LUZ

λ = Comprimento de onda (Angstrom A)

v =Frequência (Hertz)

$\lambda.v = c = (3x10^{10}\ cm.s^{-1})$

$E = h.v$

h = Constante de Planck($6.6x10^{-27}$ erg.s^{-1})

Algumas Definições

- Fluxo: Energia que passa através de uma área dA por unidade de tempo. **O fluxo depende da distância da fonte ao observador.**
- Luminosidade: Energia por unidade de tempo emitida por uma fonte. A luminosidade é uma grandeza intrínseca a fonte.
- Fluxo e Luminosidade: $F = L / 4\pi R^2$
- Fluxo e Luminosidades bolométricos: São as respectivas medidas integradas em todos os comprimentos de onda (frequências).

Mais Definições

- Magnitude Aparente: Medida logaritimica do fluxo de um dado objeto como visto pelo observador.
- $m = -2.5 \log F + cte$
- Magnitude Absoluta: Medida logaritimica do fluxo de um objeto, como se o observador estivesse a uma dsitância de 10 pc da fonte.
- Brilho Superficial: Fluxo em uma pequena superficie de área dA na superfície da estrela. Assim como a luminosidade é um valor intrínseco .

Magnitudes

$$m_2 - m_1 = 2.5 \log\left(\frac{F_1}{F_2}\right)$$

ou

$$m_2 - m_1 = -2.5 \log\left(\frac{F_2}{F_1}\right)$$

$$M = m - 5 \log\left(d/10pc\right)$$

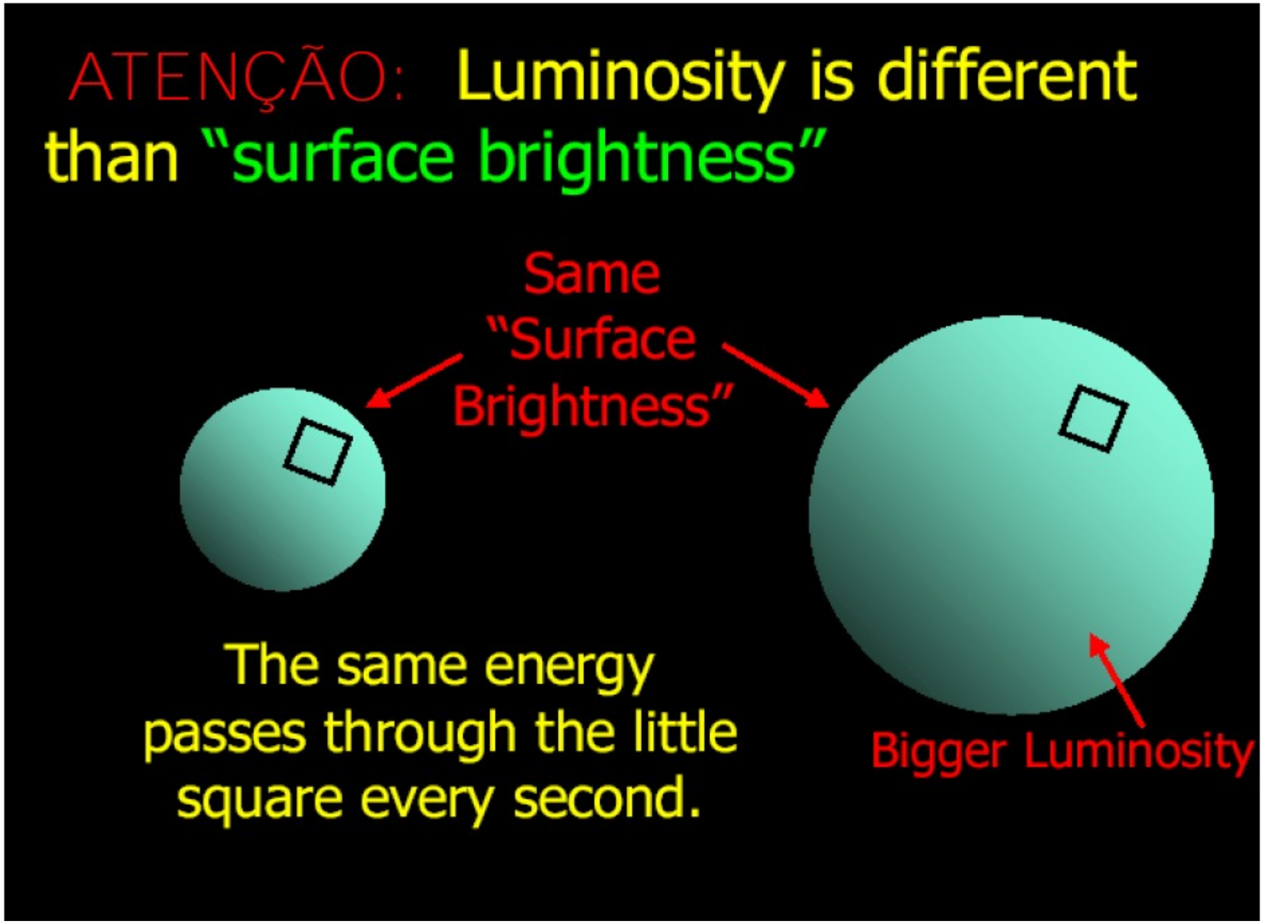
$$M = 4.74 - 2.5 \log\left(L/L_{sol}\right)$$

Definition: "Apparent Magnitude":

$$m_\nu = -2.5 \lg_{10} \frac{F_\nu}{F_{\nu,0}}$$

$$M = m + 5 - 5 \log d/\mathrm{pc}$$

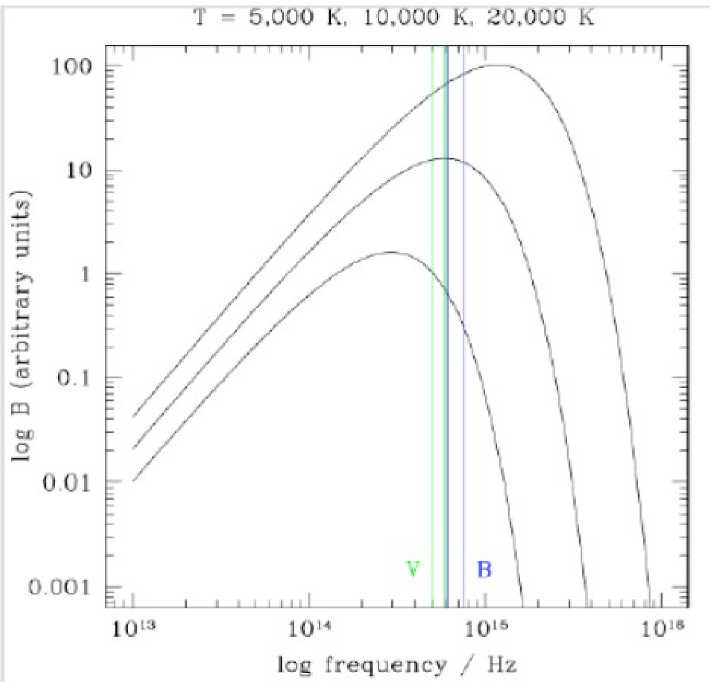
Brilho Superficial

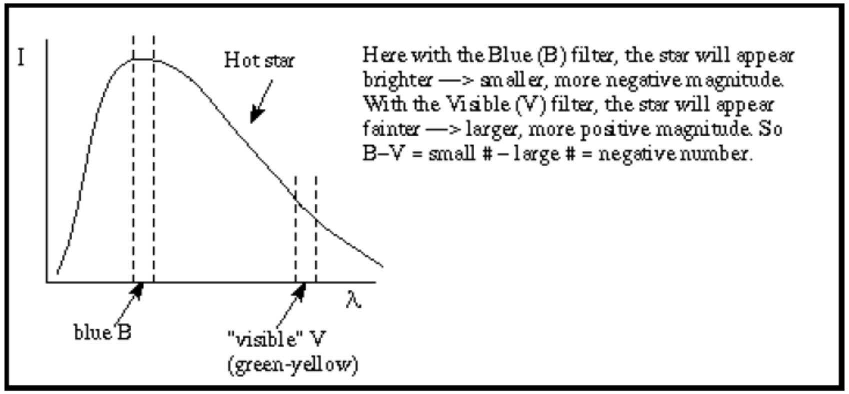


Cor

- Índice de cor é a diferença entre as magnitudes de uma estrelas em diferentes filtros (cores). B-V= m_b-m_v
- Índice de cor também é uma medida da temperatura da estrela:Radiação de corpo negro.

Cor





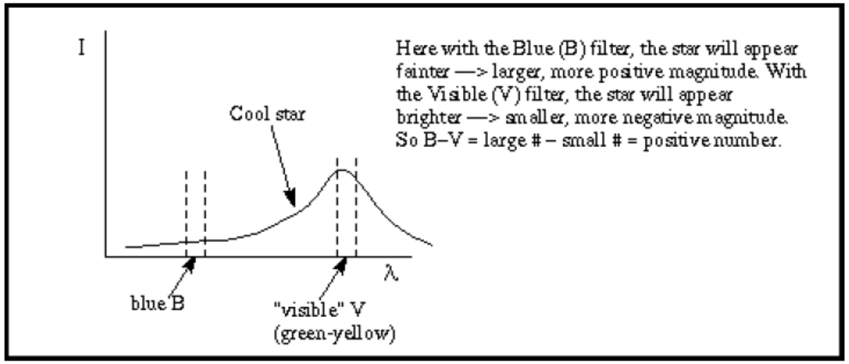
- $I_B > I_V$

Como I ↑ mag ↓

$\Rightarrow$ B < V

B - V < 0

- estrela quente



- $I_B < I_V$

Como I ↑ mag ↓

$\Rightarrow$ B > V

B - V > 0

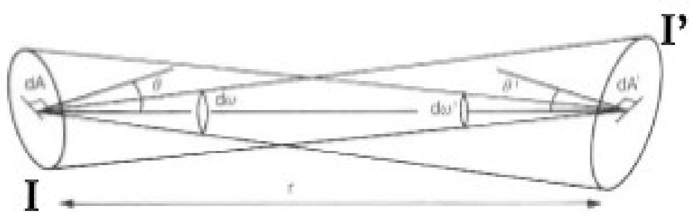
- estrela fria

Transferência Radiativa

- Intensidade especifica não muda com a distância!!

Exemplo 1: $\boxed{dE_\nu = I_\nu (dA \cos\theta) d\nu \, d\omega \, dt}$

Mostrar que a Intensidade é independente da distância*



Karttunen, p. 105

- Energia de dA → dω:
  $dE = I \cos\theta \, dA \, d\omega \, dt$

- A uma dist. **r**, dA' recebe uma energia dE' de dω':
  $dE' = I' \cos\theta' \, dA' \, d\omega' \, dt$

- Ângulos sólidos:

$$d\omega = \frac{dA' \cos\theta'}{r^2} \qquad d\omega' = \frac{dA \cos\theta}{r^2}$$

- dE = dE'

$$\Rightarrow I \cos\theta \, dA \, \frac{dA' \cos\theta'}{r^2} \, dt$$

$$= I' \cos\theta' \, dA' \, \frac{dA \cos\theta}{r^2} \, dt$$

$\Rightarrow$ **$\underline{I = I'}$**

* Sem absorção no meio

Transferência Radiativa

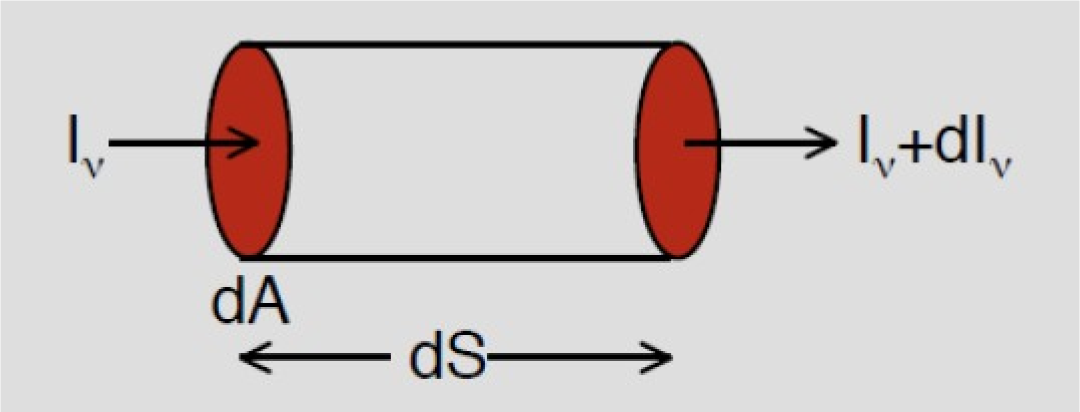
- Na ausência de matéria entre a fonte e o observador:

$$\frac{dI_\nu}{ds} = 0$$

-
-
- Na presença de matéria podem ocorrer dois processos: Absorção e emissão de luz!

Transferência Radiativa

- Se a matéria na linha entre fonte e observador emitir radiação própria essa radiação irá ser adicionada a radiação emitida pela fonte:

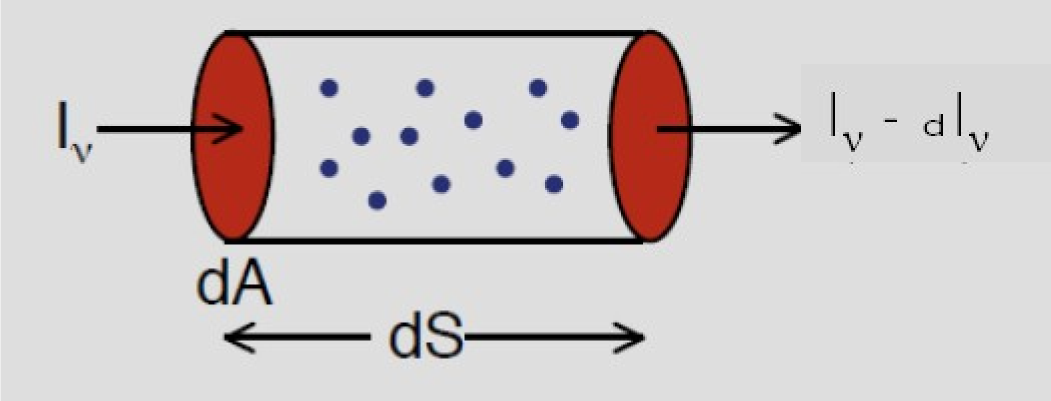


$$dE = j_\nu dV d\Omega dt d\nu$$

$$dI_\nu = j_\nu ds$$

Transferência Radiativa

- Caso a matéria na linha de visada do observador somente absorva a radiação incidente:



$$\frac{dI_\nu}{I_\nu} = -\frac{ndAds\sigma_\nu}{dA} = -n\sigma_\nu ds$$

$$dI_\nu = -n\sigma_\nu I_\nu ds \equiv -\alpha_\nu I_\nu ds$$

$$\text{number of absorbers} \times \text{cross-section} = ndAds \times \sigma_\nu$$

$$\alpha_\nu \equiv \rho\kappa_\nu$$

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

Transferência Radiativa

- No caso real temos tanto absorção quanto emissão pelo meio de forma que:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Transferência Radiativa

- Opacidade: Medida da dificuldade que uma onda eletromagnética tem de atravessar a matéria.

$$\kappa_\nu = \frac{n}{\rho}\sigma_\nu = N_A\sigma_\nu = 0.4 \text{ cm}^2 \text{ g}^{-1}$$

Transferência Radiativa

- Porundidade óptica:

τ_ν

  - **Exemplo: absorção pura,**

$$\frac{dI_\nu}{ds} = -\alpha_\nu(s)I_\nu(s)$$

Em um ponto **s** de um meio irradiado com $\mathbf{I_{(s=0)} = I_0}$,

$$I_\nu(s) = I_0 exp\left(-\int_0^s \alpha_\nu(s')ds'\right) = I_0 e^{-\tau_\nu}$$

ou seja, para absorção pura,

I_ν decresce exponencialmente com a profundidade óptica.

Transferência Radiativa

τ_ν

– A **profundidade óptica**, τ, é definida por

$$\boxed{\tau_\nu(s) = \int_0^s \alpha_\nu(s')ds' = \int_0^s n(s')\sigma_\nu ds' = n\sigma_\nu s}$$

onde a última igualdade é válida para um meio homogêneo.

– τ é importante ao descrevermos os efeitos da TR.
  - $\tau > 1 \Rightarrow$ meio é **opticamente espesso**, ou **opaco**
  - $\tau < 1 \Rightarrow$ meio é **opticamente fino,** ou **transparente**

Transferência Radiativa

$\longleftarrow \quad \tau_\nu \quad \longrightarrow$

– Pela lei exponencial,

$\Rightarrow$ a **probabilidade** de um fóton viajar uma distância τ é $\mathbf{exp(-\tau)}$

$\Rightarrow$ a prof. óptica **média** viajada é: $\langle \tau_\nu \rangle = \int_0^\infty \tau_\nu \exp(-\tau_\nu)\ d\tau_\nu = 1$

– A **distância física** correspondente a τ=1 é chamada **livre caminho médio:**

$$\tau_\nu = n\sigma_\nu l_\nu \quad \Rightarrow \quad \langle l \rangle = \frac{1}{n\sigma_\nu}$$

- Ex: **centro do Sol** - $\varrho \approx 150$ g cm^{-3} $\Rightarrow$ $n \approx 9\times10^{25}$ cm^{-3}
  $\sigma \approx 10^{-25}$ cm^2 ⇨ $\langle l \rangle \approx 1$ mm

Transferência Radiativa

A Função fonte: Podemos reescrever a equação de transferência radiativa em função da profundidade óptica da seguinte forma:

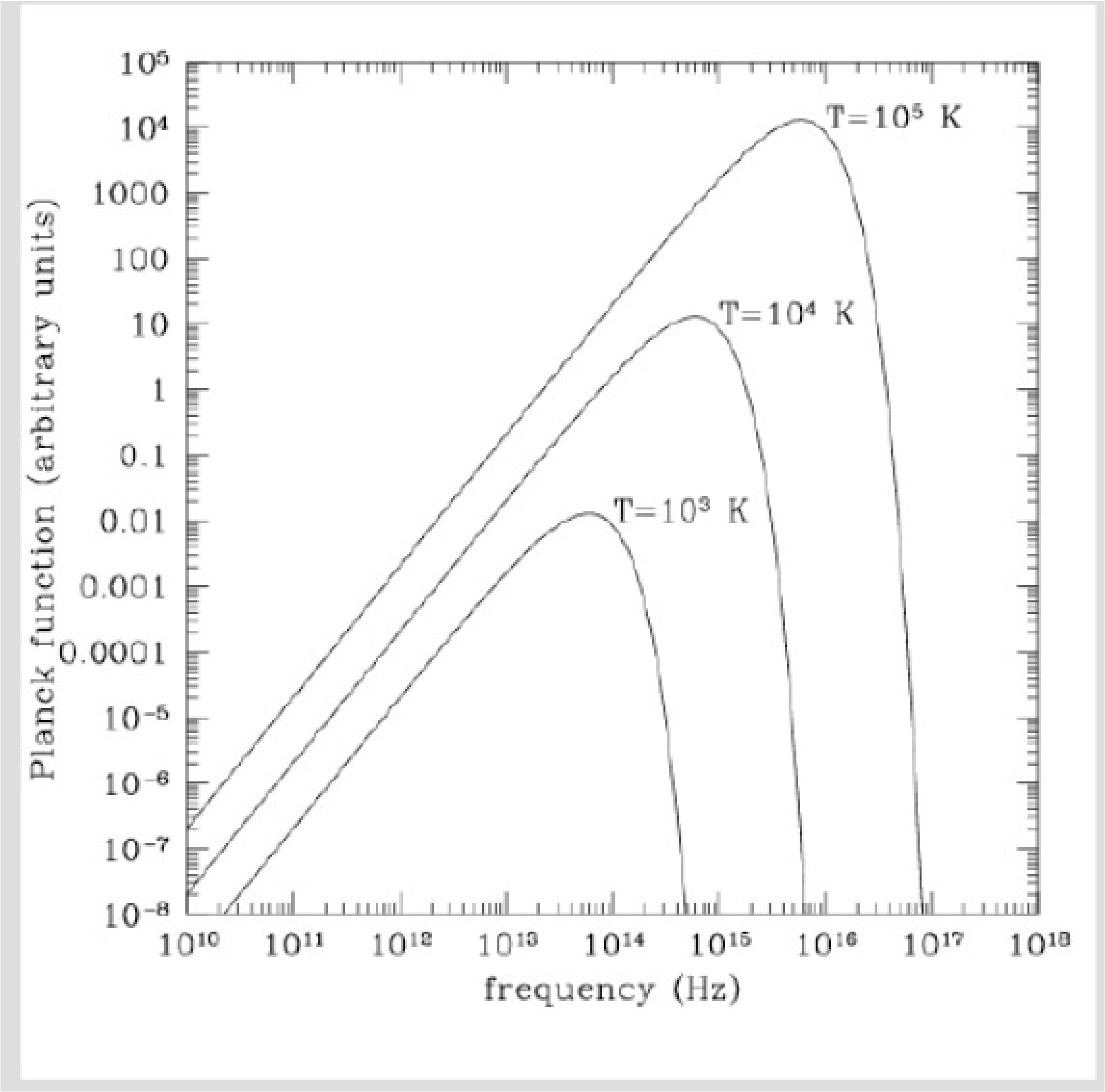
$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{\alpha_\nu ds} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

Onde S=j/α é chamada a Função fonte

Corpo Negro



$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

Thermal radiation:

$S_\nu = B_\nu$

Blackbody radiation:

$I_\nu = B_\nu$

Corpo Negro

Region	Wavelength (centimeters)	Energy (eV)	Blackbody Temp (K) T
Radio	> 10	$< 10^{-5}$	< 0.03
Microwave	$10 - 0.01$	$10^{-5} - 0.01$	$0.03 - 30$
Infrared	$0.01 - 7 \times 10^{-5}$	$0.01 - 2$	$30 - 4100$
Visible	$7 \times 10^{-5} - 4 \times 10^{-5}$	$2 - 3$	$4100 - 7300$
Ultraviolet	$4 \times 10^{-5} - 10^{-7}$	$3 - 10^{3}$	$7300 - 3 \times 10^{6}$
X-Rays	$10^{-7} - 10^{-9}$	$10^{3} - 10^{5}$	$3 \times 10^{6} - 3 \times 10^{8}$
Gamma Rays	$< 10^{-9}$	$> 10^{5}$	$> 3 \times 10^{8}$

Corpo Negro

- As regras da radiação térmica funcionam somente se o objeto é OPACO.
  - $T > 0 \rightarrow$ emissão de fotons
  - Radiação térmica é criada através da interação do foton com a matéria
  - Se os fotons não encontram resistência à propagação (ou seja, o meio é transparente), não há emissão térmica.

- Como as estrelas são opacas em essencialmente todos os comprimentos de onda $\Rightarrow$ elas emitem como corpos negros ideais.

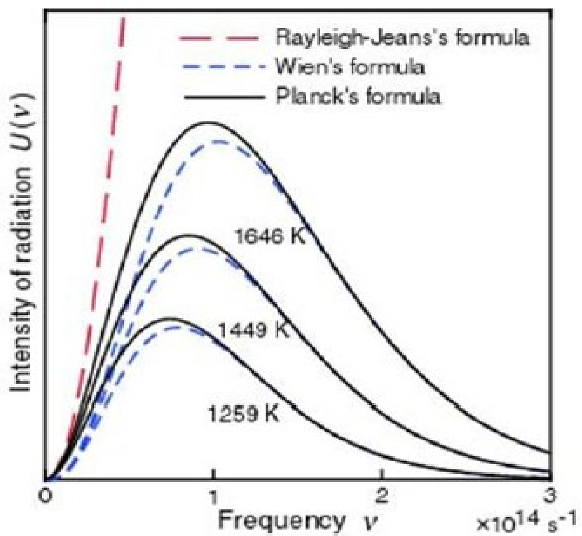
Corpo Negro

- No vermelho: domínio de Rayleigh-Jeans

$$\frac{h\nu}{kT} << 1 \qquad \exp\left(\frac{h\nu}{kT}\right) \approx 1 + \frac{h\nu}{kT}$$

$$B_\nu(\nu, T) = \frac{2k\nu^2 T}{c^2}$$

$$\boxed{B_\nu(\lambda, T) = \frac{2ckT}{\lambda^4}}$$



- No azul: domínio de Wien

$$\frac{h\nu}{kT} >> 1 \qquad \exp\left(\frac{h\nu}{kT}\right) - 1 \approx \exp\left(\frac{h\nu}{kT}\right)$$

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{kT}\right)$$

$$\boxed{B_\nu(\lambda, T) = \frac{2hc^2}{\lambda^5}\exp\left(-\frac{hc}{\lambda kT}\right)}$$

Corpo Negro

- O fluxo total emitido em todas as direções em todos os comprimentos de onda por um CN é:

$$F = \int\int d\lambda \; d\omega \cos\theta \; B_\lambda(T) = 2\pi \int_0^\infty d\lambda \int_0^\pi d\theta \cos\theta \sin\theta \; B_\lambda(T) = \pi B$$

Mas,

$$B(T) = \int_0^\infty B_\lambda \, d\lambda = \int_0^\infty B_\nu \, d\nu = \frac{A}{\pi} \times T^4, \quad A = \frac{2k^4}{c^2 h^3} \frac{\pi^5}{15}$$

Como

$$B(T) = \frac{\sigma}{\pi} T^4 \Longrightarrow$$

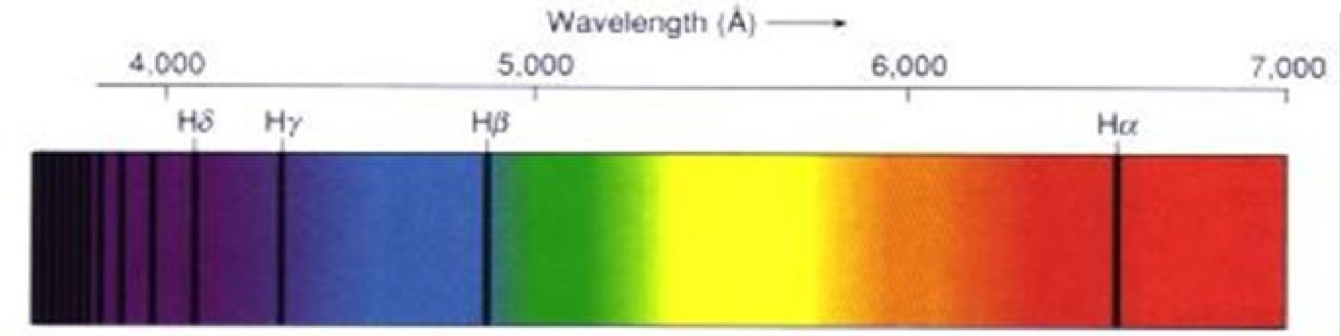
$$\boxed{F = \sigma T^4}$$

$$L = 4\pi R^2 \sigma T^4$$

Kirchoff

1. **CONTINUOUS SPECTRUM**: Any hot opaque body (e.g., hot gas/plasma) produces a continuous spectrum or complete rainbow
2. **EMISSION SPECTRUM**: A hot transparent gas will produce an emission line spectrum
3. **ABSORPTION SPECTRUM**: A (relatively) cool transparent gas in front of a source of a continuous spectrum will produce an absorption line spectrum

Kirchoff



Dark hydrogen absorption lines appear against a continuous visual spectrum, the light in the spectrum absorbed by intervening hydrogen atoms. From "Astronomy! A Brief Edition," J. B. Kaler, Addison-Wesley, 1997

- Espectros de laboratório → identificação de linhas em laboratório
- Análise de espectros → abundâncias químicas + condições físicas (temperatura, pressão, gravidade, fluxo ionizante, campos magnéticos)

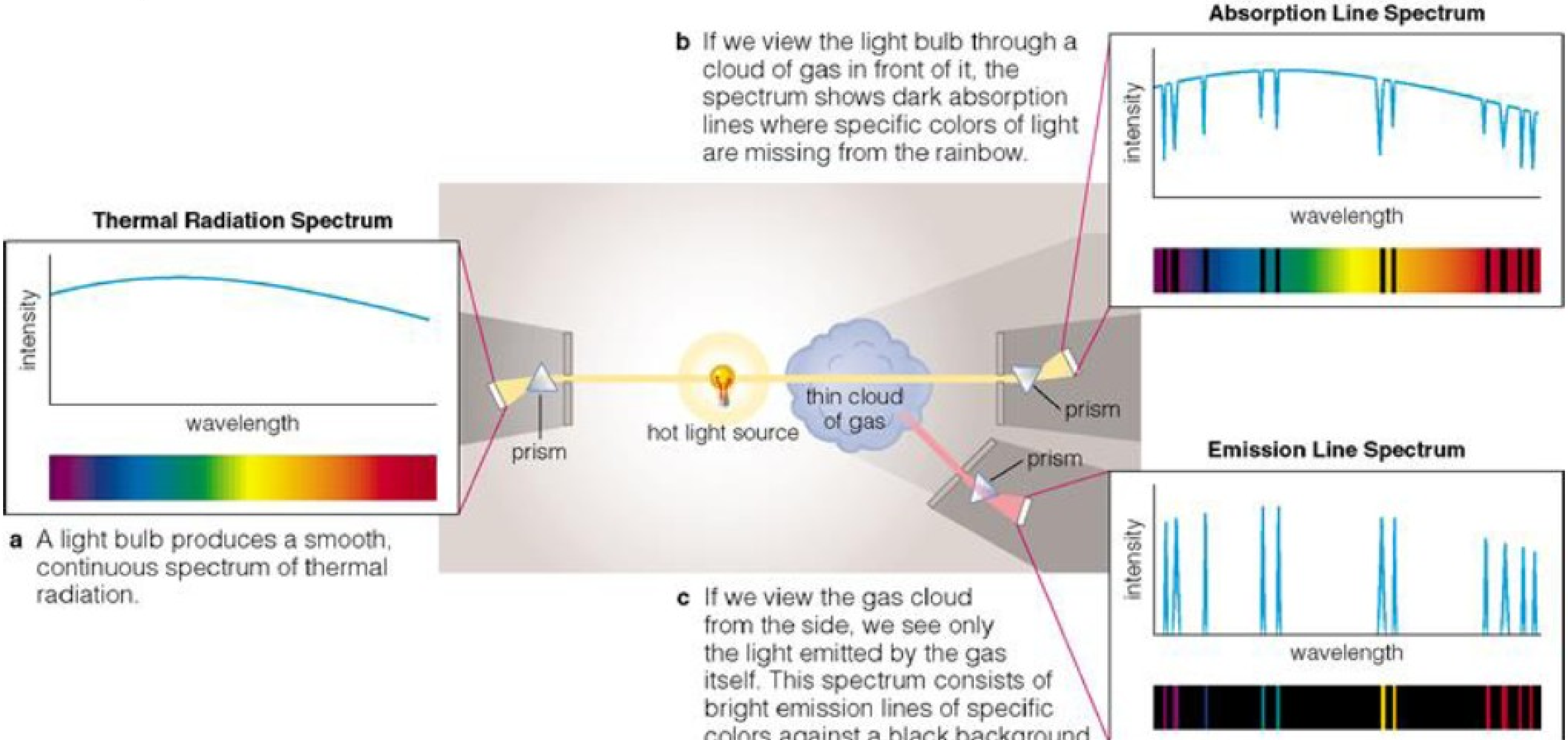
\+ Velocidades

Há três tipos básicos de espectros:

a. Espectro de radiação térmica: *Todos objetos com temperatura finita emitem radiação térmica.*

b. Espectro de Linha de Absorção:

c. Espectro de Linha de Emissão:



Atomic Processes

Radiation can be emitted or absorbed when electrons make transitions between different states:

Bound-bound: electron moves between two bound states (orbitals) in an atom or ion. Photon is emitted or absorbed.

Bound-free:

- Bound -> unbound: **ionization**
- Unbound -> bound: **recombination**

Free-free: free electron gains energy by absorbing a photon as it passes near an ion, or loses energy by emitting a photon. Also called **bremsstrahlung**.

Emission or Absorption?

It depends on whether the gas (plasma) is

Optically thick: short mean free path of photons, get absorbed and re-emitted many times, only the radiation near the surface escapes; or

Optically thin: most photons escape without being reabsorbed or scattered

(Note that a medium can be optically thick or thin for either line or continuum photons. Optical thickness is generally proportional to density.)

And then it depends on the **geometry:** if a continuum is seen though a cooler, optically thin gas, you will see an absorption spectrum; but if the gas is hotter, there will be an emission line spectrum superposed on the cont.

Alargamento de linhas

- Alargamento natural: Principio de Incerteza

$$\Delta E \Delta t \approx \frac{h}{2\pi}$$

But since E = hν, ΔE = hΔν ⇒ $$\Delta \nu \approx \frac{1}{2\pi \Delta t}$$

- Devido ao efeito Doppler os fótons emitidos não chegam ao observador com exatamente a mesma frequência, causando um alargamento nas linhas!

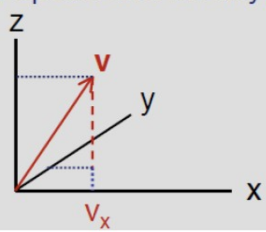
Alargamento de linhas

Doppler or thermal broadening

Atoms in a gas have random motions that depend upon the temperature. For atoms of mass m, at temperature T, the typical speed is obtained by equating kinetic and thermal energy:

$$\frac{1}{2}mv^2 = kT \qquad \text{k = Boltzmann's const}$$

Number of atoms with given speed or velocity is given by **Maxwell's law**. Need to distinguish between forms of this law for speed and for any one velocity component:



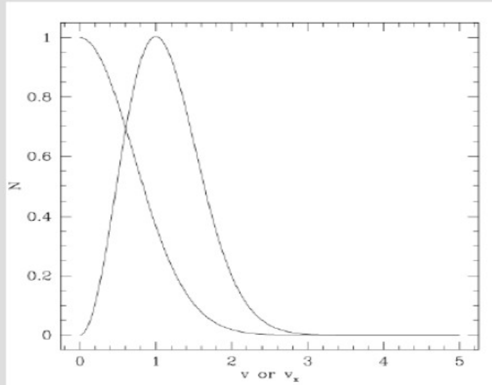
$$|\mathbf{v}|^2 = v_x^2 + v_y^2 + v_z^2$$

Distribution of *one component* of the velocity, say v_x, is relevant for thermal broadening - only care about motion along line of sight.

For one component, number of atoms dN within velocity interval dv_x is given by:

$$dN(v_x) \propto \exp\left(-\frac{mv_x^2}{2kT}\right)dv_x$$

Distribution law for speeds has extra factor of v^2:



$$dN(v) \propto v^2 \exp\left(-\frac{mv^2}{2kT}\right)dv$$

Most probable speed:

$$v_{peak} = \sqrt{\frac{2kT}{m}}$$

Average speed:

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m}}$$

Emits at frequency ν_0 ● - - - - v_x - - - -> Observed at frequency ν

Consider atom moving with velocity v_x along the line of sight to the observer.

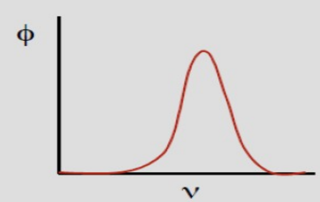
Doppler shift formula: $\dfrac{\nu - \nu_0}{\nu_0} = \dfrac{v_x}{c}$

Combine this with the thermal distribution of velocities:

$$\phi(\nu) = \frac{1}{\Delta\nu_D\sqrt{\pi}}\exp\left[-\frac{(\nu-\nu_0)^2}{(\Delta\nu_D)^2}\right]$$

…where the **Doppler width** of the line:

$$\Delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$$



Alargamento de linhas

Summary:

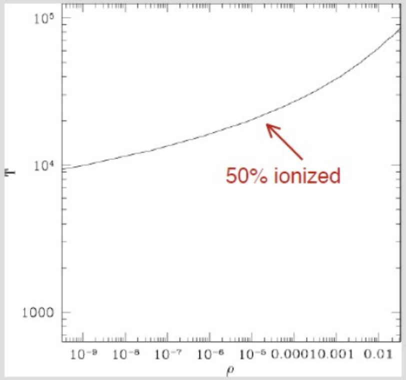
- **Strength** of different spectral lines depends upon the abundance of different elements, and on the excitation / ionization state (described in part by the Boltzmann formula).
- **Width** of spectral lines depends upon:
  - *Natural linewidth* (small)
  - *Collisional linewidth* (larger at high density)
  - *Thermal linewidth* (larger at higher temperature)

High quality spectrum gives information on composition, temperature and density of the gas.

Bremssttrahlung

Hydrogen is ionized at T ~ 10^4 K at low density.

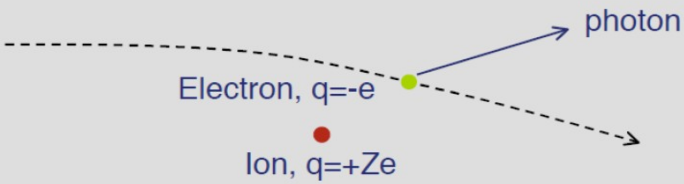
For the same mixture of chemical elements as the Sun, maximum radiation due to spectral lines occurs at T ~ 10^5 K.



At higher T, radiation due to acceleration of unbound electrons becomes most important.

Free-free radiation or **bremsstrahlung**.

- Is caused when a free electron interacts with a nucleus
- Gives a continuous energy spectrum



`Collisions' between electrons and ions accelerate the electrons. Power radiated by a single electron is given by **Larmor's formula:**

$$P = \frac{2q^2}{3c^3}|\mathbf{a}|^2$$

c.g.s. units: q is the charge, where electron charge = 4.80 x 10^{-10} esu. **a** is the acceleration, c is speed of light.

Prefer to work in SI? Larmor's formula:
…with q in Coulombs, ε_0 is the permittivity of the vacuum [10^7 / ($4\pi c^2$) C^2 N^{-1} m^{-2}]

$$P = \frac{q^2}{6\pi\varepsilon_0 c^3}|\mathbf{a}|^2$$

Power is proportional to the square of the charge and the square of the magnitude of the acceleration.

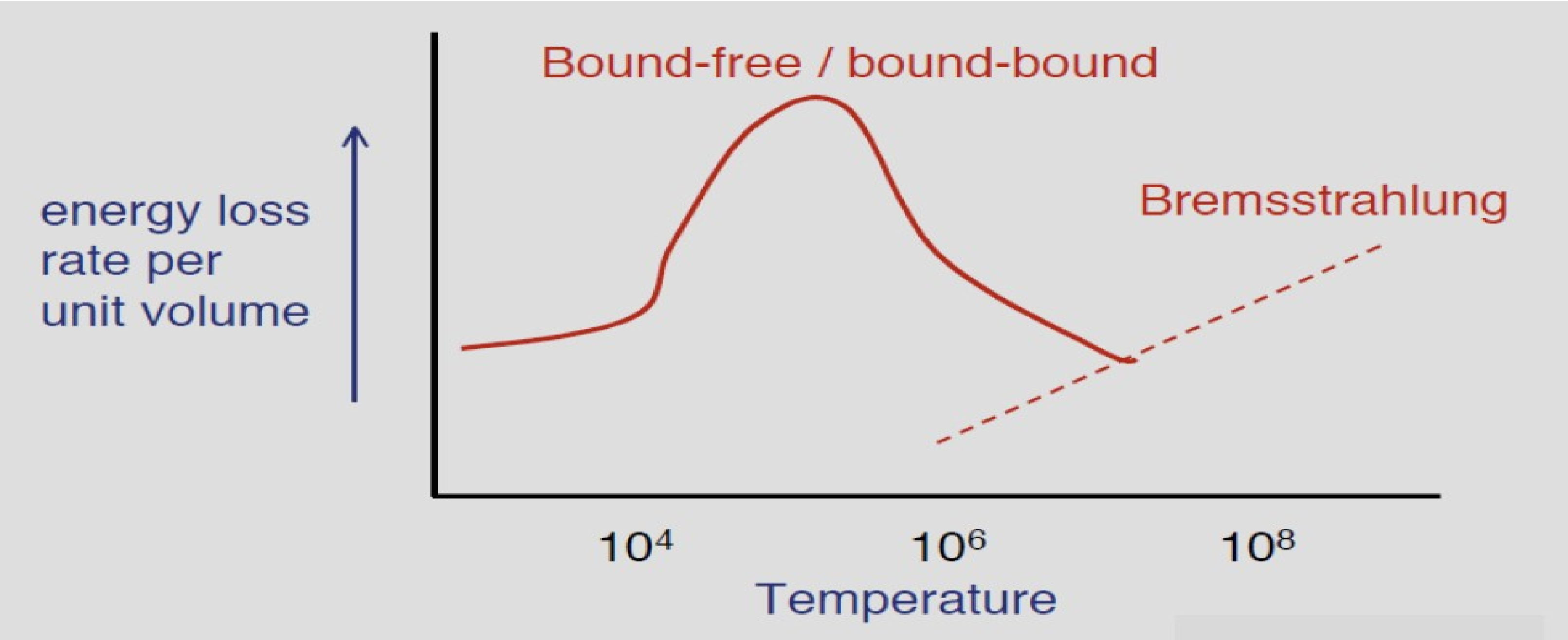
To derive spectrum of bremsstrahlung, and total energy loss rate of the plasma, need to:

- Calculate acceleration and energy loss for one electron of speed v, passing ion at impact parameter b.
- Integrate over all collisions, assuming a distribution of encounter speeds (normally a thermal / Maxwellian distribution).

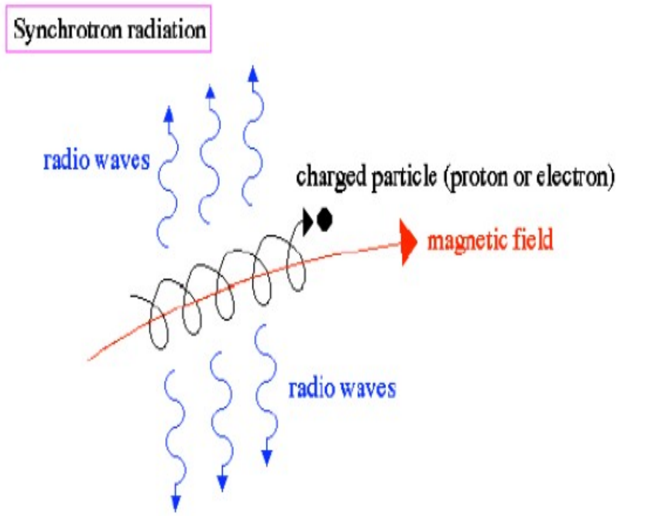
Bremssttrahlung

Bremsstrahlung loss rate increases with temperature
Atomic processes become less important as the gas
becomes fully ionized

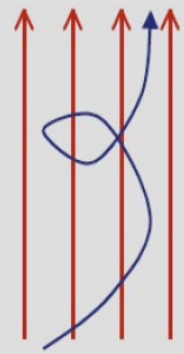
high T



Synchrotron



synchrontron radiation occurs when a charged particle encounters a strong magnetic field – the particle is accelerated along a spiral path following the magnetic field and emitting radio waves in the process – the result is a distinct radio signature that reveals the strength of the magnetic field



Electron moving perpendicular to a magnetic field feels a Lorentz force.

➡ Acceleration of the electron.

➡ Radiation (Larmor's formula).

Define the Lorentz factor: $\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$

Non-relativistic electrons: ($\gamma \sim 1$) - **cyclotron radiation**

Relativistic electrons: ($\gamma >> 1$) - **synchrotron radiation**

Same physical origin but very different spectra - makes sense to consider separate phenomena.

Cyclotron

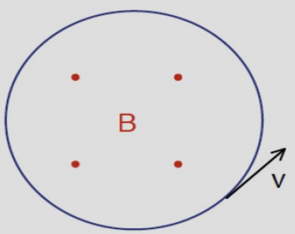
Cyclotron

Start with the non-relativistic case:

Particle of charge q moving at velocity **v** in a magnetic field **B** feels a force:

$$\mathbf{F} = \frac{q}{c}\mathbf{v}\times\mathbf{B}$$

Let v be the component of velocity perpendicular to the field lines (component *parallel* to the field remains constant). Force is constant and normal to direction of motion.



Circular motion: acceleration -

$$a = \frac{qvB}{mc}$$

…for particle mass m.

Let **angular velocity** of the rotation be ω_B. Condition for circular motion:

$$m\frac{v^2}{r} = \frac{qvB}{c}$$

$$m\omega_B v = \frac{qvB}{c}$$

$$\omega_B = \frac{qB}{mc}$$

Use c.g.s. units when applying this formula, i.e.
- electron charge = 4.80 x 10^{-10} esu
- B in Gauss
- m in g
- c in cm/s

Power given by Larmor's formula:

$$P = \frac{2q^2}{3c^3}|\mathbf{a}|^2 = \frac{2q^2}{3c^3}\times\left(\frac{qvB}{mc}\right)^2 = \frac{2q^4\beta^2B^2}{3c^3m^2}$$ where β=v/c

$$P = \frac{2q^4\beta^2B^2}{3c^3m^2}$$

Magnetic energy density is $B^2 / 8\pi$ (c.g.s.) - energy loss is proportional to the energy density.

Energy loss is largest for low mass particles, electrons radiate much more than protons (c.f. highest energy particle accelerators are proton / antiproton not electron / positron).

Cyclotron spectrum: ideally a single line at: $\nu = \frac{\omega}{2\pi} = \frac{qB}{2\pi mc}$

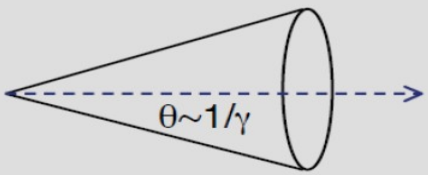
Actually get some emission at the harmonics too - 2ν, 3ν etc.

Very nice way to measure the magnetic field.

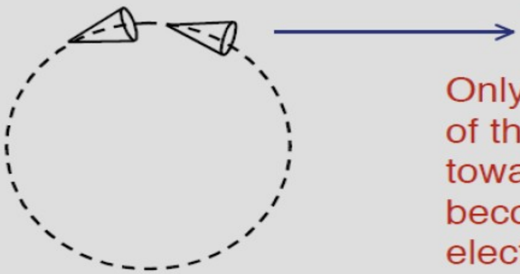
Synchrotron

If the electrons are moving at close to the speed of light, two effects alter the nature of the radiation.

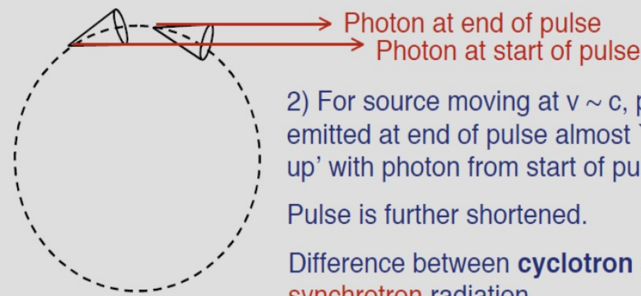
1) Radiation is **beamed**:



Particle moving with Lorentz factor γ toward observer emits radiation into cone of opening angle: $\theta \approx \gamma^{-1}$



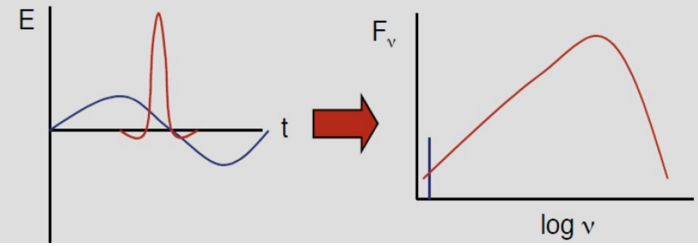
Only see radiation from a small portion of the orbit when the cone is pointed toward us - pulse of radiation which becomes shorter for more energetic electrons.



2) For source moving at v ~ c, photon emitted at end of pulse almost `catches up' with photon from start of pulse.

Pulse is further shortened.

Difference between **cyclotron** and synchrotron radiation.



Useful formulae for synchrotron radiation

For a **single particle**, spectrum extends up to a peak frequency roughly given by:

$$\nu \sim \gamma^2 \nu_c \sim \frac{\gamma^2 qB}{2\pi mc}$$

cyclotron frequency

Can produce very high frequency radiation, with a continuous spectrum (no lines).

Normally, the electrons which produce synchrotron radiation have a (wide) range of energies. If number of particles with energy between E and E+dE can be written as:

$$N(E)dE = CE^{-p}dE$$

i.e. as a power-law in energy, then it turns out that the spectrum of the resulting synchrotron radiation is *also* a power-law, but with a different index:

$$P(\nu) \propto \nu^{-s} \propto \nu^{-(p-1)/2}$$

Measure the spectral index of the radiation (s), this then gives an indication of the distribution of particle energies (p)!

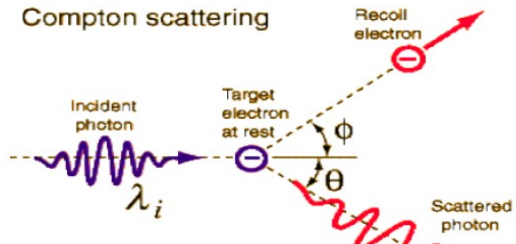
$$s = \frac{p-1}{2}$$

Compton

- Thomson scattering: elastic scattering of low-energy photons from low-energy electrons, with cross-section $\sigma_T = (8\pi/3)(e^2/mc^2)^2 = 0.665\times10^{-24}$ cm^2
- Compton scattering: low-energy photon inelastically scatters off non-relativistic electron, photon ends up with lower energy
- Inverse Compton scattering: low-energy photon inelastically scatters off relativistic electron, photon gains energy in observer rest frame

Whether the photon gives energy to the electron of vice versa

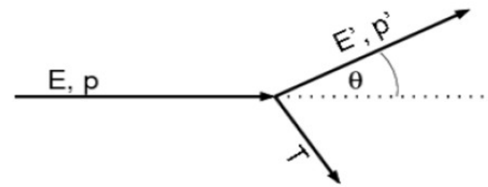
Compton Wavelength=h/mc=0.00243 nm for an electron



$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/compton.html

Compton Scattering



Thomson scattering: initial and final wavelength are identical.

But: in reality: light consists of photons

$\Longrightarrow$ Scattering: photon changes direction

$\Longrightarrow$ Momentum change

$\Longrightarrow$ Energy change!

This is a quantum picture

$\Longrightarrow$ Compton scattering.

Dynamics of scattering gives energy/wavelength change:

$$E' = \frac{E}{1+\frac{E}{m_e c^2}(1-\cos\theta)} \sim E\left(1-\frac{E}{m_e c^2}(1-\cos\theta)\right) \qquad (7.14)$$

and

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta) \qquad (7.15)$$

where $h/m_e c = 2.426\times10^{-10}$ cm (Compton wavelength).

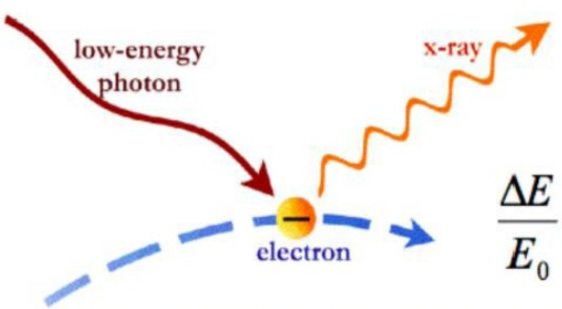
Averaging over θ, for $E \ll m_e c^2$:

$$\frac{\Delta E}{E} \approx -\frac{E}{m_e c^2} \qquad (7.16)$$

INVERSE COMPTON EMISSION

Compton scattering

- Photon $E_0=h\nu$ boosted in energy by hot e$^-$ at kT to e.g. X-rays



$$\frac{\Delta E}{E_0} = \frac{1}{m_e c^2}(4kT - E_0)$$

Examples in X-ray astronomy: active galactic nuclei (AGN), X-ray binaries

INVERSE COMPTON EMISSION

- Results depend on source geometry

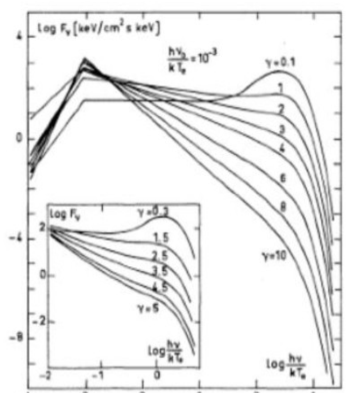


Fig. 5. The spectrum resulting from comptonization of low-frequency photons ($h\nu_0 = 10^{-3}$ kT$_e$) in a high temperature plasma clouds with different parameters γ (14)

Sunyaev & Titarchuk 1980

- ***Power law***

$$F(E) = AE^{-\Gamma}e^{-E/E_c}$$

$$I(E) = BE^{-\alpha}e^{-E/E_c}$$

A,B normalizations

F,Γ ***photon*** flux photon index

I,α ***energy*** flux, index ($\alpha=\Gamma$-1)

E_c=*kT*=cutoff energy