Non-Gaussian CMB signatures in ΛCDM cosmological models

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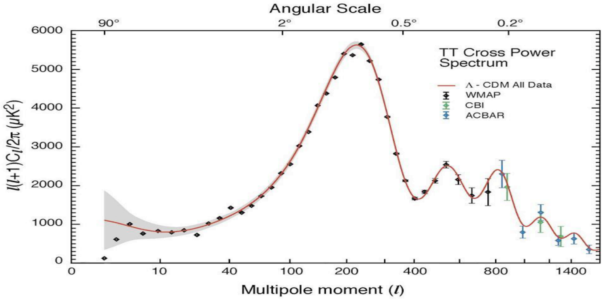
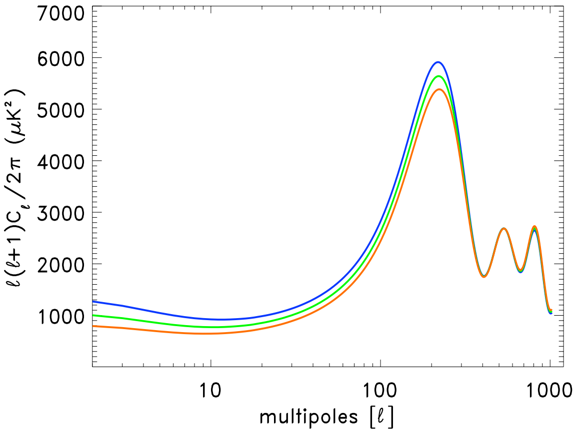
- WMAP data have confirmed the concordance cosmological model ΛCDM. This model corresponds, under some assumptions, to the *data best-fit*. In other words, there are models with slightly different hypotheses and similar (but not equal) values for the cosmological parameters that fit the WMAP data as well as the ΛCDM concordance model (see: http:\\lambda.gsfc.nasa.gov\product\map\dr4\parameters.cfm).
- One such parameter, which is fundamental for our understanding of the primordial universe, is the spectral index of primordial fluctuations n_s. The main difficulty to establish the correct spectral index value is that it is related to the Angular Power Spectra at the largest scales (i.e. low multipoles): precisely the region where the cosmic variance uncertainty dominates.



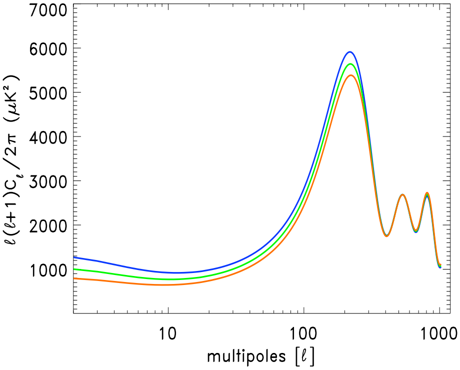
- We shall study the effects caused on the CMB statistics (i.e. Gaussianity) of three sets of Monte Carlo CMB maps produced by seeding them with slightly differ- ent Angular Power Spectra, obtained using the spectral indexes: n_s = 0.96, 0.98, 1.00.

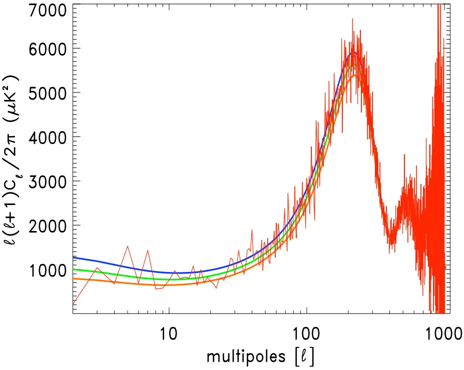


Ang. Power Spectra: WMAP data and ΛCDM models

Degenerescence problem similar APS $\simeq$ with $\neq_s$ cosmological parameters

Example: APS with $\neq_s$ spectral indexes: n_s = 0.96, 0.98, 1.00





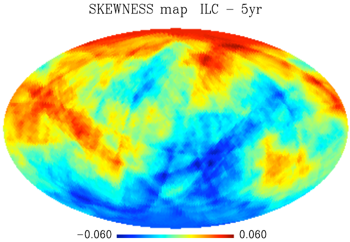
Then we generate 3 sets of MC CMB maps according to these APS

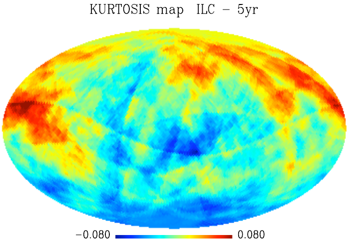
Gaussian analyses in Monte Carlo CMB maps

We use two recently defined estimators: Skewness-map & Kurtosis-map

$$\mathbf{S} = \{S_j, j = 1, \ldots, N_{\rm caps}\} \qquad \mathbf{K} = \{K_j, j = 1, \ldots, N_{\rm caps}\}$$

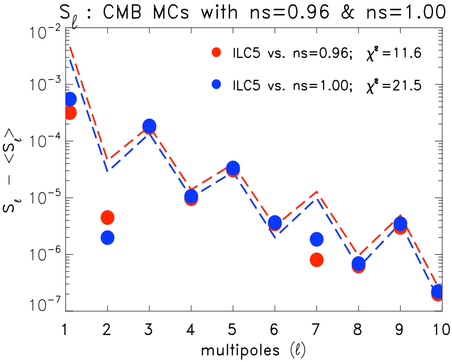
$$S_j \equiv \frac{1}{N_p\,\sigma_j^3} \sum_{i=1}^{N_p} \left(\Delta T_i - \overline{\Delta T_j} \right)^3 \qquad K_j \equiv \frac{1}{N_p\,\sigma_j^4} \sum_{i=1}^{N_p} \left(\Delta T_i - \overline{\Delta T_j} \right)^4 - 3$$

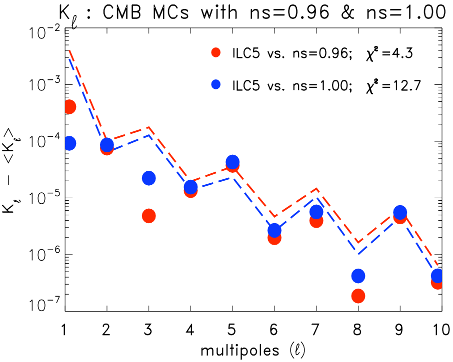




We calculate the **S**− and **K**−maps from the three sets of MC CMB maps, then compute their angular power spectra. Then use them to quantify the CONFIDENCE LEVEL of the spectra $\{S_\ell^{\rm WMAP}\}, \{K_\ell^{\rm WMAP}\}$.

Results: angular power spectra analyses of **S**− and **K**−maps





- Our results show that, in the mean, the Gaussianity property of these sets of Monte Carlo maps is different, and this fact seems to be crucial when one has to quantify the confidence level in CMB data analyses. That is to say, the statistical-significance evaluation of a result concerning a CMB map is model dependent.

Conclusions (first part)

- We analyzed the APS of **S**− and **K**−maps produced from 3 sets of MC CMB maps (seeded by ΛCDM spectra with $n_s = 0.96$, 0.98, and 1.00, respectively). Our results show that these maps contain different amounts of non-Gaussianity.

  The smaller non-Gaussian deviations appear in the $n_s = 1.00$ case.
- Differently as claimed, MCs seeded on the ΛCDM concordance model spectrum ($n_s = 0.96$) can not be considered as equivalent to simulated *Gaussian* CMB maps.
- ⇒ Gaussian confidence levels are relative to the model assumed.

References

- AB and M.J. Rebouças, Phys. Rev. D **79**, 063528 (2009), arXiv:0806.3758;
- Int. J. Mod. Phys. A **24**, 1664 (2009), arXiv:0907.0527; arXiv:0912.0269

Searching for primordial non-Gaussianity in CMB data

Density Fluctuation Field: $\delta\rho$

The statistical properties of a fluctuation field $\delta\rho(\vec{x}) \equiv [\rho(\vec{x}) - \overline{\rho}]/\,\overline{\rho}$ can be characterized by the n-point moments $\langle \delta\rho^{\,n} \rangle$ ($\langle\,\rangle$ = ensemble average). By definition $\langle \delta\rho \rangle = 0$. If the field is Gaussian, then the probability distribution for $\delta\rho$ is

$$P(\delta\rho) = \frac{1}{\sqrt{2\pi}\ \sigma}\ \exp\left[-\delta\rho^{\,2}/(2\,\sigma^2)\right],$$

the even moments $2n = 2, 4, \ldots$ are

$$\langle \delta\rho^{\,2n} \rangle = (2n-1)!!\,\langle \delta\rho^2 \rangle^n = (2n-1)!!\,\sigma^{2n}\,,$$

where $\sigma^2 \equiv \langle \delta\rho^{\,2} \rangle$ is the variance (or 2-point correlation) of the field, and the odd moments are zero. This implies that for a Gaussian field all the information is contained in the 2-point correlation function.

Primordial non-Gaussianity

The inflationary scenario establish the structure formation from primordial adiabatic density perturbations, where such perturbations can be originated by quantum fluctuations, e.g.,

- in the single scalar field (the 'inflaton') responsible for the standard slow-roll inflation,
- in a second scalar field in non-standard multi-field inflation,

producing different amounts of non-Gaussian density perturbations.

In any case, for a non-Gaussian field the lowest-order deviations from Gaussianity comes from the 3-point correlation function (equivalently the **bispectrum**, its Fourier-space counterpart.) which is not zero: $\langle \delta\rho^3 \rangle \neq 0$. Symmetric configurations of the 3-point correlation function produces non-Gaussianity of equilateral type and orthogonal type.

The non-Gaussianity achieved in multi-field inflation (or in cyclic / ekpyrotic universe models) is termed non-Gaussianity of local type.

Let us concentrate on local non-Gaussianity. It can be described by a **primordial gravitational potential** Φ, a non-Gaussian random field written in terms of a Gaussian random field Φ_L through

$$\Phi(\vec{x}) = \Phi_L(\vec{x}) + f_{NL}\Phi_{NL}(\vec{x}),$$

where

$$\Phi_{NL} = \Phi_L^2 - \langle \Phi_L^2 \rangle .$$

In this way the amplitude of the primordial non-Gaussianity is in the dimensionless parameter f_{NL}. Non-Gaussianity in the density field is then obtained from that in the gravitational potential through the Poisson eqn. And since the density field interacts with the radiation field, primordial non-Gaussianity in Φ will be encoded in the CMB data.

Primordial non-Gaussianity in CMB data

In order to simulate MC CMB maps containing non-Gaussianity of local type the sets $\{a^{\rm L}_{\ell\, m}\}$, and $\{a^{\rm NL}_{\ell\, m}\}$ are produced through the steps*:
(i) Generate the multipole moments of a purely Gaussian gravitational potential $\Phi_{{\rm L}\ \ell m}(r)$ as a function of conformal distance.
(ii) Calculate the spherical harmonic transform to derive the corresponding expression in pixel space, $\Phi_{\rm L}(r)$.
(iii) Compute $\Phi_{\rm NL}(r) = \Phi^2_{\rm L}(r) - \langle \Phi^2_{\rm L}(r) \rangle$.
(iv) Inverse transform to spherical harmonic space to obtain $\Phi_{{\rm NL}\ \ell m}(r)$.
(v) Solve the radial integral equations to obtain $a^{\rm i}_{\ell\, m}$, i=L, NL,

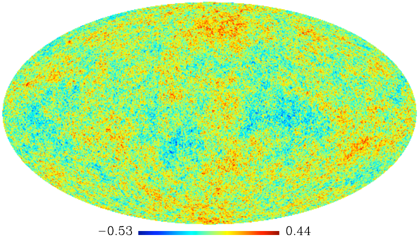
$$a^{\rm i}_{\ell\, m} = \frac{2}{\pi} \int dr\, r^2\, \Phi_{\ell m}(r) \int dk\, k^2 g^{\rm i}_{\ell}(k)\, j_\ell(kr)\,,$$

where g_ℓ and j_ℓ are the transfer function and the sph. Bessel function, respectively.

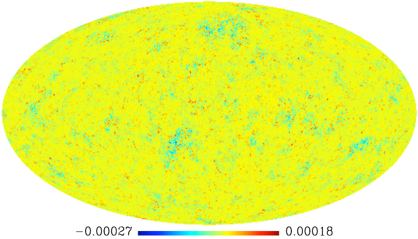
* Elsner & Wandelt, arXiv: 0909.0009

Primordial non-Gaussianity in CMB data

$\{a^{\mathrm{L}}_{\ell\, m}\} \longrightarrow$ Gaussian MC CMB:

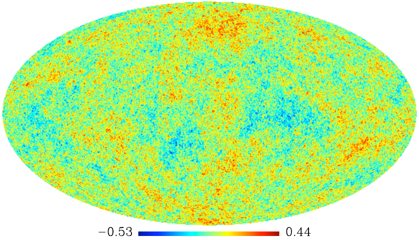


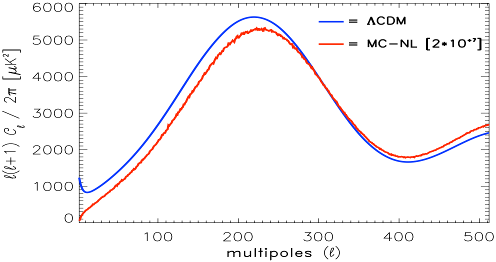
$\{a^{\mathrm{NL}}_{\ell\, m}\} \longrightarrow$ Non-Gaussian MC CMB:



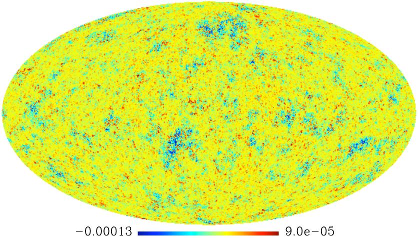
Primordial non-Gaussianity in CMB data

$\{a^{L}_{\ell\, m}\} \longrightarrow$ Gaussian MC CMB:



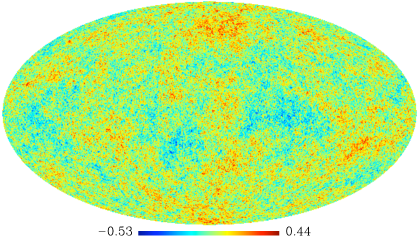


$\{a^{NL}_{\ell\, m}\} \longrightarrow$ Non-Gaussian MC CMB:

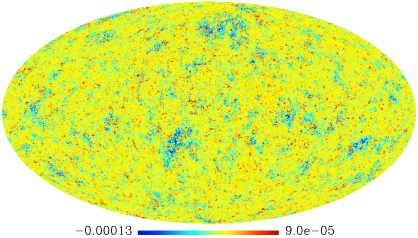


Primordial non-Gaussianity in CMB data

$\{a^{L}_{\ell\, m}\} \longrightarrow$ Gaussian MC CMB:



$\{a^{NL}_{\ell\, m}\} \longrightarrow$ Non-Gaussian MC CMB:



$$a_{\ell\, m} = a^{L}_{\ell\, m} + f_{NL}\, a^{NL}_{\ell\, m} \implies \text{MC CMB com NG}$$

Limits of Primordial non-Gaussianity in WMAP data

Limits for f_{NL} according to WMAP-7yr: $-10 \leq f_{NL} \leq 74$.

TABLE 2

SUMMARY OF THE 95% CONFIDENCE LIMITS ON DEVIATIONS FROM THE SIMPLE (FLAT, GAUSSIAN, ADIABATIC, POWER-LAW) ΛCDM MODEL EXCEPT FOR DARK ENERGY PARAMETERS

Sec.	Name	Case	*WMAP* 7-year	*WMAP*+BAO+SN[a]	*WMAP*+BAO+H_0
§ 4.1	Grav. Wave[b]	No Running Ind.	$r < 0.36$[c]	$r < 0.20$	$r < 0.24$
§ 4.2	Running Index	No Grav. Wave	$-0.084 < dn_s/d\ln k < 0.020$[c]	$-0.065 < dn_s/d\ln k < 0.010$	$-0.061 < dn_s/d\ln k < 0.017$
§ 4.3	Curvature	$w = -1$	N/A	$-0.0178 < \Omega_k < 0.0063$	$-0.0133 < \Omega_k < 0.0084$
§ 4.4	Adiabaticity	Axion	$\alpha_0 < 0.13$[c]	$\alpha_0 < 0.064$	$\alpha_0 < 0.077$
		Curvaton	$\alpha_{-1} < 0.011$[c]	$\alpha_{-1} < 0.0037$	$\alpha_{-1} < 0.0047$
§ 4.5	Parity Violation	Chern-Simons[d]	$-5.0^\circ < \Delta\alpha < 2.8^\circ$[e]	N/A	N/A
§ 4.6	Neutrino Mass[f]	$w = -1$	$\sum m_\nu < 1.3$ eV[c]	$\sum m_\nu < 0.71$ eV	$\sum m_\nu < 0.58$ eV[g]
		$w \neq -1$	$\sum m_\nu < 1.4$ eV[c]	$\sum m_\nu < 0.91$ eV	$\sum m_\nu < 1.3$ eV[h]
§ 4.7	Relativistic Species	$w = -1$	$N_{\rm eff} > 2.7$[c]	N/A	$4.34^{+0.86}_{-0.88}$ (68% CL)[i]
§ 6	Gaussianity[j]	Local	$-10 < f_{NL}^{\rm local} < 74$[k]	N/A	N/A
		Equilateral	$-214 < f_{NL}^{\rm equil} < 266$	N/A	N/A
		Orthogonal	$-410 < f_{NL}^{\rm orthog} < 6$	N/A	N/A

[a] "SN" denotes the "Constitution" sample of Type Ia supernovae compiled by Hicken et al. (2009b), which is an extension of the "Union" sample (Kowalski et al. 2008) that we used for the 5-year "*WMAP*+BAO+SN" parameters presented in Komatsu et al. (2009b). Systematic errors in the supernova data are not included. While the parameters in this column can be compared directly to the 5-year *WMAP*+BAO+SN parameters, they may not be as robust as the "*WMAP*+BAO+H_0" parameters, as the other compilations of the supernova data do not give the same answers (Hicken et al. 2009b; Kessler et al. 2009). See Section 3.2.4 for more discussion. The SN data will be used to put limits on dark energy properties. See Section 5 and Table 4.

[b] In the form of the tensor-to-scalar ratio, r, at $k = 0.002$ Mpc^{-1}.

[c] Larson et al. (2010).

[d] For an interaction of the form given by $[\phi(t)/M]F_{\alpha\beta}\tilde{F}^{\alpha\beta}$, the polarization rotation angle is $\Delta\alpha = M^{-1}\int \frac{dt}{a}\dot{\phi}$.

[e] The 68% CL limit is $\Delta\alpha = -1.1^\circ \pm 1.3^\circ$ (stat.) $\pm 1.5^\circ$ (syst.), where the first error is statistical and the second error is systematic.

[f] $\sum m_\nu = 94(\Omega_\nu h^2)$ eV.

[g] For *WMAP*+LRG+H_0, $\sum m_\nu < 0.44$ eV.

[h] For *WMAP*+LRG+H_0, $\sum m_\nu < 0.71$ eV.

[i] The 95% limit is $2.7 < N_{\rm eff} < 6.2$. For *WMAP*+LRG+H_0, $N_{\rm eff} = 4.25 \pm 0.80$ (68%) and $2.8 < N_{\rm eff} < 5.9$ (95%).

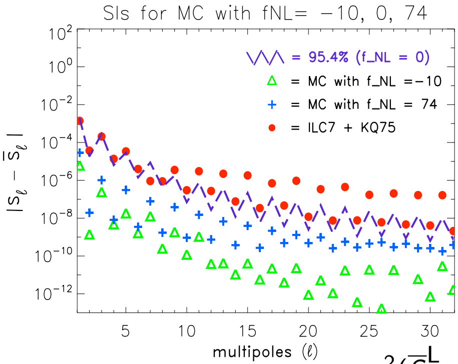
[j] V+W map masked by the *KQ75y7* mask. The Galactic foreground templates are marginalized over.

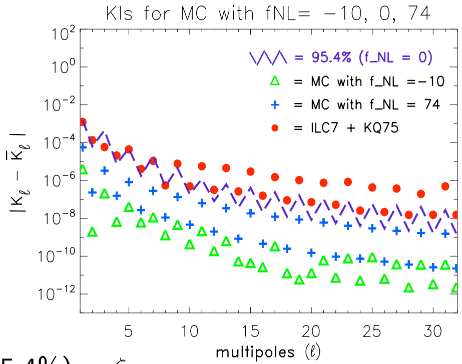
[k] When combined with the limit on $f_{NL}^{\rm local}$ from *SDSS*, $-29 < f_{NL}^{\rm local} < 70$ (Slosar et al. 2008), we find $-5 < f_{NL}^{\rm local} < 59$.

Let us use our **S** and **K** indicators to analyze two sets of 1 000 MC CMB maps each: one with $f_{NL} = -10$, the other with $f_{NL} = 74$.

Primordial non-Gaussianity analyses in CMB data

Ang. Power Spectra of **S**- & **K**-maps from MC with $\mathbf{f_{NL}} = -10$ and $\mathbf{f_{NL}} = 74$





$$\chi^2(\overline{S}_\ell^{L}, 95.4\%) = \zeta_0$$

$$\chi^2(\overline{S}_\ell^{ILC}, \overline{S}_\ell^{L}) = 4.0\,\zeta_0 \qquad \chi^2(\overline{K}_\ell^{ILC}, \overline{K}_\ell^{L}) = 3.2\,\zeta_0$$

$$\chi^2(\overline{S}_\ell^{ILC}, \overline{S}_{f_{NL}=\,74}) = 3.7\,\zeta_0 \qquad \chi^2(\overline{K}_\ell^{ILC}, \overline{K}_{f_{NL}=\,74}) = 2.8\,\zeta_0$$

Conclusions (Primordial non-Gaussian analyses)

- We study primordial Gaussian deviations of local type, in amounts consistent with the WMAP7 limits: $-10 \leq f_{NL} \leq 74$.
- Our non-Gaussian indicators reveals (and quantify) the presence of these tiny deviations from Gaussianity (i.e., $f_{NL} \neq 0$), when compared with the ΛCDM MC CMB data (i.e., $f_{NL} = 0$).
- The form of the APS, i.e. $\{S_\ell\}$, $\{K_\ell\}$, at small angular scales could be a characteristic signature of this type of non-Gaussianity.

References

- Elsner & Wandelt, arXiv: 0909.0009
- E. Komatsu & D. Spergel, astro-ph/0005036
- L. Verde et al., astro-ph/9906301; astro-ph/0011180
- M.J. Rebouças & AB, in preparation