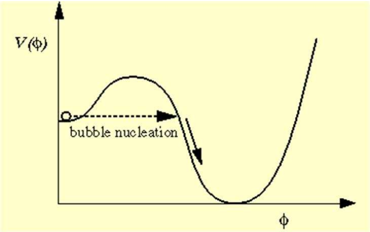
Scalar-Tensor Theories of Gravity and the Inflationary Universe

Marcos Brum, Ioav Waga

Federal University of Rio de Janeiro
Physics Institute

𝒜ℛ𝒞𝒪𝒮 - Astrophysics, Relativity and COSmology Group



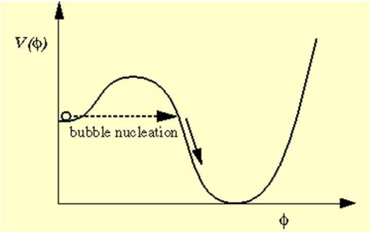
{Guth [PRD**23**, 347 (1981)]} Two requirements have to be met:

$$r \equiv \frac{\Gamma}{H^4} \ll 1. \qquad (1)$$

$$r \approx 1. \qquad (2)$$

Problems

Since H is a constant during Inflation, these conditions can't be met! So, the original scenario is not a satisfactory Inflationary Model.



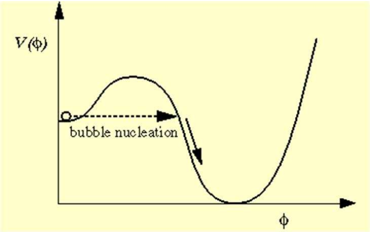
{Guth [PRD**23**, 347 (1981)]} Two requirements have to be met:

$$r \equiv \frac{\Gamma}{H^4} \ll 1. \qquad (1)$$

$$r \approx 1. \qquad (2)$$

Problems

Since H is a constant during Inflation, these conditions can't be met! So, the original scenario is not a satisfactory Inflationary Model.



{Guth [PRD**23**, 347 (1981)]} Two requirements have to be met:

$$r \equiv \frac{\Gamma}{H^4} \ll 1. \tag{1}$$

$$r \approx 1. \tag{2}$$

Problems

Since H is a constant during Inflation, these conditions can't be met! So, the original scenario is not a satisfactory Inflationary Model.

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}(M^2 + \beta\phi^2)R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \Lambda\right] \tag{3}$$

Initially the expansion is de Sitter-like[1],

$$\mathrm{H}_I^2 = \frac{\Lambda}{3\mathrm{M}^2} \tag{4}$$

When the scalar field dominates, the Hubble parameter becomes

$$\mathrm{H} = \frac{\alpha}{t}, \tag{5}$$

where

$$\alpha = \frac{1+2\beta}{4\beta} \tag{6}$$

and

$$a \propto t^\alpha \tag{7}$$

[1] F Di Marco and A Notari, [PRD**73**, 063514 (2006)]

$$\phi(x,t) = \phi(t) + \delta\phi(x,t),$$

and

$$ds^2 = -(1+2\psi)dt^2 - \chi_{,\mu} dt dx^\mu + a^2(1+2\varphi)\delta_{\mu\nu} dx^\mu dx^\nu,$$

We find a quite general prediction for the spectral index[2]:

$$\mathrm{n}_s = 4 - 2\nu \tag{8}$$

DM&N suppose that perturbations on cosmological scales leave out the horizon during the first stage. So, $\nu \simeq \frac{3}{2} + 4\beta$ and $\beta > \frac{1}{58}$,

$$\mathrm{n}_s < 0.86 \tag{9}$$

In clear disagreement with observations! ($n_S = 0.960 \pm 0.013$, Komatsu et al., $astro-ph/0803.0547$)

[2] Faraoni, *Cosmology in Scalar-Tensor Gravity*

We propose a modification of this assumption. Considering that perturbations on cosmological scales leave out the horizon during the second stage, we arrive at the result $\beta \lesssim \frac{1}{58}$, thus $a \propto t^{\alpha}$, $\alpha \gtrsim 15$.

By the gauge-invariant variable

$$\delta\phi_{\varphi} = \delta\phi - \frac{\dot{\phi}}{H}\varphi,$$

and the definition $Z(t) \equiv \frac{\phi^2(1+6\beta)}{(\phi+\dot{\phi}/H)^2}$

$$\delta\ddot{\phi}_{\varphi} + \frac{(a^3 Z)^{\cdot}}{a^3 Z}\delta\dot{\phi}_{\varphi} - \left\{ \frac{1}{a^2}\nabla^2 + \frac{H}{a^3 Z\dot{\phi}}\left[a^3 Z\left(\frac{\dot{\phi}}{H}\right)^{\cdot}\right]^{\cdot} \right\}\delta\phi_{\varphi} = 0 \tag{10}$$

Defining $v \equiv \sqrt{Z} a \delta\phi_\varphi$ and $z \equiv \sqrt{Z} \frac{a\dot{\phi}}{H}$, and the proper time $dt = a d\tau$, (10) becomes

$$v'' - \left(\frac{z''}{z} + \nabla^2 \right) v = 0 \tag{11}$$

$$\frac{z''}{z} \propto \frac{1}{\tau^2} \therefore v\left(k, \tau\right) = \frac{1}{2}\sqrt{\pi|\tau|}\left[c_1(k) H_\nu^{(1)}\left(k, |\tau|\right) + c_2(k) H_\nu^{(2)}\left(k, |\tau|\right)\right] \tag{12}$$

And the Power Spectrum is

$$P_C^{1/2}\left(k, \tau\right) = \left|\frac{H}{\dot{\phi}}\right| \frac{H}{2\pi} \frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)} \left[\frac{1}{2}\frac{k}{aH}\right]^{\frac{3}{2}-\nu} \tag{13}$$

Now we have $\nu \simeq \frac{3}{2} + 8\beta$, and we predict, for the spectral index,

$$\mathrm{n}_s > 0.72 \tag{14}$$

This is in accordance with the observed value of the scalar spectral index!

It starts from a similar lagrangian[3]

$$S = \int d^4x\sqrt{-g}\left\{M^2 f(\phi)R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \Lambda\right\}. \qquad (15)$$

where $f(\phi) = \left[1 + \beta\left(\frac{\phi}{\mathrm{M}}\right)^n\right]$. Performing a conformal transformation and re-defining the field, we are left with the potential

$$\bar{V}(\Phi) = \Lambda\left[1 - 2\beta\left(\frac{\Phi}{\mathrm{M}}\right)^n\right]. \qquad (16)$$

In the slow-roll approximation, we find the spectral index

$$n_s - 1 \simeq -0.04\left(\frac{n-1}{n-2}\right) \qquad (17)$$

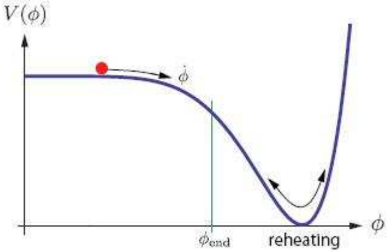
Which leads to (n $\geq$ 4)

$$0.94 \leq n_s \leq 0.96, \qquad (18)$$

in good accordance with WMAP results.

[3] T Biswas and A Notari, PRD**74**, 043508 (2006)

An alternative is the **Slow-Roll approximation** [4].

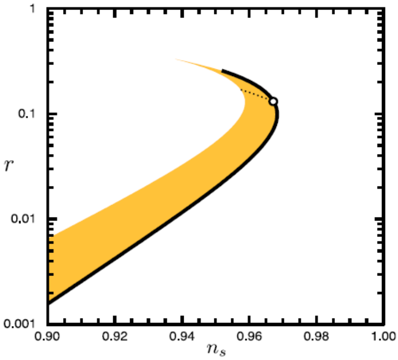


$H^2 \simeq \frac{V(\phi)}{3M_{pl}^2}$; $3H\dot{\phi} \simeq -V'(\phi)$.

Expanding the Hubble parameter,

$$H(\phi) = H_* + H'_*\phi + \frac{1}{2}H''_*\phi^2 + \frac{1}{6}H'''_*\phi^3 + \cdots \quad (19)$$

we can apply the Bootstrap tests ($astro-ph/0810.2787$). They associate the above expansion with cosmological observables, $\Delta^2_{\mathcal{R}}$, $r \equiv \Delta^2_h/\Delta^2_{\mathcal{R}}$, n_S and $\alpha \equiv \frac{dn_S}{d\ln k}$.



[4](Liddle and Lyth, *Cosmological inflation and Large-Scale Structure*)

Under the Slow-Roll approximation in STG[5] we can calculate the first two terms in the expansion of $H(\phi)$ (H_* and H'_*). The derivation of further terms, whether possible, is more involved. Out of the approximation, things get harder. H_* has a general, model-independent form, but H'_* doesn't, because of a choice of root for a quadratic equation. If we make this choice and proceed, narrowing the range of models, we might find a difficulty similar to the above.

[5] Faraoni, *Cosmology in Scalar-Tensor Gravity*

Under the Slow-Roll approximation in STG[5] we can calculate the first two terms in the expansion of $H(\phi)$ (H_* and H'_*). The derivation of further terms, whether possible, is more involved. Out of the approximation, things get harder. H_* has a general, model-independent form, but H'_* doesn't, because of a choice of root for a quadratic equation. If we make this choice and proceed, narrowing the range of models, we might find a difficulty similar to the above.

[5] Faraoni, *Cosmology in Scalar-Tensor Gravity*

→ Inflation is viable in the context of Scalar-Tensor Theories of Gravity!

→ The Bootstrap Tests are not easily implemented - if possible - in Scalar-Tensor Gravity, though they provide an interesting test to Slow-Roll Inflation in GR.

→ Inflation is viable in the context of Scalar-Tensor Theories of Gravity!

→ The Bootstrap Tests are not easily implemented - if possible - in Scalar-Tensor Gravity, though they provide an interesting test to Slow-Roll Inflation in GR.