SUSY Dark Matter and non-standard Cosmologies

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Evidence for DM

Various astrophysical sources have confirmed the existence of Dark Matter (DM)

- Binding of Galaxies in Clusters (F. Zwicky, 1933)
- Rotation curves of Galaxies (V.C. Rubin and W.K. Ford, 1970)
- Bindings of hot gases in clusters
- Gravitational Lensing observations
- Large Scale Structure simulations
- High z - Supernovae
- Observations of colliding clusters of Galaxies

The most direct and accurate evidence comes from WMAP by measuring anisotropies of the CMB power spectrum

$$\simeq 73\% \text{ DarkEnergy, } \simeq 23\% \text{ DarkMatter, } \simeq 4\% \text{ Baryons}$$

DM abundance accurately measured!

$$\Omega_{DM} h_0^2 = 0.111^{+0.011}_{-0.015} \, 2\sigma$$

Candidates for Dark Matter

**CDM** is established by various observations!

**HDM**, like SM neutrinos is ruled out!

- Thermal, or non-thermal, relics created in the early Universe may constitute part or all of the observed CDM

**WIMPs**

- Weakly Interacting Massive Particles with masses in the Electroweak range, $\sim \mathcal{O}(100)\, \text{GeV}$, naturally yield $\Omega_{\text{DM}} h_0^2 \sim 0.1$, (**WIMP miracle**)

- LSP - neutralinos in SUSY theories
- Heavy $\nu$ - like particles
- KK excitations in ED theories
- Lightest T-odd particles in little Higgs models
- Other exotic
**superWIMPs**

Interact with smaller strength (gravitationally,...) than WIMPs

- Gravitino (superpartner of graviton)
- KK gravitons
- Axino (superpartner of axion)
- ...

**Other Candidates**

- Axion \( m_a \simeq 10^{-6} \text{ eV} - 10^{-2} \text{ eV} \)
- Wimpzillas, Cryptons
- Q - balls
- BH remnants and other very massive astrophysical objects
- Moduli fields of String Theory
- ...

**SUPERSYMMETRY**, with conserved R - parity, offers an ideal WIMP candidate! The LSP neutralino (if the lightest sparticle). Also gravitino, axino or superpartner of a sterile neutrino.
SUSY is a symmetry between fermions and bosons and a natural extension of relativistic symmetries!

- Resolves the gauge hierarchy problem
- Induces radiative Electroweak Symmetry Breaking
- Unifies gauge couplings at large scales $\sim 10^{16}$ GeV
- Consistent with precision experiments
- Indispensable ingredient of Superstring Theories

Every known particle has a partner (sparticle) with spin differing by $\frac{1}{2}$ and same mass. SUSY is broken and mass degeneracy is lifted!

$$m_{sp}^2 = m_p^2 + M_{SUSY}^2$$

Theory dictates $M_{SUSY} < \mathcal{O}(\text{TeV})$ and sparticles will (?) be discovered at LHC
Three leading candidates in Supersymmetry!

- **Neutralino LSP, \( \tilde{\chi} \)**
  It qualifies as a WIMP. It is neutral, uncolored, interacting weakly. In SUSY models with R-parity conservation it is stable, if the Lightest Supersymmetric Particle. The best motivated SUSY DM candidate, likely to be seen at the LHC.

- **Gravitino, \( \psi_\mu \)**
  A spin-3/2 Rarita-Schwinger field, partner of the graviton, emerging in the gauged Supersymmetry (Supergravity). It is the gauge field of the local supersymmetric transformations, which are broken at an unknown scale \( M_s \), resulting to a gravitino mass

\[
m_{3/2} \sim \frac{M_s^2}{M_{Planck}}
\]

It interacts only gravitationally and may be a DM, if the lightest.

- **Axino, \( \tilde{a} \)**
  Superpartner of the axion, interacting feebly with the rest of the particles, like its axion partner.
Neutralino DM

Neutralinos are Majorana spin-1/2 fermions. They can annihilate in pairs to SM particles. They are produced by the inverse process if $A, B$ have sufficient energies (High temperatures $T \gg m_{\tilde{\chi}}$)

Their annihilation cross section $\sigma_A$ controls the change of their number density through Boltzmann equation

$$\frac{dn}{dt} = -3Hn - \langle v \sigma_A \rangle (n^2 - n_{eq}^2)$$

$H$ Hubble expansion rate, $n_{eq}$ = equilibrium number density, $v$ = Möller velocity
$Y = n/s$ : Number density to entropy density

$$\frac{dY}{dT} = \frac{2\pi^2}{45} T^2 h H^{-1} < v \sigma_A > (Y^2 - Y_{eq}^2)$$

- Entropy conservation is assumed after freeze-out, $sa^3 = \text{constant}$

$$s = \frac{2\pi^2}{45} h(T) T^3 \quad , \quad h(T) = \text{entropy d.o.f}$$

**What if we have late entropy production?**

- Hubble expansion rate radiation dominated at freeze-out

$$H = \left( \frac{8\pi G_N}{3} \rho_r \right)^{1/2} \quad , \quad \rho_r = \frac{\pi^2}{30} g_{\text{eff}}(T) T^4$$

**What if Universe not radiation dominated at freeze-out?**
For high temperatures $\tilde{\chi}$ are in thermal equilibrium $\tilde{\chi}\tilde{\chi} \leftrightarrow AB$ with the rest and their density is diluted because of expansion.

As temperature drops $\tilde{\chi}\tilde{\chi} \rightarrow AB$ and their number is reduced. Eventually the system reaches the freeze-out temperature, $T_F$, $\langle v\sigma_A \rangle n = H$.

For $T < T_F$ density too low for annihilation to track the expansion, $\langle v\sigma_A \rangle n << H$ and their total number is locked!
Co-annihiliation channels important if a sparticle, like stau $\tilde{\tau}$, has small mass difference $\Delta M$ with the neutralino $\tilde{\chi}$.

The key parameter is $\Delta M = m_{\tilde{\tau}} - m_{\tilde{\chi}}$.

$T_F \sim 0.1 \, m_{\tilde{\chi}} \implies$ The effect important for $\Delta M \leq O(0.1) \, m_{\tilde{\chi}}$. 

Supersymmetry is parametrized by unknown parameters $m_0, m_{1/2}$ which set $M_{SUSY}$ and determine sparticle mass spectrum and relic densities (Public codes available to calculate accurately neutralino relic density, MicroOmegas, DarkSUSY, ...)

WMAP data constrains the SUSY parameter space. The potential of discovering supersymmetry is confined to the narrow green bands.
Predictions for the neutralino relic density of millions of points in the minimal SUGRA model. Few satisfy the WMAP cosmological data.
Gravitino

The spin-3/2 gravitino $\tilde{G}$ field is the gauge fermion of local SUSY transformations and partner of the graviton. It acquires a mass $m_{3/2}$ after SSB of local SUSY (superHiggs effect)

$$m_{3/2} \simeq \frac{M_S^2}{M_{Planck}}$$

It couples gravitationally to matter with couplings $\sim M_{Planck}^{-1}$

$A_\mu$, $\lambda$ gauge-boson and its partner (gaugino)

$f_L$, $\tilde{f}_L$ fermion and its partner (sfermion)
\( \tilde{G} \)'s are produced by inelastic scattering processes during the reheating of the Universe after inflation (Bolz, Brandenburg and Buchmuller 2001, Pradler and Steffen 2006).

\[
\frac{dn_{\tilde{G}}}{dt} + 3H n_{\tilde{G}} = C_{\tilde{G}} , \quad C_{\tilde{G}} = \text{collision terms}
\]

to obtain the gravitino "yield" for \( T \leq T_R \),

\[
Y_{\tilde{G}} \equiv \frac{n_{\tilde{G}}}{n_{\gamma}} \simeq 1.9 \times 10^{-12} \left( 1 + \frac{m_{\tilde{g}}^2}{3 m_{3/2}^2} \right) \frac{T}{10^{10}} \text{ GeV}
\]
Unstable Gravitino

Decays to radiation  \( \tilde{G} \rightarrow \gamma + \tilde{\gamma} \)

\[ \tau \simeq 4 \times 10^8 \left( \frac{100 \text{ GeV}}{m_{3/2}} \right)^3 \]

Decays to hadrons  \( \tilde{G} \rightarrow g + \tilde{g} , q + \tilde{q} \)

\[ \tau \simeq 6 \times 10^7 \left( \frac{100 \text{ GeV}}{m_{3/2}} \right)^3 \]

- For  \( m_{3/2} = 10^2 \text{ GeV} \rightarrow 10 \text{ TeV} \) gravitino decays during and after primordial Nucleosynthesis with disastrous effects for BBN!
- Their overproduction dissociates light nuclei \( \gamma +^3 \text{He} \rightarrow D + p, \gamma + D \rightarrow n + p \)

**Gravitino Problem!**
For late decaying particles $X$ bounds on their lifetimes $\tau_X$ and abundances $Y_X$ are imposed by BBN. For gravitino these translate to bounds on $T_R$ and its mass $m_{3/2}$:

$$T_R = 10^5 - 10^7 \text{ GeV} \quad \text{for} \quad m_{3/2} = 10^2 \text{ GeV} - 3 \text{ TeV}$$

In contradiction with thermal Leptogenesis scenarios which require $T_R \simeq 10^9 \text{ GeV}$ and Inflation models with $T_R > 10^7 \text{ GeV}$!

Kawasaki M, Kohri K and Moroi T, 2005
Gravitino DM

If the gravitino is the LSP the gravitino problem may be avoided but the next to LSP particles (NLSP), neutralino $\tilde{\chi}$ or stau $\tilde{\tau}$, decay late!

- $\tilde{\chi}$ as NLSP is disfavoured by BBN bounds
- BBN bounds are weaker for the $\tilde{\tau}$!

Catalyzed BBN, Pospelov M, 2006

Bound-state formation of long-lived negatively charged particles with the primordial nuclei enhances $^6\text{Li}$ production by almost seven orders of magnitude putting severe upper limits on $\tilde{\tau}$ abundances, prior to their decays!

In CBBN cross-section is enhanced by 7 orders of magnitude!
The decays of the NLSP particles $\tilde{\chi} \rightarrow \tilde{G}\gamma$, $\tilde{\tau} \rightarrow \tilde{G}\tau$ produce gravitinos non-thermally. Their total density is given by:

$$\Omega_{\tilde{G}} = \Omega_{\text{thermal}} + \frac{m_{3/2}}{m_{NLSP}} \Omega_{NLSP}$$

Gravitino Dark Matter, with stau NLSP, is inconsistent with $T_R > 10^7 \text{ GeV}$ in the popular supersymmetric models.
Axino DM

Axinos, $\tilde{a}$, have masses in the $keV - GeV$ range and their couplings are suppressed by the PQ breaking scale $f_a$. Interact more weakly than a WIMP but not as weakly as a gravitino!

**Thermal production:**

Not in thermal equilibrium, are produced thermally via scattering, like gravitinos!


$$\Omega_\tilde{a}^{(th)} h_0^2 \approx 5.5 \, g_s^6 \ln \left( \frac{1.108}{g_s} \right) \left( \frac{10^{11} \text{ GeV}}{f_a} \right)^2 \left( \frac{m_\tilde{a}}{100 \text{ MeV}} \right) \left( \frac{T_R}{10^4 \text{ GeV}} \right)$$

- The lifetime for decays $\tilde{\chi} \rightarrow \gamma \tilde{a}$ are $\sim 0.01 \text{ sec}$, before BBN! The axino less constrained than gravitino!
- The NLSP decays, $\tilde{\chi} \rightarrow \gamma \tilde{a}$, produce axinos non-thermally

$$\Omega_\tilde{a} h_0^2 = \Omega_\tilde{a}^{(th)} h_0^2 + \frac{m_\tilde{a}}{m_{NLSP}} \Omega_{NLSP} h_0^2$$
Falls within the WMAP range, for temperatures $T_R \leq 10^8$ GeV
Theoretical predictions for Dark Matter are sensitive to the cosmological parameters controlling the evolution of the Universe at early eras. In alternative cosmological scenarios DM relic densities are modified in a drastic way!

- Low reheat temperature $T_R$.
- Late entropy production that dilutes relic densities.
- Non-thermal DM production, via decays of heavy particles, that increases relic densities.
- Modified Friedmann equations:
  - Scalar tensor gravities, Extra Dimensions - Braneworld models, Strings, ...
- Modified expansion rate, by adding more energy to Universe carried by Quintessence, String dilaton, ...
- ...

...
Low reheat temperature $T_R$

Low reheat temperature can resolve gravitino crisis. Realized in particular SUSY models with $T_R \sim O(\text{TeV})$.

- $T_R > 1 \text{ MeV}$ for BBN
- $T_R > 100 \text{ GeV}$ for EW phase transition
- $T_R > 10^9 \text{ GeV}$ for Leptogenesis
- $T_R < 10^7 \text{ GeV}$ for Gravitino production
- $T_R > 500 \text{ GeV}$ for SUSY WIMP DM

$T_R \sim O(\text{TeV})$ disfavoured by Leptogenesis

( BAU through EW phase transition is an alternative but requires large CP-violating phases and EDM put stringent constraints ! )
Late entropy production

The decays of out of equilibrium long-lived massive particles $X$ increase entropy

$$\frac{dS}{dt} = \frac{\Gamma_X \rho_X a^3}{T}$$

Boltzmann eq. for a particle species is modified ($Y \equiv n/s$)

$$\frac{dY}{dt} = -s \langle v \sigma_A \rangle (Y^2 - Y_{eq}^2) - \frac{\Gamma_X \rho_X a^3}{sT}$$

$Y$ drops faster as time increases, WIMP relic densities are diluted!

$$\Omega_{actual} = \frac{S(T_F)}{S(T_0)} \Omega_0 < \Omega_0$$

Valid if entropy is produced later than freeze-out $T_F >> T_s$ (Watch if $X$ decays to WIMPs since their relic density is increased!)

Late entropy has been implemented for the gravitino DM to relax the upper bounds put on $T_R$ by Nucleosynthesis
Scalar - Tensor Gravity, (Brans-Dicke)

The scalar-tensor action in Jordan frame

\[ S = \frac{1}{16\pi} \int d^4x \sqrt{-\tilde{g}} \left[ \tilde{\phi}^2 \tilde{R} + 4 \omega(\tilde{\phi}) g^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - 4 \tilde{V}(\tilde{\phi}) \right] \]

Passing to Einstein frame \[ \Rightarrow \]

\[ 3H^2 = 8\pi G_N \left[ \rho + \frac{1}{2 G_N} \dot{\phi}^2 + V(\phi) \right] \]

Depending on the potential \( V(\phi) \) the neutralino relic density can be increased by a factor \( 10^3 \)!

( Catena, Fornengo, Masiero, Pietroni and Rosati 2004 )
Brane world, Randall-Sundrum

$$3H^2 = 8\pi G_N \left[ \rho + \frac{\rho^2}{96 \pi G_N M_5^6} \right]$$, \quad M_5 = \text{Planck scale in } 5-D$$
\[ H^2 = H^2_{st} \left( 1 + \frac{\rho}{\rho_t} \right) , \quad \rho_t \equiv 96 \pi G_N M_5^6 \]

Transition temperature: \( \rho_t \equiv \frac{\pi^2}{30} g_{\text{eff}} T_t^4 \)

- \( T >> T_t \implies \text{Brane World Cosmology} \quad H \sim \rho \)
- \( T << T_t \implies \text{Standard Cosmology} \quad H \sim \sqrt{\rho} \)

Standard Cosmology is recovered for low temperatures!
For successful Nucleosynthesis, \( T_t > 1 \text{ MeV}, \ (M_5 > 10 \text{ TeV}) \)

The non-standard expansion affects processes before BBN like inflation and WIMP production!
Inflation:

\[ \varepsilon \sim \left( \frac{V'}{V} \right)^2 \frac{\rho_t (\rho_t + V)}{(\rho_t + V)^2}, \quad \eta \sim \frac{V''}{V} \frac{\rho_t}{(\rho_t + V)} \]

Enhance slow-roll and e-foldings in any inflation model if \( V > \rho_t \)

WIMP production:

\[ \frac{\Omega_{\text{brane}}}{\Omega_{\text{stand}}} \sim \left( \frac{T_F}{T_t} \right)^2, \quad T_F >> T_t \]

The neutralino relic density is enhanced (Nikei, Okada, Sato 2006)
Modified expansion rate

If Universe is not radiation dominated at decoupling and extra component $\rho_\phi$ contributes to matter - energy density

$$3 H^2 = 8\pi G_N (\rho_r + \rho_\phi)$$

The decoupling occurs earlier (freeze-out temperature larger)

$$n \langle v \sigma_A \rangle = H$$

The number to entropy density ratio $Y \equiv n/s$,

$$\frac{dY}{dT} = \xi < v \sigma_A > (Y^2 - Y_{eq}^2) \left[ \frac{45 G_N}{\pi} g_{eff} \right]^{-1/2} h$$

$$\xi \equiv \left( 1 + \frac{\rho_\phi}{\rho_r} \right)^{-1/2} < 1 \implies Y \text{ drops slower as } T \text{ decreases!}$$
\( \xi \) accounts for the modified expansion rate, depends on temperature and is model dependent!

\( \xi \) enhances relic densities since effectively \( \langle \nu \sigma_A \rangle \) is decreased by the factor \( \sim \xi \)!

The predicted density, \( \Omega \), and the conventionally obtained density, \( \Omega_0 \), are related by

\[ \Omega \sim \Omega_0 / \xi(T_F) \]

**Quintessence models**

The maximal relative enhancement to the relic density, compatible with BBN bounds

\[ \Delta \Omega \equiv (\Omega - \Omega_0) / \Omega_0 \sim 10^6 \]

Profumo and Ullio, 2003
Supercritical String Cosmology - SSC

- The situation of modified expansion rate is encountered in SSC models.

The bosonic part of the $4-D$ effective Lagrangian in the string frame is given by:

$$M^{-2} \sqrt{-G} \left[ e^{-2\phi} \left( -R_G + 4(\partial_\mu \phi)^2 - 2Q^2 \right) - V(\phi) \right] + \sqrt{-G} L_{matter}$$

Central charge deficit

$$\delta c = -3Q^2$$

Equations of motion:

$$\tilde{\beta}_\phi \equiv R_G + 2Q^2 + 4(\partial_\mu \phi)^2 - 4\Box \phi - \frac{e^{2\phi}}{2} V' = 0$$

$$\tilde{\beta}_{\mu\nu} \equiv -R^G_{\mu\nu} + 2\nabla_\mu \nabla_\nu \phi + G_{\mu\nu} \frac{e^{2\phi}}{4} (2V + V') - \frac{e^{2\phi}}{2M^2} \tilde{T}_{\mu\nu} = 0$$

- In the non-critical string the r.h.s are non-vanishing!

$$\tilde{\beta}_i = -\tilde{\mathcal{G}}_i$$
Supercritical String Cosmology - SSC

The rolling dilaton provides with a smoothly evolving Dark Energy and it couples to matter density $\rho$ (not to radiation!)

$$\frac{d\rho}{dt} + 3H(\rho + p) + \frac{<v\sigma_A>}{m}(\rho^2 - \rho^2_{eq}) = \dot{\phi}(\rho - 3p)$$

Boltzmann equation for $Y$ is modified

$$\frac{dY}{dT} = \xi <v\sigma_A>(Y^2 - Y^2_{eq}) \left[ \frac{45G_N}{\pi} g_{\text{eff}} \right]^{-1/2} h + \frac{\phi Y}{T}$$

$$\Omega \simeq R \times \Omega_0$$

$\Omega_0$ = Density obtained in standard cosmology.
$R$ = Cosmological factor.
For an LSP neutralino $R \sim \mathcal{O}(1/10)$ for dilaton solutions which are in agreement with an accelerating Universe, $q_0 = -0.61$, and smooth evolution of the DE in the regime $0 < z < 1.6$.

- LSP relic density is diluted by a factor of ten and regions of MSSM with too much DM are allowed!

- For a baryon $R \sim \mathcal{O}(1)$, leaving the predictions for conventional matter relic abundances unaffected!

The phenomenological consequences for LHC studied in:

The effect of the factor $R$ for the neutralino relic density, $\Omega_{\tilde{\chi}} h_0^2$ in the mSUGRA.

The cosmologically allowed co-annihilation region (left) moves to higher $m_0$ values (right), SSC region.
**Co-annihilation region:**

The dominant decay chain for a squark is

\[ \tilde{q}_L \rightarrow q \tilde{\chi}_2^0 \rightarrow q \tau \tilde{\tau} \rightarrow q\tau\tau \tilde{\chi}_1^0 \]

**Dominant signal:** \( 2\tau + \text{jet} + \not{E}_T \)

with low-energy \( \tau \)'s.

**SSC region:**

Three possible \( \tilde{\chi}_2^0 \) decays

\[ \tilde{\chi}_2^0 \rightarrow h_0 + \tilde{\chi}_1^0, \quad \tilde{\chi}_2^0 \rightarrow Z + \tilde{\chi}_1^0, \quad \tilde{\chi}_2^0 \rightarrow \tau \tilde{\tau} \]

**Possible signals**

\[
\begin{align*}
Z + \text{jet} + \not{E}_T \\
h_0 + \text{jet} + \not{E}_T \\
2\tau + \text{jet} + \not{E}_T
\end{align*}
\]

with high-energy \( \tau \)'s ( for \( m_{1/2} > 500 \text{ GeV} \)).
Signals of SSC in \( pp \) collisions at LHC. \( Z \) and/or Higgses, \( h_0 \), are expected in the final state.
Extensions of the Standard Model, and in particular SUSY, predict well-motivated DM candidates.

The predicted DM abundances are sensitive to the cosmological assumptions for the evolution of the Universe during and after inflation.

Astrophysical and accelerator searches unite to test particle physics and probe the Universe before the era of Nucleosynthesis.

We are entering into a very exciting era in Astroparticle Physics!