Modified Gravity and its mapping to Scalar-tensor theories

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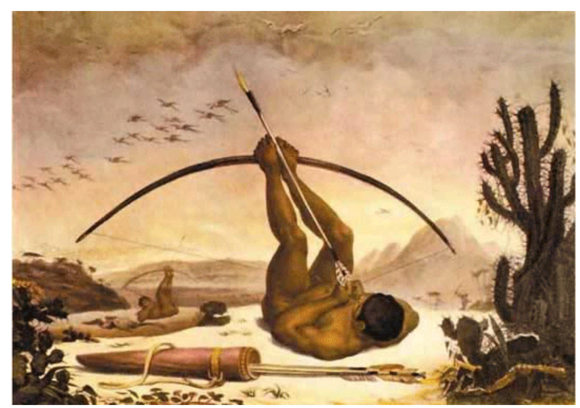


I Workshop “Challenges of New Physics in Space”, Campos do Jordão, 2009

When the observational data is analysed from the point of view of General Relativity (GR), we arrive at the astonishing conclusion that about *95%* of the content of the universe is unknow and is called of dark matter and dark energy.

An interesting and reasonable alternative is to suppose that GR is not valid at any length scale and that the required modifications are (erroneously?) interpreted as exotic fluids.

In this seminar we will focus on the current approach to this so-called modified gravity theories including the mapping onto scalar-tensor theories and its shortcomings.

SUMMARY

- **Modified Lagrangean:**

  mathematical and physical

  consequences
- **Mappings**
  - **Standard approach**
  - **Curvature fluid**
  - **Scalar field**
  - **(actual?) constraints on** $f(R)$
- **Conclusions**

General Relativity

$$S = \int d^4x \, \sqrt{-g} R$$

$$R = R\left[g_{\mu\nu}, \frac{\partial}{\partial x^\alpha} g_{\mu\nu}, \frac{\partial^2}{\partial x^\alpha x^\beta} g_{\mu\nu} \right]$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -T_{\mu\nu}$$

Bianchi Identity

$$\nabla^{\nu}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = 0$$

$$\nabla^{\nu}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -T_{\mu\nu}\right) = 0$$

$$\boxed{\nabla^{\nu}T_{\mu\nu} = 0}$$

Bianchi Identity

$$\nabla^{\nu}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)=0$$

$$\nabla^{\nu}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-T_{\mu\nu}\right)=0$$

$$+\Lambda\, g_{\mu\nu}$$

$$\boxed{\nabla^{\nu}T_{\mu\nu}=0}$$

Modified Theories of Gravity

$$S = \int d^4x \sqrt{-g} \left[f(R) + \mathcal{L}_{(m,r)} \right]$$

$$\frac{\delta}{\delta g_{\mu\nu}}$$

$$\begin{aligned} f_R \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) &= -T^{(m,r)}_{\mu\nu} + \\ &+ \frac{1}{2} g_{\mu\nu} \left(f - R f_R \right) + \nabla_\mu \nabla_\nu f_R - \\ &- g_{\mu\nu} \nabla_\alpha \nabla^\alpha f_R \end{aligned}$$

$$f_R \equiv \frac{df}{dR}$$

I

Energy-momentum Conservation and the Bianchi Identity

$$\nabla^\mu \left(\begin{array}{rcl} f_R \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) & = & -T^{(m,r)}_{\mu\nu} + \\ & + & \frac{1}{2} g_{\mu\nu} \left(f - R f_R \right) + \nabla_\mu \nabla_\nu f_R - \\ & - & g_{\mu\nu} \nabla_\alpha \nabla^\alpha f_R \end{array} \right)$$

$$\nabla^\mu g_{\mu\nu} = 0$$

$$(\Box \nabla_\nu - \nabla_\nu \Box) f_R = R_{\mu\nu} \nabla^\mu f_R$$

$$\boxed{T^{\mu\nu}{}_{;\nu} = 0}$$

Koivisto, CQG (2006)

Energy-momentum Conservation and the Bianchi Identity

$$\nabla^{\mu}\left(\begin{array}{rcl} f_R\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right) & = & -T^{(m,r)}_{\mu\nu}+ \\ & + & \frac{1}{2}g_{\mu\nu}\left(f-Rf_R\right)+\nabla_{\mu}\nabla_{\nu}f_R- \\ & - & g_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}f_R \end{array}\right)$$

$$\nabla^{\mu}g_{\mu\nu}=0$$

$$(\Box\nabla_{\nu}-\nabla_{\nu}\Box)f_R=R_{\mu\nu}\nabla^{\mu}f_R$$

$$\Rightarrow \quad \boxed{T^{\mu\nu}{}_{;\nu}=0}$$

Koivisto, CQG (2006)

II 4^{th}-order differential equation for the metric

IIa Extra degrees of freedom for the initial conditions

Einstein's Equations for an homogeneous universe:

$$\begin{cases} 3f_R H^2 = (\rho_m + \rho_r) + \frac{1}{2}(f_R R - f) - 3H\dot{f}_R \\ -2f_R \dot{H} = \left(\rho_m + \frac{4}{3}\rho_r\right) + \ddot{f}_R - H\dot{f}_R \end{cases}$$

$$\begin{cases} f_R \equiv \dfrac{df}{dR} \\ \dot{f}_R \equiv \dfrac{d}{dt}\dfrac{df}{dR} = \dfrac{dR}{dt}\dfrac{d^2 f}{dR^2} \\ \ddot{f}_R \equiv \dfrac{d^2 R}{dt^2}\dfrac{d^2 f}{dR^2} + \left(\dfrac{dR}{dt}\right)^2 \dfrac{d^3 f}{dR^3} \end{cases}$$

$$R = R\left[g_{\mu\nu}, \frac{\partial}{\partial x^\alpha} g_{\mu\nu}, \frac{\partial^2}{\partial x^\alpha x^\beta} g_{\mu\nu}\right]$$

IIb Well-behaved GR-limit

$$f(R) = R + \epsilon\Delta(R) \quad \Rightarrow \quad \lim_{\epsilon \longrightarrow 0} R(t) \qquad ?$$

Ilab

Ostrogradski's theorem:

M. Ostrogradski: Mem. Ac. St. Petersbourg VI 4, 385 (1850)
Woodard, astro-ph/0601672

Linear instablility in Lagrangeans with more than one time derivative

$f(R)$ **: higher-order diff. eq.**

1D : higher-order diff. eq.

=

higher-order Lagrangean

$$L(\dot{q}, \ddot{q}, \dddot{q})$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{q}} = 0$$

+ SURFACE TERMS

$$\begin{cases} Q_1 \equiv q & P_1 \equiv \dfrac{\partial L}{\partial \dot{q}} - \dfrac{d}{dt}\dfrac{\partial L}{\partial \ddot{q}} \\ Q_2 \equiv \dot{q} & P_2 \equiv \dfrac{\partial L}{\partial \ddot{q}} \end{cases}$$

$$\boxed{\ddot{q} = \ddot{q}(Q_1, Q_2, P_2)}$$

$$L \qquad \longrightarrow \qquad H(Q_1, Q_2, P_1, P_2)$$

$$H(Q_1, Q_2, P_1, P_2) = P_1 Q_2 + P_2\, \ddot{q}(Q_1, Q_2, P_2) - L$$

<u>not</u> bounded from below!!!

Perturbed “Harmonic Oscillator”

$$\lambda \dddot{x} + \ddot{x} + w^2 x = 0$$

$$\lambda \dddot{x} + \ddot{x} + w^2 x = 0$$

$$\lambda w \ll 1$$

a_0

$\tilde{w}$

$$x(t) = A \exp\left(-\frac{t}{\lambda}\right) + R \exp\left[\lambda w^2 \left(\frac{1}{2} - \lambda w^2\right) t\right] \cos\left[w\left(1 - \frac{5}{8}\lambda^2 w^2\right) t + \theta\right]$$

(x_0, v_0)

$$\boxed{\lambda \dddot{x} + \ddot{x} + w^2 x = 0}$$

$$\lambda w \ll 1$$

a_0

$$\begin{aligned} x(t) &= A \exp\left(-\frac{t}{\lambda}\right) + \\ &+ R \exp\left[\lambda w^2 \left(\frac{1}{2} - \lambda w^2\right) t\right] \cos\left[\overbrace{w\left(1 - \frac{5}{8}\lambda^2 w^2\right)}^{\tilde{w}} t + \theta\right] \end{aligned}$$

(x_0, v_0)

Harm.Osc. — Linear Lagrangean:

$$x(t) = x_{\infty}(t) + x_{\rm ho}(t)$$

determination of the integration constants can be done analytically

f(R) — Non-linear Lagrangean:

$$R(t) = R_{\lambda}(t) \rightarrow \infty \quad (\text{as } \lambda \rightarrow 0)$$

perturbative calculation of the integration const.

The singularity is reached at a finite time in the past!

Still, in both cases the singularity will be present if the initial conditions do not have the exact values that yield a vanishing divergent "term".

Verified numerically

Frolov, 0803.2500
Appleby, 0803.1081

Is this an intrinsic divergence ?

No, we may evade Ostrogradski's theorem:

$$\ddot{q} \neq \ddot{q}(Q_1, Q_2, P_2)$$

$$\Longrightarrow \qquad L \not\longrightarrow H$$

no Legendre transformation!

e.g., $$f(R) = R + \alpha R^2$$

Mapping onto GR + scalar field: the standard approach

$$S = \int d^4x \sqrt{-g}\, [f_R(Q)\underbrace{(R-Q)} + f(Q)]$$

Legendre multiplier
$f_{RR} \neq 0 \Rightarrow Q = R$

$$S = \int d^4x \sqrt{-g}\, \left[\chi R - \chi^2 V(\chi)\right]$$

$$\chi \equiv f_R \equiv \frac{df}{dR}$$

$$V(\chi) \equiv \frac{1}{2\chi^2}\left\{Q(\chi)\chi - f[Q(\chi)]\right\}$$

Conformal transformation

$$\tilde{g}_{\mu\nu} \equiv \chi \, g_{\mu\nu} \equiv \exp\left(\frac{2}{\sqrt{6}}\phi\right) g_{\mu\nu}$$

$$S = \int d^4x \, \sqrt{-\tilde{g}} \, \left[\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + \tilde{S}_{matter}$$

$$\tilde{S}_{matter} \equiv \tilde{S}_{matter}\left[\tilde{g}_{\mu\nu} \exp\left(\frac{2}{\sqrt{6}}\phi\right), \Psi, A_\mu, \cdots\right]$$

Jordan Frame

$$f_R\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right) = -T^{(m,r)}_{\mu\nu}+$$

$$+ \quad \frac{1}{2}g_{\mu\nu}\left(f-Rf_R\right)+\nabla_\mu\nabla_\nu f_R -$$

$$- \quad g_{\mu\nu}\nabla_\alpha\nabla^\alpha f_R$$

Einstein Frame

$$\tilde{R}_{\mu\nu}-\frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R}=-\tilde{T}^{(m,r)}_{\mu\nu}-\tilde{T}^{(\phi)}_{\mu\nu}$$

- **Particles do not follow geodesics in the new metric!!**
- $T_{\mu\nu}^{(m,r)}$ **is not conserved**
- **The mapping is not valid if** $f_R = 0$

$$\tilde{g}_{\mu\nu} \equiv \chi\, g_{\mu\nu} \equiv \exp\left(\frac{2}{\sqrt{6}}\phi\right) g_{\mu\nu}$$

$$\chi \equiv f_R \equiv \frac{df}{dR}$$

5th force

- Particles do <u>not follow geodesics</u> in the new metric!!
- $T_{\mu\nu}^{(m,r)}$ is <u>not conserved</u>
- The mapping is <u>not valid</u> if $f_R = 0$

interaction

$$\tilde{g}_{\mu\nu} \equiv \chi\, g_{\mu\nu} \equiv \exp\left(\frac{2}{\sqrt{6}}\phi\right) g_{\mu\nu}$$

$$\chi \equiv f_R \equiv \frac{df}{dR}$$

Let's face it:

$$
\begin{aligned}
f_R\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) &= -T^{(m,r)}_{\mu\nu} + \\
&+ \frac{1}{2}g_{\mu\nu}\left(f - Rf_R\right) + \nabla_\mu\nabla_\nu f_R - \\
&- g_{\mu\nu}\nabla_\alpha\nabla^\alpha f_R
\end{aligned}
$$

$$
f_R \equiv \frac{df}{dR}
$$

Let's face it:

$$
\begin{aligned}
f_R\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) &= -T^{(m,r)}_{\mu\nu} + \\
&+ \frac{1}{2}g_{\mu\nu}\left(f - Rf_R\right) + \nabla_\mu\nabla_\nu f_R - \\
&- g_{\mu\nu}\nabla_\alpha\nabla^\alpha f_R
\end{aligned}
$$

$$
f_R \equiv \frac{df}{dR}
$$

$$-\tilde{T}_{\mu\nu}^{(m,r)}$$

$$\begin{aligned}
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R &= -\frac{1}{f_R} T_{\mu\nu}^{(m,r)} + \\
&+ \frac{1}{2} g_{\mu\nu} \left(f - R f_R \right) + \nabla_\mu \nabla_\nu f_R - \\
&- \frac{1}{f_R} g_{\mu\nu} \nabla_\alpha \nabla^\alpha f_R
\end{aligned}$$

$$-T_{\mu\nu}^{(c)}$$

$$3f_R H^2 = (\rho_m + \rho_r) + \frac{1}{2}(f_R R - f) - 3H\dot{f}_R$$

$$\boxed{3H^2 = \frac{1}{f_R}(\rho_m + \rho_r) + \frac{1}{f_R}\left[\frac{1}{2}(f_R R - f) - 3H\dot{f}_R\right]}$$

$$-2f_R\dot{H} = \left(\rho_m + \frac{4}{3}\rho_r\right) + \ddot{f}_R - H\dot{f}_R$$

$$\boxed{-2\dot{H} = \frac{1}{f_R}\left(\rho_m + \frac{4}{3}\rho_r\right) + \frac{1}{f_R}\left[\ddot{f}_R - H\dot{f}_R\right]}$$

$$\left\{\begin{aligned} 3H^2 &= \frac{1}{f_R}\rho_m + \frac{1}{f_R}\rho_r + \frac{1}{2f_R}\left(f_R R - f\right) - 3H\frac{\dot{f}_R}{f_R} \\ -2\dot{H} &= \left(\frac{1}{f_R}\rho_m + \frac{4}{3f_R}\rho_r\right) + \frac{1}{f_R}\left(\ddot{f}_R - H\dot{f}_R\right) \end{aligned}\right.$$

$\rho_m + \cancel{p_m}$

$\rho_r + p_r$

$\rho_c + p_c$

$w_c \equiv \dfrac{p_c}{\rho_c}$

$$\rho_c \equiv \frac{1}{2f_R}(f_R R - f) - 3H\frac{\dot{f}_R}{f_R}$$

$$p_c \equiv \frac{\ddot{f}_R}{f_R} + 2H\frac{\dot{f}_R}{f_R} - \frac{1}{2f_R}(f_R R - f)$$

$$\boxed{w_c(t) \equiv \frac{p_c(t)}{\rho_c(t)} = w_c(a)}$$

The curvature is a perfect fluid !

... but can we actually assume that

$$f' > 0 \quad ?$$

$$\nabla_\mu v_\nu = -v_\mu \dot{v}_\nu + \frac{1}{3}\theta h_{\mu\nu} + \sigma_{\mu\nu} + w_{\mu\nu}$$

$$\tilde{T}_{\mu\nu} = \tilde{\rho} v_\mu v_\nu + \tilde{p} h_{\mu\nu} + \tilde{q}_{(\mu} v_{\nu)} + \tilde{\pi}_{\mu\nu}$$

$$\begin{aligned} \dot{\sigma}_{\mu\nu} \quad + \quad \theta\sigma_{\mu\nu} &= \\ &= h^{\alpha}{}_{\mu}\nabla_{(\alpha}\dot{v}_{\nu)} + \dot{v}_{(\mu}\dot{v}_{\nu)} + \tilde{\pi}_{\mu\nu} + \frac{1}{3}h_{\mu\nu}\left[2\tilde{\rho} - \frac{2}{3}\theta^2 + 2\sigma^2\right] \end{aligned}$$

Maeda, PRD 39 (1989) 3158

$$\dot{\sigma}_{\mu\nu} + \theta\sigma_{\mu\nu} = -\frac{f''}{f'}\dot{R}\sigma_{\mu\nu}$$

$$\theta \equiv 3\frac{\dot{a}}{a}$$

$$\sigma_{\mu\nu} = C_{\mu\nu}\frac{1}{f' a^3}$$

Maeda, PRD 39 (1989) 3158

$$\dot{\sigma}_{\mu\nu} + \theta\sigma_{\mu\nu} = -\frac{f''}{f'}\dot{R}\sigma_{\mu\nu}$$

$$\theta \equiv 3\frac{\dot{a}}{a}$$

$$\sigma_{\mu\nu} = C_{\mu\nu}\frac{1}{f' a^3}$$

... but recall that we have used

$$\tilde{T}_{\mu\nu} = \tilde{\rho} v_\mu v_\nu + \tilde{p} h_{\mu\nu} + \tilde{q}_{(\mu} v_{\nu)} + \tilde{\pi}_{\mu\nu}$$

III

Time-varying G ?

$$
\begin{aligned}
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \quad &= \quad -\frac{1}{f_R} T^{(m,r)}_{\mu\nu} + \\
&+ \quad \frac{1}{2} g_{\mu\nu} \left(f - RF \right) + \nabla_\mu \nabla_\nu f_R - \\
&- \quad \frac{1}{f_R} g_{\mu\nu} \nabla_\alpha \nabla^\alpha f_R
\end{aligned}
$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \quad = \quad -\tilde{T}^{(m,r)}_{\mu\nu} + T^{(c)}_{\mu\nu}$$

$$\nabla^\mu \tilde{T}^{(m,r)}_{\mu\nu} = \nabla^\mu \left(\frac{1}{f_R} T^{(m,r)}_{\mu\nu} \right) \neq 0$$

$$\nabla^\mu T^{(m,r)}_{\mu\nu} = 0$$

$$\nabla^\mu G_{\mu\nu} = 0$$

$$\overbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}^{} = \tilde{T}^{(m,r)}_{\mu\nu} + T^{(c)}_{\mu\nu}$$

$$\nabla^\mu T^{(c)}_{\mu\nu} \neq 0$$

IV **Lack of conservation!**

V

Cosmological constraints

$$R \gg R_c \Rightarrow f(R) \sim R$$ **early universe**

$$R \ll R_c \Rightarrow f(R) \sim R - 2\Lambda$$ **present universe**

$$S = \int d^4x \, \sqrt{-g} \; f(R)$$

Let's face it again:

$$
\begin{aligned}
f_R\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) &= -T^{(m,r)}_{\mu\nu} + \\
&+ \frac{1}{2}g_{\mu\nu}\left(f - Rf_R\right) + \nabla_\mu\nabla_\nu f_R - \\
&- g_{\mu\nu}\nabla_\alpha\nabla^\alpha f_R
\end{aligned}
$$

$$f_R \equiv \frac{df}{dR}$$

$$3f_R H^2 = (\rho_m + \rho_r) + \frac{1}{2}(f_R R - f) - 3H\dot{f}_R$$

$$\boxed{3H^2 = \frac{1}{f_R}(\rho_m + \rho_r) + \frac{1}{f_R}\left[\frac{1}{2}(f_R R - f) - 3H\dot{f}_R\right]}$$

$$3f_R H^2 = (\rho_m + \rho_r) + \frac{1}{2}(f_R R - f) - 3H\dot{f}_R$$

$$3H^2 = \frac{1}{f_R}(\rho_m + \rho_r) + \frac{1}{f_R}\left[\frac{1}{2}(f_R R - f) - 3H\dot{f}_R\right]$$

$$3H^2 = \rho_m + \rho_r + \underbrace{\frac{1}{2}(f_R R - f) - 3H\dot{f}_R + 3H^2(1-F)}_{\equiv \rho_c}$$

The curvature is a well-defined perfect fluid

→ $\sigma_{\mu\nu}(t)$: **finite!**

Leach *et al.*, gr-qc/0603012

VI

Avoiding ghosts

Faraoni, astro-ph/0610734

$$f(R) = R + \epsilon \Delta(R)$$

$$\epsilon \to 0$$

$$\begin{aligned} f_R\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) &= -T^{(m,r)}_{\mu\nu} + \\ &+ \frac{1}{2}g_{\mu\nu}\left(f - Rf_R\right) + \nabla_\mu\nabla_\nu f_R - \\ &- g_{\mu\nu}\nabla_\alpha\nabla^\alpha f_R \end{aligned}$$

Tr ↓

$$\Box R + \frac{\Delta'''}{\Delta''}\nabla^\alpha R \nabla_\alpha R + \frac{\epsilon\Delta' - 1}{3\epsilon\Delta''}R = \frac{-1}{3\epsilon\Delta''}T + \frac{\Delta}{3\Delta''}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$R = T + R_1$$

< 0 : *ghosts*!

$$\ddot{R}_1 - \nabla^2 R_1 - \frac{2\Delta'''}{\Delta''}\dot{T}\dot{R}_1 + \frac{1}{3\Delta''}\left(\frac{1}{\epsilon} - \Delta'\right)R_1 +$$

$$+ \frac{2\Delta'''}{\Delta''}\vec{\nabla}T \cdot \vec{\nabla}R_1 = \ddot{T} - \nabla^2 T - \frac{T\Delta' + \Delta}{3\Delta''}$$

$$\Box R + \frac{\Delta'''}{\Delta''}\nabla^{\alpha} R \nabla_{\alpha} R + \frac{\epsilon\Delta' - 1}{3\epsilon\Delta''} R = \frac{-1}{3\epsilon\Delta''} T + \frac{\Delta}{3\Delta''}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$R = T + R_1$$

< 0 : *ghosts*!

$$\ddot{R}_1 - \nabla^2 R_1 - \frac{2\Delta'''}{\Delta''}\dot{T}\dot{R}_1 + \frac{1}{3\Delta''}\left(\frac{1}{\epsilon} - \Delta'\right) R_1 + \\ + \frac{2\Delta'''}{\Delta''}\vec{\nabla} T \cdot \vec{\nabla} R_1 = \ddot{T} - \nabla^2 T - \frac{T\Delta' + \Delta}{3\Delta''}$$

$$\Delta(R) = -\frac{\mu^4}{R}$$

$$\Delta''(R) = -2\frac{\mu^4}{R^2} < 0$$

$$t_c \sim 10^{-26} s$$

Dolgov, Kawasaki (phys. Lett. 573B, 1 (2003)

VIII

Palatini's approach

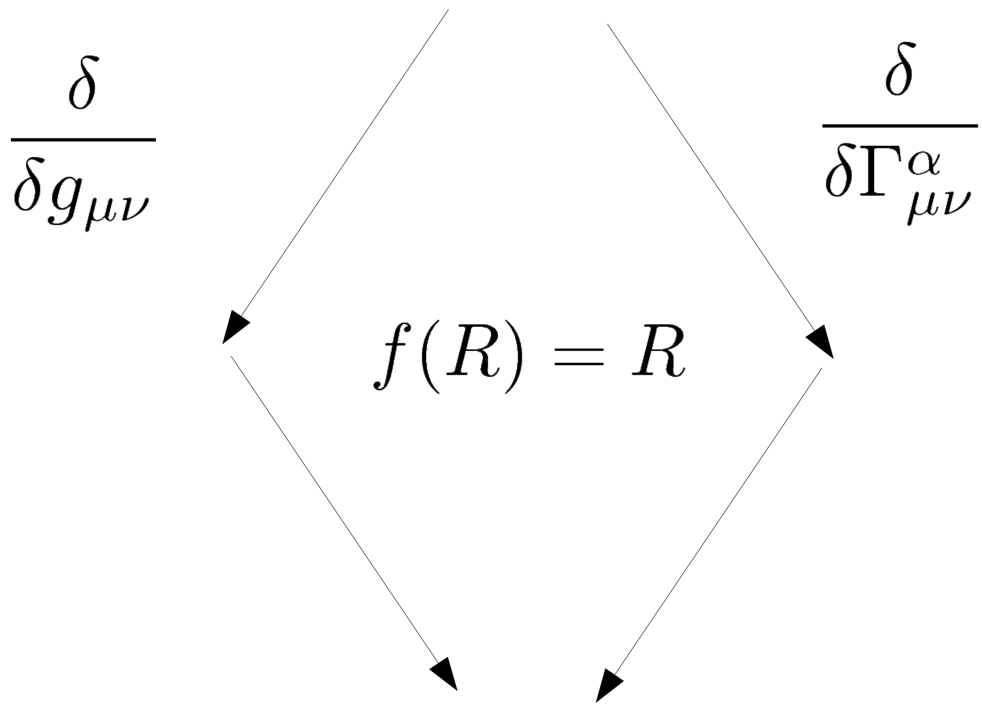
$$S = \int d^4x \sqrt{-g}\, f(R)$$

$$\frac{\delta}{\delta g_{\mu\nu}} \qquad \frac{\delta}{\delta \Gamma^{\alpha}_{\mu\nu}}$$

VIII

Palatini's approach

$$S = \int d^4x \sqrt{-g}\, f(R)$$



Einstein's equation

For a general $f(R)$,

the field ϕ

has no dynamics!

<u>Cauchy's problem</u>

Faraoni, 0806.0766

CONCLUSIONS

There are many constraints:

- **Singularities**
  - **Due to the higher-order diff. equations**
  - **“Physical” ones: actual or not?**
- **Cosmological history**
- **Solar system constraints**
- **Perturbation growth?**

QUESTIONS

- $G[R(t)]$ **?!**
- **Can** $f(R)$ **replace** ***dark matter*** **?**
- **How to map a modified-gravity theory onto a perfect-fluid approach? Is there (one) correct answer?**

A final challenge:

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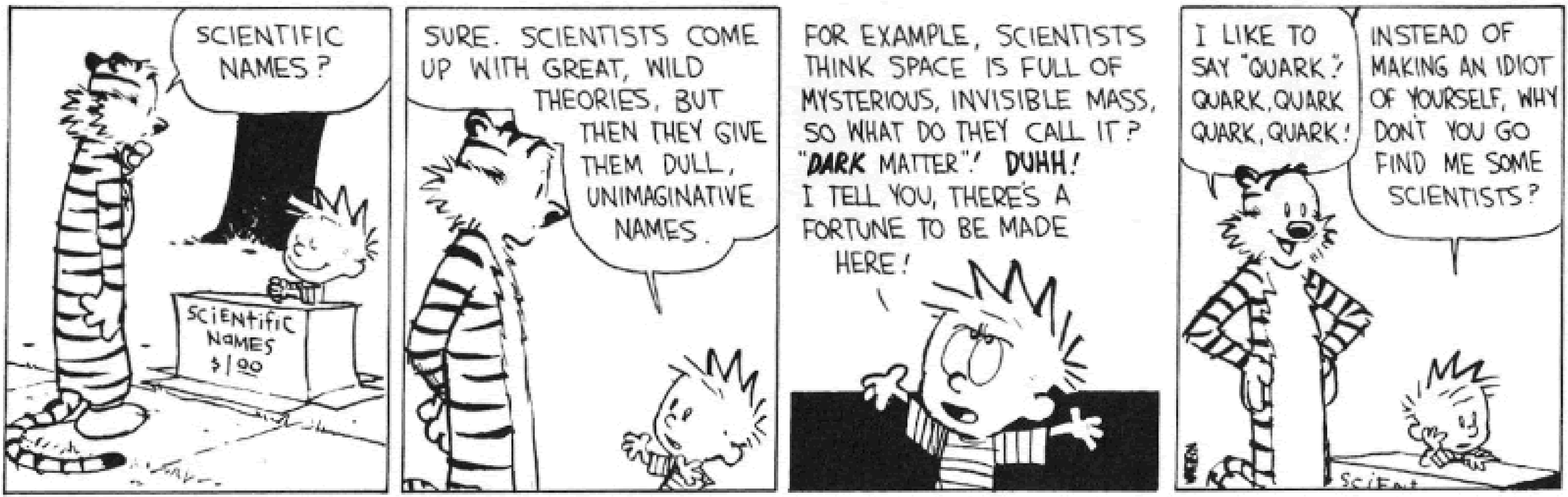

... find nice names !

Mapeamento para campo escalar

$$S = \int d^4x \sqrt{-g} f(R)$$

$$= \int d^4x \sqrt{-g} \left[f'(\phi)(R - \phi) + f(\phi) \right]$$

$$= \int d^4x \sqrt{-g} \left[\chi R - V(\phi) \right]$$

$$V(\phi) \equiv \phi\chi - f \qquad \chi = f'$$

Transformada de Legendre

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,[f'R - V(\phi)]$$

$$\frac{\delta}{\delta\phi}\ \Big\downarrow$$

$$R\frac{d\psi}{d\phi} - \frac{dV}{d\phi} = (R-\phi)f''(\phi) = 0$$

$$f'' \neq 0 \Longrightarrow \phi = R$$