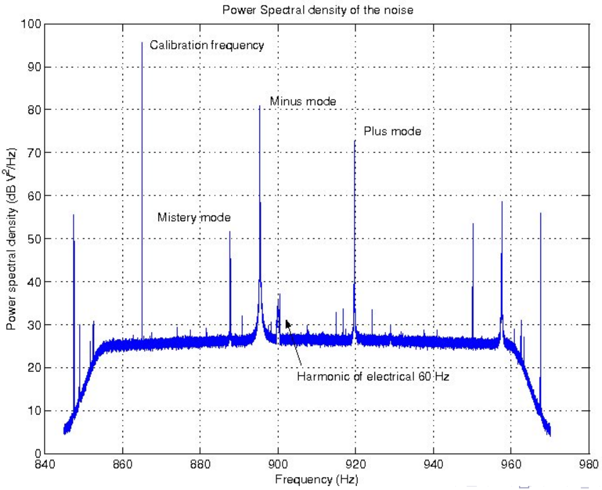
$$\Delta\nu = 0.9445\ \mathrm{mHz}$$

- The maximum annual Doppler shift variation

$$\Delta\nu = 178.7\ \mathrm{mHz}$$

Power Spectral Density of the Noise

- Analysis of the mystery mode behavior



The Data Analysis (Digital Lockin)

- Digital lockin:
  - For each 1200 s of data $\mathbf{z} = \mathbf{x} + i\mathbf{y}$,
  - We obtain the analytic function $\mathbf{z}_p$, the preenvelope, with the help of the Hilbert transform

$$\mathbf{z}_p = \mathcal{H}[\mathbf{z}].$$

  - Then we demodulate the frequency of this peak down to zero obtaining the complex envelope,

$$\tilde{\mathbf{z}} = \mathrm{e}^{-2\pi i (f_{\text{mysterymode}} - f_{\text{ref}})t}\mathbf{z}_p.$$

  - We are interested in signals with maximum frequency of 0.1787 Hz.
  - then we filter the signal with a low pass filter with a cutoff frequency of 0.2 Hz.
- We have tracked the mystery mode peack for 100 days.

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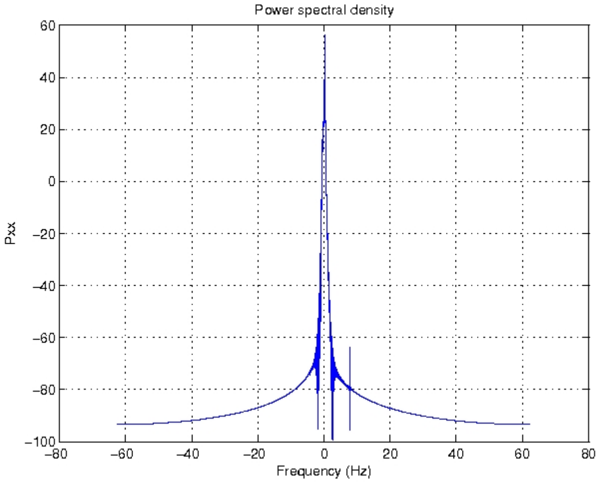
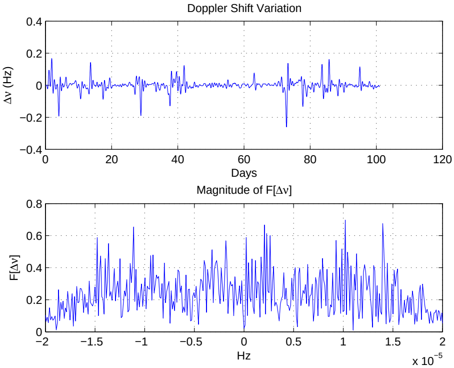
Power Spectral Density of the complex envelope $\tilde{z}$



Doppler shift variation of the Mystery Mode Peak

- For each 1200 s of data we obtain a new temporal series with the frequency of the mystery mode peak
- If this peack where a GW signal we should observe two peaks one with the 3.169×10^{-8} Hz and another with 1.658×10^{-5} Hz in the magnitude of the Fourier transform of $\Delta\nu$



Outline

Bayesian Estimation

- As you have seen the Doppler shift is of the form

$$\nu = A\cos(wt+\alpha) + B\cos(\Omega t+\beta) + C$$

where A, B, α and β depend on n_x, n_y, n_z and C a noise

- Basically these parameters are extimated with the following algorithm
- Let us start with the Bayes theorem

Bayes Theorem

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

- In the case of data analysis (simple problem of radioative decay)

$$p(\lambda|t) = \frac{e^{-\lambda t}p(\lambda)}{N}$$

Parameters Estimation

Algorithm

Put the prior $p(H) = 1$ in the region of interest of the parameters.
Notice that if we have enouth data the prior is irrelevant

while there exist the n^{th} amount of data to process **do**

$$p(H|D_n) = p(D_n|H)p(H|D_{n-1})/N$$

end

here, N is a normalization constant, H stands form the set of parameters A, α, B, β, C, and $p(H|D_0) = p(H)$ is the prior.

Conclusions

The Challenge Continues!

Thank You

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