Scalar-Tensor theories of Gravity and the Inflationary Universe

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Abstract

A model of Cosmic Inflation recently proposed by Di Marco and Notari [1] attempts to solve some of the problems of the original scenario [2]. In their model, the phase transition that drives the inflationary expansion is completed, and it is attractive because it is based on a minimum of extra hypothesis to those of the original work. Gravitation is considered to be described by a Scalar-Tensor theory, whose action is

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}(M^2+\beta\phi^2)R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \Lambda\right].$$

In the proposed model, the inflationary expansion occurs in two stages: the first, de Sitter, is driven by the constant energy of the false vacuum (Λ). In the second, the dynamics is dominated by the scalar field ϕ, and the scale factor evolves as a power-law. This second stage, in which the Hubble parameter decreases, is of great importance for the completion of the phase transition. The relevant perturbations of the scalar field's energy density are generated during the de Sitter stage, though their spectral index is in disagreement with observations. We investigate the changes that occur if these perturbations are generated during the second stage of the inflationary evolution. We obtain the necessary conditions for the spectral index to be in accordance with the observations. Another proposal for Inflation, similar in spirit to the work of Di Marco and Notari, was made by Biswas and Notari [3]. Though similar in spirit, it opens extra possibilities that will be explored here. We also intend to adapt the bootstrap tests recently proposed by Latham Boyle and Paul Steinhardt [4], which compares not only the model to the observational data, but also it's predictive power!

Introduction

Inflation was designed to solve the Horizon and Flatness problems, two questions left by the Standard Big Bang Theory. It is also in charge of generating the perturbations that seeded the Structure formation and are responsible for the tiny anisotropies of the CMB. These three points will serve to constrain Inflation.

Horizon Problem

The Universe is, on large scales, homogeneous and isotropic. But, following the standard Big Bang evolution, these regions would not have had causal contact in order to achieve homogeneity.

Flatness Problem

Any tiny deviation from flatness at early epochs would lead to a Universe which would hardly be flat!

Inflationary solution - General Relativity - Guth's model

The Universe was dominated by a scalar field (*inflaton*). As the temperature cooled through a critical temperature T_c, the field became trapped in its False Vacuum State, with very small Γ, the vacuum decay rate.

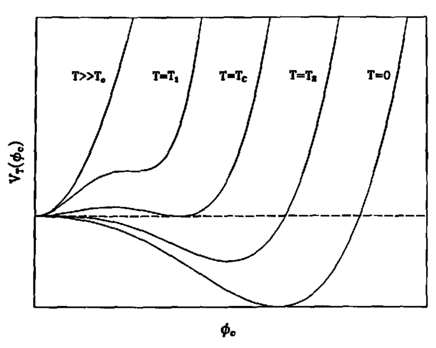

Fig. 1: Behavior of the potential as the temperature drops

The energy density of the False Vacuum was a constant, leading to exponential expansion. In some regions, the scalar field tunnels to the True Vacuum state, then forming regions (bubbles) of True Vacuum inside the Universe, still dominated by the False Vacuum.
The Universe keeps cooling until $T_s \ll T_c$, at which bubbles of True Vacuum begin to collide, releasing the energy initially stored in their walls, reheating the Universe to $T_r \approx T_c$.
In order to have sufficient Inflation, solving the above mentioned problems and leaving time for the released energy from the walls to be thermalized, we must have the inequality

$$r \equiv \frac{\Gamma}{H^4} \ll 1. \tag{1}$$

For Inflation to get to an end, the True Vacuum regions must be both numerous and large enough, so we need

$$r \approx 1. \tag{2}$$

Since H is a constant during Inflation, these conditions can't be met! So, the original scenario is not a satisfactory Inflationary Model.

Inflation and Scalar-Tensor Gravity - Di Marco and Notari's model modified

In the Scalar-Tensor theory, Gravity will be described by the following action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}(M^2+\beta\phi^2)R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \Lambda\right]. \tag{3}$$

The field equations derived from this action give

$$H^2 = \frac{1}{3M^2}\left(\rho_\phi + \Lambda\right), \tag{4}$$

where Λ is the vacuum energy density, assumed constant. Initially, the scalar field is subdominant, and so the Hubble parameter is constant, given by

$$H_I^2 = \frac{\Lambda}{3M^2}. \tag{5}$$

While the Universe expands exponentially, driven by the vacuum energy, the field energy density also grows exponentially and, at a certain moment, it begins to dominate the dynamics of the expansion. At this stage, the Hubble parameter is not a constant anymore,

$$H = \frac{\alpha}{t}, \tag{6}$$

where

$$\alpha = \frac{1+2\beta}{4\beta}. \tag{7}$$

The first stage is necessary in order to have sufficient Inflation and solve the above Cosmological problems. The second is needed so Inflation ends successfully. These simple features make Scalar-Tensor Gravity a good background to study Inflation!

Constraints on the model

The amount of Inflation can be parametrized by the number of e-folds

$$N \equiv \int_t^{t_{end}} H(\tilde{t})d\tilde{t} = \ln\frac{a(t_{end})}{a(t)}. \tag{8}$$

To solve the Cosmological problems, the initial nucleation rate must be very small, and this value has to hold during the whole first stage ($r_0 \approx 10^{-7}$). When Inflation ends, observations show this number is $r_{end} = \frac{9}{4\pi}$.
The second stage begins when $r \approx r_0$ and $N = N_{II}$, so

$$N_{II} \simeq -\frac{\alpha}{4}\ln\left[\frac{4\pi}{9}r_0\right]. \tag{9}$$

Perturbations

Observations show that perturbations on scales between $50h^{-1}$Mpc and $3000h^{-1}$Mpc have a flat spectrum, that means, the amplitude of the energy density perturbations are independent on the amplitude of the field itself, when these perturbations re-enter the horizon, after Inflation. Considering that perturbations on the relevant scales were produced during the second stage, in contrast with [1]

$$N_{II} > N_{3000h^{-1}\mathrm{Mpc}}. \tag{10}$$

$N_{3000h^{-1}\mathrm{Mpc}} \approx 63.3$. Using 9 and 7, we find

$$\beta < 0.016 \tag{11}$$

Considering fluctuations associated with the scalar field,

$$\phi(x,t) = \phi(t) + \delta\phi(x,t),$$

and scalar type perturbations in the metric,

$$ds^2 = -(1+2\psi)dt^2 - \chi_{,\mu}dtdx^\mu + a^2(1+2\varphi)\delta_{\mu\nu}dx^\mu dx^\nu,$$

the power spectrum of the gauge-invariant combination [5] and [6] $\delta\phi_\varphi = \delta\phi - \frac{\dot{\phi}}{H}\varphi$ is

$$P_C^{1/2}(k,\tau) = \left|\frac{H}{\dot{\phi}}\right|\frac{H}{2\pi}\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\left[\frac{1}{2}\frac{k}{aH}\right]^{\frac{3}{2}-\nu}. \tag{12}$$

The spectral index is then

$$n_S \equiv 1 + \frac{d\ln P_C}{d\ln k} = 4 - 2\nu = 1 - 10\beta \tag{13}$$

From the value of β (11), we get

$$n_S > 0.83 \tag{14}$$

In good agreement with observations!

Inflation and Scalar-Tensor Gravity - Biswas and Notari's model

The work of Biswas and Notari [3] starts from a similar, though more general, lagrangian

$$S = \int d^4x\sqrt{-g}\left\{M^2 f(\phi)R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \Lambda\right\}. \tag{15}$$

where $f(\phi) = \left[1 + \beta\left(\frac{\phi}{M}\right)^n\right]$.
Performing the conformal transformation $\bar{g}_{\mu\nu} = f(\phi)g_{\mu\nu}$ the gravitational action becomes the usual one,

$$S = \int d^4x\sqrt{-\bar{g}}\left[M^2\bar{R} - K(\phi)\left(\bar{\partial}\phi\right)^2\right]. \tag{16}$$

Re-defining the field $\sqrt{K(\phi)}d\phi = d\Phi$ to get a canonical kinetic term in the action, we are left with the potential

$$\bar{V}(\Phi) = \Lambda\left[1 - 2\beta\left(\frac{\phi}{M}\right)^n\right]. \tag{17}$$

In the slow-roll approximation, using the slow-roll parameter $\eta = M^2\frac{d^2V}{d\Phi^2}$, we find the spectral index

$$n_s - 1 \simeq 2\eta \simeq -0.04\left(\frac{n-1}{n-2}\right) \tag{18}$$

Which leads to the prediction ($n \geq 4$)

$$0.94 \leq n_s \leq 0.96, \tag{19}$$

in excellent agreement with the WMAP results.
So we have two possible inflationary models based on scalar-tensor gravity. Now we are going to look at a new way of testing the inflationary mechanism.

Proving Inflation: a bootstrap approach

The bootstrap approach to test Inflation, suggested by Boyle and Steinhardt [4], is valid whenever Inflation is driven by a single scalar field and the Hubble parameter can be expanded in a Taylor series around the point $\phi_* = 0$:

$$H(\phi) = H_* + H'_*\phi + \frac{1}{2}H''_*\phi^2 + \frac{1}{6}H'''_*\phi^3 + \cdots \tag{20}$$

Each term in the expansion can be related to observables in the following manner:

$$H_* = \frac{\pi\left(\Delta^2_{\mathcal{R}}(k)\right)^{1/2}}{2}(2r)^{1/2} \tag{21}$$

$$H'_* = \frac{\pi\left(\Delta^2_{\mathcal{R}}(k)\right)^{1/2}}{8}(-r) \tag{22}$$

$$H''_* = \frac{\pi\left(\Delta^2_{\mathcal{R}}(k)\right)^{1/2}}{32}(2r)^{1/2}\left[r + 4(n_S - 1)\right] \tag{23}$$

$$H'''_* = \frac{\pi\left(\Delta^2_{\mathcal{R}}(k)\right)^{1/2}}{128}\left[64\alpha_S - 3r^2 - 20r(n_S - 1)\right] \tag{24}$$

Each mode k_* of the Fourier decomposition of perturbations must satisfy the closure condition

$$\ln\left(\frac{a_0H_0}{k_*}\right) = N_{bef}(k_*) - N_{aft}. \tag{25}$$

If the only known observable is the spectrum of scalar perturbations $\Delta^2_{\mathcal{R}}$, the best estimate for the Hubble parameter is $H(\phi) = H_* + H'_*\phi$. So we derive the value of the observable r. If this is in agreement with observations, that means the expansion up to first order is good, so $H''_* = H'''_* = 0$, and equations 23 and 24 become predictions of the model for these observables. If these predictions are in good agreement with the values observed, this will be great confirmation that the Universe underwent a period of inflationary expansion. If the calculated value for the tensor-to-scalar-ratio is not in agreement with observations, we must take the expansion to second order $H(\varphi) = H_* + H'_*\varphi + \frac{1}{2}H''_*\varphi^2$. Now, a curve will result, relating the values for the tensor-to-scalar-ratio and the spectral index.

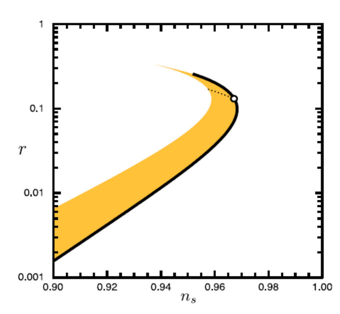

Fig. 2: The first test is indicated by the open circle. This becomes the dotted line if the radiation era doesn't follow immediately from the end of Inflation. The second test corresponds to the black solid line, transforming into the shaded region if the transition Inflation-radiation is not instantaneous.

Conclusions

- We modified the work of Di Marco and Notari, putting it in accordance with observations (14);
- Inflation may be successful in the context of Scalar-Tensor Theories of Gravity;
- If Inflation pass the tests proposed by [4], it will become much harder to account its successes to other alternatives.

Acknowledgments

CNPq

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