# Radiative Processes in the Interstellar Medium

Moshe Elitzur

University of Kentucky

UK
UNIVERSITY OF KENTUCKY

Radiative transfer – generalities

Line formation

Shocks & PDR

Masers

Dust emission

IR from AGN

# Thermodynamic Equilibrium

- Equilibrium distributions – result of interactions
- No photon-photon interactions!

No “spontaneous” black-body!

$$\frac{dI_\nu}{d\ell} = \varepsilon_\nu - \kappa_\nu I_\nu$$

$$d\tau_\nu = \kappa_\nu d\ell \qquad S_\nu = \varepsilon_\nu / \kappa_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu$$

Equilibrium: $I_\nu = S_\nu$

$S_\nu$ - matter

$I_\nu$ - radiation

Radiative transfer equation is only <u>HALF</u> of a pair!

Unlike radiation, matter-matter interactions <u>DO</u> exist!

# Principle of Detailed Balance

Every microscopic physical process has a reverse counterpart. The rates for the process and its inverse are related such that if they were the only ones taking place, the corresponding distribution would be the equilibrium thermodynamic result.

Deviations from thermodynamic equilibrium – competing processes

# Elastic Collisions – Kinetic Temperature

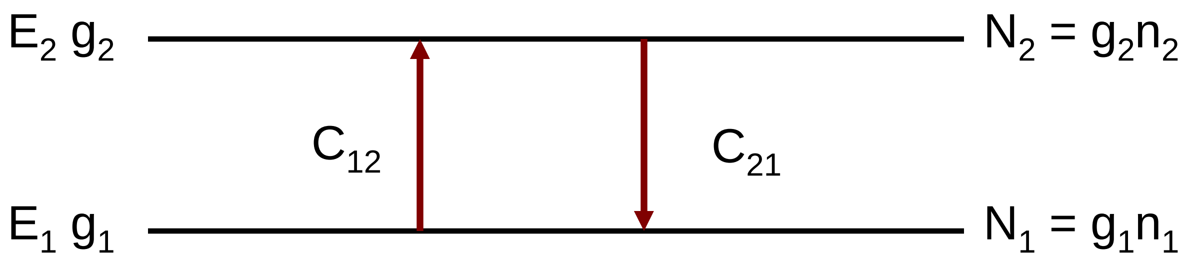
$$dN(v) = N\, f(v)\, dv$$

$$f(v) \propto v^2 \exp(-mv^2/2kT)$$

Redistribution of bulk energy

x-section for elastic >> inelastic

# Inelastic Collisions – Excitation Temperature




$C_{ij} = NK_{ij}$ $K_{ij} = \langle \sigma_{ij} v \rangle$

steady-state: $N_1 C_{12} = N_2 C_{21}$

detailed balance: $\frac{n_2}{n_1} = e^{-\Delta E / kT}$

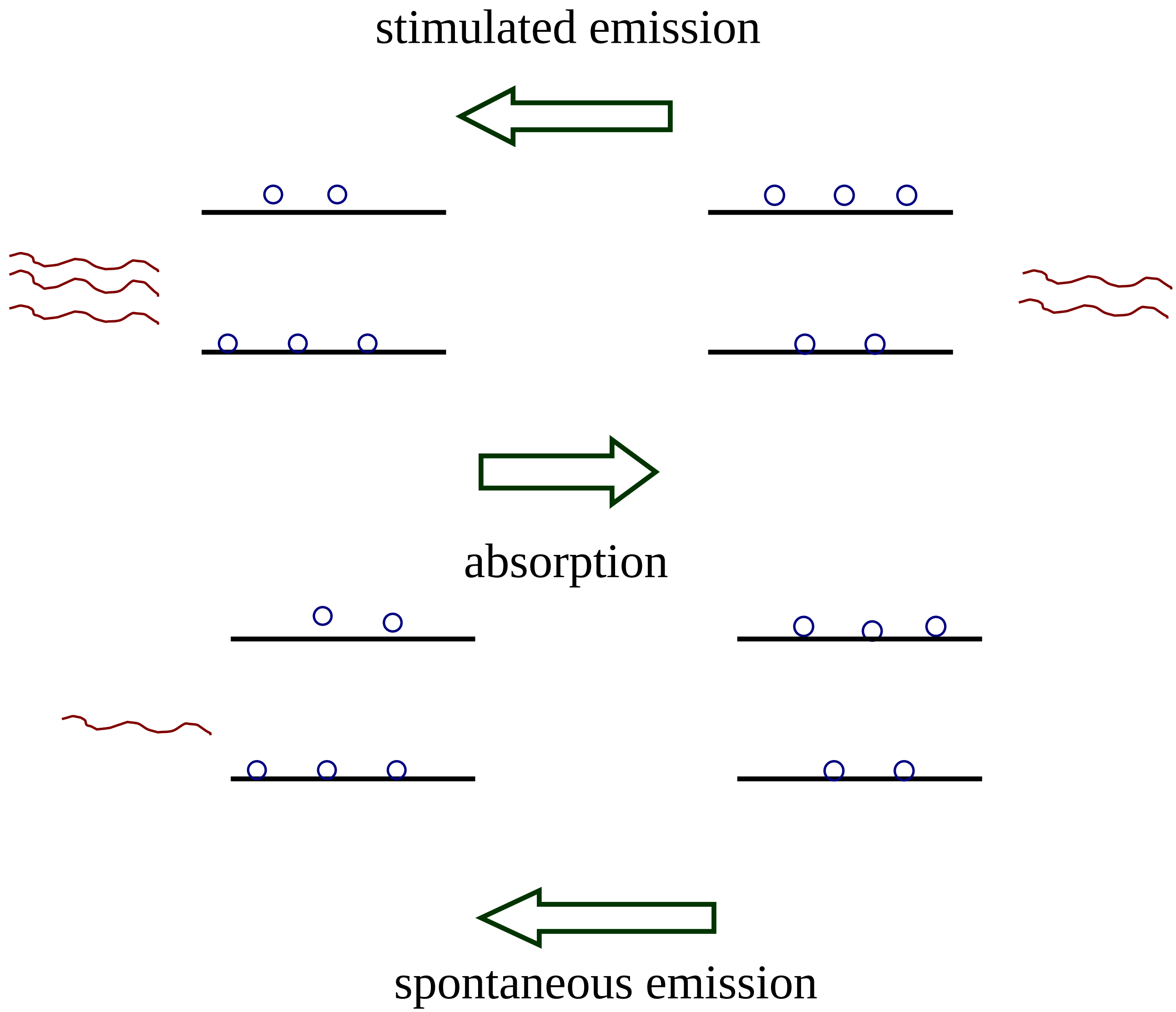
$g_1 K_{12} = g_2 K_{21} e^{-\Delta E/kT}$

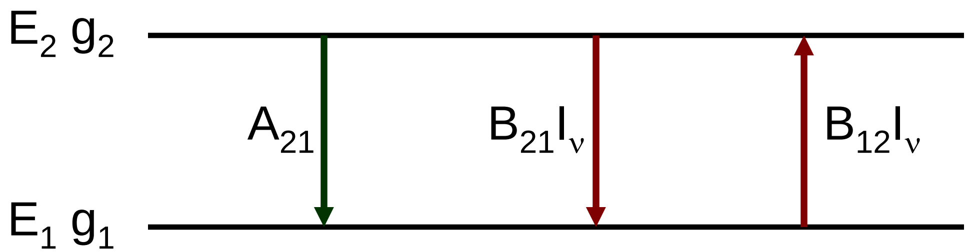
Reflection of Maxwellian

Excitation Temperature:

$$n_2/n_1 = \exp(-\Delta E/kT_x)$$

# Line Radiative Interactions






$$g_1 B_{12} = g_2 B_{21}$$

$$A_{21} = B_{21} \cdot 2h\nu^3/c^2$$

$$B_{21} \propto \mu^2_{21}$$

$$\varepsilon_\nu = A_{21} n_2 g_2 h\nu\phi(\nu)/4\pi$$

$$\kappa_\nu = B_{21}(n_1 - n_2) g_2 h\nu\phi(\nu)/4\pi$$

$$S = \frac{\varepsilon_\nu}{\kappa_\nu} = \frac{A_{21}}{B_{21}} \times \frac{1}{n_1/n_2 - 1}$$

$$S = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT_x) - 1}$$

$$\boxed{S = B_\nu(T_x)}$$

# Line Brightness

$$B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

Brightness Temperature

$$I_\nu = B_\nu(T_b)$$

$$\frac{dB_\nu(T_b)}{d\tau_\nu} = B_\nu(T_x) - B_\nu(T_b)$$

$$T_b \leq T_x$$

constant $T_x$; R-J domain, $B_\nu(T) = 2kT/\lambda^2$:

$$T_b = T_x(1 - e^{-\tau})$$

What sets the scale for $T_x$?

# If collisions always dominated…

Elastic $\Rightarrow$ Maxwellian velocity distribution; T

Inelastic $\Rightarrow$ Boltzman distribution; $T_x = T$

Radiative equilibrium:

$$I_\nu = S_\nu = B_\nu(T_x) = B_\nu(T)$$

… which, of course, they don't!

HEATING

kinetic energy redistribution; T

$\Downarrow$

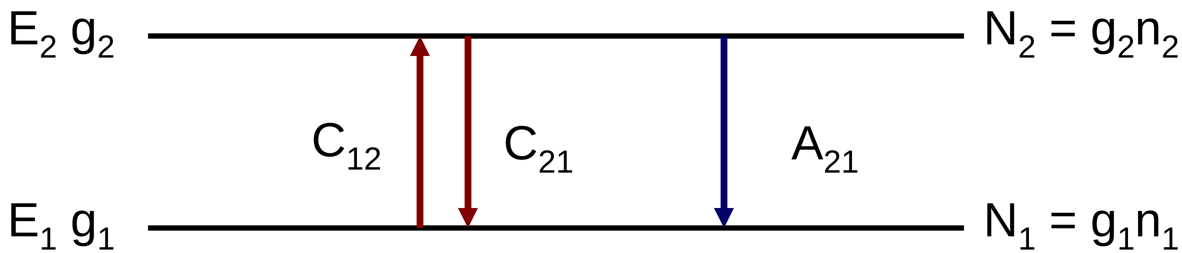
line excitations; internal energy redistribution

$\Downarrow$

line emission

COOLING

# Negligible Radiation




Steady State: $N_1 C_{12} = N_2(C_{21} + A_{21})$

$$\frac{n_2}{n_1} = \frac{g_2}{g_1}\frac{C_{12}}{C_{21} + A_{21}} = \frac{e^{-\Delta E/kT}}{1 + A_{21}/C_{21}}$$

Introduce $N_{cr} = A_{21}/K_{21}$ then

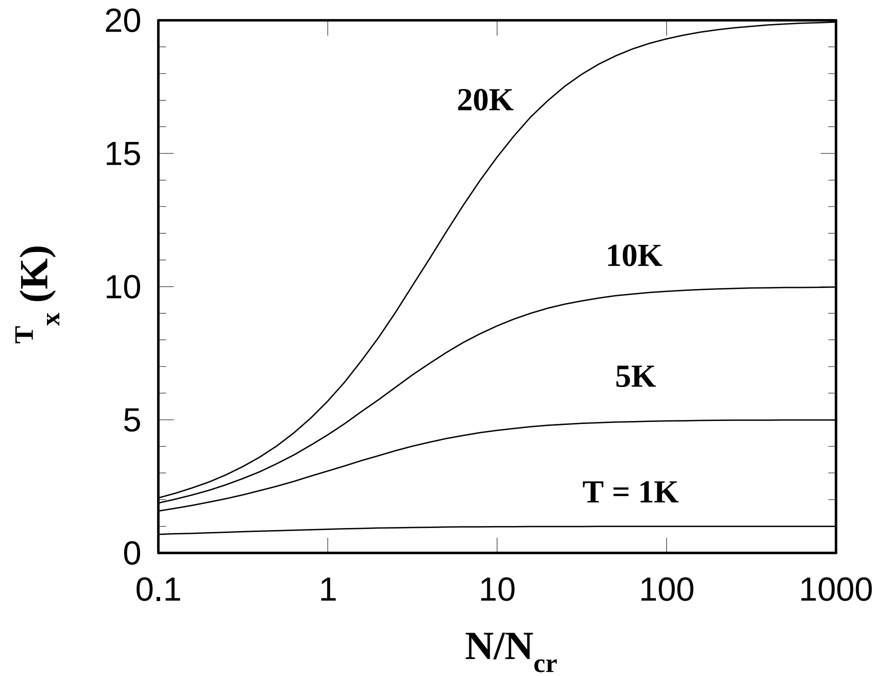
$$n_2/n_1 = e^{-\Delta E/kT}/(1 + N_{cr}/N)$$

Thermalization: $N >> N_{cr}$ …sort of

$$T_x = \frac{T}{1 + (kT/\Delta E)\ln(1 + N_{cr}/N)}$$

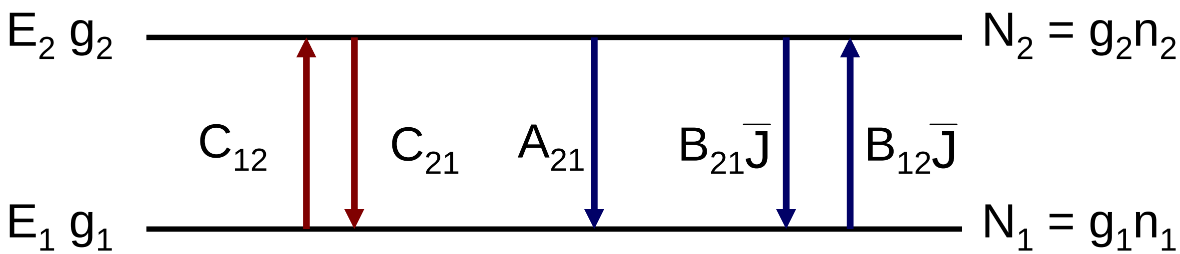
$$\approx T\left(1 + \frac{kT}{\Delta E}\frac{N_{cr}}{N}\right)^{-1}$$

CO (J = 1→0): $\lambda$ = 2.6 mm, $\Delta E/k$ = 5.5°




Thermalization: $N >> N_{cr} kT/\Delta E$

# The Complete Rate Equation




Steady State: $N_1(C_{12} + B_{12}\bar{J}) = N_2(C_{21} + A_{21} + B_{21}\bar{J})$

$$S = (1 - \varepsilon_c)\,\bar{J} + \varepsilon_c B_\nu(T)$$

$$\varepsilon_c = \frac{1}{1 + (N_{cr}/N)[1 + (c^2/2h\nu^3)B_\nu(T)]}$$

Effect of particles on the brightness: $BI_\nu n(v = c[\nu - \nu_0]/\nu_0)$

$$n\, f(v)\, dv = n\, \phi(\nu)\, d\nu$$

Effect of radiation on level populations: $B\int I_\nu (d\Omega/4\pi) n\phi(\nu) d\nu = B\bar{J}n$

$$\bar{J} = \int J_\nu \phi(\nu) d\nu \qquad J_\nu = \int I_\nu\, d\Omega/4\pi$$

The spectral shape of $I_\nu$ is not necessarily $\phi(\nu)$!

$$\frac{dI_\nu}{d\ell} = \varepsilon_\nu - \kappa_\nu I_\nu$$

$\kappa_\nu I_\nu \ll \varepsilon_\nu$ i.e. $I_\nu \ll S$ :

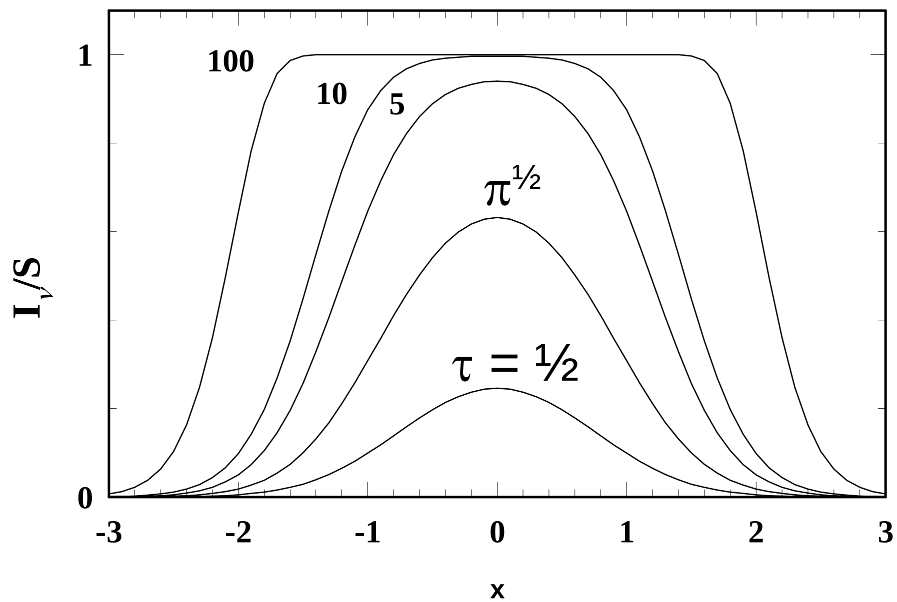
$d\, I_\nu = \varepsilon_\nu\, d\ell = S\, d\tau_\nu \propto \phi(\nu)$

But – saturation when $I_\nu$ approaches S

$d\tau_\nu = \phi(x)\, d\tau \qquad \phi(x) = \pi^{-½} \exp(-x^2) \qquad x = (\nu - \nu_0)/\Delta\nu_D$

$$\frac{dI_\nu}{d\tau} = \phi(x)(S - I_\nu)$$

Constant S: $I_\nu = S[1 - e^{-\tau\phi(x)}]$



radiative transfer: $\vec{e} \cdot \nabla I_\nu = \varepsilon_\nu - \kappa_\nu I_\nu$

$J_\nu = \int I_\nu \, d\Omega / 4\pi$ $\vec{F}_\nu = \int I_\nu \vec{e} \, d\Omega$

Flux divergence relation

$$\nabla \cdot \vec{F}_\nu = 4\pi(\varepsilon_\nu - \kappa_\nu J_\nu)$$

Line radiation: $\kappa_\nu = \phi(x)\,\kappa$ $\varepsilon_\nu = \phi(x)\,\varepsilon$

Line flux: $F = \int F_\nu d\nu$

$$\nabla \cdot \vec{F} = 4\pi \Delta\nu_D (\varepsilon - \kappa \bar{J})$$

Line Luminosity

$$L = 4\pi \Delta\nu_D \int \varepsilon \, \beta_l \, dV$$

$$\beta_l = 1 - \bar{J}/S$$

Radiative rate terms:

$$\frac{dN_2}{dt} = -A_{21}N_2 + \bar{J}(B_{12}N_1 - B_{21}N_2)$$

$$= -A_{21}N_2\left[1 - \bar{J}\,\frac{B_{21}}{A_{21}}\,\frac{n_1 - n_2}{n_2}\right]$$

$$= -A_{21}N_2\underbrace{\left[1 - \frac{\bar{J}}{S}\right]}_{\beta_1}$$

$$\boxed{\frac{dN_2}{dt} = -\beta_1 A_{21}N_2}$$

$$A \rightarrow \beta A$$

$$N_{cr} \rightarrow \beta N_{cr}$$