Semiclassical Corrections and their Cosmological Implications

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I Challenges of New Physics in Space

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Why semiclassical approach?

General Relativity (GR) is a complete theory of classical gravitational phenomena. However its perfectness is spoiled by singularities which are present in the most important solutions such as Schwarzschild and cosmological ones.

At very short distances / great cuvature, gravity should be described by another theory, more general than the GR.

Indeed, we expect a new theory being close to GR in the low energy / weak field limit.

The most probable origin of assumed deviation from GR are quantum effects.

The expected scale of quantum effects is associated to Planck units,

- length l_P, time t_P and mass M_P,

$$l_P = G^{1/2}\,\hbar^{1/2}\,c^{-3/2} \approx 1.4 \cdot 10^{-33}\,cm;$$

$$t_P = G^{1/2}\,\hbar^{1/2}\,c^{-5/2} \approx 0.7 \cdot 10^{-43}\,sec;$$

$$M_P = G^{-1/2}\,\hbar^{1/2}\,c^{1/2} \approx 0.2 \cdot 10^{-5} g \approx 10^{19}\,GeV\,.$$

We assume the existence of the fundamental units does indicate some fundamental physics at the M_P scale.

It may be Quantum Gravity, String theory, ...

We do not know what it really is.

So, what is certain?

Quantum Field Theory (QFT) and curved space definitely are.

Therefore the first step should be to consider **QFT of matter fields in curved space.**

Different from quantum theory of the metric, QFT of matter in curved space is well defined.

This theory is renormalizable and free of conceptual problems.

However, deriving the most relevant observables is, in general, an unsolved problem.

Formulating QFT in curved space.

The first step: choose the **action** for gravity and matter.

We assume the vacuum part of the action includes **Einstein-Hilbert term** with a cosmological constant **(CC)**.

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{g}\,\{\,R + 2\Lambda\,\}\,.$$

Why do we need other terms?

Standard criteria for the action of external metric:

• Locality. • Renormalizability. • Simplicity.

$$S_{vac} = S_{EH} + S_{HD}\,,$$

$$S_{HD} = \int d^4x\sqrt{-g}\,\{a_1C^2 + a_2E + a_3\Box R + a_4R^2\}\,,$$

where $\quad C^2 = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + (1/3)\,R^2\,, \qquad E = R^2_{\mu\nu\alpha\beta} - 4R^2_{\alpha\beta} + R^2\,.$

Quantum corrections to the vacuum action.

$$e^{i\Gamma[g_{\mu\nu}]} = \int \mathcal{D}\phi \, e^{iS[\phi;\, g_{\mu\nu}]} \,.$$

$\Gamma[g_{\mu\nu}]$ **is the Effective Action of gravity, which is a well-defined diffeomorphism invariant quantity**.

There are no gravitational anomalies in $4d$.
As a consequence, $\Gamma[g_{\mu\nu}]$ can not include odd powers of metric derivatives.

An odd-power behavior in the gravitational solutions, would be a direct indication to a kind of a “new physics”, e.g. scalar field.

It can not be a vacuum quantum effect on purely metric background!

Particular application to cosmology.

Quantum corrections to CC in the late universe, without scalar fields, may only start from H^2

$$\rho_\Lambda(H) = \rho_\Lambda(H_0) + \nu M^2 \cdot \left(H^2 - H_0^2\right) , \qquad (*)$$

where H_0 is the present-day Hubble parameter.

Quantum physics below the GUT scale is irrelevant in the case of ρ_Λ .

Cosmological models based on eq. (*) were developed in
I.Sh., J. Solà, C. España-Bonet, P. Ruiz-Lapuente, Ph.L. B574 (2003) 149; JCAP 02 (2004);
I.Sh., J.Solà, H.Stefancic, JCAP 0501 (2005).
J.Fabris, I.Sh., J.Sola, JCAP (2007).
Partially based on the ideas of:
I.Sh., J.Solà, JHEP 02 (2002);
A.Babic, B.Guberina, R.Horvat, H.Stefancic, PRD 65 (2002).

Conformal anomaly.

Consider massless conformal fields.

The theory includes $g_{\mu\nu}$ and matter fields Φ.

k_Φ is the conformal weight of the field.

The local conformal Noether identity is

$$\left[-2\,g_{\mu\nu}\,\frac{\delta}{\delta g_{\mu\nu}} + k_\Phi\,\Phi\,\frac{\delta}{\delta\Phi}\right]\,S(g_{\mu\nu},\,\Phi)\,=\,0$$

It produces $T^\mu_\mu = 0$ for the vacuum (on shell for matter fields)

$$-\,\frac{2}{\sqrt{g}}\,g_{\mu\nu}\,\frac{\delta S_{vac}(g_{\mu\nu})}{\delta g_{\mu\nu}}\,=\,T^\mu_\mu\,=\,0\,.$$

At the quantum level $S_{vac}(g_{\mu\nu})$ is replaced by the effective action, $\Gamma_{vac}(g_{\mu\nu})$.

For free field only 1-loop order is relevant.

$$\Gamma_{div} \,=\, \frac{1}{\varepsilon}\,\int d^4x\sqrt{g}\,\left\{\beta_1 C^2 + \beta_2 E + \beta_3 \Box R\right\}\,.$$

For the **global** conformal symmetry the renormalization group tells us

$$< T_\mu^\mu >= \left\{\beta_1 C^2 + \beta_2 E + a' \Box R\right\}\,,$$

where $a' = \beta_3$. In the local case a' is **ambiguous**.

Duff, Class.Quant.Grav. (1994);
Asorey, Gorbar & I.Sh., CQG **21** (2003).

Consider the theory including massless fields,

N_0 scalars, $N_{1/2}$ spinors, N_1 vectors.

Conformal Anomaly

$$T =< T^{\mu}_{\mu} >= -(wC^2 + bE + c\,\Box R + \beta F^2_{\mu\nu})\,,$$

where

$$\begin{pmatrix} w \\ -b \\ c \end{pmatrix} = \frac{1}{360(4\pi)^2} \begin{pmatrix} 3N_0 + 18N_{1/2} + 36N_1 \\ N_0 + 11N_{1/2} + 62N_1 \\ 2N_0 + 12N_{1/2} - 36N_1 \end{pmatrix}$$

Anomaly-induced action of vacuum

One can use conformal anomaly to construct the equation for the 1-loop EA

$$\frac{2}{\sqrt{-g}}\, g_{\mu\nu} \frac{\delta\,\bar{\Gamma}_{ind}}{\delta g_{\mu\nu}} = \frac{1}{(4\pi)^2}\,\left(wC^2 + bE + c\Box R \right)\,.$$

The solution has the form (*Riegert; Fradkin & Tseytlin, PLB-1984*).

$$\Gamma_{ind} = S_c[g_{\mu\nu}] - \frac{3c+2b}{36(4\pi)^2}\,\int d^4x\sqrt{-g}\,R^2(x)$$

$$+\frac{w}{4}\int_x\int_y C_x^2 G(x,y)(E-\frac{2}{3}\Box R)_y + \frac{b}{8}\int_x\int_y (E-\frac{2}{3}\Box R)_x G(x,y)(E-\frac{2}{3}\Box R)_y\,,$$

where $\int\limits_x = \int d^4x\sqrt{-g(x)}$ and

$$\Delta_4^x\, G(x,y) = \delta(x,y)\,, \qquad \Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}\,R\Box + (\nabla_\mu R)\,\nabla^\mu\,.$$

The most useful local form

$$\Gamma_{ind} \;=\; S_c[g_{\mu\nu}] \;-\; \frac{3c+2b}{36(4\pi)^2}\int_x R^2(x)$$

$$+\int_x \Big\{ \frac{1}{2}\,(\varphi\Delta_4\varphi - \psi\Delta_4\psi) + \frac{w}{8\pi\sqrt{-b}}\,(\psi-\varphi)\,C^2 + \frac{\sqrt{-b}}{8\pi}\,\varphi\,(E - \frac{2}{3}\Box R)\Big\}\,.$$

Here φ and ψ are auxiliary scalar fields which are called to parametrize the non-local sectors of the induced EA of vacuum.

The conformal invariant functional $S_c[g_{\mu\nu}]$ is an undefined component of the EA. – the "integration constant" of the equation for Γ_{ind}.

A.Jacksenaev, I.Sh., Phys.Lett.B (1994).

Similar expression introduced by Mazur & Mottola, PRD (1998).

Anomaly is known to be, in part, responsible for the Hawking radiation

S.M. Christensen, S.A. Fulling, PRD (1977).

The **vacuum states of the black hole** (Boulware, Unruh & Hartle-Hawking)

can be classified through the choice of initial conditions for the two auxiliary fields

R. Balbinot, A. Fabbri, I. Sh., PRL+NPB (1999).

Let us stress:

This task can not be accomplished by using only one field φ.

Another application of the anomaly-induced action is the Starobinsky Model (1980).

Modified version

J.Fabris, A.Pelinson, I.Sh., NPB (1999);
I.Sh. IJMPhD (2002);
I.Sh. & J. Sola, PLB (2002);
A.Pelinson, I.Sh., Takakura, NPB (2003).

Cosmological Model based on the action

$$S_{total} \,=\, -\,\frac{M_P^2}{16\pi}\,\int d^4x\sqrt{-g}\,(R+2\Lambda) + S_{matter} + S_{vac} + \bar{\Gamma}_{ind}\,.$$

Equation of motion for $a(t),\ k=0,$

$$\frac{\ddddot{a}}{a}+\frac{3\dot{a}\dddot{a}}{a^2}+\frac{\ddot{a}^2}{a^2}-\left(5+\frac{4b}{c}\right)\frac{\ddot{a}\dot{a}^2}{a^3}$$

$$-\frac{M_P^2}{8\pi c}\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}-\frac{2\Lambda}{3}\right)=0\,,$$

Has particular solutions (Starobinsky, PhLB-1980)

$$a(t)=a_0\cdot\exp(Ht)\,,$$ **where**

$$H^2=-\frac{M_P^2}{32\pi b}\left(1\pm\sqrt{1+\frac{64\pi b}{3}\,\frac{\Lambda}{M_P^2}}\right)\,.$$

For $0<\Lambda\ll M_P^2$ there are two solutions:

$$H\approx\sqrt{\Lambda/3}\,;\qquad (IR)$$

$$H\approx\sqrt{-\frac{M_P^2}{16\pi b}-\frac{\Lambda}{3}}\approx\frac{M_P}{\sqrt{-16\pi b}}\,.\qquad (UV)$$

The criterion for a stable (UV) inflation

$$c > 0 \quad \Longleftrightarrow \quad N_1 \, < \, \frac{1}{3}\, N_{1/2} \, + \, \frac{1}{18}\, N_0 \, ,$$

in agreement with Starobinsky (1980).

Suppose at **UV** $(H \gg M_F)$ there is **SUSY**, e.g. **MSSM** with a particle content

$$N_1 = 12 \, , \qquad N_{1/2} = 32 \, , \qquad N_0 = 104 \, .$$

This provides **stable inflation**, because $c > 0$

$$N_1 \, < \, \frac{1}{3}\, N_{1/2} \, + \, \frac{1}{18}\, N_0 \, .$$

Similar for any realistic SUSY model. Inflation **independent on initial data**.

Fine!

But why should inflation end?

Already for MSM $(N_{1,\,1/2,\,0} = 12, 24, 4)$ **inflation is unstable**, $c < 0$.

Natural interpretation:

All **sparticles** are heavy $\Rightarrow$ **decouple**, when $H \ll M_{sparticles}$.

According to calculations (E. Gorbar & I.Sh., 2003) the transition $c > 0 \Longrightarrow c < 0$ is smooth, giving a hope for a graceful exit.

Finally, it has been shown that the stable version of Starobinsky inflation slows down due to the massed of the quantum fields.

Simple test of the model.

Late Universe, $k = 0$, $H_0 = \sqrt{\Lambda/3}$. Only photon is active

$$N_0 = 0\,, \qquad N_{1/2} = 0\,, \qquad N_1 = 1\,.$$

$c < 0 \quad \Longrightarrow \quad$ **today inflation is unstable.**

Stability for the small $H = H_0$ case:

$$H \to H_0 + const \cdot e^{\lambda t}$$

The solutions for λ are

$$\lambda_1 = -4H_0\,, \qquad \lambda_{2/3} = -\frac{3}{2}\,H_0 \pm \frac{M_P}{\sqrt{8\pi|c|}}\,i\,.$$

$\Lambda > 0$ **protects us from the higher derivative "quantum" terms!**

Most recent result of our group

Quantum correction to photon sector

Classical action:

$$S_{em} = -\frac{1}{4} \int d^4x \sqrt{g} \; F_{\mu\nu} \, F^{\mu\nu}$$

possesses local conformal invariance,

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} \, e^{2\sigma(x)} \,, \quad A_\mu \to A'_\mu = A_\mu \,,$$

Why it is important that the local conformal symmetry is violated at quantum level?

1) **Broken conformal symmetry changes the equation of state (EOS) of radiation. Modified EOS for the radiation may lead to interesting new cosmological models.**
Modified EOS changes the expansion of the Universe, entropy production, affects the red-shift dependence of CMB, etc.

2) **The broken conformal symmetry may affect the rate of photons creation in the reheating period, leading to potentially observable consequences.**

3) **The conformal symmetry violation can be important for creation of initial seeds of magnetic field of the galaxies.**
Turner & Widrow, PRD (1988).
Dolgov, ZhETP (1981); PRD (1993).
Calzetta, Kandus & Mazzitelli, PRD (1998).
Bassett et al, PRD (2001).
Lambiase & Prasanna, PRD (2004).
Bamba & Sasaki, JCAP (2007).

4) **Quantum corrections can move the pole in the photon propagator and make the speed different from the one in the classical case.**

Similar effect occurs in curved space,

Khriplovich, PLB (1995) 251
Lafrance & Myers, PRD (1995).
Shore, NPB (1996,2001); Cont.Phys.(2003).
Dolgov & Novikov, PLB (1998).
Rong-Gen Cai, NPB (1998).
Mohanty & Prasanna, NPB (1998).
Scharnhorst, Ann.Phys. (1998).
Cougo-Pinto, Farina et al, PLB (1999).
Novello et al, PRD (2000).
Ferrer, de la Incera & Romeo, PLB (2001).
Bruneton, PRD (2007).
Balakin, Bochkarev & Lemos, PRD (2008).

The two main questions are as follows:

1) **What is the mechanism of violation of the local conformal symmetry at the quantum level?**

2) **To which extent the finite quantum corrections and, in particular, violation of local conformal symmetry, are universal?**

In other words, do we have an ambiguity in the quantum terms?

As a first step we present the electromagnetic part of the anomaly-induced eff. action

$$\Gamma = S_c[g_{\mu\nu}, A_\lambda] \; + \; \int d^4x\sqrt{g}\,\left\{ \frac{\tilde{\beta}}{4\pi\sqrt{-b}}(\psi - \varphi)\,F_{\mu\nu}^2 \right.$$

$$\left. + \frac{\sqrt{-b}}{8\pi}\,\varphi\,\left(E - \frac{2}{3}\Box R\right) + \frac{1}{2}\,(\varphi\Delta_4\varphi - \psi\Delta_4\psi) \right\}.$$

The presence of the A_μ-independent term $\varphi\big[E - (2/3)\Box R\big]$ is relevant, because only this term violates the conformal symmetry in the whole expression,

$$\sqrt{g}(E - \frac{2}{3}\Box R) = \sqrt{\bar{g}}(\bar{E} - \frac{2}{3}\bar{\Box}\bar{R} + 4\bar{\Delta}_4\sigma)\,.$$

What about quantum effects of massive fields?

The one-loop effective action (EA) in the case of $g_{\mu\nu}$ and A_μ background can be defined via the path integral

$$e^{i\Gamma^{(1)}[g_{\mu\nu},\, A_\mu]} = \int D\psi D\bar{\psi}\, e^{iS_{QED}}\,,$$

or

$$\bar{\Gamma}^{(1)} = -\frac{1}{2}\,\text{Log Det}\,\hat{H}\,,$$

where

$$\hat{H} = i\big(\gamma^\mu \nabla_\mu - iM - ie\gamma^\mu A_\mu\big)$$

We use heat-kernel method and the Schwinger-DeWitt technique in curved space QED.

Reducing the problem to the derivation of Log Det $\hat{\mathcal{O}}$,

$$\hat{\mathcal{O}} \,=\, \widehat{\Box} + 2\hat{h}^{\mu}\nabla_{\mu} + \hat{\Pi}\,.$$

Multiply $\hat{H}$ by an appropriate conjugate $\hat{H}^{*}$,

$$\hat{\mathcal{O}} \,=\, \hat{H} \cdot \hat{H}^{*}$$

and use the relation

$$\text{Log Det } \hat{H} \,=\, \text{Log Det } \hat{\mathcal{O}} \,-\, \text{Log Det } \hat{H}^{*}\,.$$

The simplest choice,

$$\hat{H}_1^* = -\,i\left(\gamma^\mu \nabla_\mu + iM - ie\gamma^\mu A_\mu\right).$$

According to G. De Berredo-Peixoto, M.Ph.L.A 16 (2001),

$$\text{Log Det } \hat{H} \;=\; \text{Log Det } \hat{H}_1^*\,,$$

then $$\text{Log Det } \hat{H} \;=\; \frac{1}{2}\,\text{Log Det } \left(\hat{H}\hat{H}_1^*\right).$$

Our result for the one-loop Euclidean effective action is

$$\bar{\Gamma}^{(1)}_{\sim F^2} = \frac{e^2}{2(4\pi)^2}\int d^4x\sqrt{g}\;F_{\mu\nu}\;\left[\frac{2}{3\epsilon} + k_1^{FF}\right]\;F^{\mu\nu}\,,$$

where $$k_1^{FF} \;=\; k_1^{FF}(a) \;=\; Y\left(2 - \frac{8}{3a^2}\right) - \frac{2}{9}\,.$$

$$Y = 1 - \frac{1}{a}\ln\left(\frac{2+a}{2-a}\right)\,, \qquad a^2 = \frac{4\Box}{\Box - 4m^2}\,.$$

This expression represents a complete one-loop contribution.
Goncalves, de B.-Peixoto & I.Sh., hep-th/0904.4171

Is this result universal and unambiguous?

An aternative choice:

$$\hat{H}_2^* = -\,i\left(\gamma^\mu \nabla_\mu + iM\right).$$

This operator does not depend on A_μ, hence

$$\text{Log Det}\,\hat{H}\Big|_{FF} = \text{Log Det}\,\left(\hat{H}\hat{H}_2^*\right)\Big|_{FF}.$$

If the relation

$$\text{Det}\,(\hat{A}\cdot\hat{B}) = \text{Det}\,\hat{A}\cdot\,\text{Det}\,\hat{B}$$

holds in this case, we are going to meet the equal expressions,

$$\frac{1}{2}\,\text{Log Det}\,\left(\hat{H}\hat{H}_1^*\right)\Big|_{FF} = \text{Log Det}\,\left(\hat{H}\hat{H}_2^*\right)\Big|_{FF},$$

In reality, we meet

$$\frac{1}{2}\,\text{Log Det}\left(\hat{H}\hat{H}_1^*\right)\Big|_{FF} = \text{Log Det}\left(\hat{H}\hat{H}_2^*\right)\Big|_{FF}$$

for divergencies, but

$$\frac{1}{2}\,\text{Log Det}\left(\hat{H}\hat{H}_1^*\right)\Big|_{FF} \neq \text{Log Det}\left(\hat{H}\hat{H}_2^*\right)\Big|_{FF}$$

for nonlocal finite parts.

This is nothing else, but the Multiplicative Anomaly

Kontsevich & Vishik, hep-th/9406140/9404046.
Elizalde, Vanzo & Zerbini, Com.Math.Phys. 194 (1998);
Cognola, Elizalde and Zerbini, Com.Math.Phys. 237 (2003); NPB 532 (1998); ...

Conclusions.

1) The semiclassical approach to Quantum Gravity is interesting and worthwhile to study.

2) In the framework of this method we can learn a lot of relevant and interesting things about cosmology and related areas, such as black hole physics, and maybe even astronomy.

3) We have confirmed the existence of the Multiplicative Anomaly in QED, both in flat and curved space-time cases.

Different from other calclations, we are safe from the renormalization ambiguity problem.

End of the story & Great Thanks to Ana!

$$\bar{\Gamma}^{(1)}\Big|_{AA} = -\frac{e^2}{2(4\pi)^2}\int d^4x\sqrt{g}\,\Big\{F_{\mu\nu}\left[\frac{2}{3\,\epsilon}+k_2^{FF}(a)\right]F^{\mu\nu}$$

$$+2\nabla_\mu A^\mu\Big[Y\big(\frac{8}{3a^2}-2\big)+\frac{2}{9}\Big]\nabla_\nu A^\nu$$

$$+\nabla_\mu A^\nu\Big[\frac{16\,Y}{3a^2}+\frac{4}{9}\Big]\nabla_\nu A^\mu+\mathcal{O}(R\cdot A\cdot A)\Big\}\,,$$

where

$$k_2^{FF}(a) = Y\Big(1+\frac{4}{3a^2}\Big)+\frac{1}{9}\,,$$

and $\mathcal{O}(R\cdot A\cdot A)$ are terms proportional to scalar curvature.
Here ϵ is the parameter of dimensional regularization

$$\frac{1}{\epsilon} = \frac{2}{4-d}+\ln\Big(\frac{4\pi\mu^2}{m^2}\Big)-\gamma\,,\qquad \gamma = 0.5772\ldots\,.$$