Gravitational Waves From Compact Binary Sources

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Gravitational Waves from Binary Sources

Compact binary systems (eg pulsars, neutron stars and black holes) are the most likely sources of gravitational waves to be detected. Yet, up to February 2010 nothing was found by all currently operating detectors.

Binary systems can be described as axially symmetric rotating solutions of Einstein's equations, as described by Hermann Bondi et al in the early 60's.

We briefly review Bondi's results and their implications to gravitational wave detectors design and analysis. S. Fairhust et al, ArXiv:0908.4006v1
Joint LIGO-GEO-Virgo Team, ArXiv 1002.103 v1

LIGO-Virgo for Gravitational Waves Searches from Coalescing Binaries ArXiv: 0911.2738 v1

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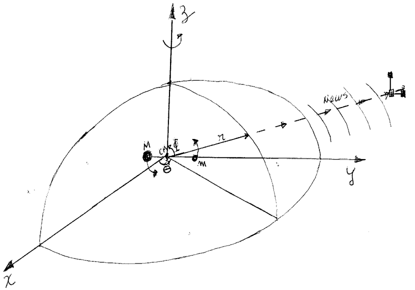
The News Function

The gravitational field of a binary system is described by a general rotating geometry with axial symmetry. Using spherical coordinates (r, θ, ϕ) and the retarded time
$u = t - r$



the general metric expression is

$$ds^2 = g_{00} du^2 + 2 g_{01} du dr + 2 g_{02} du d\theta - g_{22} d\theta^2 - g_{33} d\phi^2$$

$$g_{00} = -A^2 r^2 e^{2\alpha} + \frac{B}{r} e^{2\beta}, \quad g_{01} = e^2 \beta, \quad g_{02} = A r^2 e^{2\alpha},$$

$$g_{22} = -r^2 e^2 \alpha, \quad g_{33} = -r^2 sin^2 \phi e^{-2\alpha}$$

$(A, B, \alpha, \beta$ are functions of u and $\theta)$

Replacing in the vacuum Einstein's equations $R_{\mu\nu} = 0$, and imposing boundary conditions at two arbitrary instants $u = u_0$ constant, Bondi found an exact radiating solution which depends on a single partial integration constant $\mathcal{N}(u, \theta)$, called the "$\mathcal{N}$ews function", which determines the the loss of the *source mass* at any instant $u = u_0$.

The Petrov classification shows that the solution produces gravitational waves if and only if

$$\mathcal{N}(\theta, u) \neq 0$$

H. Bondi et al, Proc. Roy. Soc. A269, 21, (1962)

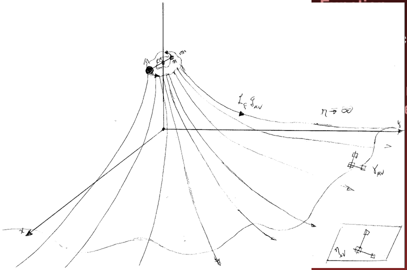
V. D. Zhakarov, Grav. Waves in Einstein's theory. J. Willey, (1972)

The BMS Group

For an asymptotic observer in a flat space-time it was found that the metric is invariant under the *asymptotic isometry group*:

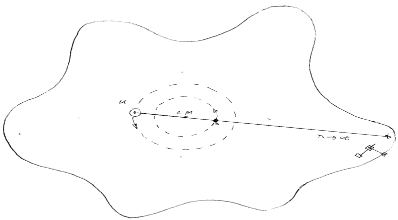
$$\begin{cases} \pounds_\xi g_{\mu\nu}\rfloor_{r\to\infty} = \xi_{(\mu;\nu)} = 0 \\ R_{\mu\nu\rho\sigma}\rfloor_{r\to\infty} = 0 \end{cases}$$

Surprisingly, this system defines a much larger group than the Poincaré group, called the BMS group.



H. Bondi, et al, Pro. Roy. Soc. A269, 21 (1962),
R. K. Sachs, Proc. Roy. Soc. A270, 103 (1962),

The BMS group contains the usual Lorentz subgroup, but the translations are replaced by the supertranslations which depend on the rotation angle θ of the binary system



The Poincaré group occurs only in the cases where the binary system collapse into a spherically symmetric configuration, when there are no news functions, and no gravitational waves.

P. J. McCarthy, J. Math. Phys. 13, 1837, (1972)

The de Rham Linear Wave Equation

The asymptotic isometry group of the axial Bondi metric defines a flat space-time metric

$$\gamma_{\mu\nu} \quad \text{such that} \quad \overset{(\gamma)}{R}_{\mu\nu\rho\sigma} = 0$$

which is not the Minkowski space-time.
Likewise, the gravitational linear wave equation is different from the one defined by the Minkowski metric.

The following procedure is similar to the Hartle-Brill-Isaacson, high frequency gravitational wave equations, except that the space is flat: G. de Rham, *Varietés Differentiables*, Hermann, Paris (1960).

V. Zhakarov, *Gravitational Waves in Einstein's Theory*, John Willey (1972)

Consider a perturbation of a background metric $\gamma_{\mu\nu}$

$$g_{\mu\nu} = \gamma_{\mu\nu} + \epsilon h_{\mu\nu}$$

Defining the wave tensor $\Psi_{\mu\nu} = \gamma_{\mu\nu} - \frac{\epsilon}{2} h \gamma_{\mu\nu}$ and replacing in the vacuum Einstein's equations $R_{\mu\nu}(g) = 0$, we obtain the **de Rham wave equation**

$$\Box^2_\gamma \Psi_{\mu\nu} \equiv \gamma^{\alpha\beta}\Psi_{\mu\nu;\alpha\beta} + 2 \overset{(\gamma)}{R}_{\alpha\mu\nu\beta} \Psi^\alpha_\beta + \overset{(\gamma)}{R}_{\mu\alpha} \Psi^\alpha_\nu + \overset{(\gamma)}{R}_{\nu\alpha} \Psi^\alpha_\mu = 0$$

In particular, for a flat metric $\overset{(\gamma)}{R}_{\mu\nu\alpha\beta} = 0$, we recover the linear wave equation calculated with $\gamma_{\mu\nu}$

$$\Box^2_\gamma \Psi_{\mu\nu} \equiv \gamma^{\alpha\beta}\Psi_{\mu\nu;\alpha\beta} = 0$$

whose solutions are *different* from those of the Minkowski wave equation $\Box^2_\eta \Psi_{\mu\nu} = 0$

Personal views

In reality we live in a gravitational environment which is not described by the Minkowski metric. However, at the length scale of particle physics, the long wavelength gravitational waves and the supertranslations do not play a significant role.

The BMS invariant metric $\gamma_{\mu\nu}$ depends on the knowledge of the predominant gravitational wave source for Earth or near Earth observations. This can be accomplished with the help of the SDSS and the existing gravitational wave detectors.

The binary system produces two asymptotic synchronized effects: gravitational waves and supertranslations. The combined effect may cancel the difference of arms lengths in the interferometers.