TIDES AND EXOPLANETS

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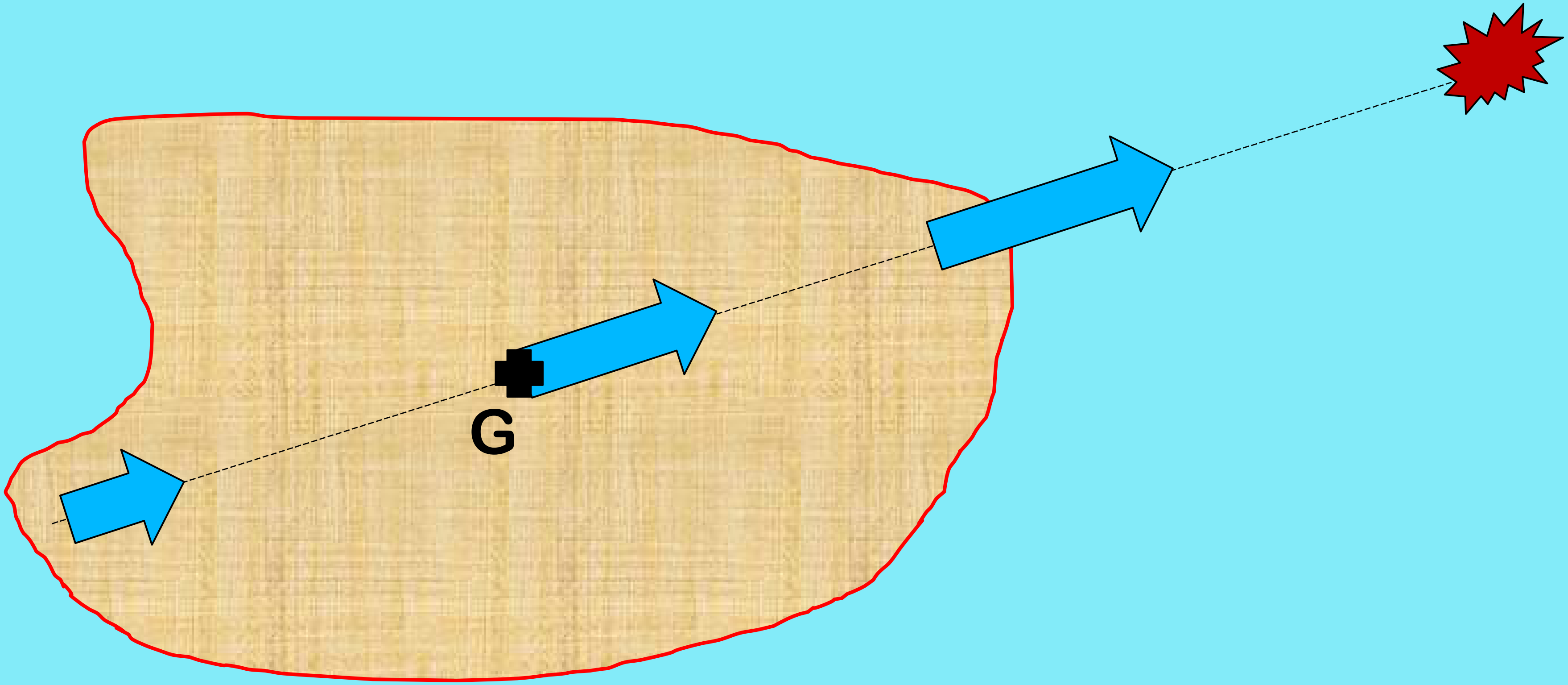
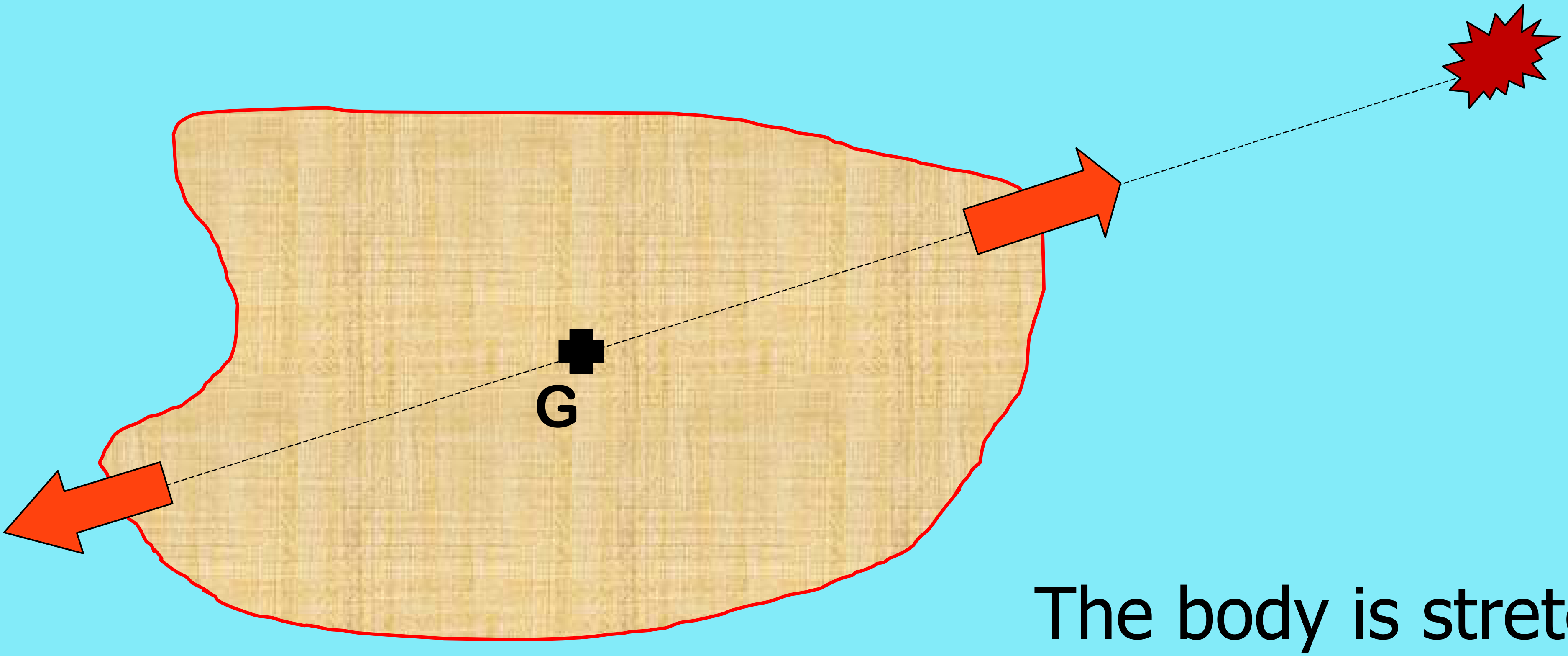
Lectures
CELTA
Inverness/Skye
2022

TIDE = Variation in the surface level of one celestial body caused by the gravitational attraction of other celestial bodies.

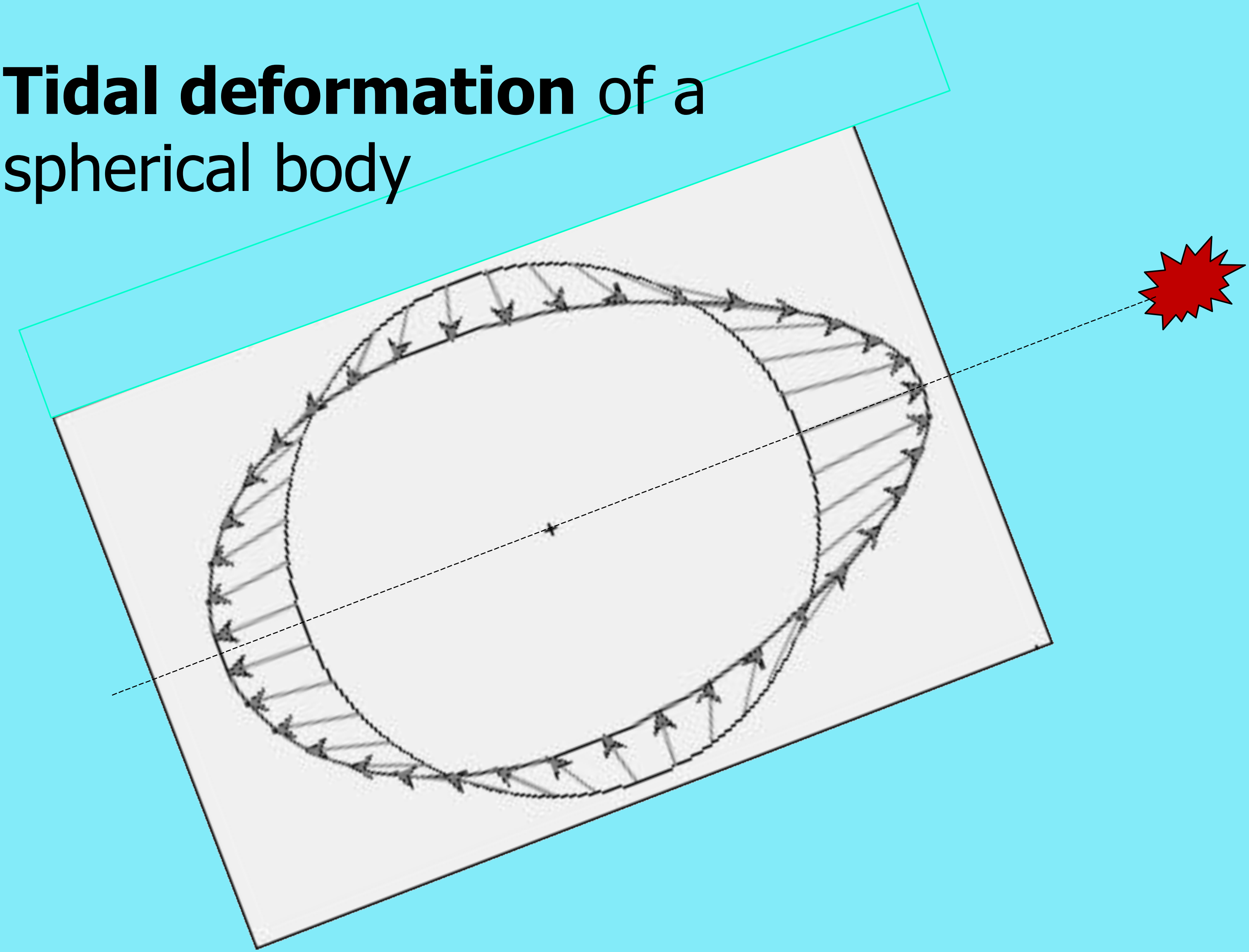


In a frame fixed in the body:

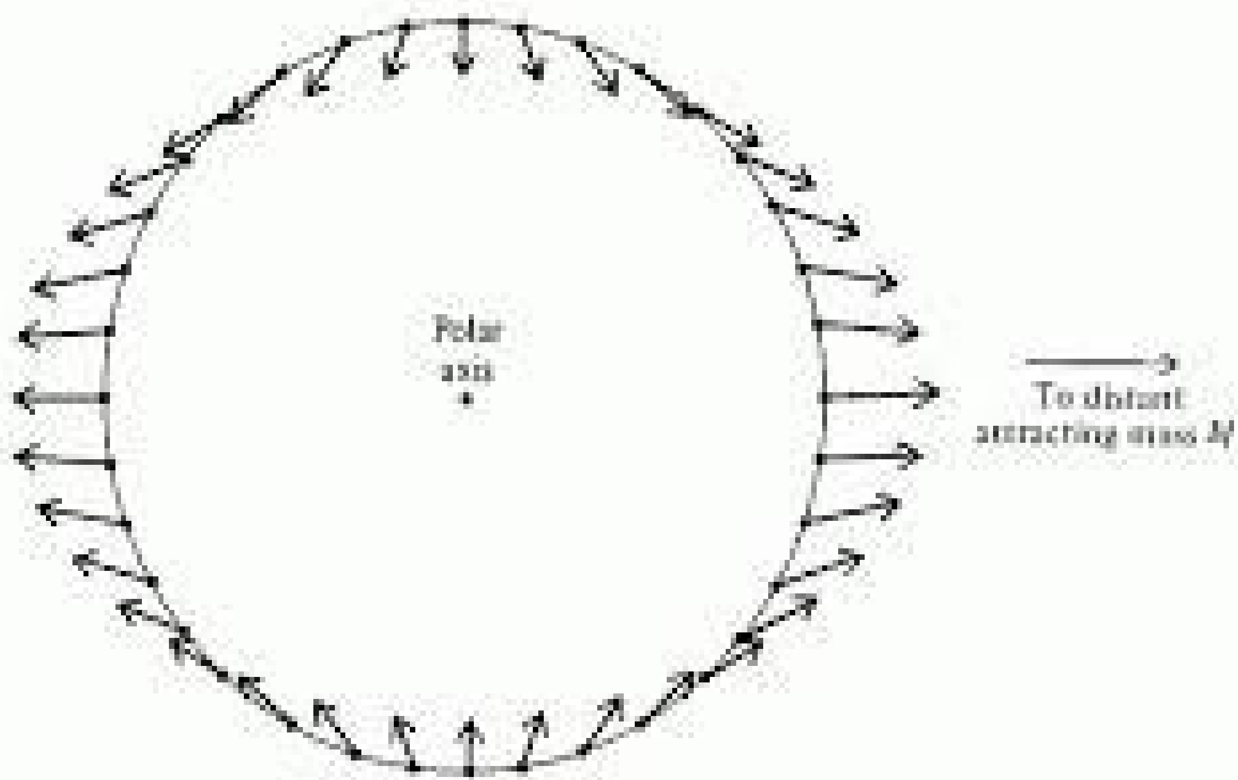


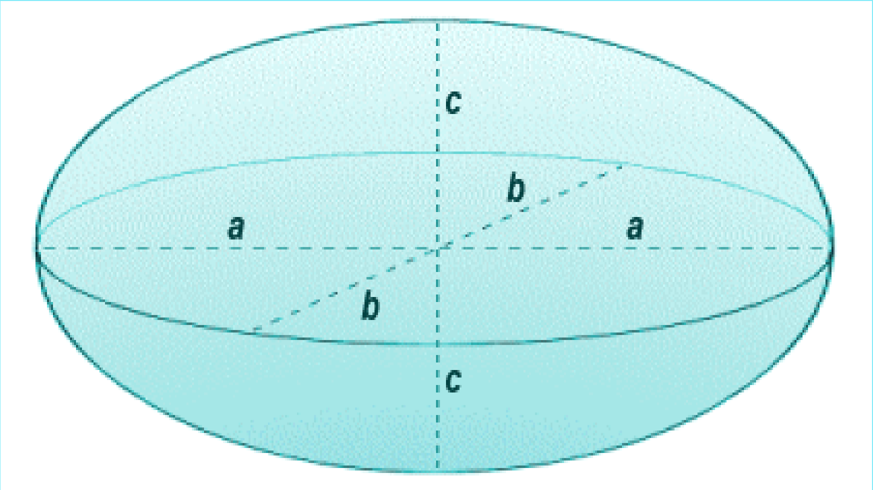
The body is stretched along the direction of the disturbing forces

Tidal deformation of a spherical body



Approximation: ellipsoid





FLATTENINGS

Equatorial $$\varepsilon_\rho = \frac{a-b}{R_e}$$

Polar $$\varepsilon_z = 1 - \frac{c}{R_e}$$

Eqn. Ellipsoid

$$\rho = R_e\left(1 + \frac{1}{2}\varepsilon_\rho \sin^2\theta \cos 2\varphi - \varepsilon_z \cos^2\theta\right)$$

N.B.

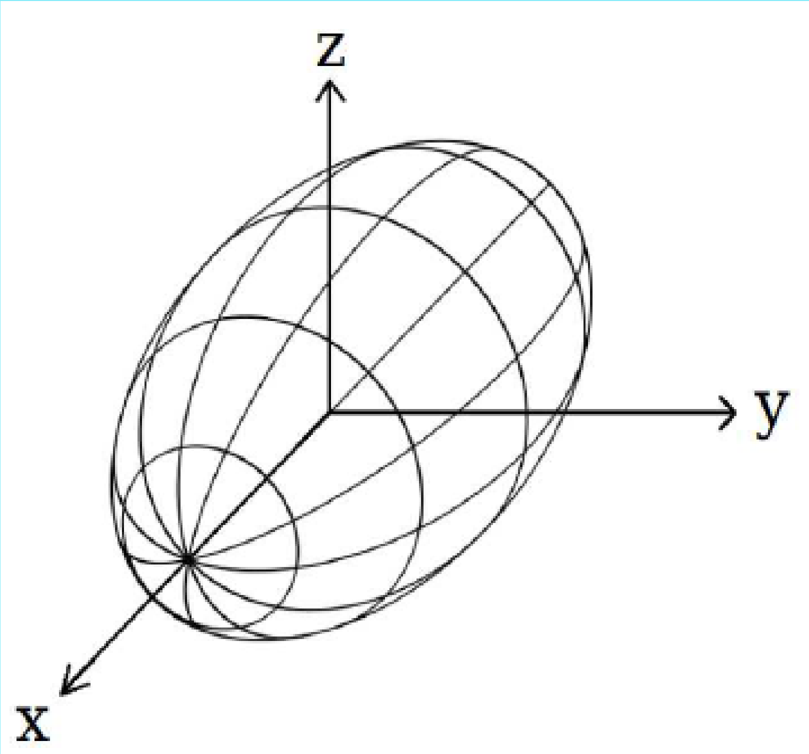
$$R_e = \sqrt{ab}$$

$$R = R_e\left(1 - \tfrac{1}{3}\varepsilon_z\right)$$

$$a = R_e(1 + \varepsilon_\rho/2)$$
$$b = R_e(1 - \varepsilon_\rho/2)$$
$$c = R_e(1 - \varepsilon_z)$$

In case of a perfect fluid (inviscid homogeneous)

Jeans spheroid (b=c) ($\Omega = 0$)



$$\epsilon_J = \frac{15}{4}\frac{M}{m}\left(\frac{R}{r}\right)^3$$

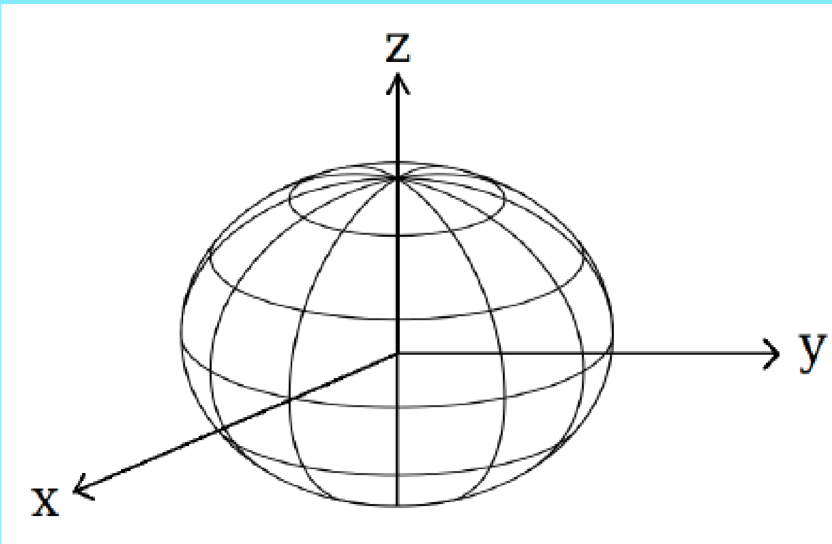
m planet mass
M star mass
R planet radius
r mutual distance

$$\varepsilon_\rho = \frac{a-b}{R_e}$$

Primary	Companion	ϵ_J	$a-b$
Earth	Moon	2.1×10^{-7}	1.34 m
Earth	Sun	9.6×10^{-8}	0.6 m
Mercury	Sun	1.7×10^{-6}	4.1 m
Venus	Sun	2.6×10^{-7}	1.5 m
Jupiter	Sun	3.0×10^{-9}	0.2 m
Jupiter	Io	8.5×10^{-7}	61 m
Moon	Earth	2.8×10^{-3}	50 m
Io	Jupiter	4.9×10^{-3}	8.2 km
Titan	Saturn	1.5×10^{-4}	0.38 km
planet CoRoT 7b	star CoRoT 7	8×10^{-3}	85 km

if free rotating body

MacLaurin spheroids



$$\epsilon_M = \frac{5R^3\Omega^2}{4mG}.$$

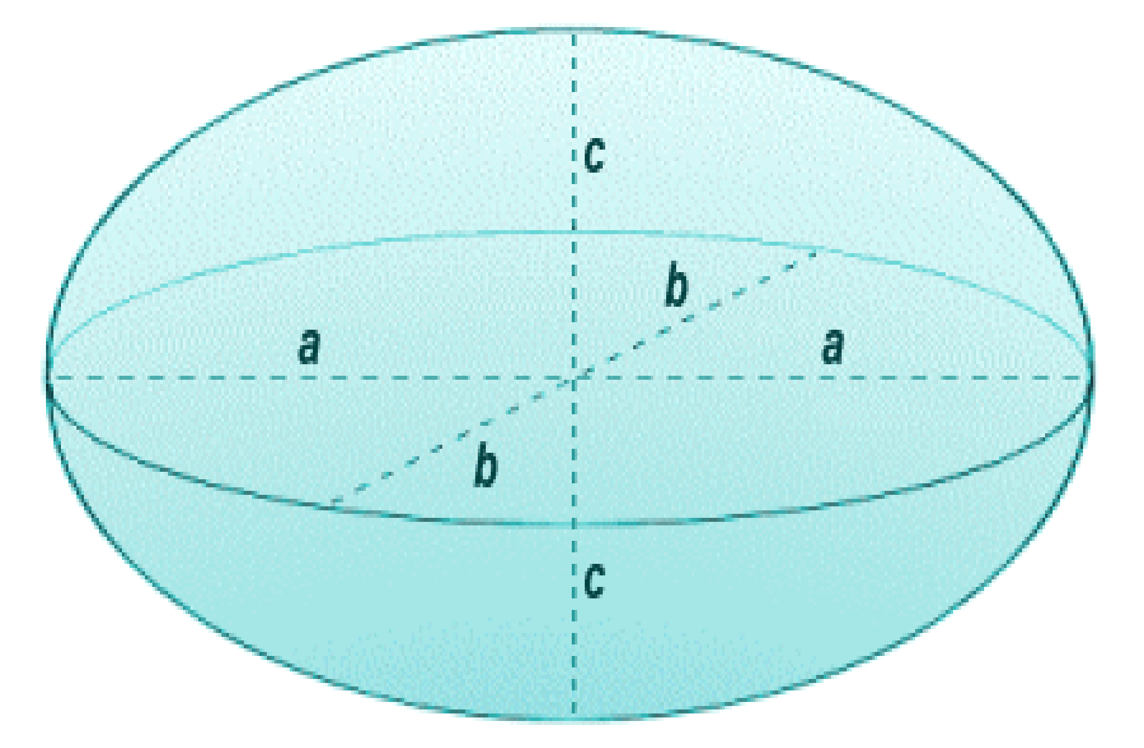
m planet mass
R planet radius
Ω planet rotation vel.

c=polar radius

$$\varepsilon_z = 1 - \frac{c}{R_e}$$

General case

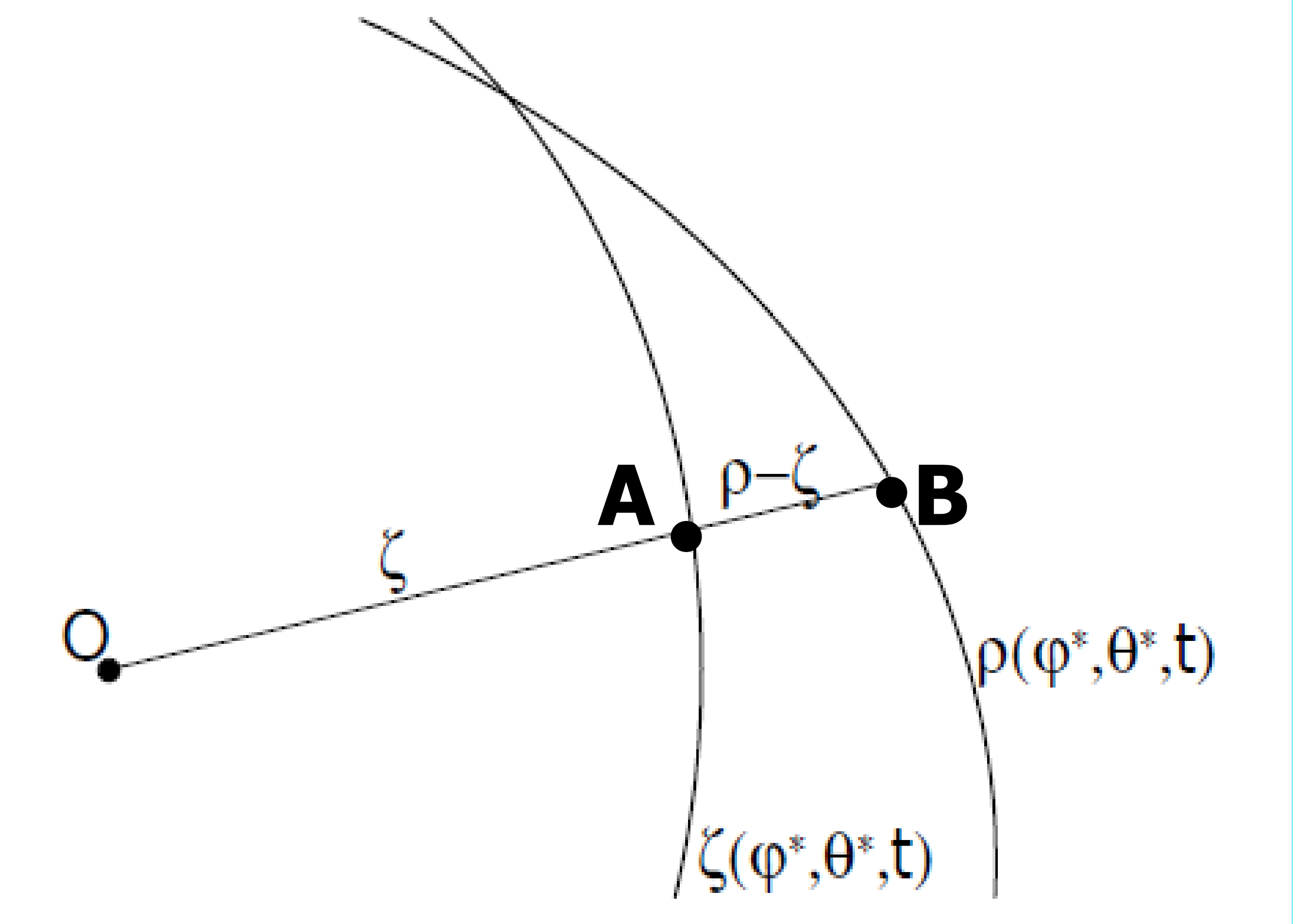
- tide + $\Omega \neq 0$ (any)



$$\varepsilon_z = \varepsilon_M + \tfrac{1}{2}\varepsilon_J$$

$$\varepsilon_\rho = \varepsilon_J$$

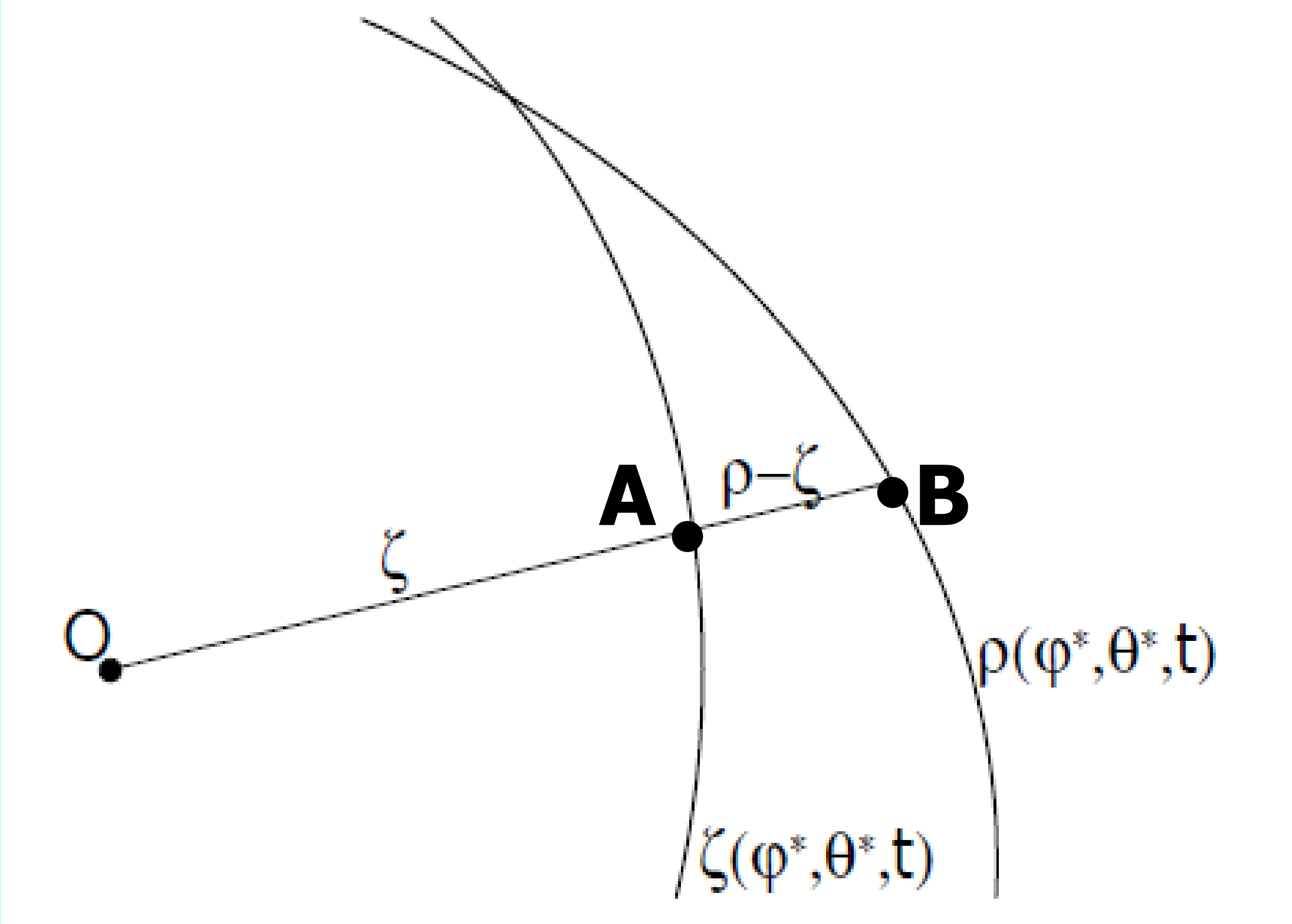
Moving Viscous Bodies:



ζ ….. Actual Surface of the body at time

ρ ….. Surface of instantaneous equilibrium (VIRTUAL)

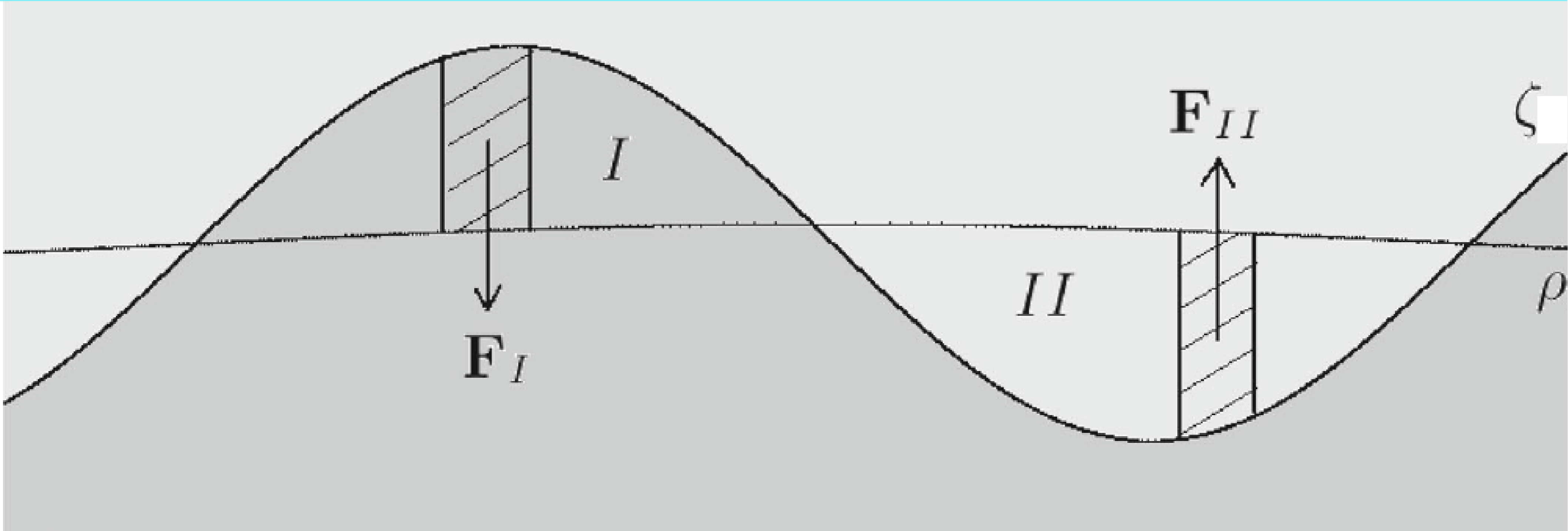
Newtonian CREEP



Relaxation factor γ

$$\dot{\zeta} = \gamma(\rho - \zeta).$$

AB decreases exponentially $e^{-\gamma t}$



ζ.... Actual surface of the body

ρ..... Equilibrum surface (static tide)

F_i... Forces ("weight")

$|P_i|=h_i\ w$ ..... Pressure

Navier-Stokes equation

Stokes approximation without Inertia and external forces

$$\text{grad } p = \eta \, \mathbf{\Delta u}$$

where $\mathbf{\Delta u} = (\Delta u_x \; \Delta u_y \; \Delta u_z)'$

η = viscosity

Consider only the radial displacement:

$$\Delta u_r = \frac{\partial^2 V_r}{\partial \zeta^2} + \frac{2}{\zeta}\frac{\partial V_r}{\partial \zeta} - \frac{2V_r}{\zeta^2} = \frac{w}{\eta}$$

Solution:

$$V_r(\zeta) = C_1\zeta + \frac{C_2}{\zeta^2} - \frac{w}{4\eta}\zeta^2$$

Boundary conditions:

$V_r(\rho) = 0$ $r=\rho$ is the equilibrium

$V_r''(\rho) = 0$ = linear approximation (i.e. Newtonian **creep**)

→ $C_1 = \rho w/6\eta$

$C_2 = \rho^4 w/12\eta$

$$V_r(\zeta) = -\gamma(\zeta - \rho)$$

$$\gamma = \frac{wR}{2\eta} = \frac{3gm}{8\pi R^2\eta}$$

NB. In Darwin's theory (1879,1880) the relaxation factor is different:

$$\wp = 4\gamma/19.$$

Ref:

Happel and Brener, Low Reynolds number Hydrodynamics, Kluwer, 1973

Darwin, 1879

Folonier & FM, CMDA, **129**(4), 2017

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S.F.M. Cel.Mech.Dyn.Ast. **116**,109,2013; **122**,359,2015 (astro-ph 1204.3957 and 1505.05384)

S.F.M. et al. (astro-ph 1707-09229)

Table 1 Typical values of the relaxation factor adopted in applications.

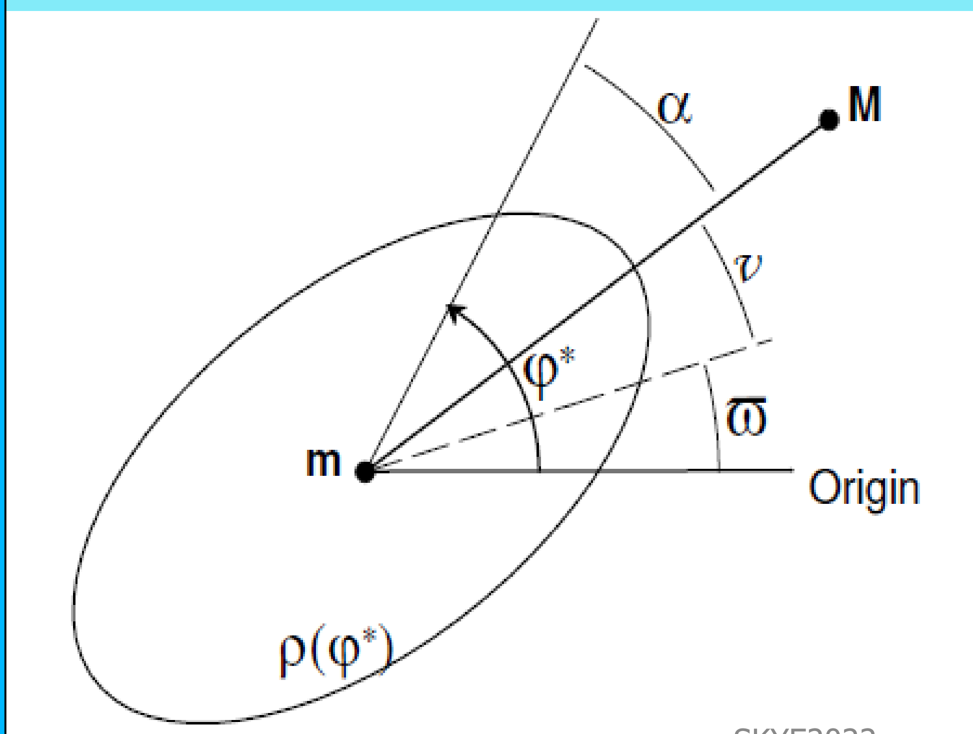
Body	γ (s^{-1})	$2\pi/\gamma$	η (Pa s)
Moon	$2.0 \pm 0.3 \times 10^{-9}$	36,000 d	$2.3 \pm 0.3 \times 10^{18}$
Titan	$2.9 \pm 0.2 \times 10^{-8}$	2500 d	$1.1 \pm 0.1 \times 10^{17}$
Solid Earth	$0.9 - 3.6 \times 10^{-7}$	200-800 d	$4.5 - 18 \times 10^{17}$
Io	$4.9 \pm 1.0 \times 10^{-7}$	730 d	$1.2 \pm 0.3 \times 10^{16}$
Europa	$1.8 - 8.0 \times 10^{-7}$	90–400 d	$4 - 18 \times 10^{15}$
Neptune	2.7–19	< 2 s	$1.2 - 4.8 \times 10^{10}$
Saturn	> 7.2	< 0.9 s	$< 15 \times 10^{10}$
Jupiter	23 ± 4	~ 0.3 s	$4.7 \pm 0.9 \times 10^{10}$
hot Jupiters	8–50	0.1–0.8 s	$5 \times 10^{10} - 10^{12}$
solar-type stars	> 30	< 0.2 s	$< 2 \times 10^{12}$

The CREEP TIDE theory

$$\dot{\zeta} + \gamma\zeta = \gamma\rho$$

O.D.E. for $\zeta(t)$

$$= \gamma R\left(1 + \frac{1}{2}\epsilon_\rho \sin^2\hat{\theta}\cos(2\hat{\varphi} - 2\omega - 2v) - \epsilon_z\left(\cos^2\hat{\theta} - \frac{1}{3}\right)\right)$$



$$\hat{\varphi} \approx \Omega t$$

$$\epsilon_\rho = \frac{a_e - b_e}{R_e} = \frac{15}{4}\left(\frac{M}{m}\right)\left(\frac{R_e}{r}\right)^3$$

$$\epsilon_z = \frac{\epsilon_\rho}{2} + \frac{5}{4}\frac{\Omega^2 R^3}{Gm}$$

Expand r.h.s. using for r(t) and v(t) their Keplerian expressions.

Introduce the Cayley functions (a.k.a. Hansen)

$$E_{q,p}(e) = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{a}{r}\right)^3 \cos\left(qv + (p-q)\ell\right) d\ell$$

Examples

$$E_{2,-1} = \frac{7}{2}e - \frac{123}{16}e^3 + \frac{489}{128}e^5 - \frac{1763}{2048}e^7$$

$$E_{2,0} = 1 - \frac{5}{2}e^2 + \frac{13}{16}e^4 - \frac{35}{288}e^6$$

$$E_{2,1} = -\frac{1}{2}e + \frac{1}{16}e^3 - \frac{5}{384}e^5 - \frac{143}{18432}e^7$$

& cttes.

$$\bar{\epsilon}_\rho = \frac{15}{4}\left(\frac{M}{m}\right)\left(\frac{R_e}{a}\right)^3$$

$$\bar{\varepsilon}_Z = \varepsilon_M$$

$$\dot{\zeta}+\gamma\zeta = \gamma R\Big(1+\frac{1}{2}\overline{\epsilon}_\rho \sin^2\widehat{\theta}\sum_{k\in\mathbb{Z}} E_{2,k}\cos\left(2\widehat{\varphi}+(k-2)\ell-2\omega\right)$$

$$-\overline{\epsilon}_z(\cos^2\widehat{\theta}-\frac{1}{3})-\frac{1}{2}\overline{\epsilon}_\rho(\cos^2\widehat{\theta}-\frac{1}{3})\sum_{k\in\mathbb{Z}} E_{0,k}\cos k\ell\Big)$$

Hypotheses (restrictive)

homogeneous bodies

orthogonal case $\theta^* = \pi/2$

(equator = orbital plane)

$$\dot{\zeta}+\gamma\zeta=\gamma R\Big(1+\frac{1}{2}\overline{\epsilon}_\rho\sin^2\widehat{\theta}\sum_{k\in\mathbb{Z}}E_{2,k}\cos\left(2\widehat{\varphi}+(k-2)\ell-2\omega\right)$$

$$-\overline{\epsilon}_z(\cos^2\widehat{\theta}-\frac{1}{3})-\frac{1}{2}\overline{\epsilon}_\rho(\cos^2\widehat{\theta}-\frac{1}{3})\sum_{k\in\mathbb{Z}}E_{0,k}\cos k\ell\Big)$$

$\hat{\varphi} \approx \Omega\, t$

$\ell = nt + cte$

Working hypothesis: $d\Omega/dt \approx 0$
(SFM, 2013-2015; Darwin theories)

→ Nonhomogeneous 1st order EDO with constant coefficients

Solution

$$\zeta = C(\hat{\varphi},\hat{\theta}).e^{-\gamma t} + R + \delta\zeta$$

$$\delta\zeta = R\sum_{k\in\mathbb{Z}} \Big(\mathcal{C}_k \sin^2\widehat{\theta}\cos\overline{\sigma}_k \cos(\Theta_k - \overline{\sigma}_k) + \mathcal{C}''_k(\cos^2\widehat{\theta} - \frac{1}{3})\cos\overline{\sigma}''_k \cos(k\ell - \overline{\sigma}''_k)\Big)$$

where

$$\mathcal{C}_k = \frac{1}{2}\overline{\epsilon}_\rho E_{2,k}$$

$$\mathcal{C}''_k = -\frac{1}{2}\overline{\epsilon}_\rho E_{0,k} - \delta_{0,k}\overline{\epsilon}_z$$

and

$$\Theta_k = 2\widehat{\varphi} + (k-2)\ell - 2\omega$$

$$\Theta''_k = k\ell.$$

$$\sigma_k = \arctan\left(\frac{kn+\nu}{\gamma}\right)$$

phases of the forced terms

$\nu = 2\Omega - 2n$

$$\delta\zeta = R_e \sum_{k\in\mathbb{Z}} \Big(\mathcal{C}_k \sin^2\widehat{\theta} \cos\overline{\sigma}_k \cos(\Theta_k - \overline{\sigma}_k) + \mathcal{C}''_k \cos^2\widehat{\theta} \cos\overline{\sigma}''_k \cos(\Theta''_k - \overline{\sigma}''_k) \Big)$$

Superposition of tidal bulges

ellipsoids with prolatenesses:

$$\epsilon_k = \frac{15}{4} E_k(e) \cos\sigma_k \left(\frac{M}{m}\right)\left(\frac{R}{a}\right)^3$$

N.B

$$\mathcal{C}_k = \frac{1}{2}\overline{\epsilon}_\rho E_{2,k}$$
$$\mathcal{C}''_k = -\frac{1}{2}\overline{\epsilon}_\rho E_{0,k} - \delta_{0,k}\overline{\epsilon}_z$$

$$\sigma_k = \arctan\left(\frac{kn+\nu}{\gamma}\right).$$

$$\delta U_k = -\frac{3GmR^2}{5r^3}\,\mathcal{C}_k \cos\overline{\sigma}_k \sin^2\theta \cos(2\varphi - \beta_k)$$

$$-\frac{GmR^2}{5r^3}\mathcal{C}''_k \cos\overline{\sigma}''_k (3\cos^2\theta - 1)\cos\beta''_k$$

where

$$\begin{aligned}\beta_k &= (2-k)\ell + 2\omega + \overline{\sigma}_k \\ \beta''_k &= k\ell - \overline{\sigma}''_k.\end{aligned}$$

ℓ = mean anomaly

Finally,

$$\delta \mathrm{f} = -M.\mathrm{grad}_{\mathrm{r}}\delta U$$

We obtain (sums over k)

$$F_{1k} = -\frac{3GMmR^2}{5r^4}\left(3C_k \cos\overline{\sigma}_k \cos(2v-(2-k)\ell-\overline{\sigma}_k) - C_k'' \cos\overline{\sigma}_k'' \cos(k\ell-\overline{\sigma}_k'')\right)$$

$$F_{2k} = 0$$

$$F_{3k} = -\frac{6GMmR^2}{5r^4} C_k \cos\overline{\sigma}_k \sin(2v-(2-k)\ell-\overline{\sigma}_k)$$

and

Torque

$$M_{1k} = 0$$

$$M_{2k} = \frac{6GMmR^2}{5r^3} C_k \cos\overline{\sigma}_k \sin(2v-(2-k)\ell-\overline{\sigma}_k)$$

$$M_{3k} = 0$$

(sum over k)

Tidal Evolution - 1

Rotation of the Primary: $C\dot{\Omega} = M_2$

Important: $M_Z = -M_2$

N.B. neglect variation of C

From: $C\dot{\Omega} = M_2$

$$\dot{\Omega} = -\frac{3GM\overline{\epsilon}_\rho}{2a^3} \sum_{k\in\mathbb{Z}} E_{2,k} \cos\overline{\sigma}_k \sum_{j+k\in\mathbb{Z}} E_{2,k+j} \sin(j\ell + \overline{\sigma}_k),$$

where
$$\sin 2\sigma_k = \frac{2\gamma(\nu + kn)}{\gamma^2 + (\nu + kn)^2}$$

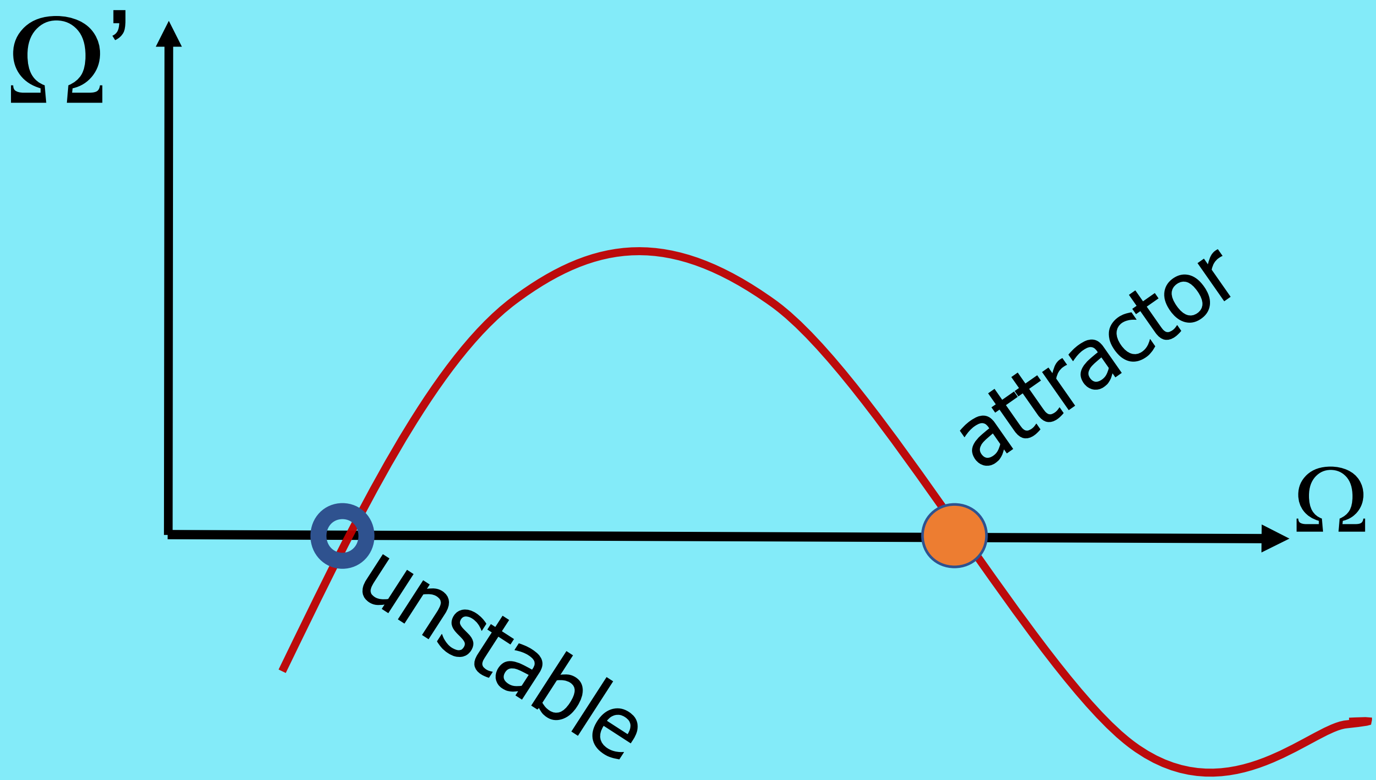
ν is the semi-diurnal frequency = $2\Omega - 2\dot{\lambda}$

FIRST-order non-linear o.d.e.

Instead of the analytical study,

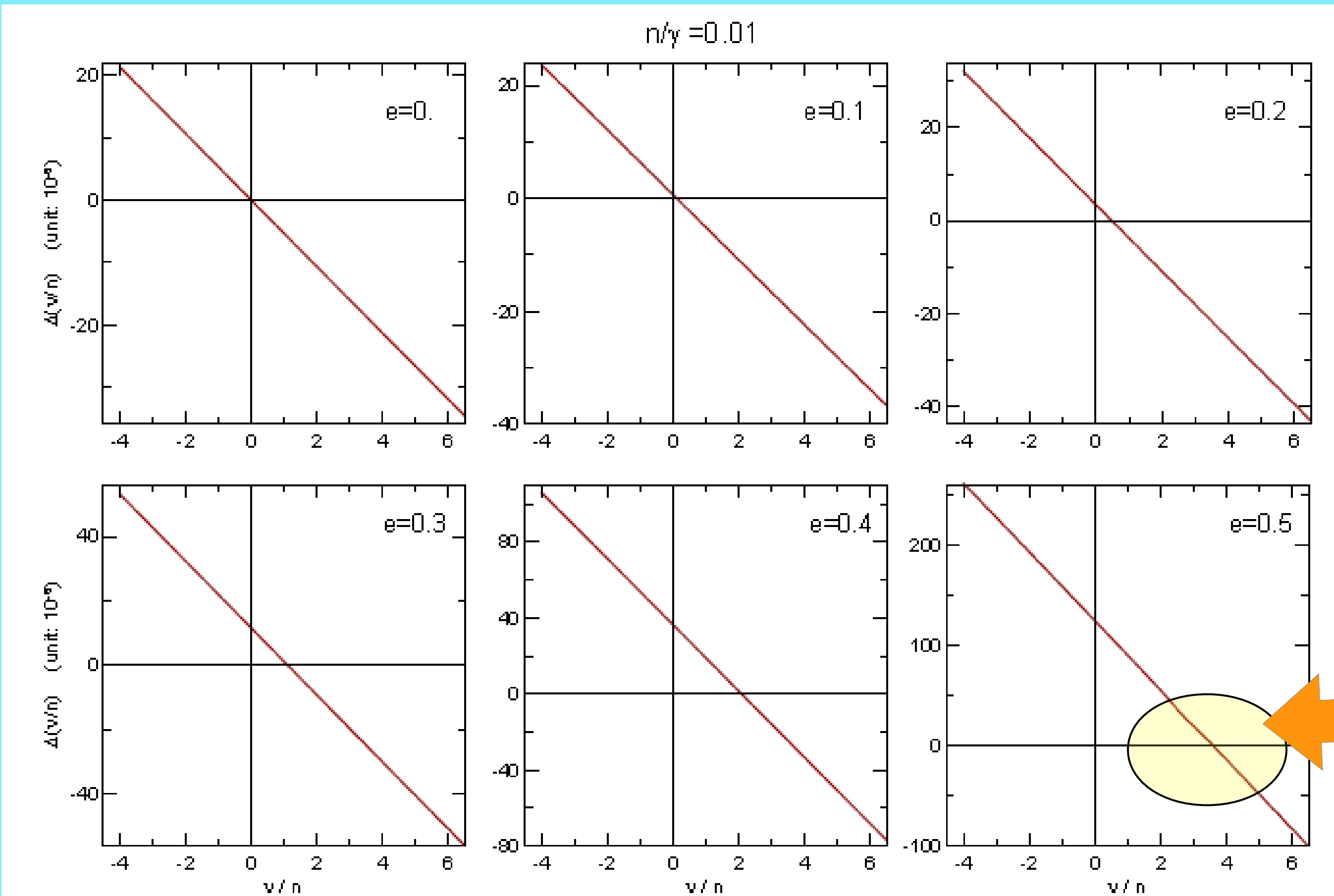
We use here the map

$$\Omega(\ell) \longrightarrow \Omega(\ell + 2\pi) - \Omega(\ell)$$



The intersections with the axis Ω'=0are stationary solutions

Case $\gamma >> n$
(ex: hot Jupiters)



The intersections with the axis $\Omega'=0$ are attractors

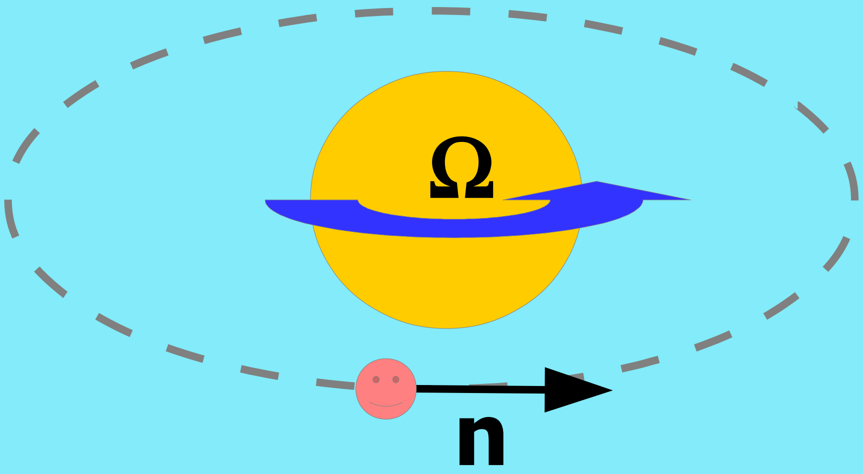
N.B.

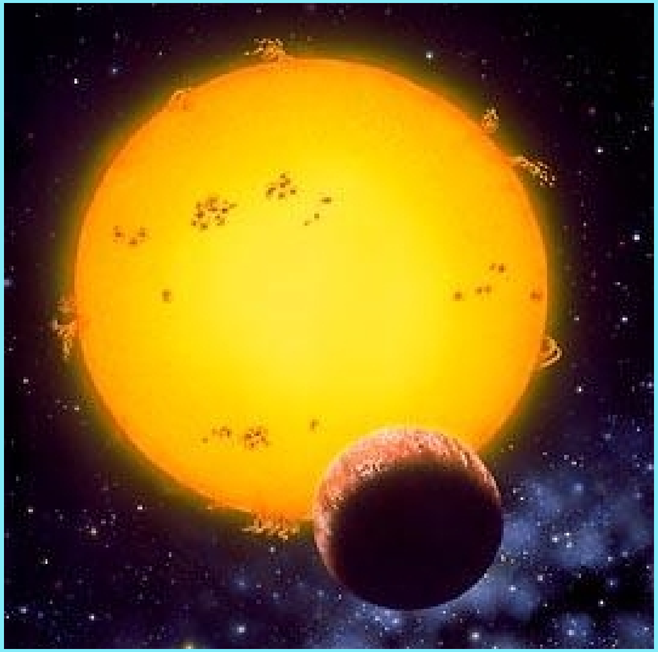
These attractors are **supersynchronous**

$$\Omega = n + 6ne^2$$

Ref: SFM, DDA 2014 (astro-ph 1204.3957) & CMDA (2015); Correia et al. A&A 2013.

- Synchronous (1:1 resonance) $\Omega = n \quad (\nu = 0)$
- Supersynchronous rotation $\Omega > n \quad (\nu > 0)$



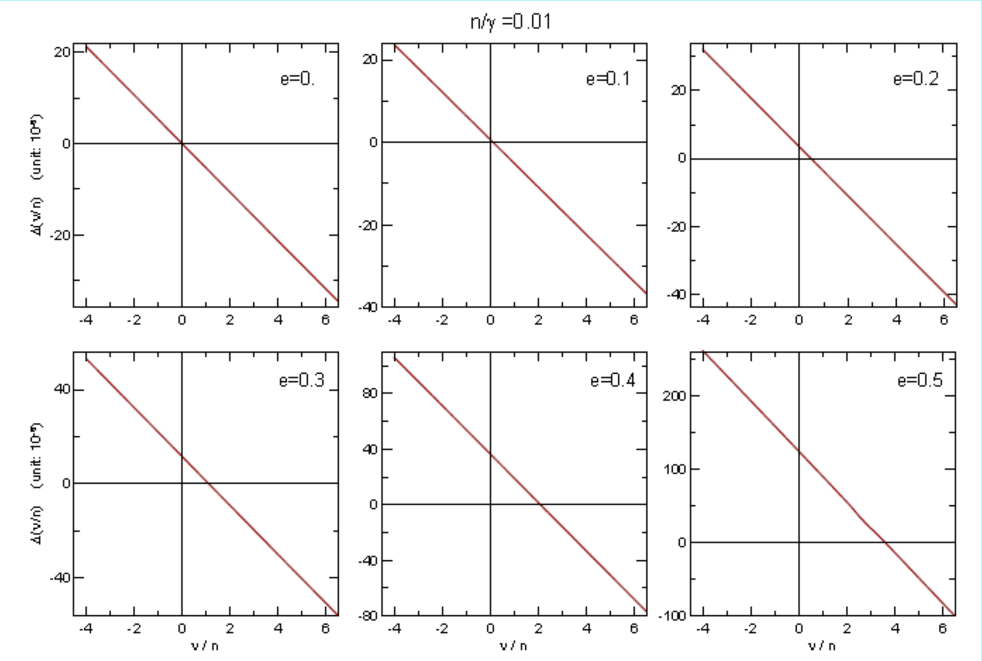


n = mean motion (mean angular orbital velocity)

$\nu = 2\Omega - 2n$ = frequency of the semi-diurnal tide

Application: HOST STARS

(high γ)



Examples:

CoRoT 15b BD (m=63.3 Jup) around a F7V star
Orbital period: 3.06 d
Star rotation: 2.9 – 3.1 d

KELT 1b BD (=27.4 Jup) around a F5 star
Orbital period: 1.217 d
Star rotation: (1.348 ± 0.4) sin I (d)

Solar-type stars are affected by wind braking

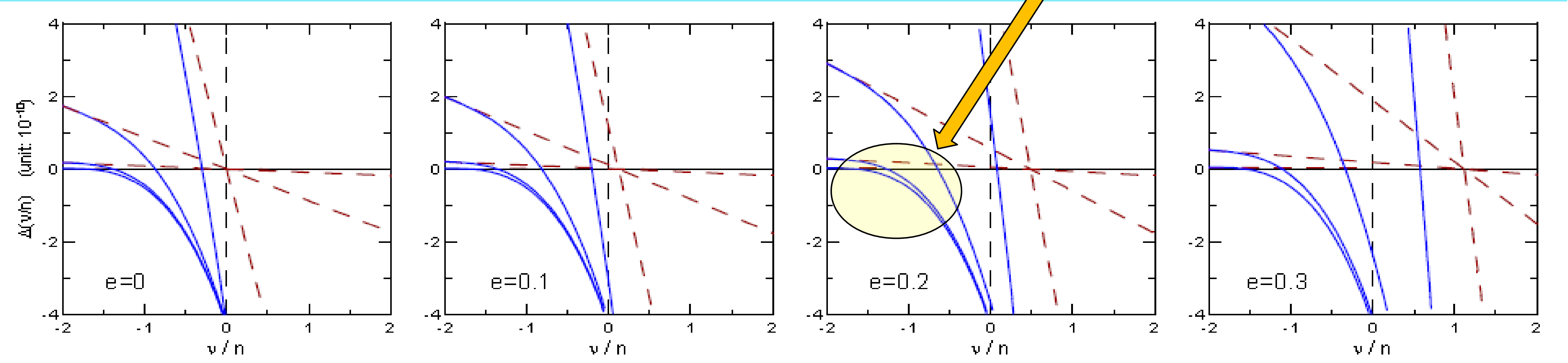
$$\dot{\Omega} = -f_P B_W \Omega^3$$

Ref: Bouvier et al. 1997
Skumanich, 1972

where

$$B_W = 2.7 \times 10^{47} \frac{1}{C} \sqrt{\left(\frac{R}{R_\odot} \frac{M_\odot}{m}\right)} \quad \text{(cgs units)}$$

subsynchronous atractors



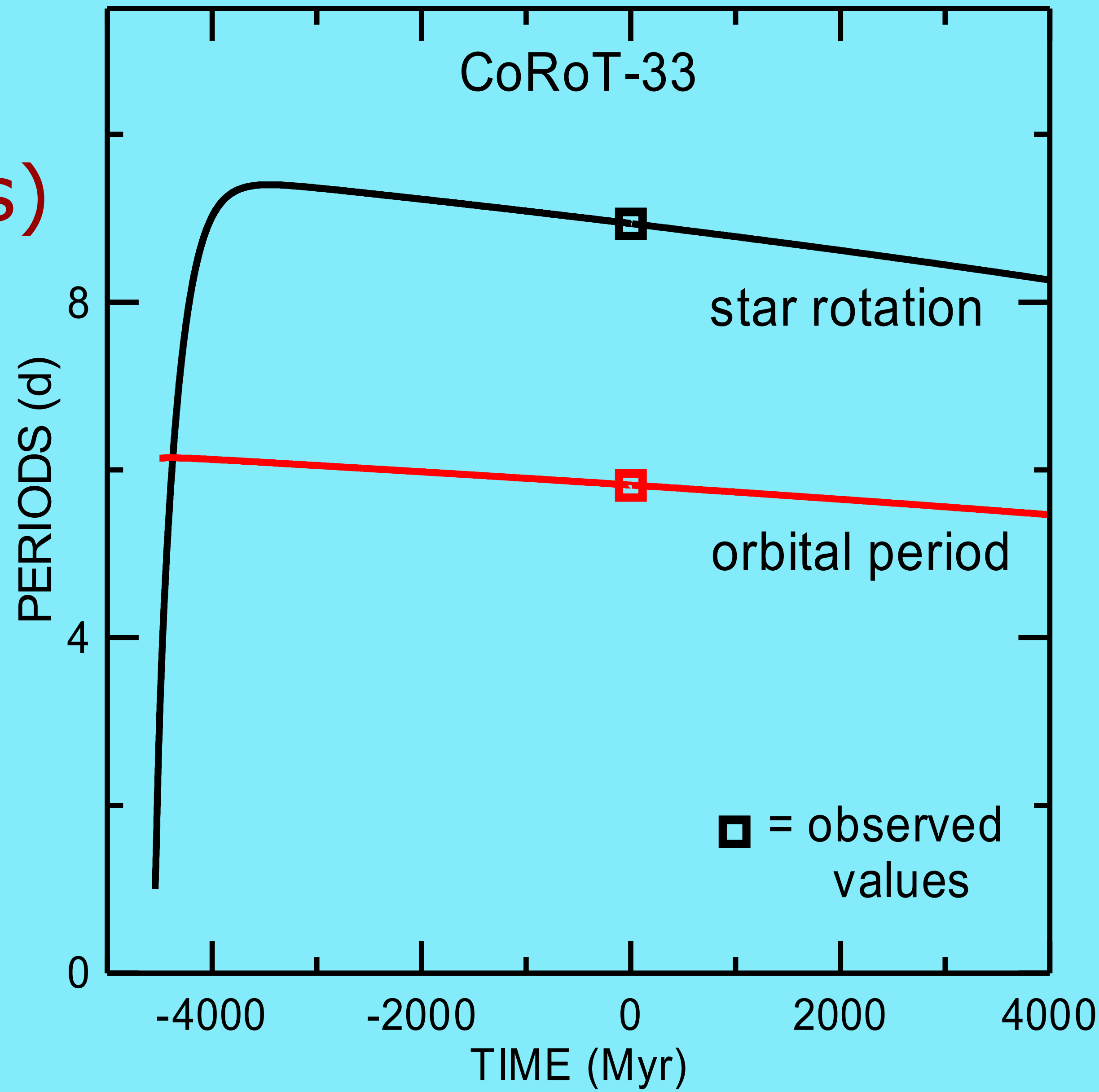
$n/\gamma = 10^{-6}$ to 10^{-3} [brown: no brake; blue: with brake]

CoRoT 33:

A paradigm (subsynchronous)

G9V
M~0.86 Sun
age >4.6 Gyr

CoRoT-33b
M~59 Jup (BD)
P~5.8 days
e~0.07
$\gamma \sim$ **36** s^{-1}



After ~1 Gyr the tidal interaction is stronger than the wind braking

- and the stellar rotation accelerates.
- Ref:SFM et al. Astrophys. J. (2015) and Corot Legacy Book (2016).

SYNCHRONIZATION OF STIFF BODIES

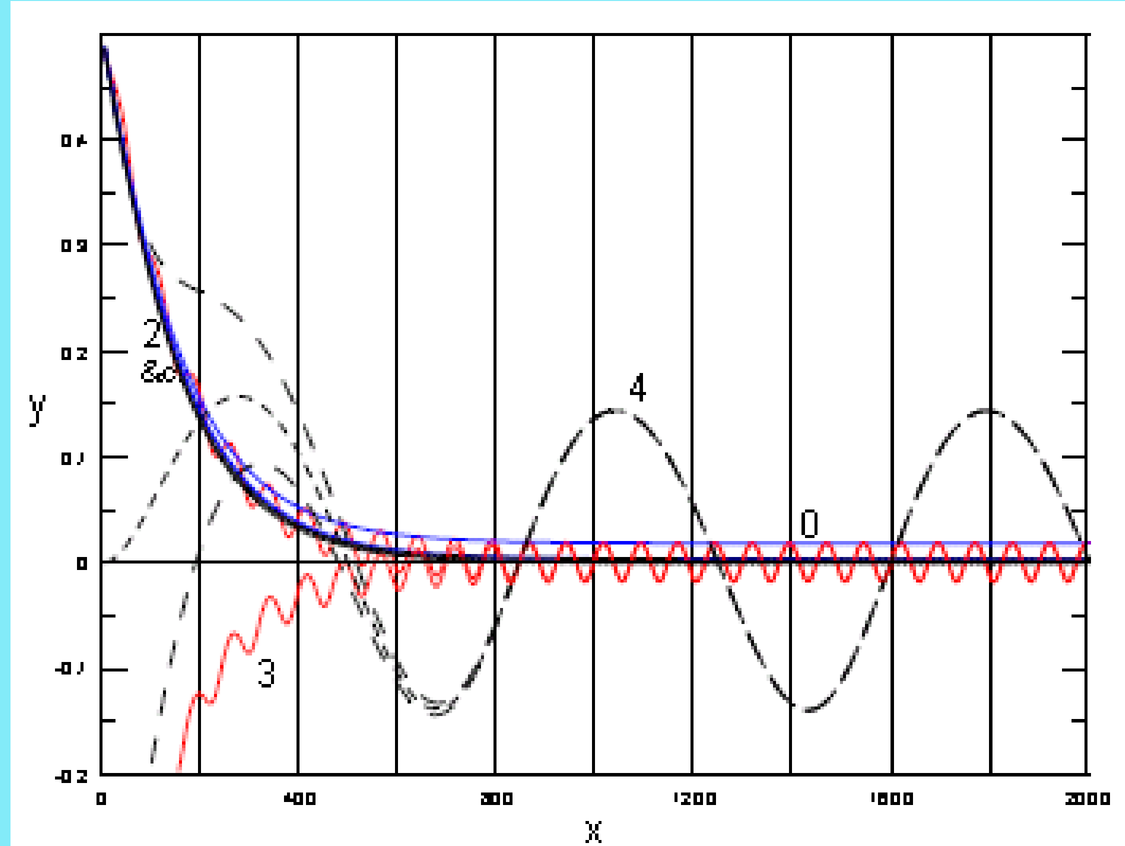
Simulations
near ν=0

(normalized variables)

$$y = \nu/\gamma$$

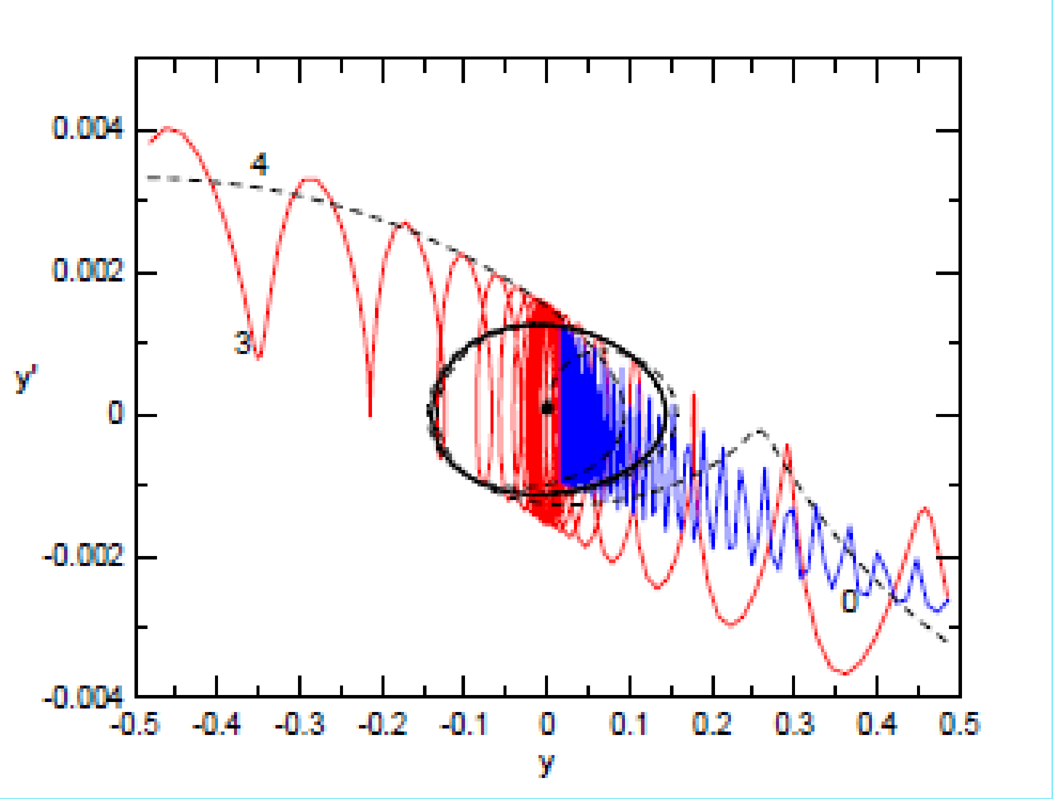
$$x = (n/\gamma)(t - t_0)$$

Parameter $\log_{10} n/\gamma$



The attractor (plane Ω–Ω′)

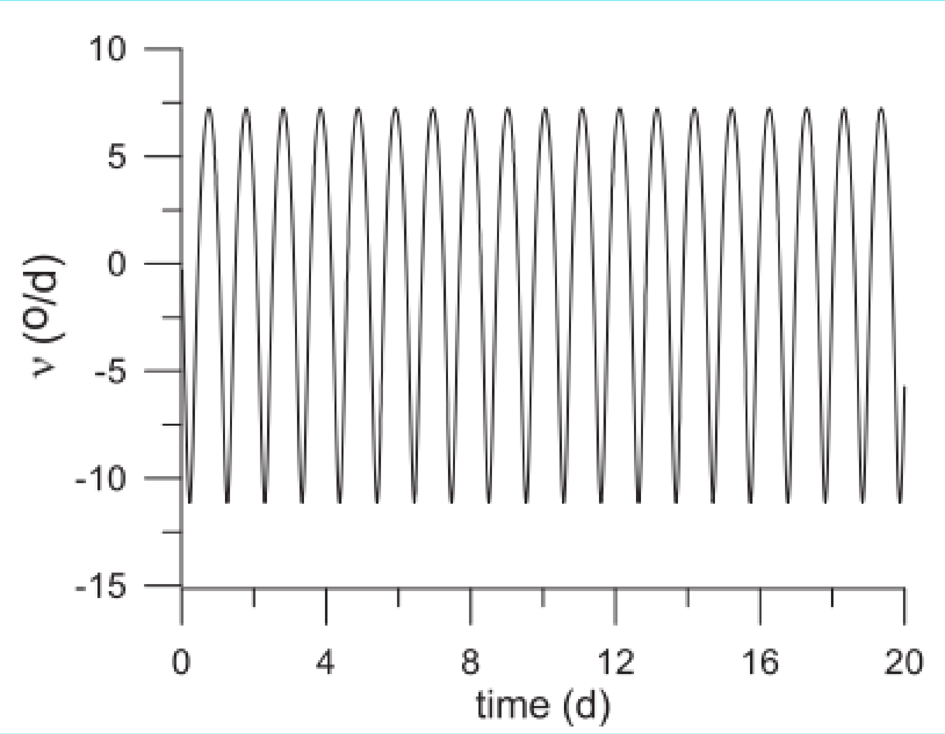
$$y = \nu/\gamma$$



Parameter: $\log_{10} n/\gamma$

<u>Forced oscillation about synchronism</u>

Example: Semi-diurnal frequency of an Earth-like exoplanet in stationary rotation (a=0.02 AU, e=0.2)



Ferraz-Mello et al. 2020

Working hypothesis $d\Omega/dt \sim 0$ not satisfied.
In this case a different theory is necessary.

Parametric version of the creep tide theory
(Folonier et al. 2018, Ferraz-Mello et al, 2020)
The variable $\zeta(\phi,\theta)$ is substituted by a set of 3 parameters defining the orientation and the shape of the tidally deformed ellipsoid.

The equations can be integrated together with the equation for $d\Omega/dt$ without the need of a working hypotheses for Ω.

Tidal Evolution - 2

Variation of the semi-major axis

$$\dot{a} = \frac{2}{na}\frac{\partial R}{\partial \ell}$$

$$R = -(1+M/m)\delta U$$

Compare to the work done by the disturbing force:

$$\dot{W} = \delta \mathbf{f} \cdot \mathbf{V} = -M.\mathrm{grad}_{\mathbf{r}}\delta U \cdot \mathbf{V} = -Mn\frac{\partial \delta U}{\partial \ell}.$$

$$\dot{a} = \frac{2a^2}{GmM}\dot{W}.$$

N.B. In the 2-body problem: $W = -GmM/2a$

Variation of the semi-major axis

$$\dot{a} = \frac{2}{na}\frac{\partial R}{\partial \ell} \qquad R = -(1 + M/m)\delta U$$

Compare to the work done by the disturbing force:

$$\dot{W} = \delta \mathbf{f} \cdot \mathbf{V} = -M.\mathrm{grad}_{\mathbf{r}}\delta U \cdot \mathbf{V} = -Mn\frac{\partial \delta U}{\partial \ell}.$$

$$\dot{a} = \frac{2a^2}{GmM}\dot{W}.$$

N.B. In the 2-body problem: $W = -GmM/2a$

Variation of the eccentricity

$$\dot{e} = -\frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial \mathcal{R}}{\partial \omega} + \frac{1-e^2}{na^2 e}\frac{\partial \mathcal{R}}{\partial \ell}$$

or

$$\dot{e} = \frac{1-e^2}{e}\left(\frac{\dot{a}}{2a} - \frac{\dot{\mathcal{L}}}{\mathcal{L}}\right)$$

where $\mathcal{L} = \frac{GMm}{na}\sqrt{1-e^2})$

$$\dot{\mathcal{L}} = M_z$$

LOW-ECCENTRICITY FORMULAS

$$[\langle \dot{a} \rangle]_i = \frac{3k_{2i} n m_j R_i^5}{m_i a^4} \left((1 - 5e^2) \frac{\gamma_i \nu_i}{\gamma_i^2 + \nu_i^2} - \frac{3e^2}{4} \frac{\gamma_i n}{\gamma_i^2 + n^2} + \frac{e^2}{8} \frac{\gamma_i (\nu_i + n)}{\gamma_i^2 + (\nu_i + n)^2} + \frac{147e^2}{8} \frac{\gamma_i (\nu_i - n)}{\gamma_i^2 + (\nu_i - n)^2} \right) + \mathcal{O}(e^4).$$

$$[\langle \dot{e} \rangle]_i = -\frac{3k_{2i} n e m_j R_i^5}{4 m_i a^5} \left(\frac{\gamma_i \nu_i}{\gamma_i^2 + \nu_i^2} + \frac{3}{2} \frac{\gamma_i n}{\gamma_i^2 + n^2} + \frac{1}{4} \frac{\gamma_i (\nu_i + n)}{\gamma_i^2 + (\nu_i + n)^2} - \frac{49}{4} \frac{\gamma_{(} \nu_i - n)}{\gamma_i^2 + (\nu_i - n)^2} \right) + \mathcal{O}(e^3).$$

see SFM 2022 (IAU Symp. 364)

$\nu = 2\Omega - 2\dot{\lambda}$

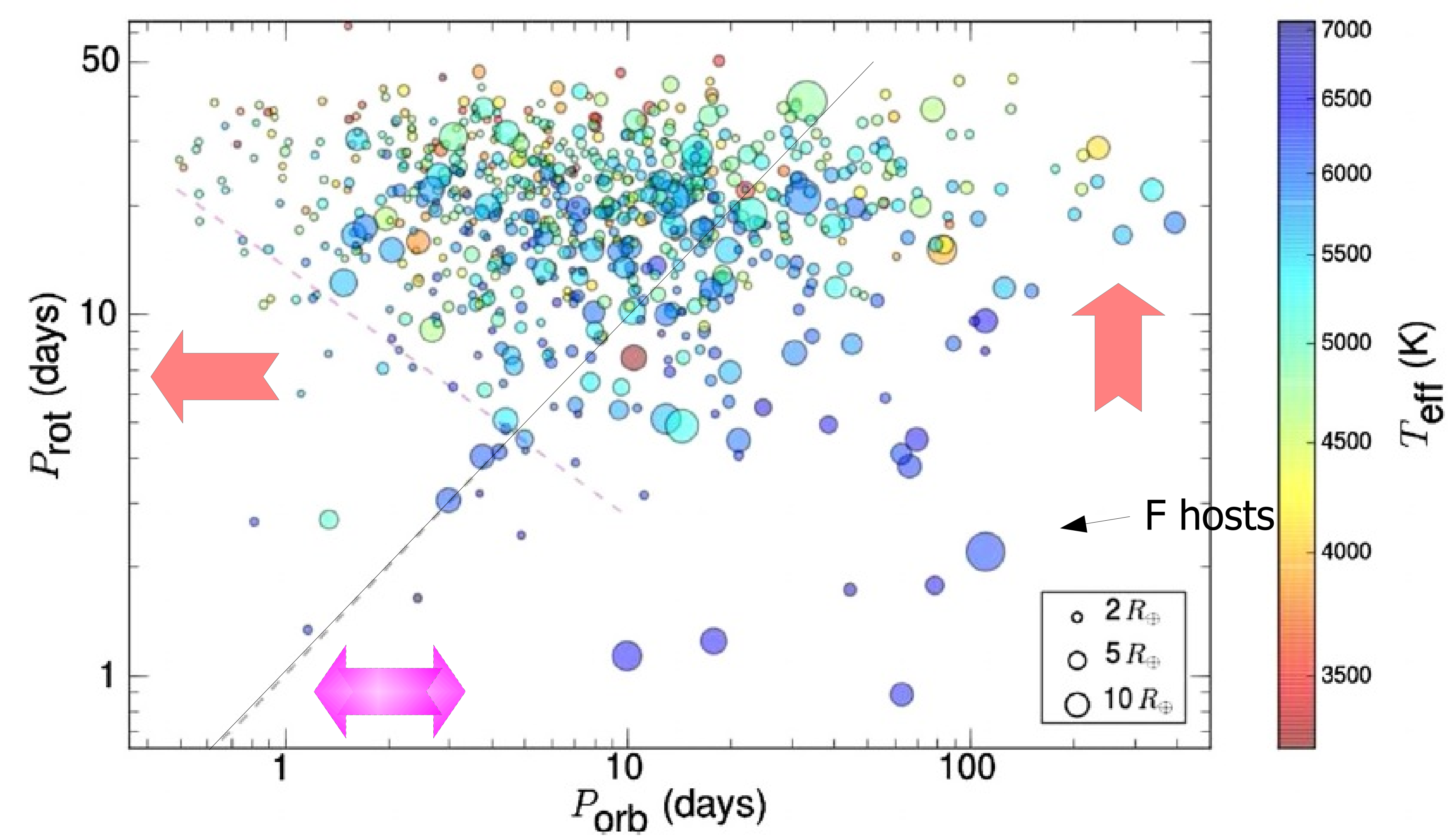
Role of $\nu = 2\Omega - 2\dot{\lambda}$

$$[\langle \dot{a} \rangle]_i = \frac{3k_{2i} n m_j R_i^5}{m_i a^4} (1 - 5e^2) \frac{\gamma_i \nu_i}{\gamma_i^2 + \nu_i^2} + \mathcal{O}(e^2)$$

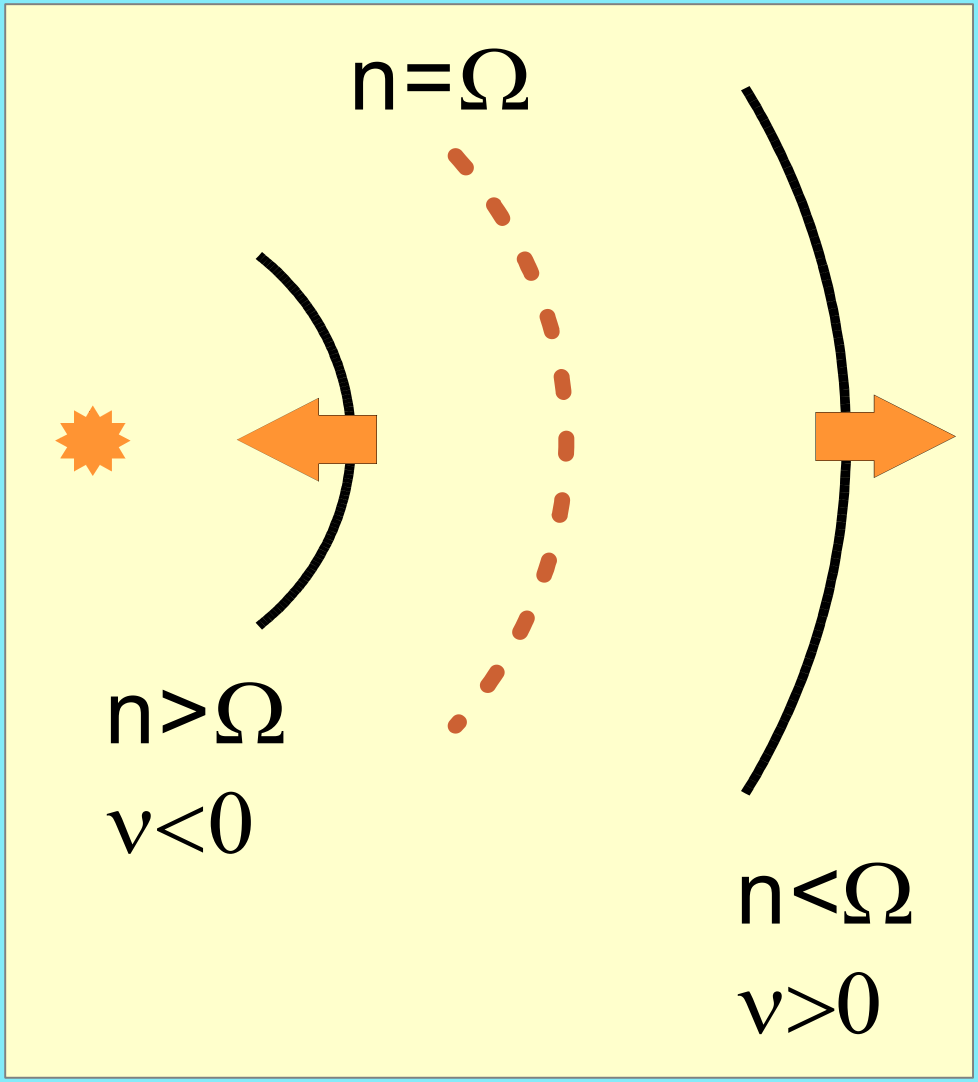
Planetary satellites $\quad n>0 \rightarrow da/dt > 0$

Exoplanets $\quad$ both signs are possible
(example > close-in resonant chains)

Kepler KOI population (2013)

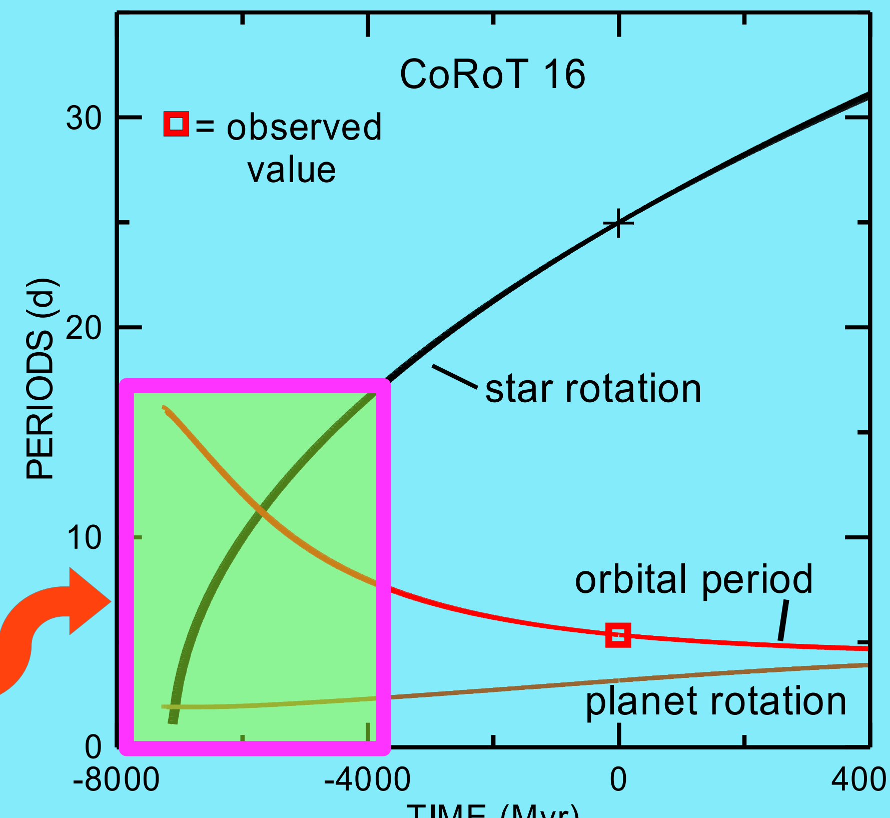


Ref: McQuillan et al. ApJL 2013



<u>N.B.</u>:
This is true only for small eccentricities

watch here



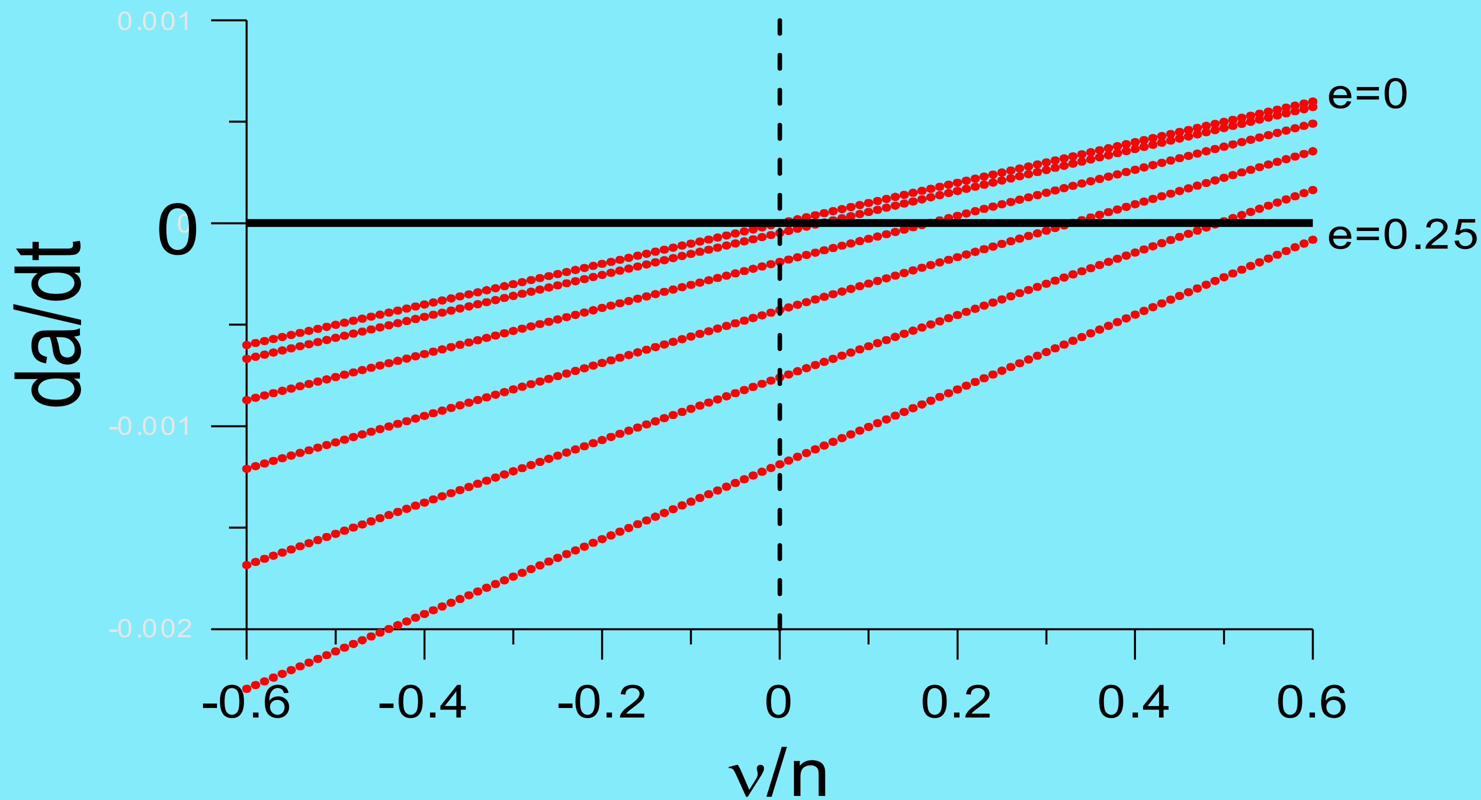
Current e = 0.33

$$[\langle\dot{a}\rangle]_i = \frac{3k_{2i}nm_jR_i^5}{m_ia^4}\left((1-5e^2)\frac{\gamma_i\nu_i}{\gamma_i^2+\nu_i^2} - \frac{3e^2}{4}\frac{\gamma_i n}{\gamma_i^2+n^2}\right.$$
$$\left. + \frac{e^2}{8}\frac{\gamma_i(\nu_i+n)}{\gamma_i^2+(\nu_i+n)^2} + \frac{147e^2}{8}\frac{\gamma_i(\nu_i-n)}{\gamma_i^2+(\nu_i-n)^2}\right) + \mathcal{O}(e^4).$$

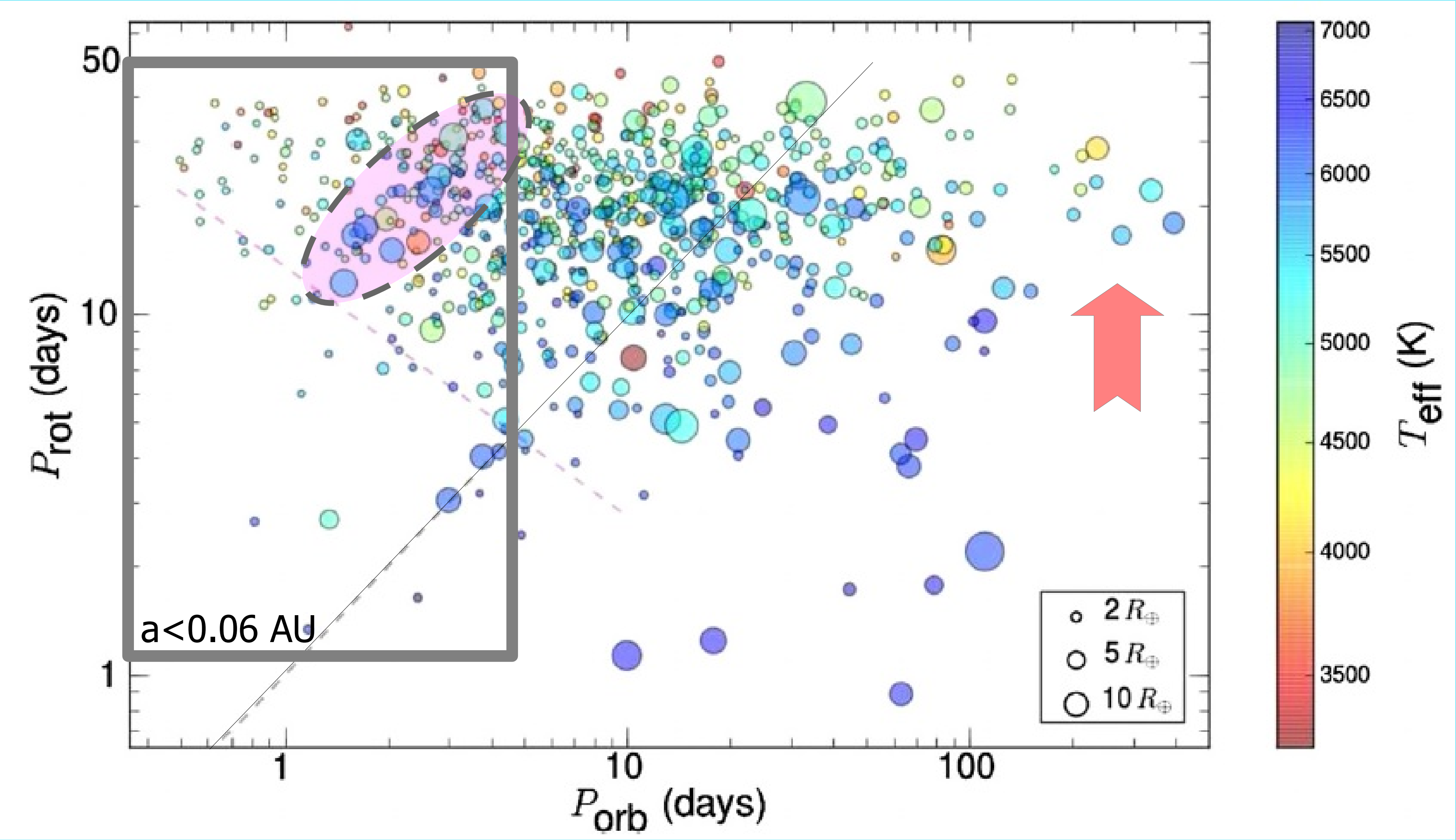
i=star (<u>tide in the star</u>)
j=planet

$\nu = 2\Omega - 2n$

$$[\langle\dot{a}\rangle]_i = \frac{3k_{2i}nm_jR_i^5}{m_ia^4}\left((1-5e^2)\frac{\gamma_i\nu_i}{\gamma_i^2+\nu_i^2} - \frac{3e^2}{4}\frac{\gamma_i n}{\gamma_i^2+n^2} + \frac{e^2}{8}\frac{\gamma_i(\nu_i+n)}{\gamma_i^2+(\nu_i+n)^2} + \frac{147e^2}{8}\frac{\gamma_i(\nu_i-n)}{\gamma_i^2+(\nu_i-n)^2}\right) + \mathcal{O}(e^4).$$

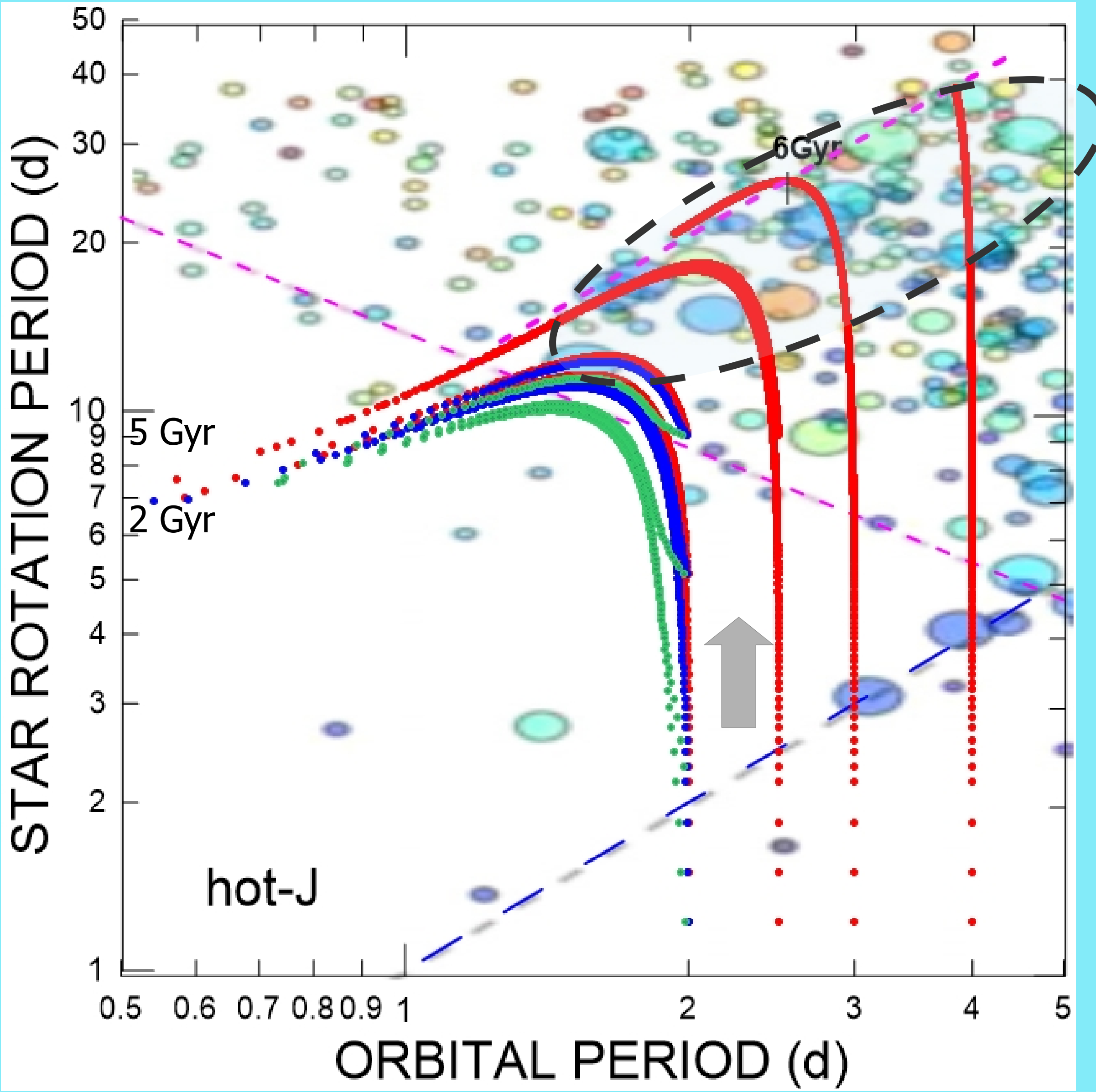


Accumulation of close-in hot Jupiters



N.B. Rjup ~ 11 $R_\oplus$

Validation of the Tidal Model and the Wind Braking of host G-stars via Evolutionary Tracks in P-P diagram

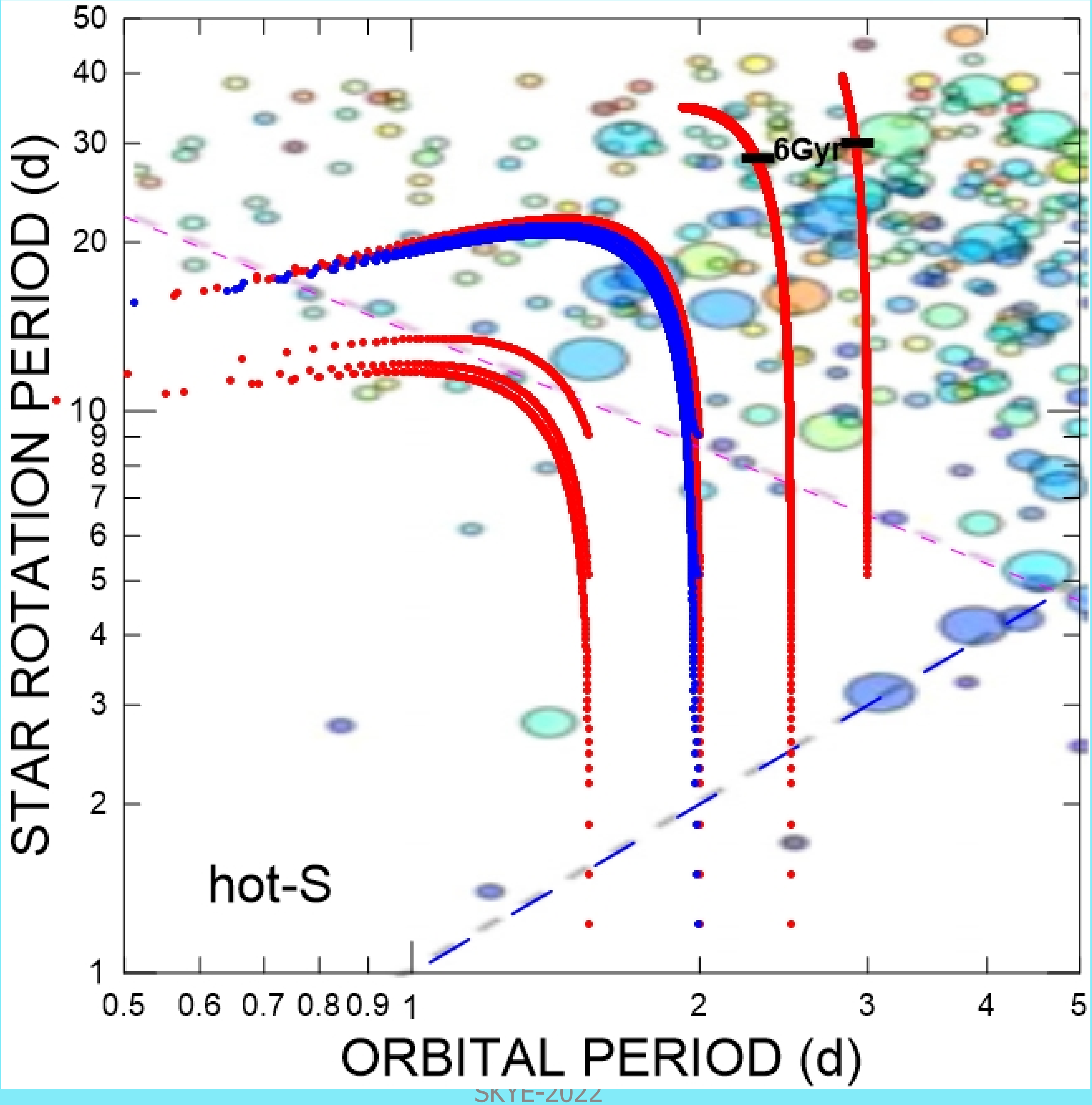


Accumulation of Hot Jupiters

Evolutionary tracks of Hot Jupiters around Sun-like star

γ=50 /s
Red: e=0
Blue: e=0.1
Green: e=0.2

Evolutonary tracks of Hot Saturns

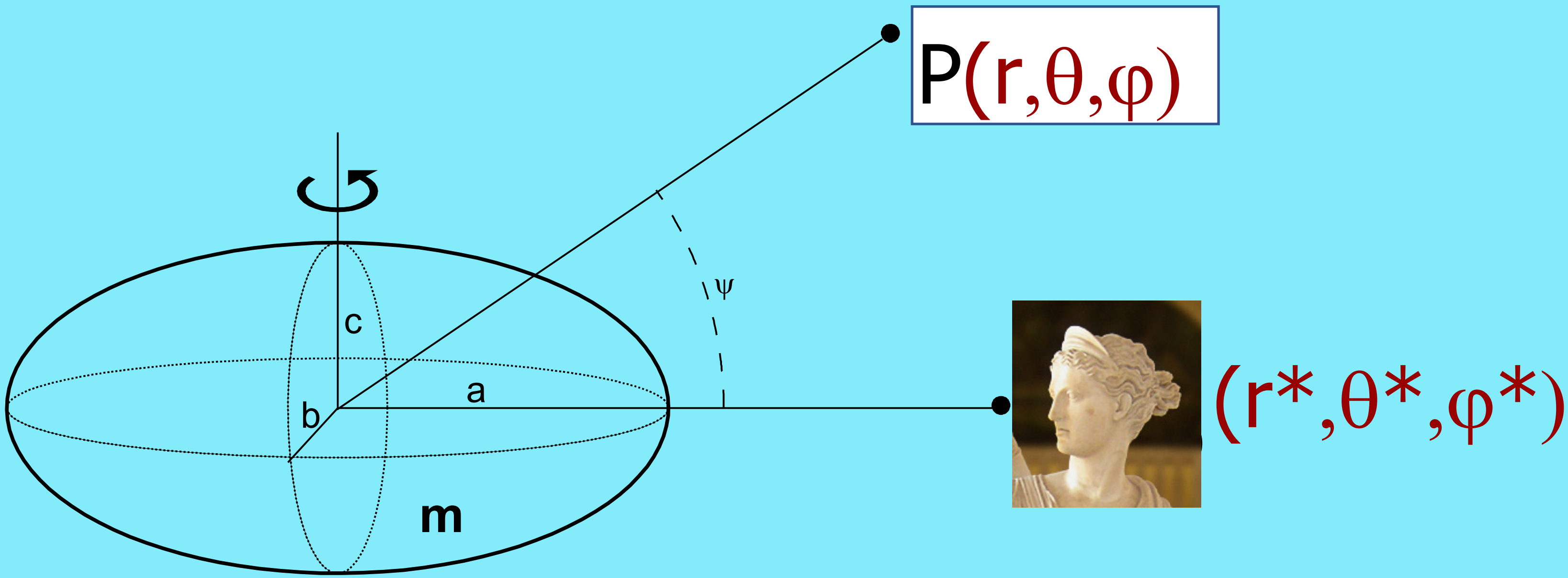


MIGNARD's theory

The Moon and the Planets, **30**, 301 (1979)

Reproduces Darwin CTL theories
Weak Friction Approximation
constant time lag
checked to 3rd. order in e,i

!!!



Potential of the Jeans ellipsoid of prolateness ε_ρ

$(U = U_0 + U_2 + \ldots)$

$$U_2(r) = -k_2 \frac{GmR_E^2}{15r^3} \varepsilon_\rho (1 + 3\cos 2\Psi)$$

or

$$U_2(\mathbf{r}) = -\, k_2 \frac{Gm^* R_E^5}{2r^{*5} \cdot r^5} \left[3(\mathbf{r} \cdot \mathbf{r}^*)^2 - r^2 \cdot r^{*2} \right]$$

where $k_2 = 3/2$

Mignard's modification

$\mathbf{r}^* \rightarrow$

$$\mathbf{r}_1^* = \mathbf{r}^*(t - \Delta t) + \boldsymbol{\omega}\Delta t \times \mathbf{r}^*;$$

N.B. $\mathbf{r}^*(t-\Delta t) = \mathbf{r}^* - \mathbf{v}^*\Delta t$

e.g. Earth
$\Delta t \sim 10$ min

$$V(\mathbf{r},\mathbf{r}^*) = 3\frac{k_2 G m^* R_E^5}{r^5 r^{*5}} \cdot \Delta t \left\{ (\mathbf{r}\cdot\mathbf{r}^*)[\mathbf{r}^* \cdot (\boldsymbol{\omega}\times\mathbf{r}) + \mathbf{r}\cdot\mathbf{v}^*] - \frac{(\mathbf{r}^*\cdot\mathbf{v}^*)}{2r^{*2}}[5(\mathbf{r}\cdot\mathbf{r}^*)^2 - r^2 r^{*2}] \right\}.$$

$$\mathbf{F} = -\operatorname{grad}_r V,$$

$$\mathbf{F} = 3\frac{k_2 G m^* R_E^5}{r^5 r^{*5}} \Delta t \left\{ 5\frac{\mathbf{r}}{r^2} \left[(\mathbf{r}\cdot\mathbf{r}^*)[\mathbf{r}^*\cdot(\boldsymbol{\omega}\times\mathbf{r}) + \mathbf{r}\cdot\mathbf{v}^*] - \frac{(\mathbf{r}^*\cdot\mathbf{v}^*)}{2r^{*2}}\cdot[5(\mathbf{r}\cdot\mathbf{r}^*)^2 - r^2 r^{*2}] \right] - [\mathbf{r}^*\cdot[\mathbf{r}^*\cdot(\boldsymbol{\omega}\times\mathbf{r}) + \mathbf{r}\cdot\mathbf{v}^*] + (\mathbf{r}^*\times\boldsymbol{\omega} + \mathbf{v}^*)(\mathbf{r}\cdot\mathbf{r}^*)] + \frac{(\mathbf{r}^*\cdot\mathbf{v}^*)}{r^{*2}}[5\mathbf{r}^*(\mathbf{r}\cdot\mathbf{r}^*) - \mathbf{r}r^{*2}] \right\},$$

Or, after identification of **r** and **r***:

$$\mathbf{F} = -\frac{3kGM^2R^5}{r^{10}}\left[2\mathbf{r}(\mathbf{r}\cdot\mathbf{v}) + r^2(\mathbf{r}\times\boldsymbol{\Omega} + \mathbf{v})\right]\tau$$

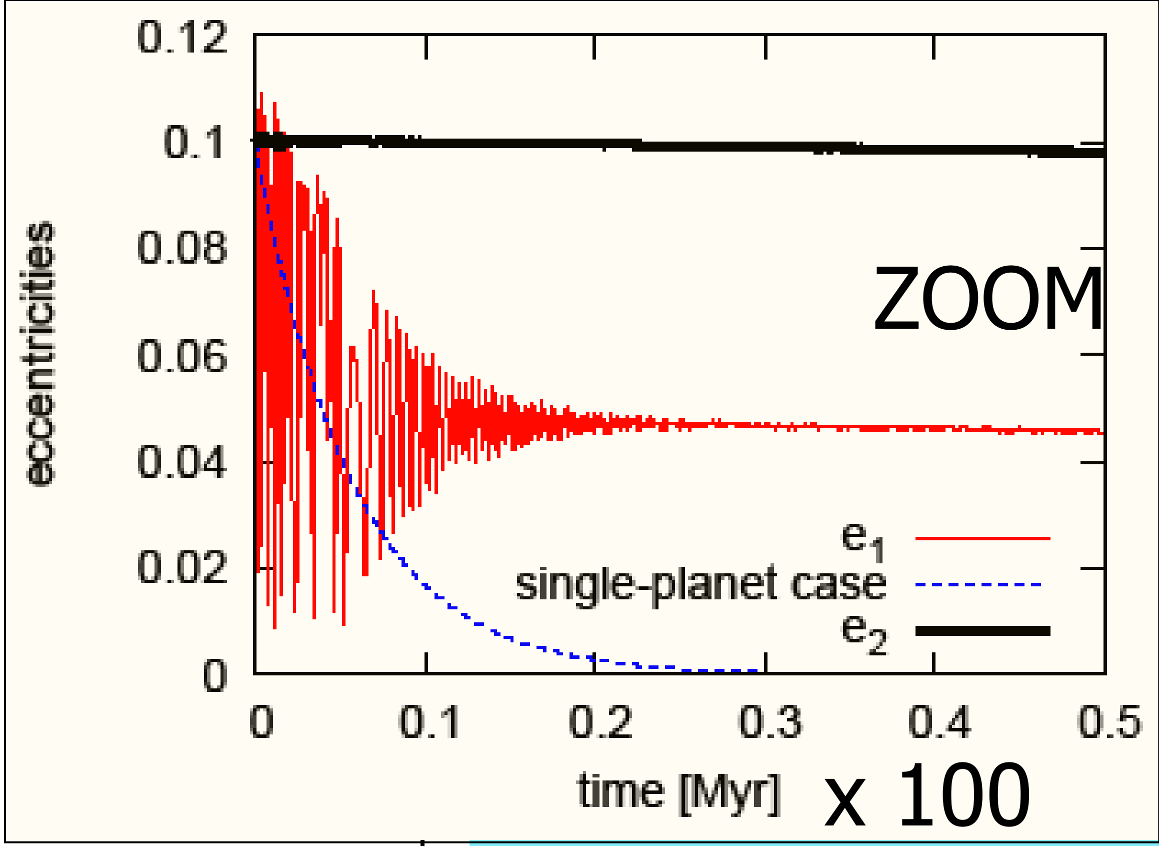
This expression was also obtained, later, by a completely different approach, by Hut et al. (1998). See also, Eggleton, 2004

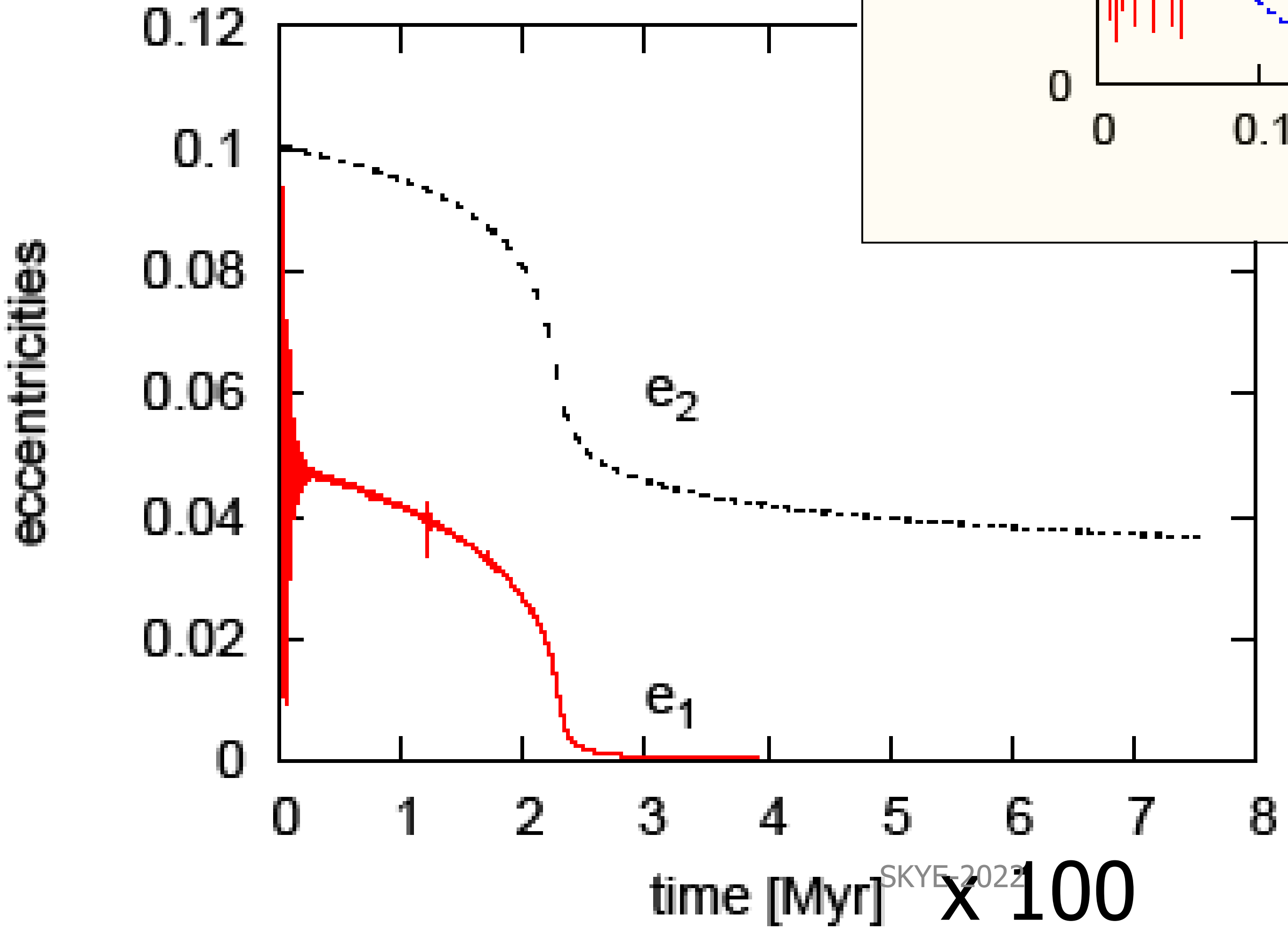
Application:

TWO hot exoplanets
with masses **5 m_Earth** & **1 m_Jupiter**
semi-axes **0.04** and **0.1 AU** resp.
Tides only in the star—inner planet system.

Rodríguez et al. MNRAS **415**, 2349-2358 (2011)

Evolution of the Eccentricities

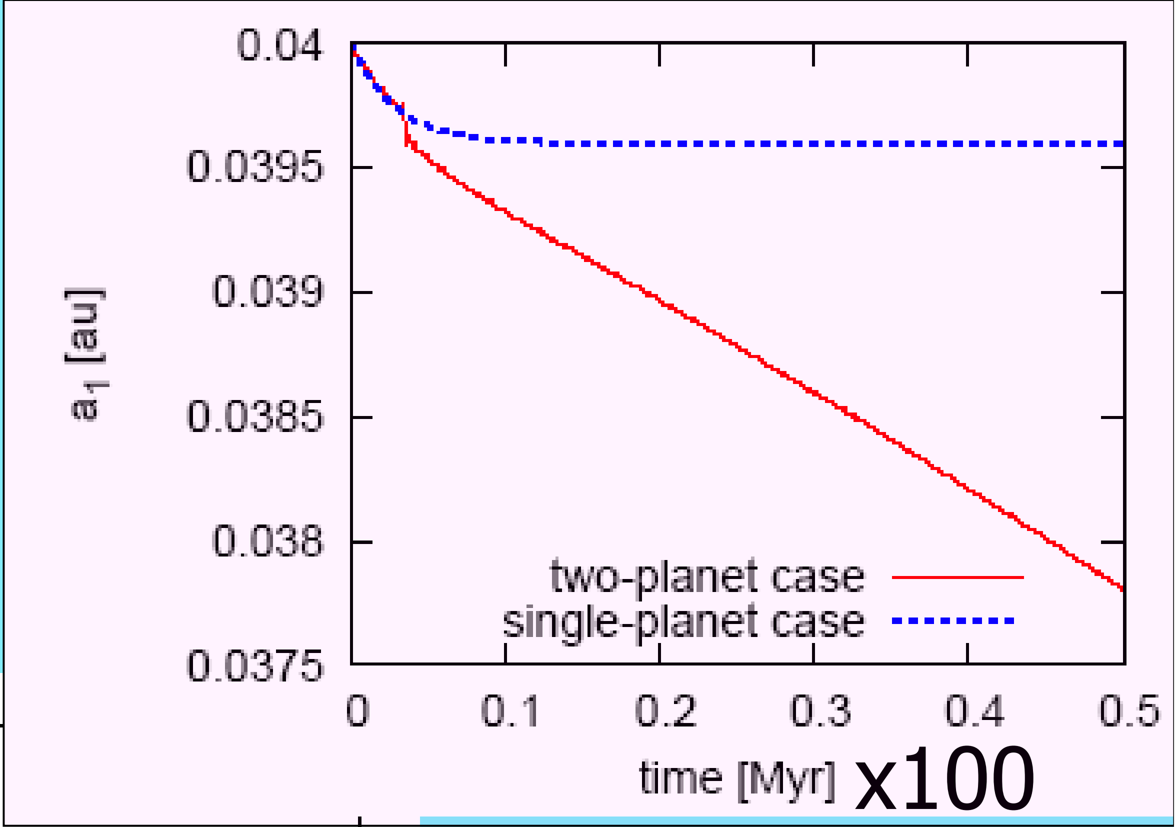




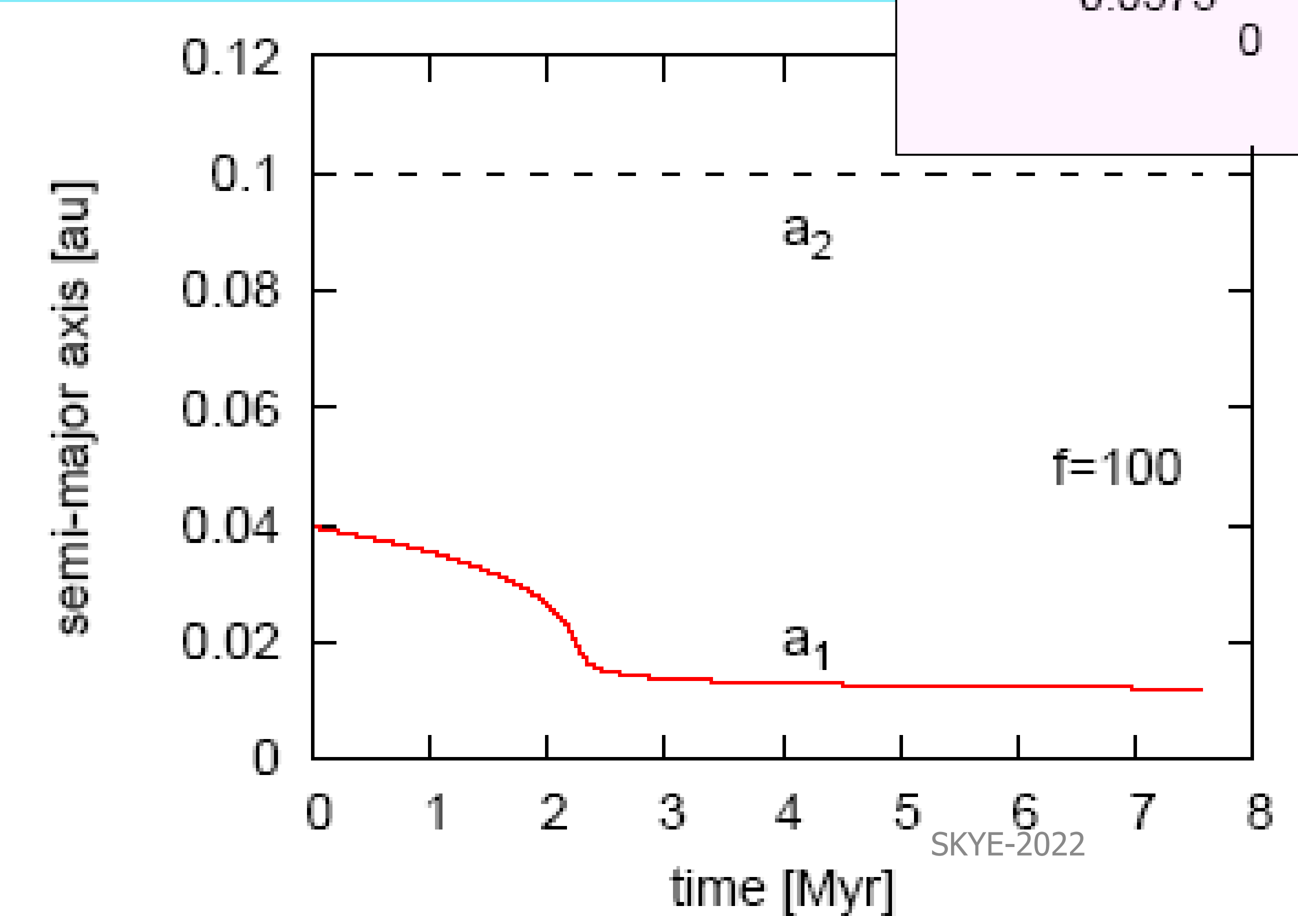
(Mardling´s 1st stationary eccentricity

(dominated by the tide on the planet)

Evolution of the Semi-major axes



x100

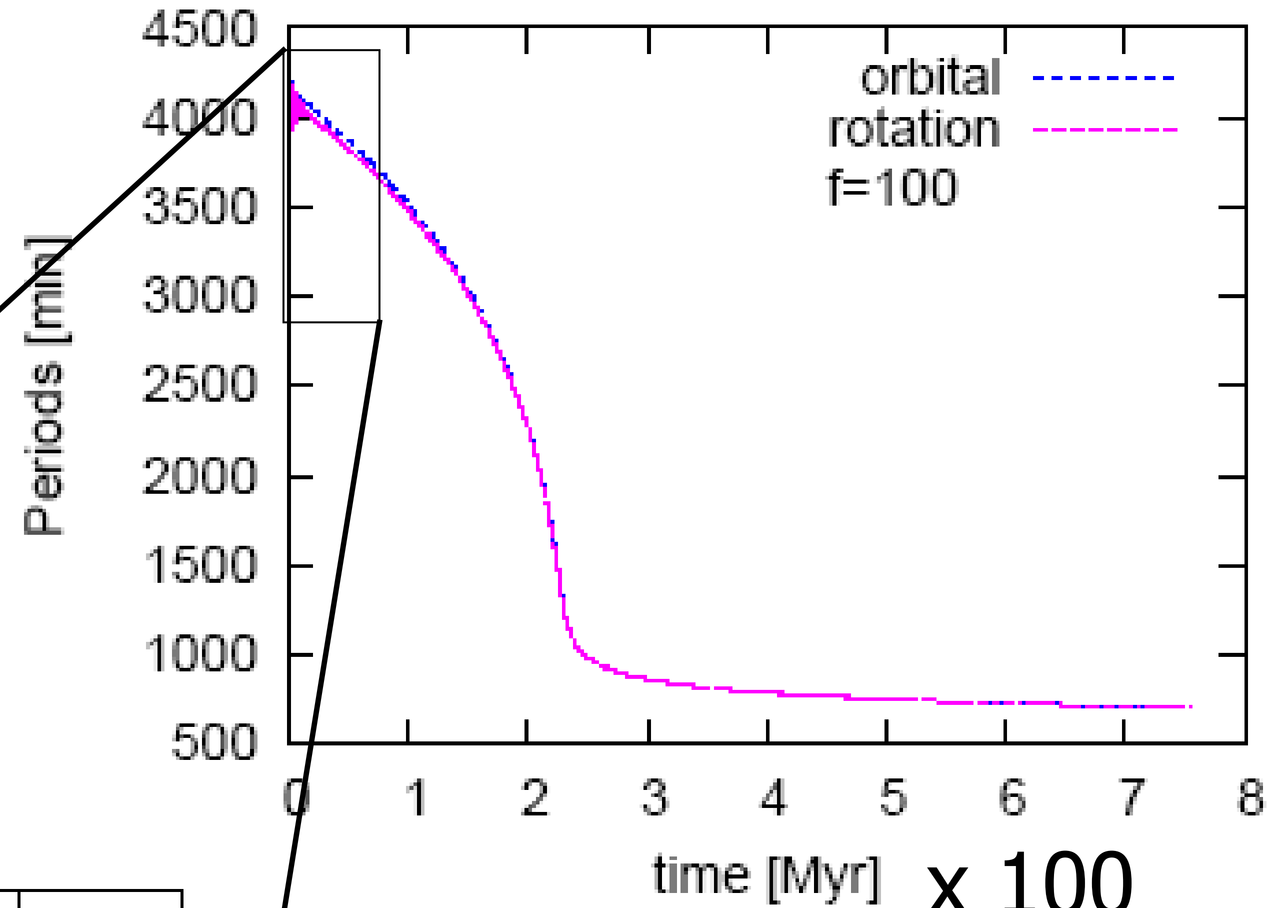


x100

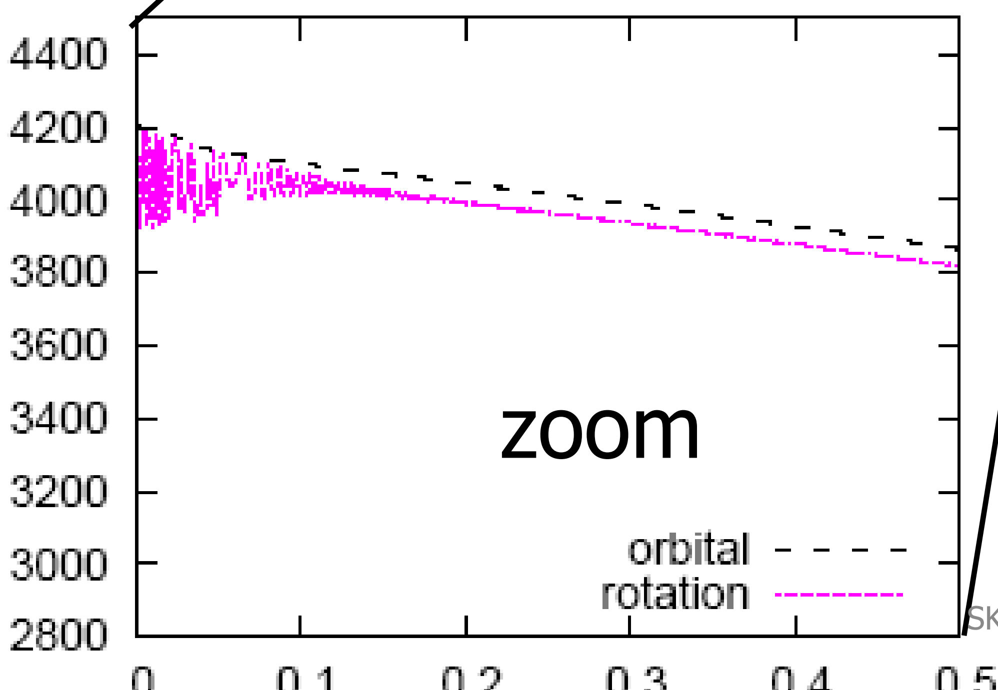
Price to pay: SCALING !

(to be handled with care)

Evolution of the periods (inner planet)



x 100



zoom

The inner planet rotation is trapped in a super synchronous state.

Happel and Brener, Low Reynolds number Hydrodynamics,1973.
Darwin, 1879
====================
S.F.M. Cel.Mech.Dyn.Ast.**116**,109,2013; **122**,359,2015
(astro-ph 1204.3957 and 1505.05384)
S.F.M. et al. (astro-ph 1707-09229)
Folonier et al. CMDA 2018,
Ferraz-Mello et al, EJPST, 2020
SFM 2022 (IAU Symp. 364)

Mignard, Moon and Planets, **30**, 301 (1979)
Rodríguez et al. MNRAS **415**, 2349-2358 (2011)

Bouvier et al. A&A, 1997.
SFM et al. Astrophys. J. (2015) and Corot Legacy Book (2016).

http://www.astro.iag.usp.br/~sylvio/

End of Lecture

Example:

CoRoT-3

F3V

M~1.37 Sun

age~[1.3-2.8] Gyr

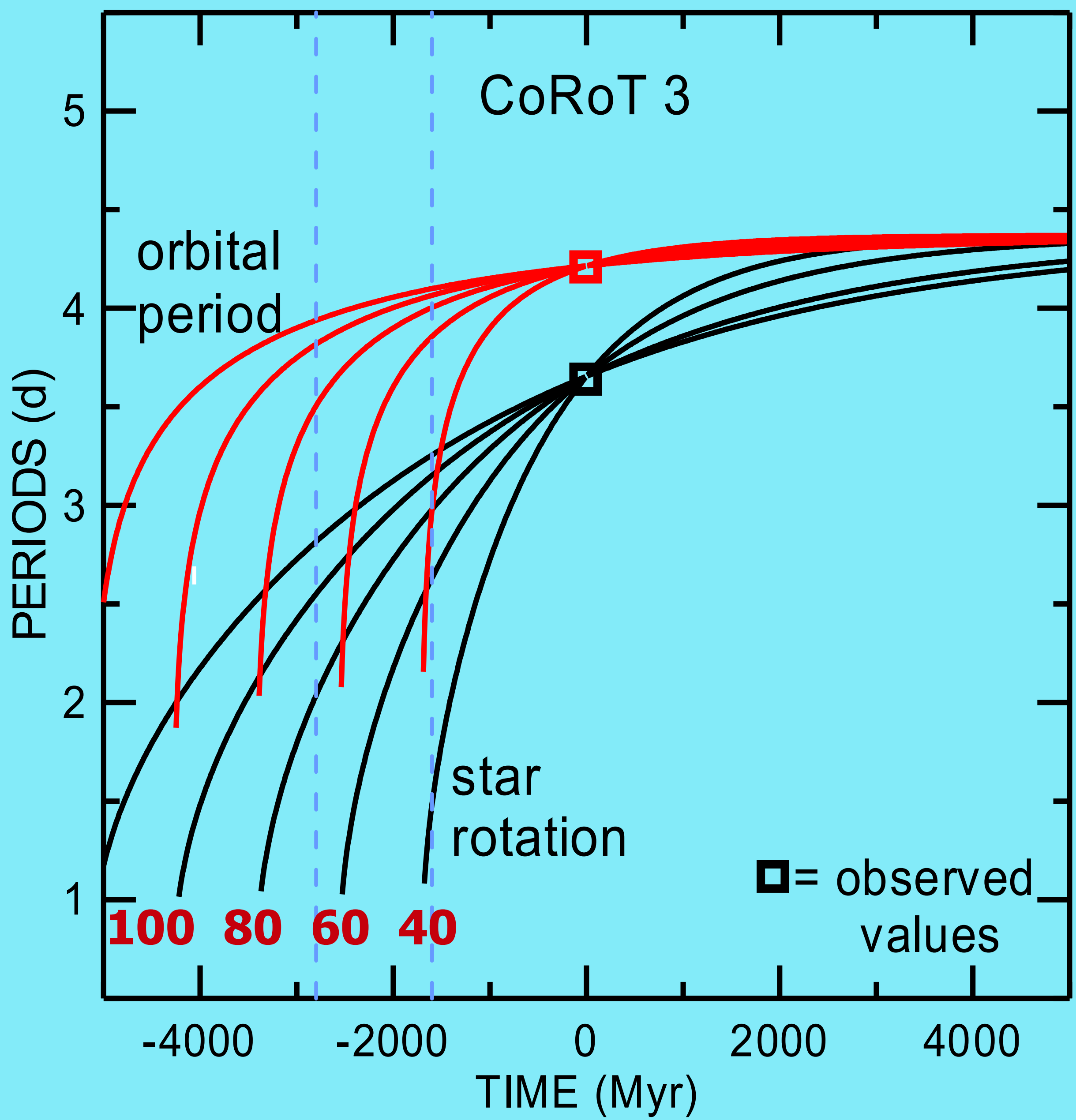
CoRoT-3 b

M~22 Jup (BD)

P~4.25 days

e~0.012

γ~ [**40-80**] s^{-1}



N.B. Magnetic braking insignificant (if any)

Evolutionary Tracks in P-P plot