Workshop “Challenges of New Physics in Space”

Campos do Jordão, April 2009

Teleparallelism and Quantum Mechanics

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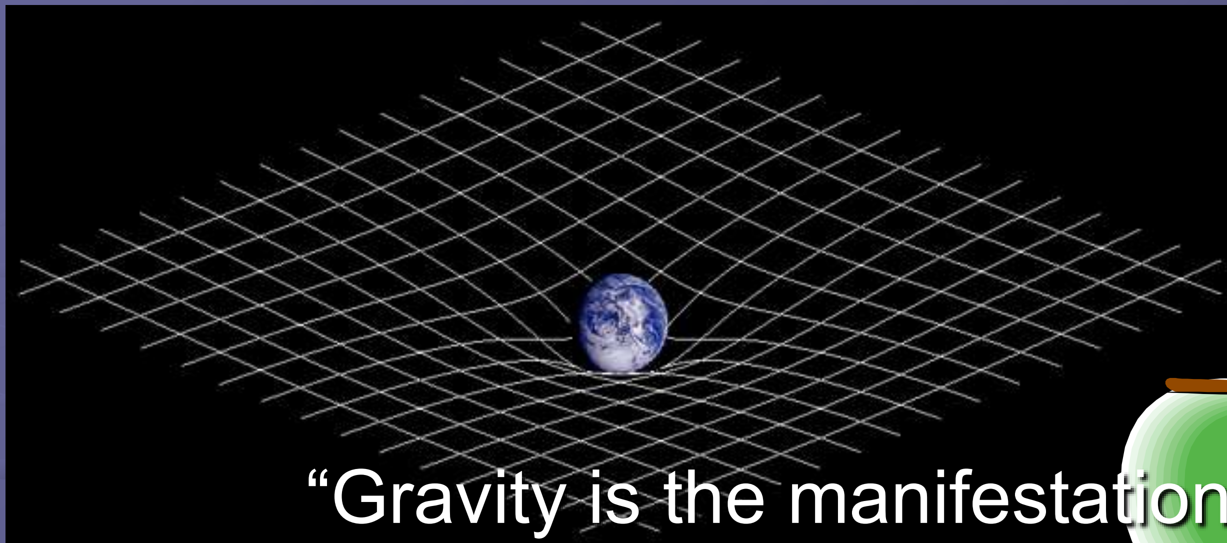


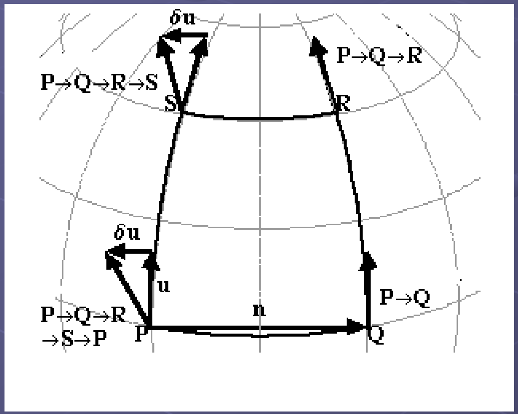
“Gravity is the manifestation of space-time curvature”

Curvature:

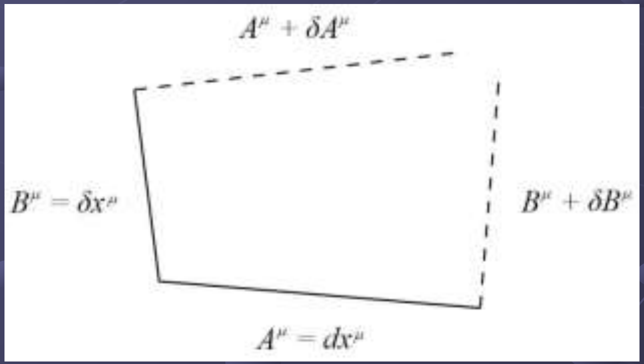
$$R^{\rho}{}_{\theta\mu\nu} = \partial_{\mu}\Gamma^{\rho}{}_{\theta\nu} - \partial_{\nu}\Gamma^{\rho}{}_{\theta\mu} + \Gamma^{\rho}{}_{\sigma\mu}\Gamma^{\sigma}{}_{\theta\nu} - \Gamma^{\rho}{}_{\sigma\nu}\Gamma^{\sigma}{}_{\theta\mu}$$

Torsion: $T^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\nu\mu} - \Gamma^{\rho}{}_{\mu\nu} = 0$ (GR)

Curvature



Torsion



General Relativity: $\Gamma^{\rho}{}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho}(\partial_{\mu}g_{\rho\nu} - \partial_{\nu}g_{\rho\mu} - \partial_{\rho}g_{\mu\nu})$

Einstein-Cartan Theory

Curvature:

$$R^{\rho}{}_{\theta\mu\nu} = \partial_{\mu}\Gamma^{\rho}{}_{\theta\nu} - \partial_{\nu}\Gamma^{\rho}{}_{\theta\mu} + \Gamma^{\rho}{}_{\sigma\mu}\Gamma^{\sigma}{}_{\theta\nu} - \Gamma^{\rho}{}_{\sigma\nu}\Gamma^{\sigma}{}_{\theta\mu}$$

Source: Energy and Momentum

Torsion: $$T^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\nu\mu} - \Gamma^{\rho}{}_{\mu\nu}$$ New Physics!

Source: Spin

Teleparallelism

or

Teleparallel Equivalent of General Relativity

Curvature:

$$R^{\rho}{}_{\theta\mu\nu} = \partial_{\mu}\Gamma^{\rho}{}_{\theta\nu} - \partial_{\nu}\Gamma^{\rho}{}_{\theta\mu} + \Gamma^{\rho}{}_{\sigma\mu}\Gamma^{\sigma}{}_{\theta\nu} - \Gamma^{\rho}{}_{\sigma\nu}\Gamma^{\sigma}{}_{\theta\mu} = 0$$

(TEGR)

Torsion: $T^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\nu\mu} - \Gamma^{\rho}{}_{\mu\nu}$

Torsion in TEGR is associated with the same degrees of freedom as curvature in GR

Alternative and Equivalent Theory

Foundations of Teleparallelism

- *Tetrad field*

$$g = g_{\mu\nu} dx^\mu dx^\nu \xrightarrow{h_a = h_a{}^\mu \partial_\mu} \eta = \eta_{ab} dx^a dx^b$$

$$\eta_{ab} = g_{\mu\nu} h_a{}^\mu h_b{}^\nu$$

Physical meaning: The tetrad field establishes a class of observers

Foundations of Teleparallelism

- *Gauge theory for the translation group T(4)*

  Energy and momentum, the sources of gravity, are conserved if the physical system is invariant under translations in space-time (Noether's Theorem)

- *Fundamental field:* gauge potential $B_\mu = B^a{}_\mu P_a$

  ($P_a = \partial_a$, the generators of infinitesimal translations)

- Gauge potential as the non-trivial part of the tetrad field:

Inertial effects

$$h^a{}_\mu = \partial_\mu x^a + B^a{}_\mu$$

Gravity

Foundations of Teleparallelism

- *Torsion tensor:*

$$T^a{}_{\mu\nu} = \partial_\mu h^a{}_\nu - \partial_\nu h^a{}_\mu$$
$$= \partial_\mu B^a{}_\nu - \partial_\nu B^a{}_\mu$$

Field strength of the theory

Weitzenböck connection: $\Gamma^\rho{}_{\mu\nu} = h_a{}^\rho \partial_\mu h^a{}_\nu$ $\qquad R^\rho{}_{\theta\mu\nu}=0$

- *Lagrangian density:*

$$L= h/2k^2 \,(T^\rho{}_{\mu\nu} T_\rho{}^{\mu\nu} + 1/4\ T^\rho{}_{\mu\nu} T^{\nu\mu}{}_\rho - 1/2 T_{\rho\mu}{}^\rho T^{\nu\mu}{}_\nu)$$
$$= -\tfrac{1}{2}\,k^{-2}(-g)^{1/2} R - \partial_\mu(2h\, k^{-2} T^{\nu\mu}{}_\nu)$$

$h = \det(h^a{}_\mu) = (-g)^{1/2}$ and $k = 8\pi G/c^4$.

Teleparallelism and Quantum Mechanics

Hamiltonian Formulation of TEGR

Starting point
for quantization

$$i\hbar \{\ \ \} \longrightarrow [\ \ ]$$

$$i\hbar\dot{C} = [C,H]$$

$$h^a{}_\mu \qquad \text{and} \qquad \Pi^{a\mu} = \delta L/\delta h'_{a\mu}$$

$$L = h_{a\mu}\Pi'^{a\mu} - H$$

- *Hamiltonian density:*

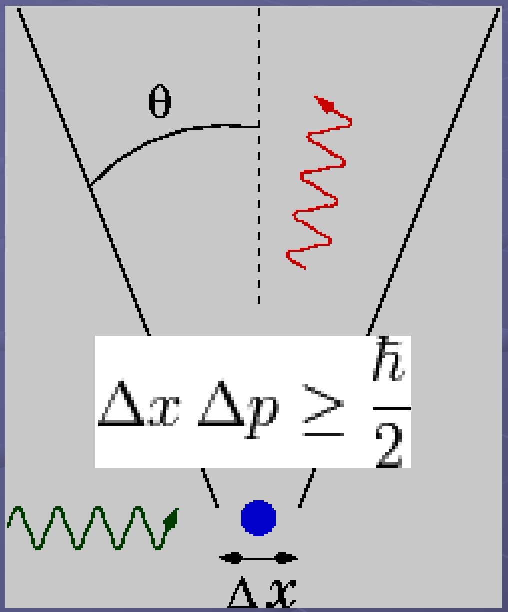
J. W. Maluf, J. F. R. Neto, 2001

Lagrange multipliers

$$H = h_{a0}C^a + \alpha_{ik}X^{ik} + \beta_k X^k \quad \approx 0$$

First class constraints → Non-polynomial dependency on the canonical variables

Teleparallelism and Quantum Mechanics



Uncertainty Principle
(*non-locality*)

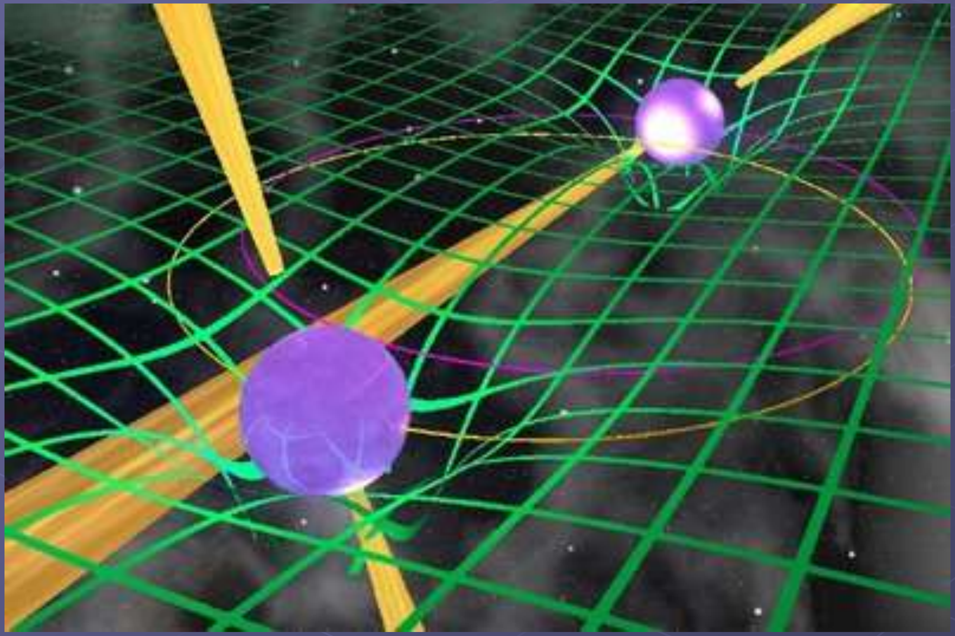
QM

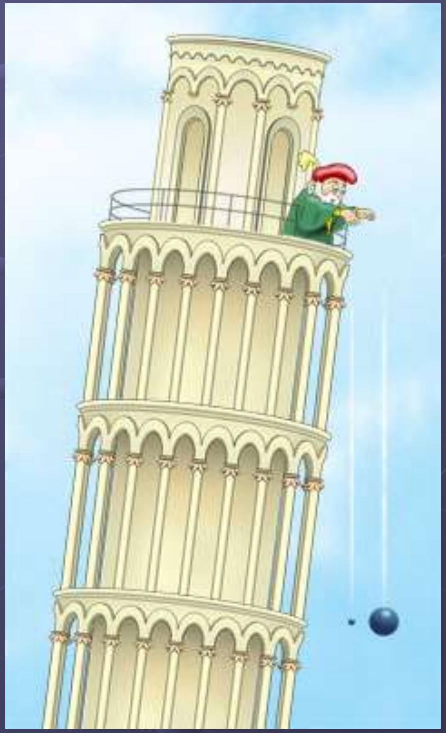
Equivalence Principle
(*local* equivalence of gravity and inertia)

GR

Teleparallelism and Quantum Mechanics

- Gravity and Universality





Weak Equivalence Principle: $m_g = m_i$

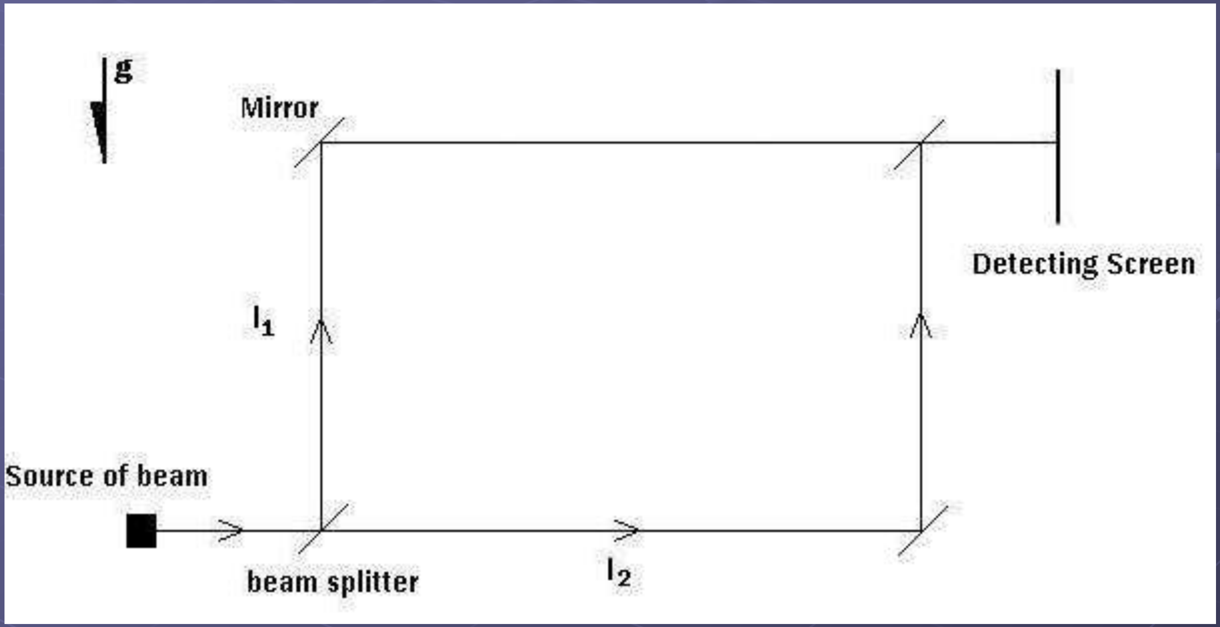
⇕

No mass-dependency in the equations

Teleparallelism and Quantum Mechanics

- COW experiment

R. Colella, A. W. Overhauser and S. A. Werner, 1974



$$\Delta\phi = \frac{m^2 g l_1 l_2 \lambda \sin(\theta)}{\hbar^2}$$

Mass-dependency

Teleparallelism – suitable description
– not dependent on EP

R. Aldrovandi, J. G. Pereira, K. H. Vu, 2005

Teleparallelism and Quantum Mechanics

- *Equation of motion – particle of mass "m" in a gravitational field*

$$du_a/ds = T^b{}_{ac} u_b u^c$$

- If $m_g \neq m_i$

$$P^\mu{}_\nu \, (\partial_\mu x^a + \alpha \, B^a{}_\mu) \, du_a/ds = \alpha T^a{}_{\nu\mu} u_a u^\mu$$

$(P^\mu{}_\nu = \delta^\mu{}_\nu - u^\mu u_\nu$ and $\alpha = m_g/m_i)$ Solvable for the gauge potential

- Correspondent relativistic equation:

$$du_\mu/ds - \omega^\lambda{}_{\mu\nu} u_\lambda u^\nu = (m_g - m_i)/m_g \, P^\nu{}_\mu \partial_\nu x^a \, du_a/ds$$

Deviation of geodesic motion

Comments and conclusions

- GR and TEGR are alternative and empirically equivalent geometrical formulations for describing Gravity, in which curvature, in one side, and torsion, in the other, plays the fundamental role.
- TEGR can be formulated as a gauge-like theory for T(4).
- The Hamiltonian formulation of Teleparallelism can be carried out, but technical and conceptual difficulties for the construction of the quantum theory remain here, as in GR.
- Conceptually, the TEGR may have some advantages over GR, since its interpretation does not depend no strongly on the Equivalence Principle and on the Universality of Gravity.

Comments and conclusions

- What is the role of torsion in the description of Gravity?

- Unnecessary geometrical entity? (GR)

- Necessary to describe the spin interaction with the gravitational field (relevant to small scale gravitational phenomena)? (EC)

- Associated with the same degrees of freedom of curvature in the description of Gravity? (TEGR)

- Other? ...

More References...

[1] R. Aldrovandi e J. G. Pereira, “An introduction to Teleparallel Gravity”.

[2] H. I. Arcos, V. C. Andrade, J. G. Pereira, “Torsion and Gravitation: A New View”, International Journal of Modern Physics D, Cingapura, v. 13, n. 5, p. 807-818, 2004.

[3] J. W. Maluf, J. F. da Rocha Neto, Phys. Rev. D64, 084014 (2001).

[4] J. W. Maluf, S. C. Ulhoa, F. F. Faria, “The Pound-Rebka experiment and torsion in the Schwarzschild spacetime” [arXiv:0903.2565v1]


THANK
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