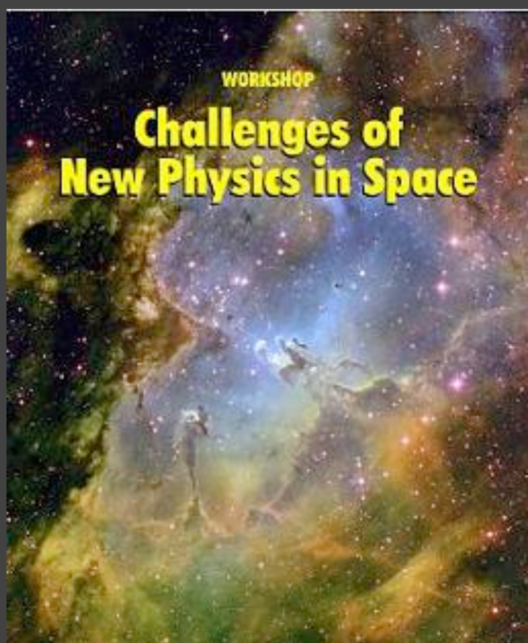
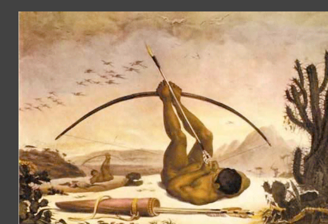
I Workshop Challenges of New Physics in Space

Viable Singularity-Free f(R) Gravity Without a Cosmological Constant.



Work in collaboration with Vinícius Miranda, Sérgio E. Jorás and Miguel Quartin



ARCOS
Astrofísica, Relatividade e COSmologia

Instituto de Física
Universidade Federal do Rio de Janeiro

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Ioav Waga

What is causing the cosmic acceleration?

Main Possibilities

A new exotic component with negative pressure (DE) or modified gravity?

New Component

$$G_{\mu\nu} = \kappa T^{(m)}_{\mu\nu} + T_{\mu\nu}(\varphi)$$

Modified Gravity

$$G_{\mu\nu} + L_{\mu\nu}(g_{\mu\nu}) = \kappa T^{(m)}_{\mu\nu}$$

f(R) Gravity

$$S_{JF} = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}f(R) + \mathcal{L}_m\left(g_{\mu\nu},\Psi_m\right)\right]$$

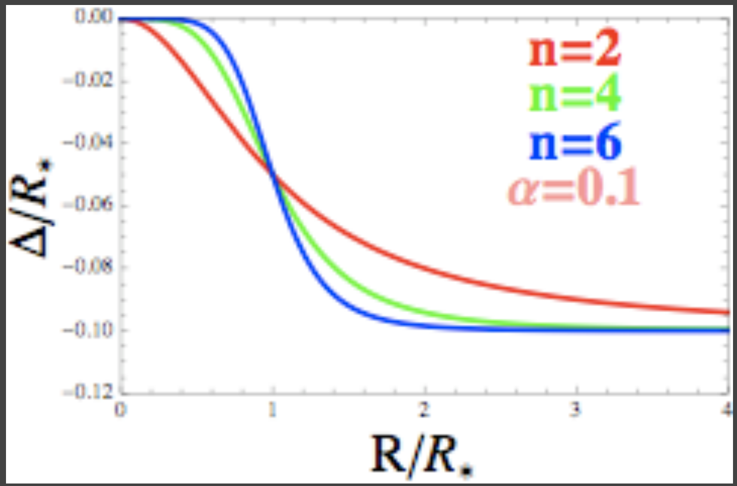
- $f(R) \rightarrow$ simplest modification to the E-H Lagrangian ; in general $f(R, R^{\alpha\beta}R_{\alpha\beta}, R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}, \ldots)$
- $f(R)$ is a special case of a scalar-tensor theory (Brans-Dicke with w=0).
- An accelerated expansion appears naturally in these models.
- Starobinsky (PLB 91,99,1980) showed that an accelerated expansion can be curvature driven if $f(r) = R + \alpha R^2$.
- Higher order terms, like the above, are generically predicted in high energy corrections to gravity.
- More recently the same idea was explored by Capozzielo&Cardone (IJMP D12, 1963, 2003) and Carrol et al. (PRD 043528, 2004) for late time acceleration. They considered $f(r) = R - \alpha R^{-n}$.
- This $f(R)$ Lagrangian doesn't present a regular MDE ($a \propto t^{1/2}$ and not $a \propto t^{2/3}$) [Amendola et al. , PRD 75, 083504, 2007]. $\Rightarrow$ Inverse power-law $f(R)$ are incompatible with structure formation.

This kind of problem doesn't appear in the f(R) suggested by Hu&Sawicki [PRD 76, 064004 (2007)] and Starobinsky [JETPLett, 86, 157, (2007)].

$$f(R) = R + \Delta(R)$$

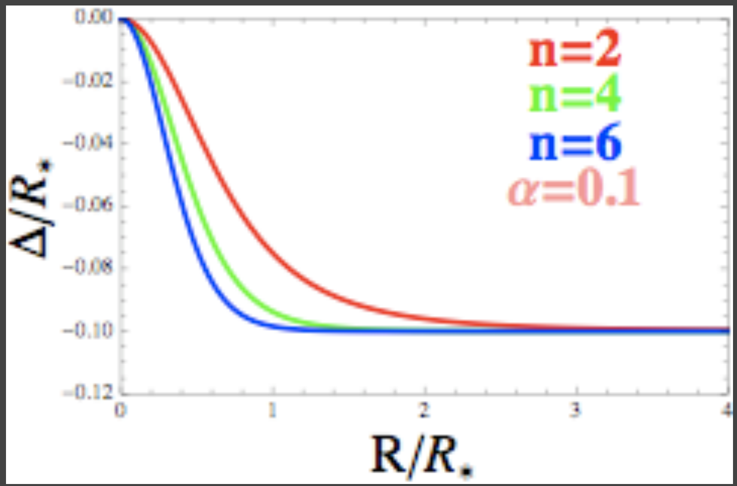
Hu & Sawicki

$$f(R) = R - \alpha R_* \left\{ 1 - \frac{1}{1+\left(\frac{R}{R_*}\right)^n} \right\}$$



Starobinsky

$$f(R) = R - \alpha R_* \left\{ 1 - \frac{1}{\left[1+\left(\frac{R}{R_*}\right)^2\right]^n} \right\}$$



$f(R) \simeq R - 2\Lambda$ for $R \gg \Lambda$

$f(0) = 0 \Rightarrow$ desapearing cosmological constant

- However both f(R) have singularity problems:

1. Frolov [PRL, 101, 061103 (2008)]
2. Kobayachi & Maeda [PRD 78, 064019 (2008)].

$$\mathcal{S}_{JF} = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}f(R) + \mathcal{L}_m\left(g_{\mu\nu},\Psi_m\right)\right]$$

By varying the action with respect to the metric, we obtain a fourth order equation for

$$f_R R_{\mu\nu} - \nabla_\mu\nabla_\nu f_R + \left(\Box f_R - \tfrac{1}{2}f\right)g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Taking the trace we get

$$\Box f_R = \frac{8\pi G}{3}T + \frac{1}{3}\left(2f - f_R R\right)$$

We now introduce a new auxiliary field Q and write the gravitational part of the action as

$$\mathcal{S}_{grav} = \int d^4x\frac{\sqrt{-g}}{16\pi G}\left[(R-Q)\,f_Q(Q) + f(Q)\right]$$

The equation of motion for Q is

$$f_{QQ}(Q)\,(Q-R) = 0$$

Therefore $Q = R$ as long as $f_{QQ}(Q) \neq 0$

We now introduce a new field

$$\chi := f_R = 1 + \Delta_R$$

And rewite the Lagrangian as a Brans-Dicke gravity theory with w=0

$$\mathcal{S}_{JF} = \int d^4x\sqrt{-g}\left[\frac{\chi R(\chi)}{16\pi G} - \chi^2 V_E(\chi)\right] + \mathcal{S}_m(g_{\mu\nu}, \Psi_m)$$

where

$$V_E\left(R(\chi)\right) = \frac{1}{16\pi G}\frac{R\Delta_R - \Delta}{(1+\Delta_R)^2} = \frac{1}{16\pi G}\frac{R(\chi)\chi - f(R(\chi))}{\chi^2}$$

The equation of motion for χ is $\Box\chi = \frac{\mathrm{d}V}{\mathrm{d}\chi} - \mathcal{F}$

Where $\mathcal{F} = -\frac{8\pi G}{3}T = \frac{8\pi G}{3}(\rho - 3p)$ and $\frac{\mathrm{d}V(R(\chi))}{\mathrm{d}\chi} := \frac{1}{3}\left(2f - \chi R\right)$

We can discuss now the mentioned f(R) singularity problems.

Frolov f(R) singularity [PRL, 101, 061103 (2008)]

$$f(R) = R - \alpha R_* \left\{ 1 - \frac{1}{[1+(\frac{R}{R_*})^2]^n} \right\}$$

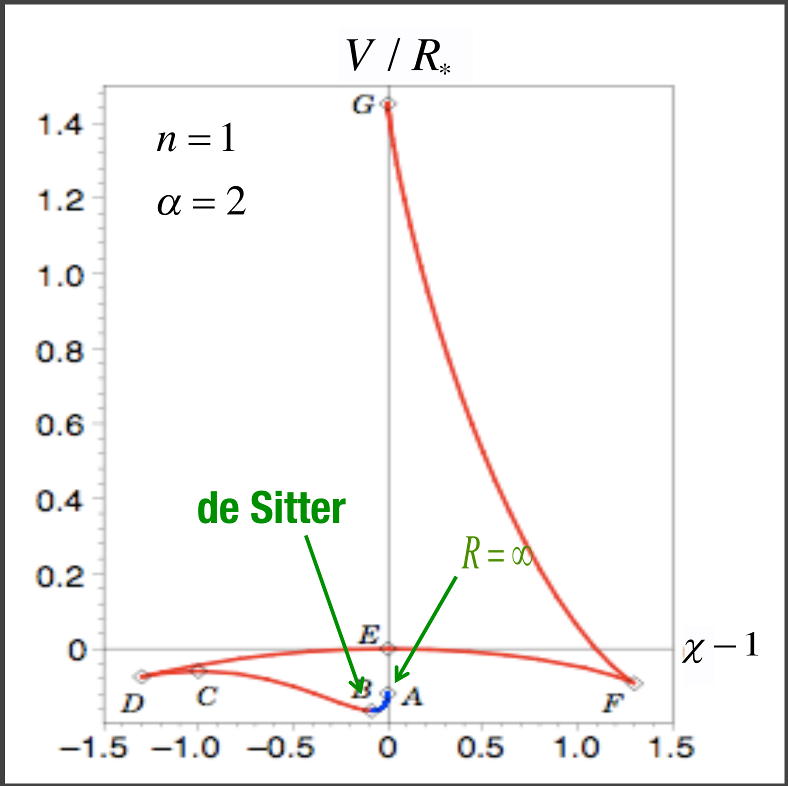
Starobinsky f(R)

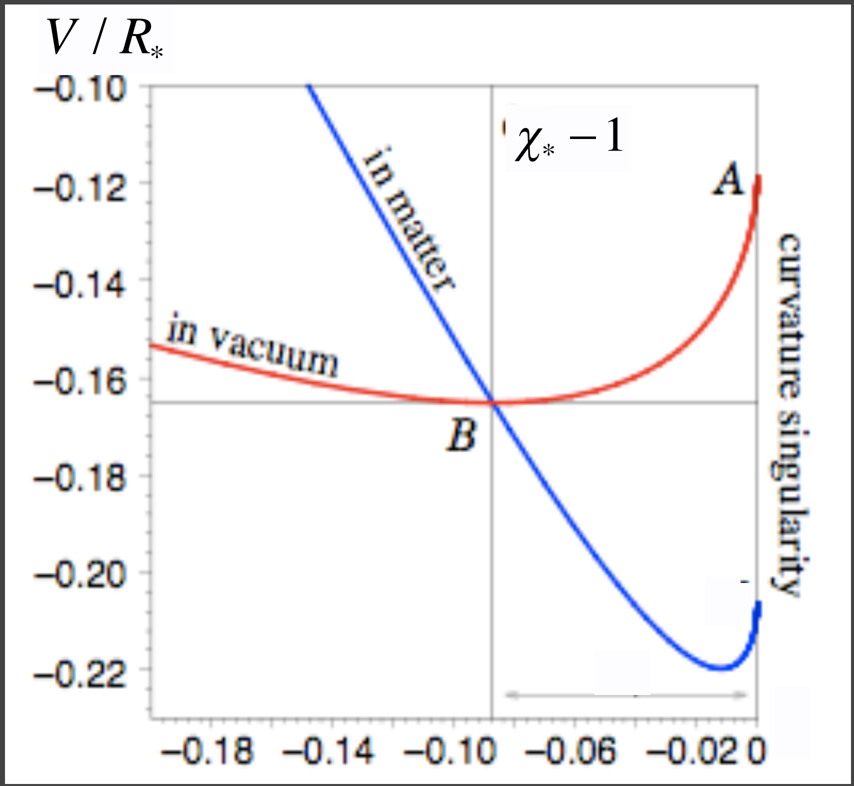
$$\frac{\mathrm{d}V(R(\chi))}{\mathrm{d}\chi} := \frac{1}{3}(2f - \chi R)$$

$$\Box\chi = \frac{\mathrm{d}V}{\mathrm{d}\chi} - \mathcal{F}$$

$$V_{eff}(\chi) = V(\chi) + \mathcal{F}(\chi_* - \chi)$$

$$\chi := f_R = 1 + \Delta_R$$





Kobayachi & Maeda singularity problem

[PRD 78, 064019 (2008)]

Spherically Symmetric Stars in f(R) Gravity

Basic Equations

Kobayachi & Maeda
PRD 78, 064019 (2008)

$$ds^2 = -N(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

Energy-momentum tensor

$$T_\mu^{\ \nu} = \mathrm{diag}\,(-\rho, p, p, p) \qquad \nabla_\nu T_\mu^{\ \nu} = 0 \implies p' + \frac{N'}{2N}(\rho + p) = 0$$

The field equations (00) and (11) give

$$\frac{\chi}{r^2}\left(-1 + B + rB'\right) = -8\pi G\rho - \chi^2 V - B\left[\chi'' + \left(\frac{2}{r} + \frac{B'}{2B}\right)\chi'\right]$$

$$\frac{\chi}{r^2}\left(-1 + B + rB\frac{N'}{N}\right) = 8\pi G p - \chi^2 V - B\left(\frac{2}{r} + \frac{N'}{2N}\right)\chi'$$

The equation of motion for χ is

$$B\left[\chi'' + \left(\frac{2}{r} + \frac{N'}{2N} + \frac{B'}{2B}\right)\chi'\right] = \frac{8\pi G}{3}(-\rho + 3p) + \frac{2\chi^3}{3}\frac{dV}{d\chi}$$

Boundary Conditions

$$N(r) = 1 + N_2 r^2 + \dots$$

$$B(r) = 1 + B_2 r^2 + \dots$$

$$\chi(r) = \chi_c \left(1 + \frac{C_2}{2} r^2 + \dots\right)$$

$$\rho(r) = \rho_c + \frac{\rho_2}{2} r^2 + \dots$$

$$p(r) = p_c + \frac{p_2}{2} r^2 + \dots$$

Here χ_c, ρ_c and p_c are the central values of the scalar field, energy density and pressure.

At the surface of the star $r = \mathfrak{R}$ we have $p(\mathfrak{R})=0$

To integrate the equations Kobayachi & Maeda considered constant density stars ($\rho=\rho_c$).

$$p' + \frac{N'}{2N}(\rho + p) = 0 \quad \Longrightarrow \quad N(r) = \left[\frac{\rho_c + p_c}{\rho_c + p(r)}\right]^2$$

They showed that, for the Starobinsky f(R), it was not possible to evolve the metric from inside the star up to large r and match de Sitter asymptoticaly.

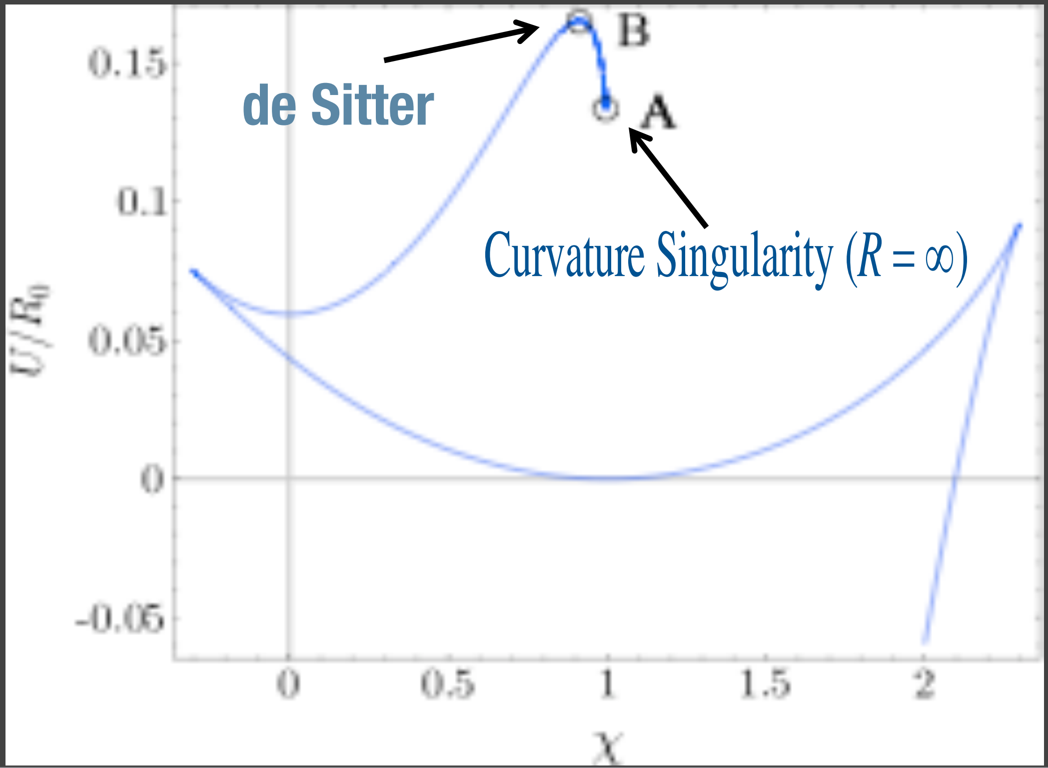
Classical mechanics picture

$(-,+,+,+) \rightarrow \Box \equiv -\frac{\partial^2}{\partial t^2} - 3H\frac{\partial}{\partial t}$

For simplicity assume Minkowski background such that $\Box \equiv \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$

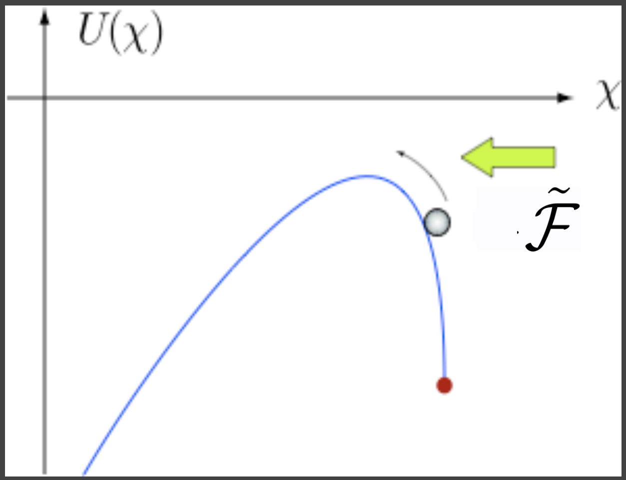
$$\Box\chi = \frac{dV}{d\chi} - \mathcal{F} \quad \Rightarrow \quad \frac{d^2\chi}{dr^2} + \frac{2}{r}\frac{d\chi}{dr} = \tilde{\mathcal{F}} + \mathcal{F}_U$$

$$\tilde{\mathcal{F}} = -\mathcal{F} = -\frac{8\pi G}{3}(\rho - 3p) \qquad \boxed{\mathcal{F}_U = -\frac{dU}{d\chi}} \qquad \boxed{U = -V}$$



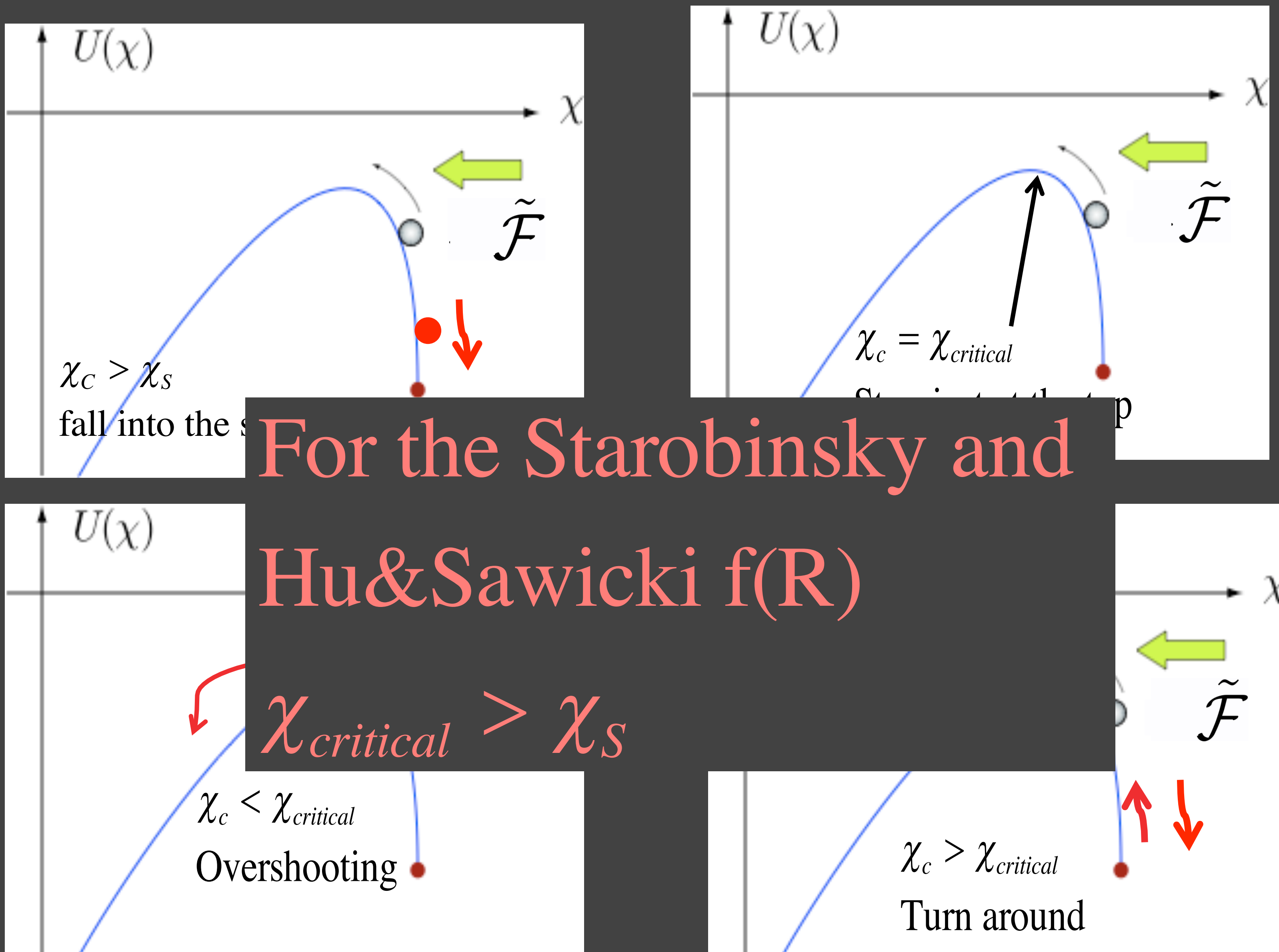
Remember

$$r > \mathcal{R} \quad \Rightarrow \quad \tilde{\mathcal{F}} = 0$$



$\chi_c \Rightarrow$ initial position

$$|\tilde{\mathcal{F}} - \frac{dU}{d\chi}|_{\chi_S} = 0$$



For the Starobinsky and Hu&Sawicki f(R)

$$\chi_{critical} > \chi_S$$

Can we fix the Frolov and Kobayachi&Maeda singularity problems?

YES WE CAN!

$$f(R) = R - R_S\beta\left\{1 - \left[1 + \left(\frac{R}{R_*}\right)^n\right]^{-\frac{1}{\beta}}\right\}$$

$$\beta = -1 \Rightarrow f(R) = R + \alpha R_*(\frac{R}{R_*})^n \qquad (R_S = \alpha R_*)$$

$$\beta = 1 \Rightarrow f(R) = R - \alpha R_*\{1 - [1 + (\frac{R}{R_*})^n]^{-1} \quad \text{(Hu\&Sawicki)}$$

$$n = 2 \Rightarrow f(R) = R - \alpha R_*\{1 - [1 + (\frac{R}{R_*})^2]^{-\frac{1}{\beta}} \quad \text{(Starobinsky)}$$

Here we will consider the special case n=1 and $\beta \to \infty$.

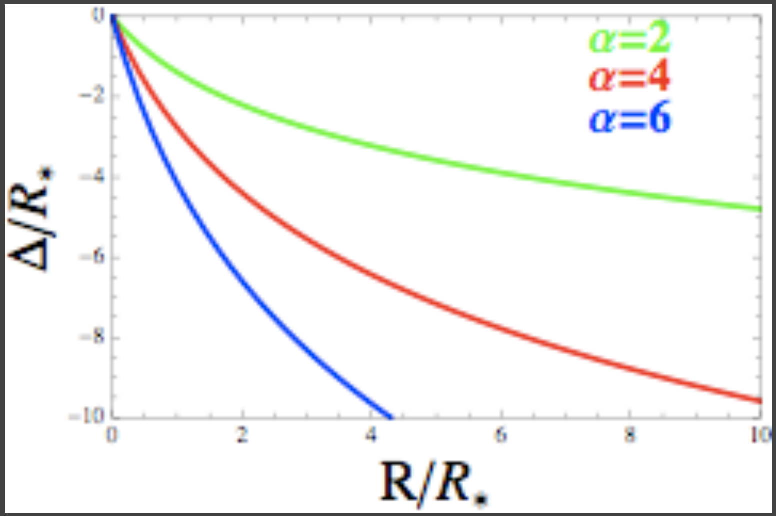
In this limit we get

$$f(R) = R - \alpha R_* \ln\left(1 + \frac{R}{R_*}\right)$$

α and R_* are positive parameters

$$f(R) = R - \alpha R_* \ln\left(1 + \frac{R}{R_*}\right)$$

$$f(R) = R + \Delta(R)$$



The above function satisfies the stability conditions:

$$a)\, f_{RR} \equiv \frac{d^2 f}{dR^2} > 0$$

$$b)\, f_R \equiv \frac{df}{dR} > 0 \quad \text{for } \alpha < \frac{\bar{R}}{R_*} + 1,$$

$\bar{R} \rightarrow$ value of the Ricci Scalar at the final de Sitter atractor

$$c)\, \lim_{R\to\infty} \frac{\Delta}{R} = 0 \;\;\&\;\; \lim_{R\to\infty} \Delta_R = 0 \quad \text{(GR is recovered at high redshifts)}$$

$$f(R) = R - \alpha R_* \ln\left(1 + \frac{R}{R_*}\right)$$

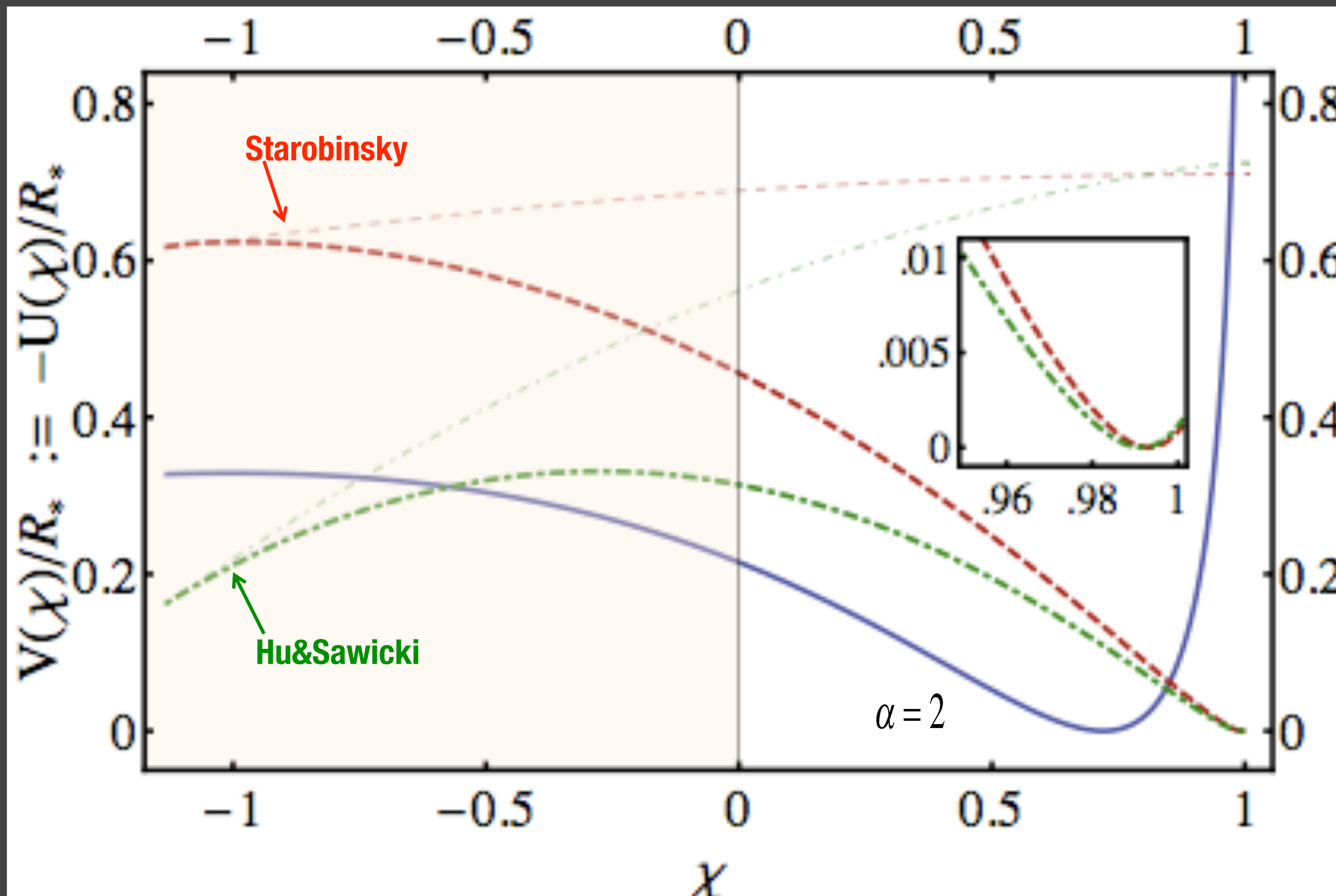
$$\Box\chi = \frac{\mathrm{d}V}{\mathrm{d}\chi} - \mathcal{F} \qquad \Box = -\frac{\partial^2}{\partial t^2} - 3H\frac{\partial}{\partial t}$$

$$\chi := f_R = 1 + \Delta_R \qquad \frac{\mathrm{d}V(R(\chi))}{\mathrm{d}\chi} := \frac{1}{3}\left(2f - \chi R\right)$$

$$\chi[R(t)] = 1 - \frac{\alpha R_*}{R(t)+R_*}.$$

$$\frac{3V(\chi)}{R_*} = -\alpha(2\chi - 3)\ln\left(\frac{\alpha}{1-\chi}\right) + (\chi - 1)\left(\frac{\chi-3}{2} - \alpha\right).$$

$$V(\chi \to 1^-) \approx \frac{\alpha R_*}{3}\ln\left(\frac{\alpha}{1-\chi}\right) \to +\infty$$


Starobinsky
Hu&Sawicki
$\alpha = 2$
$V(\chi)/R_* := -U(\chi)/R_*$
χ

To understand the necessary conditions to solve the problems it is better to go to the Einstein frame

$$\mathcal{S}_{JF} = \int d^4x\sqrt{-g}\left[\frac{\chi R(\chi)}{16\pi G} - \chi^2 V_E(\chi)\right] + \mathcal{S}_m(g_{\mu\nu}, \Psi_m)$$

$$\tilde{g}_{\mu\nu} = \chi g_{\mu\nu} = e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{pl}}} g_{\mu\nu}$$

$$\chi = e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{pl}}}$$

$$M_{Pl}^2 = \frac{1}{8\pi G}$$

$$\mathcal{S}_{EF} = \int d^4x\sqrt{-\tilde{g}}\left[\frac{\tilde{R}}{16\pi G} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_E(\phi)\right]$$

$$+ \mathcal{S}_m(\tilde{g}_{\mu\nu}e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{Pl}}}, \Psi_m)$$

$$V_E(R(\phi)) = \frac{1}{16\pi G}\frac{R\Delta_R - \Delta}{(1+\Delta_R)^2}$$

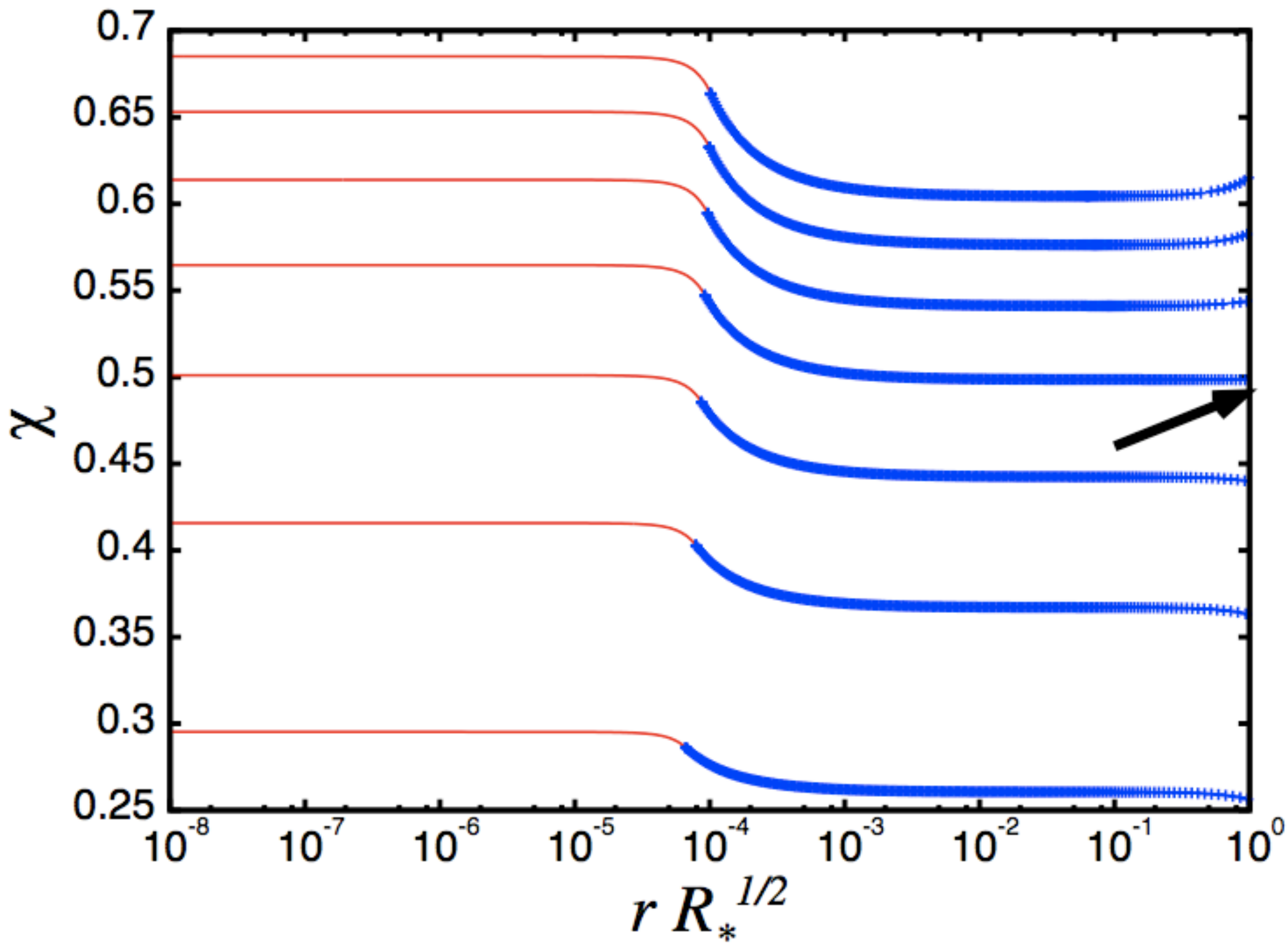
1) $\lim_{R\to\infty} R\Delta_R = \infty$

2) $\lim_{R\to\infty} \Delta = -\infty$

In Starobinsky and Hu&Sawicki models Δ_R goes faster to zero than R $\to \infty$.

$$\tilde{\Box}\phi = \frac{dV_E}{d\phi} + \frac{\tilde{T}}{\sqrt{6}M_{Pl}}$$

$$f(R) = R - \alpha R_*\left(1 + \frac{R}{R_*}\right)^n$$



0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
χ
10^-8 10^-7 10^-6 10^-5 10^-4 10^-3 10^-2 10^-1 10^0
r R_*^1/2

What about Cosmology?

Amendola et al PRD 75, 083504,2007

Viable cosmology:

a) Start with a RD universe
b) Have a sadle point MD phase
c) Have a final accelerated atractor

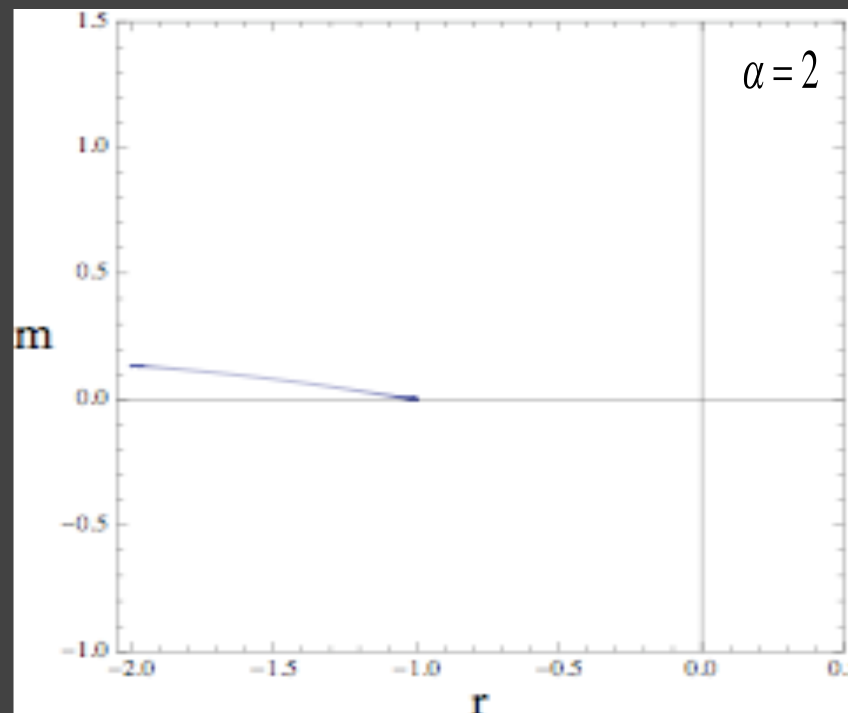
$$m(r \approx -1) \approx 0^{+} \quad \text{and} \quad \frac{dm}{dr}(r \approx -1) > -1$$

necessary condition

$$0 < m(r \approx -2) \leq 1$$

$$m := \frac{Rf_{RR}}{f_R}$$

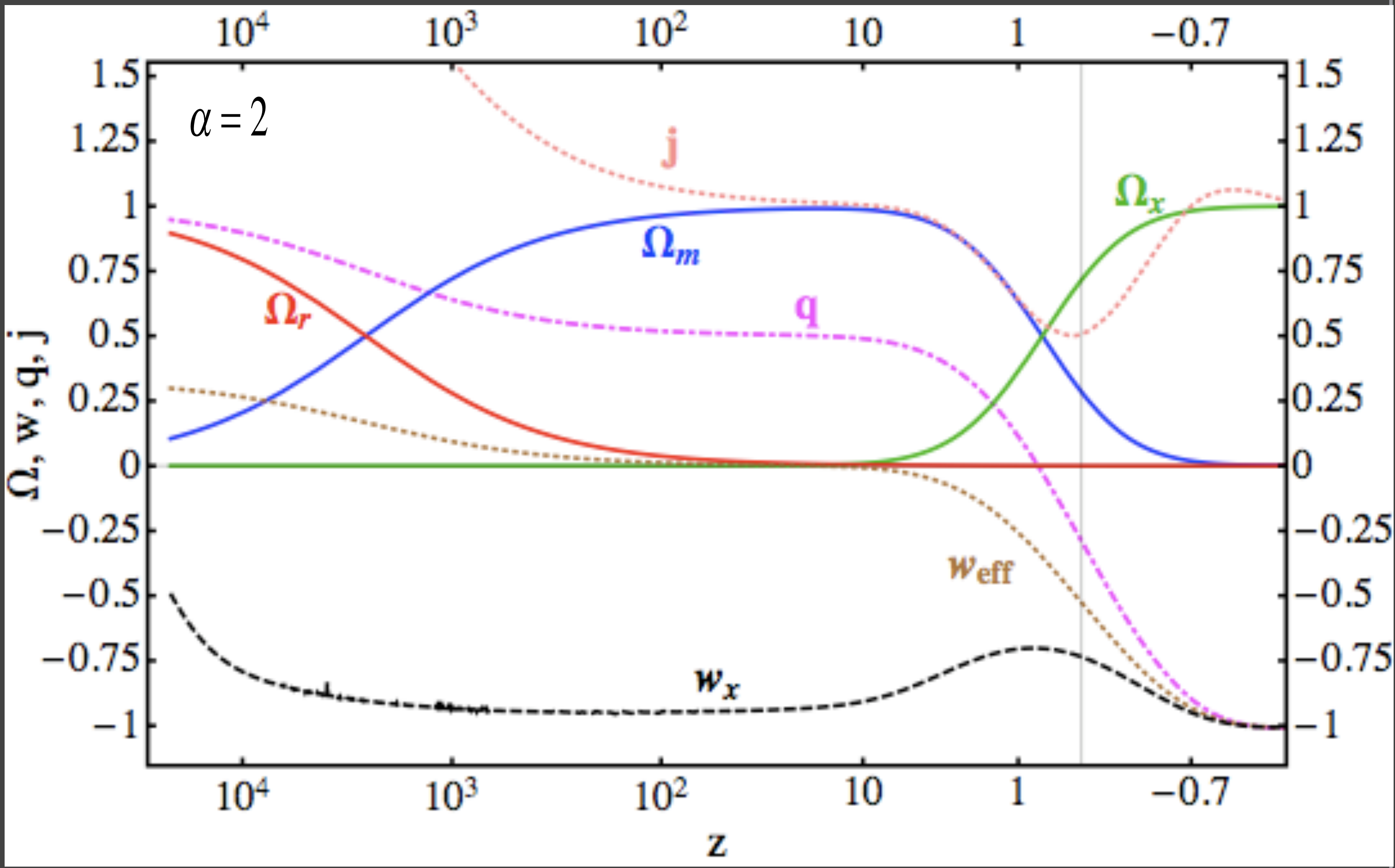
$$r := -\frac{Rf_R}{f}$$

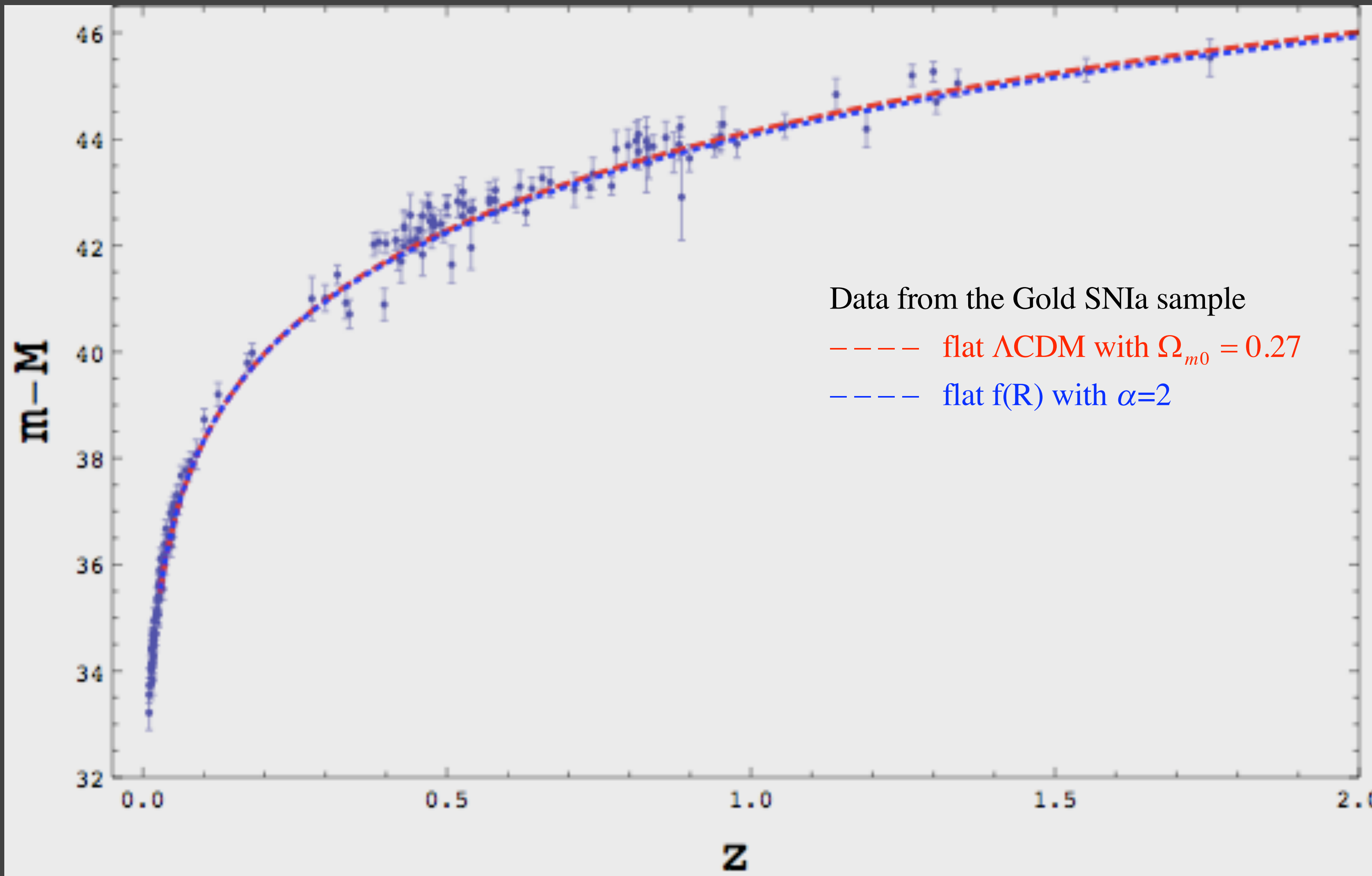


Our f(R) model satisfies all these constraints for $\alpha>1$, regardless of R_*

$$3H^2 = 8\pi G\left(\rho_m + \rho_r + \rho_x\right)$$
$$-2\dot{H} = 8\pi G\left(\rho_m + 4\rho_r/3 + (1+w_x)\rho_x\right)$$
$$8\pi G\rho_x := \left(f_R R - f\right)/2 - 3H\dot{f}_R + 3H^2(1-f_R)$$
$$8\pi G p_x := \ddot{f}_R + 2H\dot{f}_R(2\dot{H} + 3H^2)(1-f_R) + (f - f_R R)/2$$

$$w_x = \frac{p_x}{\rho_x}$$
$$w_{eff} = \frac{p_{total}}{\rho_{total}}$$
$$\Omega_i = \frac{8\pi G}{3}\rho_i$$
$$q = -\frac{\ddot{a}a}{\dot{a}^2}$$
$$j = \frac{\dddot{a}a^2}{\dot{a}^2}$$





46
44
42
40
38
36
34
32
m−M
0.0
0.5
1.0
1.5
2.0
z
Data from the Gold SNIa sample
flat ΛCDM with $\Omega_{m0} = 0.27$
flat f(R) with α=2

Conclusion & Final Remarks

- We have shown that some recent results in the literature regarding divergences in f(R) are not as general as previously thought.
- We obtained the conditions that should satisfy any f(R) in order to be singularity-free and investigated a particular model that satisfies these conditions.
- Cosmological observational constraints are under investigation.
- The model is fully compatible with the Chameleon mechanism. We are now investigating the constraints from solar system and local gravity tests.