Propagação e absorção de ondas escalares em Schwarzschild e Reissner-Nördstrom

Gabriel Bié Alves,

Sergio E. Jorás

IF - UFRJ

Apresentação

1. Formulação matemática
2. Solução Numérica: o método de Prüfer
3. *Toy model*
4. Reissner-Nördstrom
5. Conclusões

Formulação matemática

campos escalares sem massa:

$$\Box\Psi = 0$$

Métrica de Schwarzschild
em coordenadas esféricas:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\, d\Omega^2$$

$$\Psi(t, \vec{r}) = R(r)S(\theta)\mathrm{e}^{im\phi}\mathrm{e}^{-iwt}$$

equação radial:

$$r(r-2M)\frac{d^2R}{dr^2}+2(r-M)\frac{dR}{dr}+\left[\frac{w^2r^3}{r-2M}-l(l+1)\right]R=0$$

$$\frac{dr}{dr^*}=1-\frac{2M}{r}$$

$r=2M$ $\qquad$ $r\rightarrow+\infty$

$r\rightarrow-\infty$ $\qquad$ $r^*\rightarrow+\infty$

$$u(r) \equiv r\,R(r)$$

$$\boxed{\frac{d^2u}{dr^{*\,2}} + \left[w^2 - \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2M}{r^2}\right)\right]u = 0}$$

~ Equação de Schrödinger

$$x \equiv wr$$

$$x_s \equiv 2Mw$$

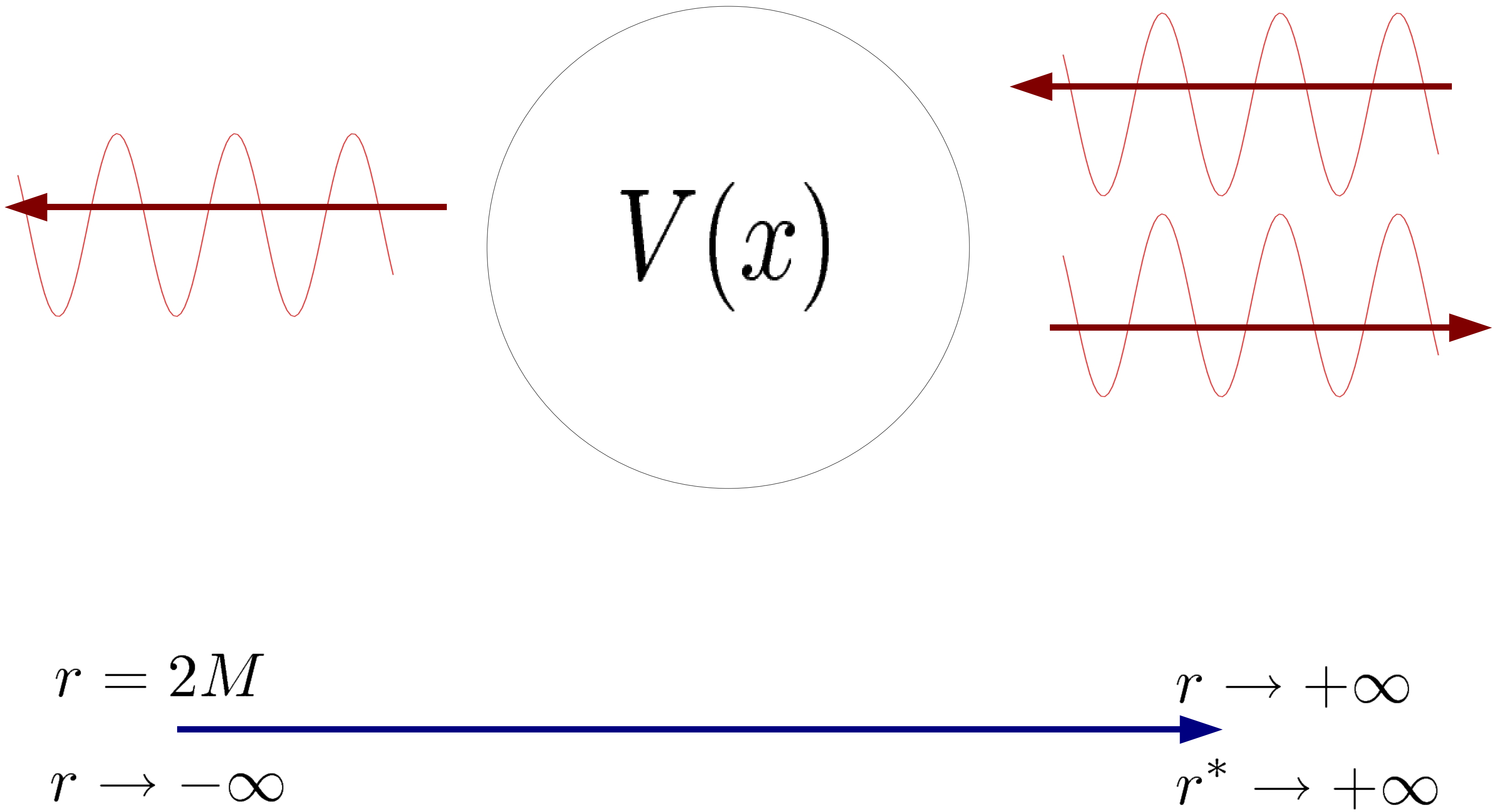
$$x^* \equiv wr^* = x + x_s \ln\left(\frac{x}{x_s} - 1\right) - x_s \ln(2x_s) + \frac{x_s}{2}$$

$$\boxed{\left[\frac{d^2}{dx^{*2}} + V(x)\right] u(x^*) = 0}$$

$$\boxed{V(x) \equiv 1 - \left(1 - \frac{x_s}{x}\right)\left(\frac{l(l+1)}{x^2} + \frac{x_s}{x^3}\right)}$$

$$V(x) \equiv 1 - \left(1 - \frac{x_s}{x}\right)\left(\frac{l(l+1)}{x^2} + \frac{x_s}{x^3}\right)$$

$$x = \mathcal{W}\left[\exp\left(\frac{x_s \ln(2x_s) - x_s + 2x^*}{2x_s}\right)\right] x_s + x_s$$


V(x)
r = 2M
r → +∞
r → −∞
r* → +∞

Solução Numérica: Método de Prüfer

Glampedakis e Andersson (2001)

$$\frac{d}{dx}\left(P(x)\frac{du}{dx}\right) + Q(x)u(x) = 0$$

$$P(x) \equiv 1$$

$$Q(x) \equiv V(x^*)$$

$$\left[\frac{d^2}{dx^{*2}} + V(x)\right] u(x^*) = 0$$

$$\begin{cases} P(x)u'(x) = r(x)\cos[\theta(x)] \\ \\ u(x) = r(x)\sin[\theta(x)] \end{cases}$$

$$G(x^*) \equiv \frac{u'}{u}$$

$$\frac{dG}{dx^*} + G^2 + V = 0$$

$$G(x^* \to -\infty) = -i$$

$$\begin{cases} \tilde{G}(x^*) \equiv \theta(x^*) - x^* \\ \dfrac{d\tilde{G}}{dx^*} + (1 - V)\sin^2(\tilde{G} + x^*) = 0 \end{cases}$$

$$\tilde{G}(x^* \to +\infty) = \zeta = \text{cte.}$$

Phase Shift

$$\delta = \zeta + l\frac{\pi}{2}$$

$$G(x^* \to -\infty) = -i$$

$$\tilde{G}(x_m^*) = \frac{i}{2} \ln \left(\frac{G(x_m^*) - i}{G(x_m^*) + i} \right) - x_m^*$$

$$\tilde{G}(x^* \to +\infty) = \zeta = \text{cte.}$$

$$\delta = \zeta + l\frac{\pi}{2}$$

$$G(x^* \to -\infty) = -i$$

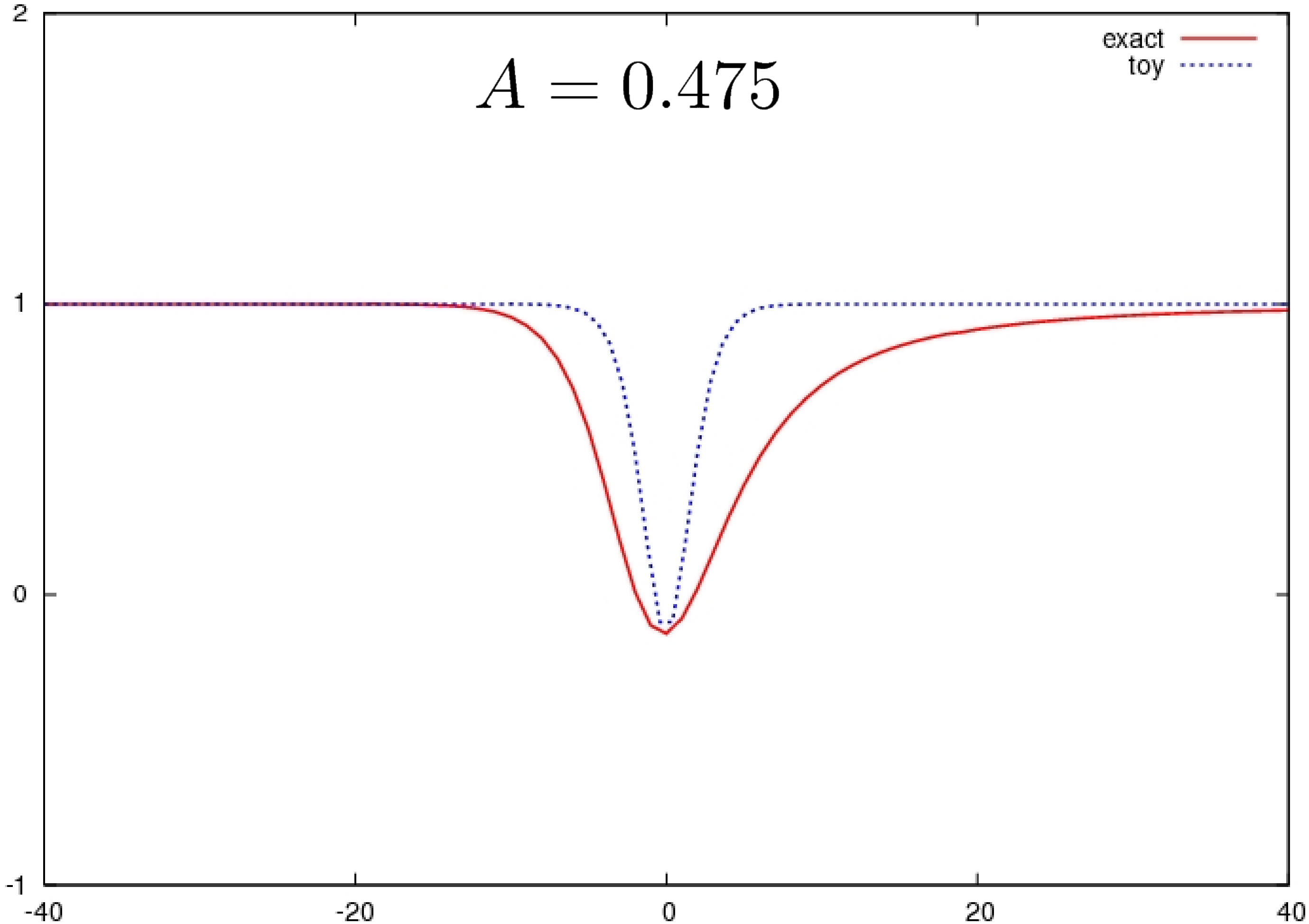
$$\tilde{G}(x_m^*) = \frac{i}{2} \ln \left(\frac{G(x_m^*) - i}{G(x_m^*) + i} \right) - x_m^*$$

$$\tilde{G}(x^* \to +\infty) = \zeta = \text{cte.}$$

$$\delta = \zeta + l \frac{\pi}{2}$$

Toy Model

$$V_{toy} = B\tanh^2(Ax^*) + 1 - B$$



$$\left[\frac{d^2}{dx^{*2}} + V(x)\right] u(x^*) = 0$$

$$V_{toy} = B\tanh^2(Ax^*) + 1 - B$$

$$\begin{cases} y \equiv \tanh(Ax^*) \\ \mu \equiv \dfrac{i}{A} \\ \nu \equiv \dfrac{\sqrt{A^2 - 4B} - A}{2A} \end{cases}$$

$$\Rightarrow \quad \boxed{u(y) = P_\nu^{-\mu}(y) + Q_\nu^{\mu}(y)}$$

$$u(x^*) \sim \begin{cases} e^{-ix^*} & , x^* \to -\infty \\ A_{in} e^{-ix^*} + A_{out} e^{+ix^*} & , x^* \to +\infty \end{cases}$$

$$\mathcal{R} = \left| \frac{A_{out}}{A_{in}} \right|^2$$

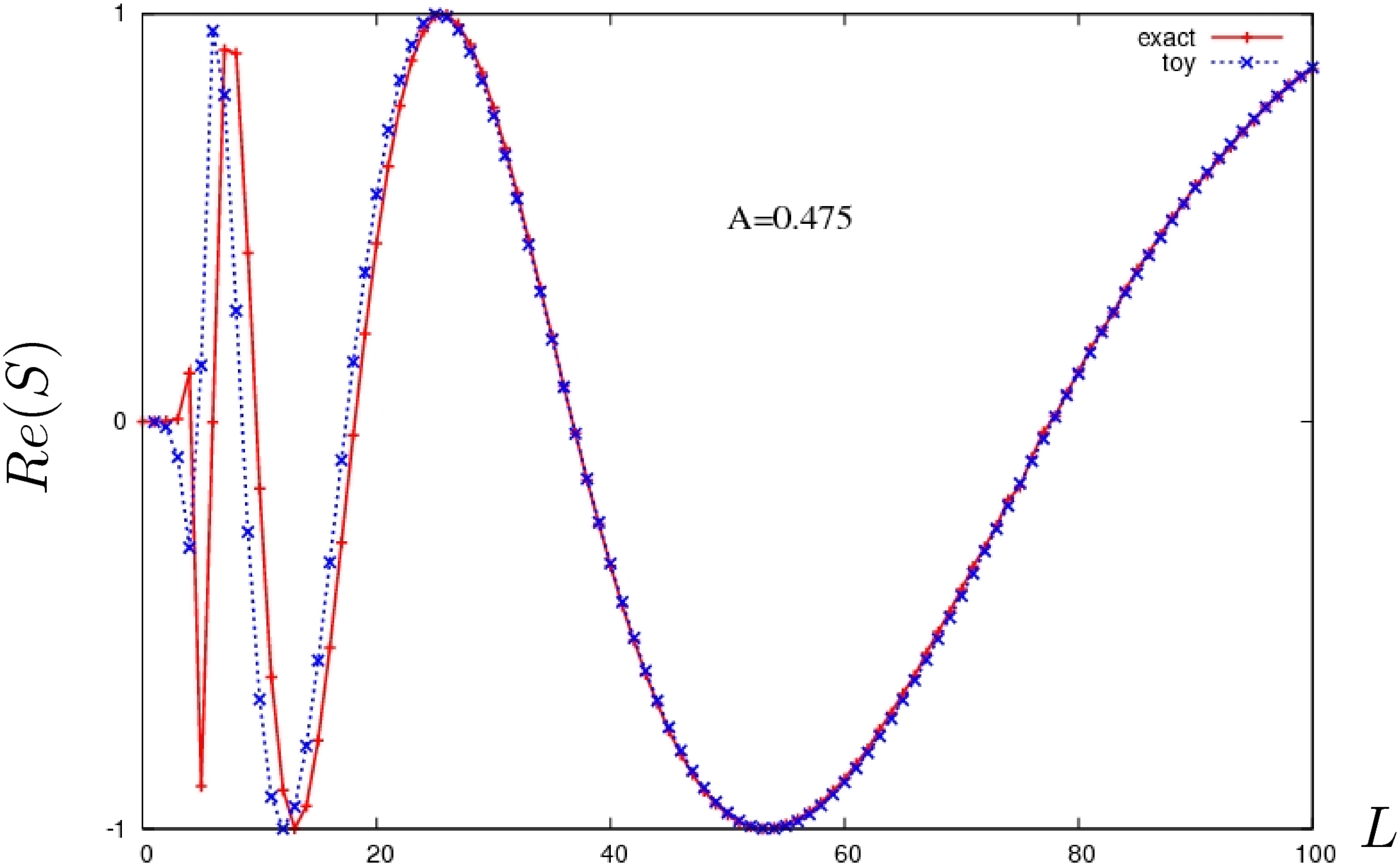
$$\mathcal{T} = 1 - \mathcal{R}$$

Matriz de Espalhamento

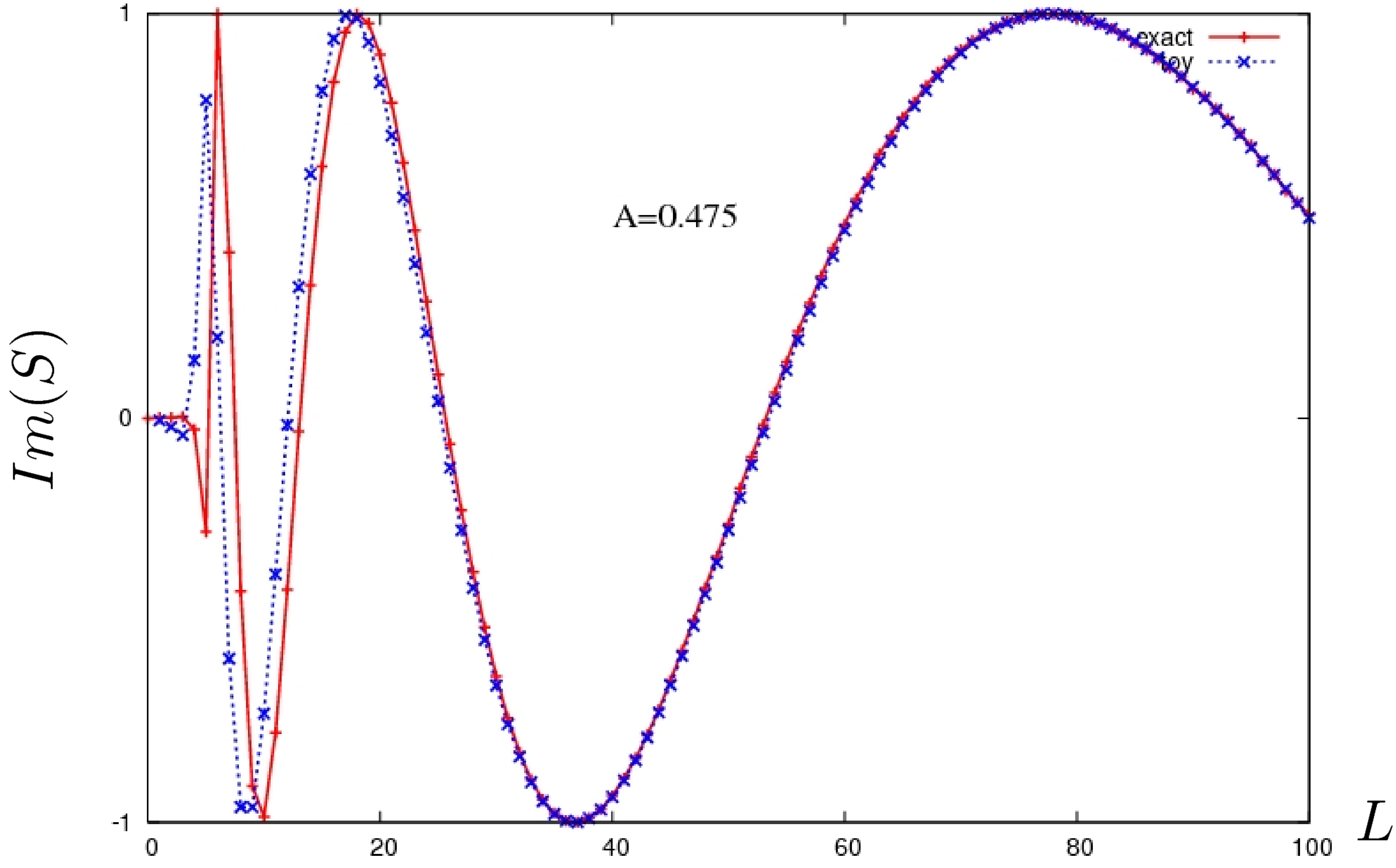
$$S \equiv \exp(2i\delta) = \frac{A_{out}}{A_{in}}$$

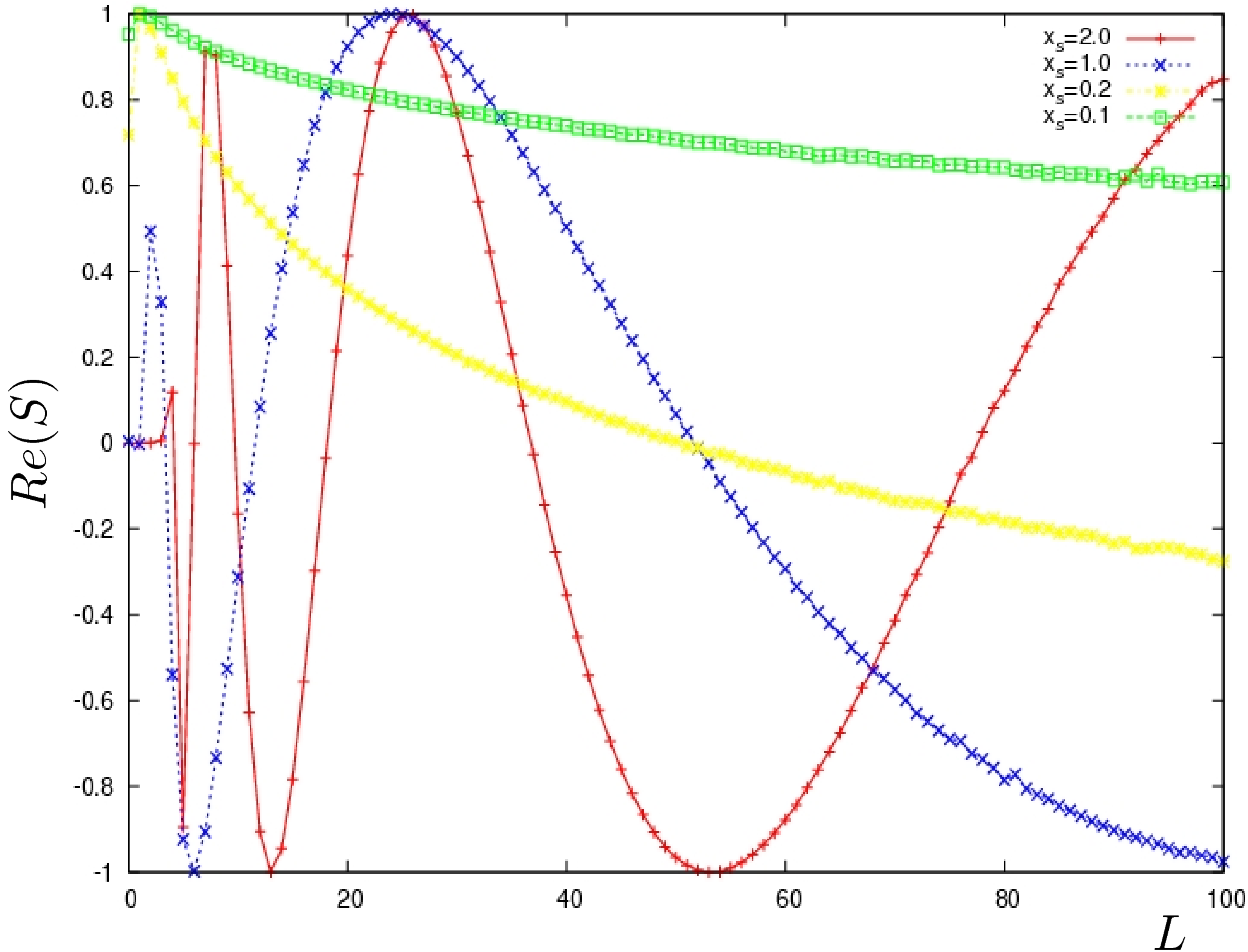
$$= \frac{1}{\pi} \frac{\cos(\mu\pi)\cos(\nu\pi)}{\sin[(\mu+\nu)\pi]} \frac{\Gamma(\nu-\mu+1)\,\Gamma(\mu)\,\Gamma(\mu+1)}{\Gamma(\mu+\nu+1)}$$

$$x_s \equiv 2Mw = 2$$

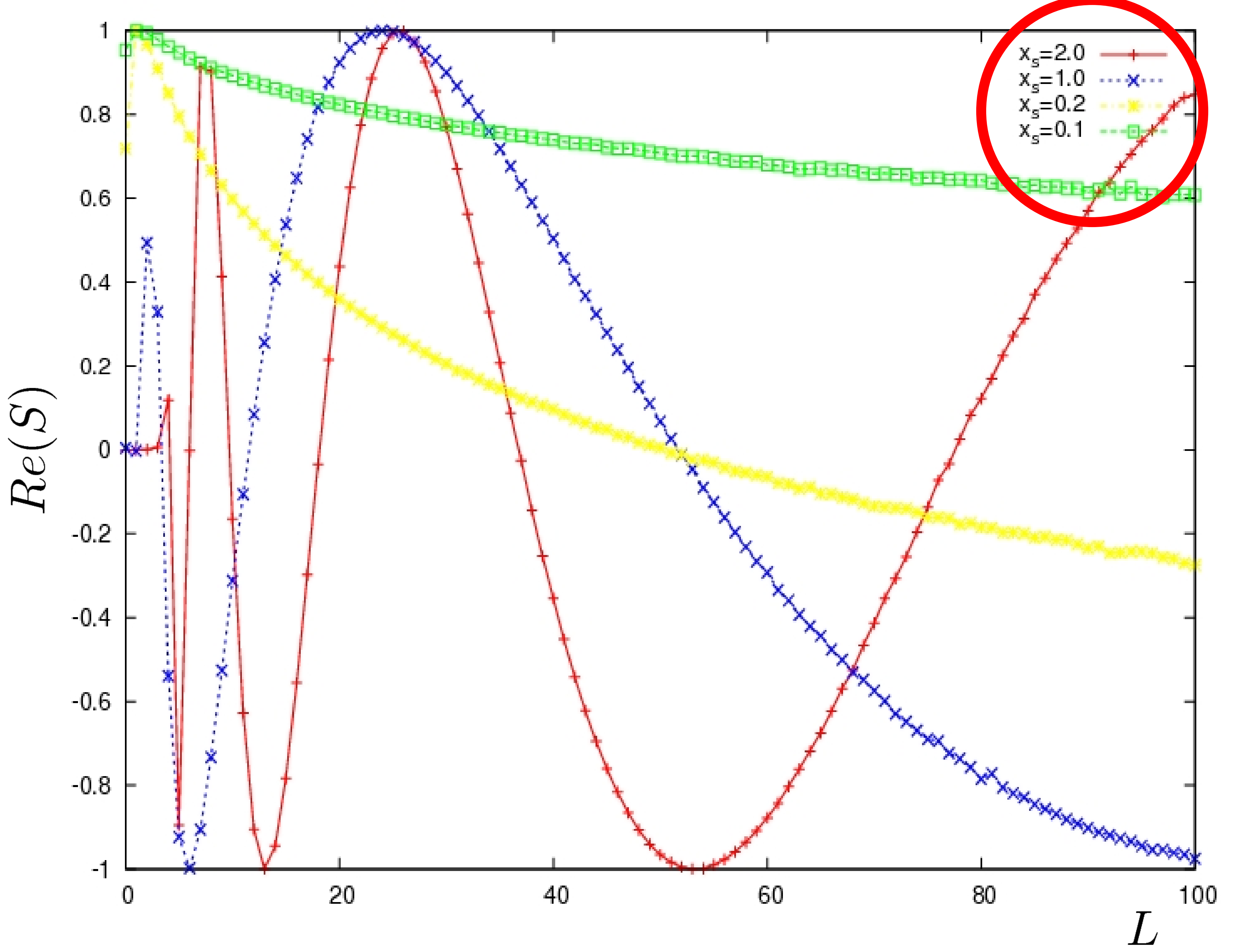


$$x_s \equiv 2Mw = 2$$

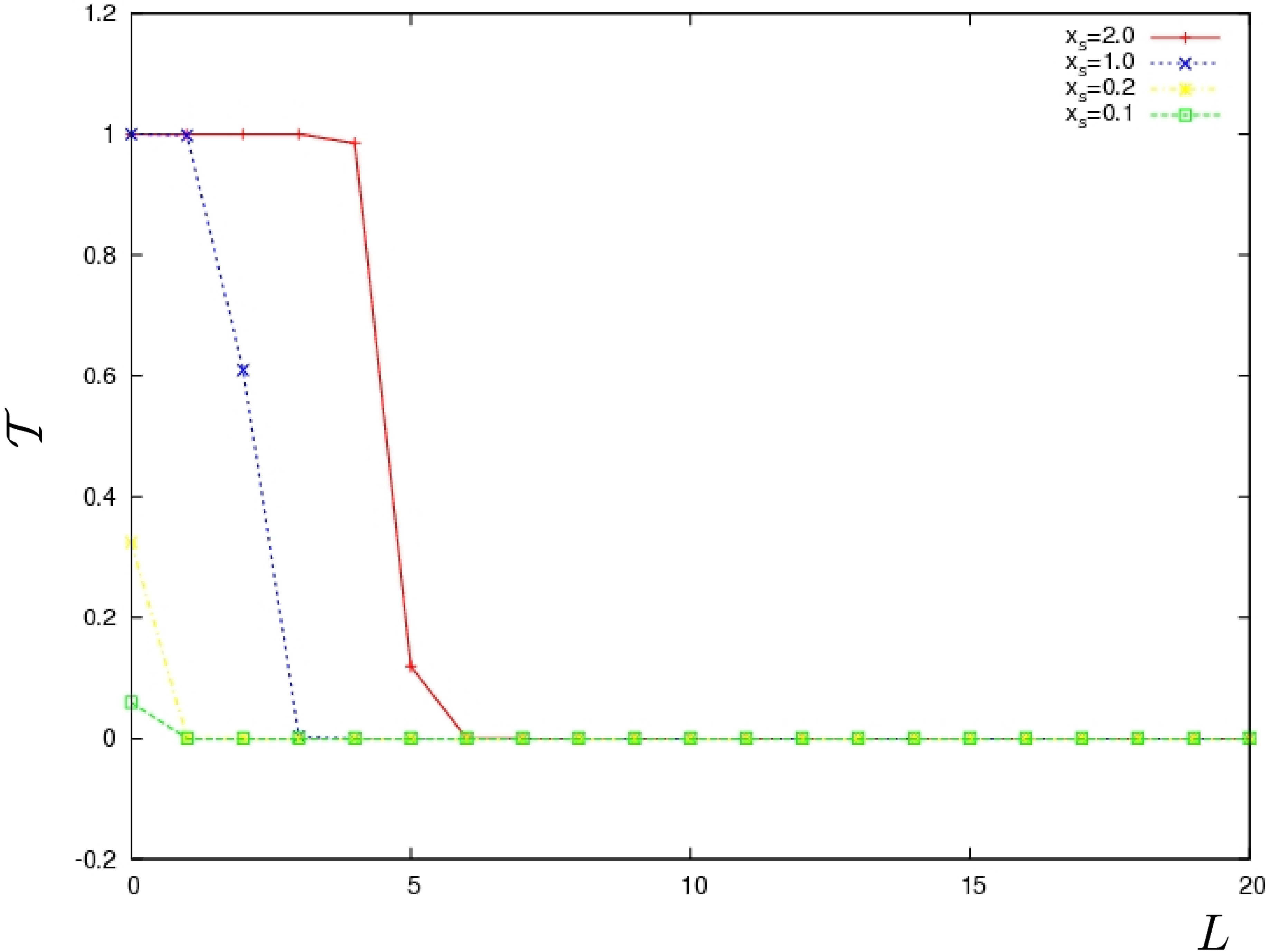




x_s=2.0
x_s=1.0
x_s=0.2
x_s=0.1
$Re(S)$
L
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100



x_s=2.0
x_s=1.0
x_s=0.2
x_s=0.1
$Re(S)$
L
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100



1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
5
10
15
20
$\mathcal{T}$
L
x_s=2.0
x_s=1.0
x_s=0.2
x_s=0.1

Reissner-Nördstrom

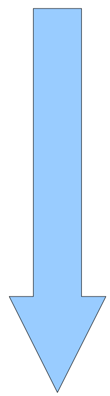
$$\Box \Psi = 0$$

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2\, d\Omega^2$$

$$f(r) \equiv \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)$$

$$r_\pm \equiv M \pm \sqrt{M^2 - Q^2}$$

$$\left[-f(r)\frac{d}{dr}\left(f(r)\frac{d}{dr}\right)+V_{ef}(r)\right]\Psi = w^2\Psi$$



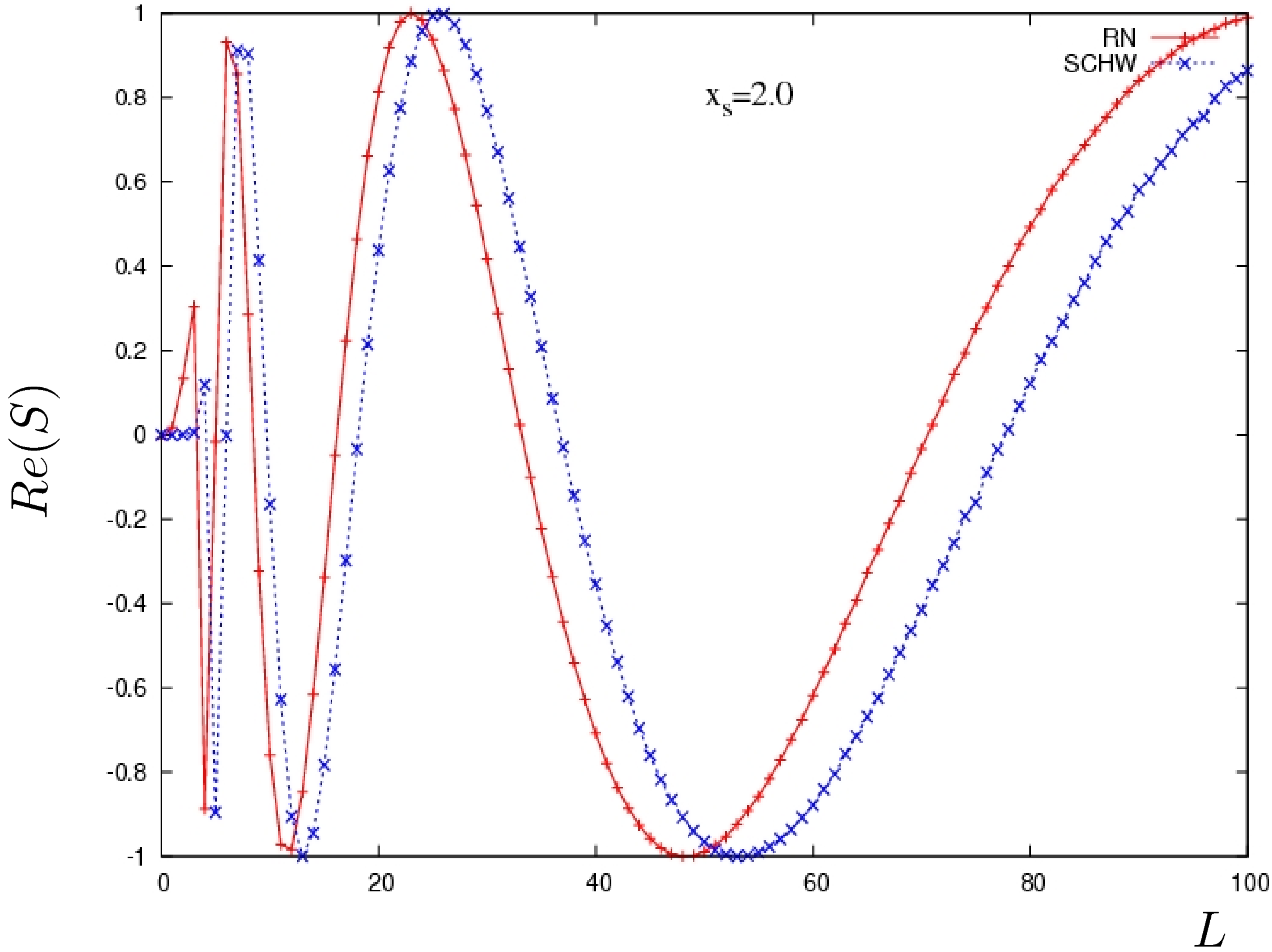
$$\frac{dr}{dr^*} = f(r)$$

$$x^* \equiv wr^*$$

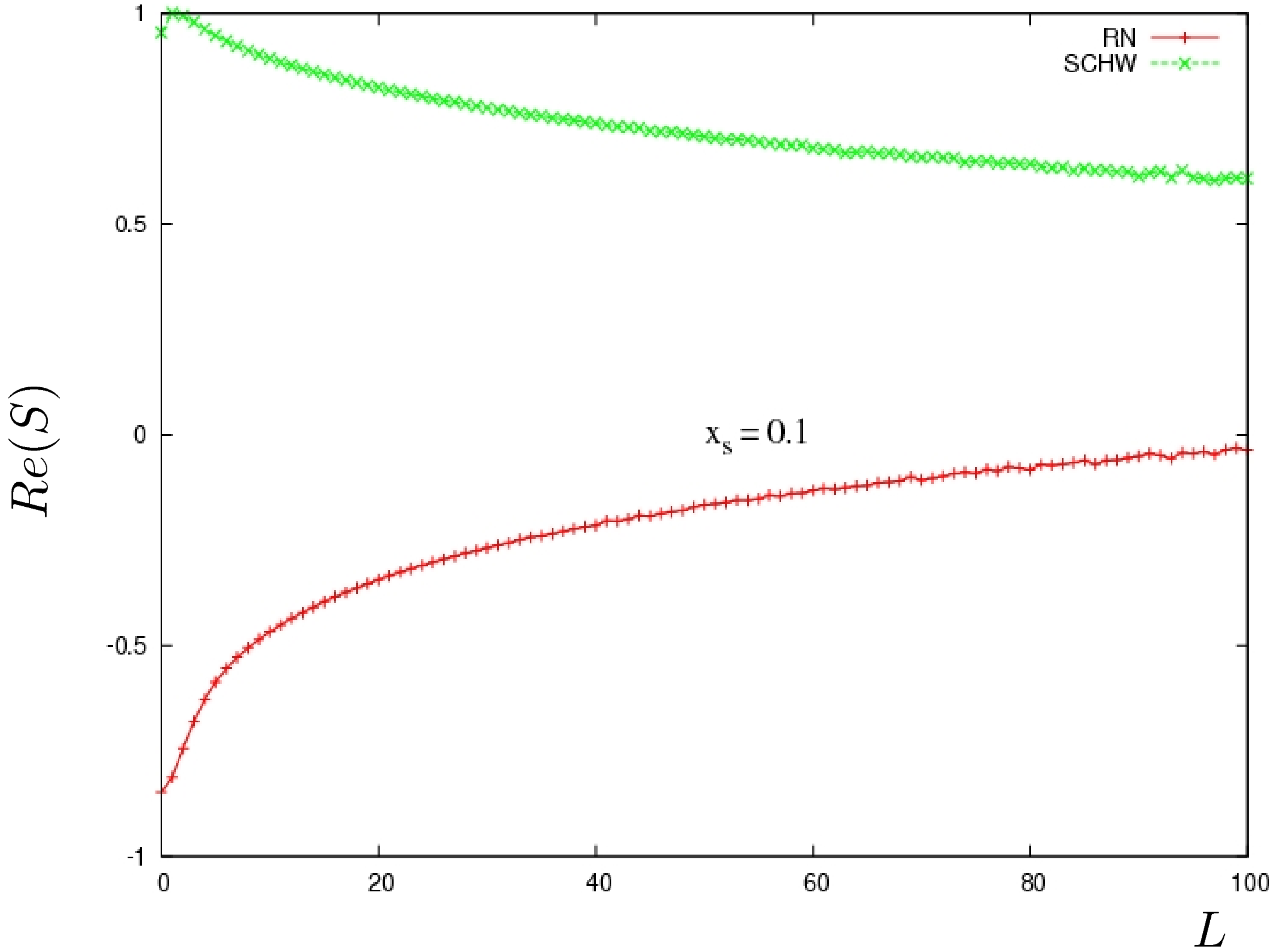
$$\left[\frac{d^2}{dx^{*2}} + V(x)\right] u(x^*) = 0$$

$$V(x) = 1 - \left(1 - \frac{x_+}{x}\right)\left(1 - \frac{x_-}{x}\right)\left(\frac{l(l+1)}{x^2} + \frac{x_s}{x^3} + \frac{x_s \epsilon^2}{2x^4}\right)$$

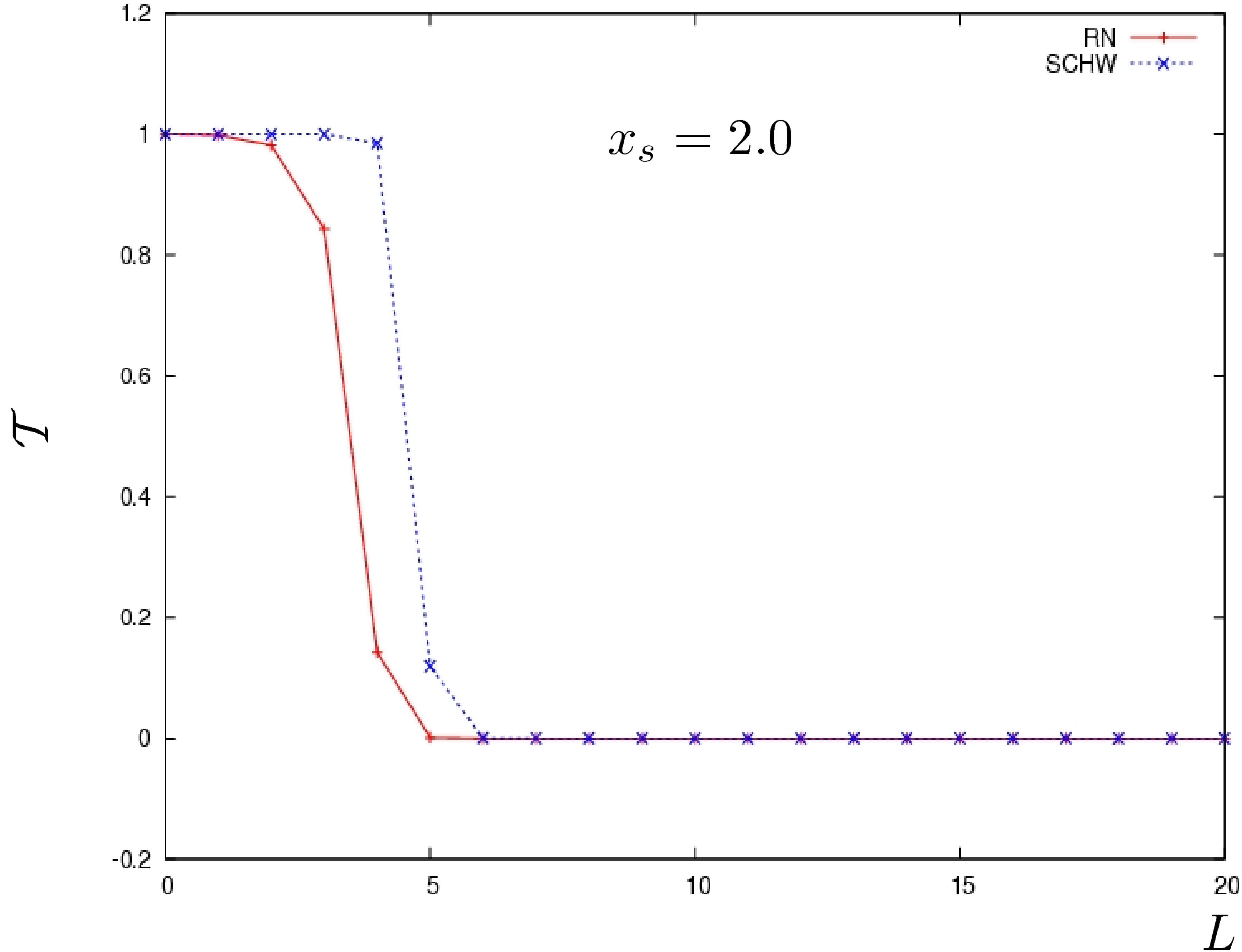
$$\epsilon \equiv \frac{Q}{M}$$

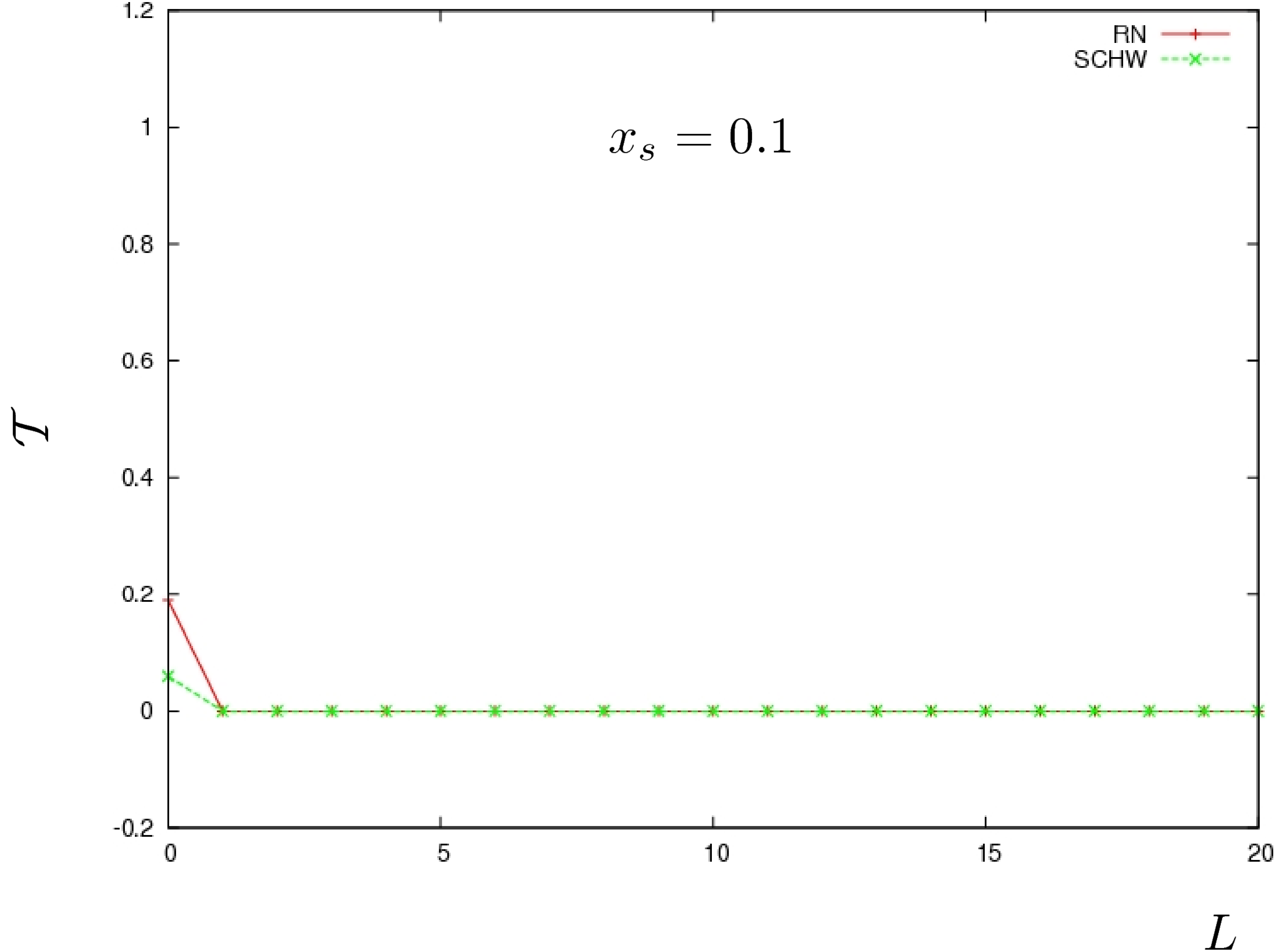


RN
SCHW
x_s=2.0
$Re(S)$
L
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100



RN
SCHW
$Re(S)$
$x_s = 0.1$
L
1
0.5
0
-0.5
-1
0
20
40
60
80
100


RN
SCHW
$x_s = 2.0$
$\mathcal{T}$
L
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
5
10
15
20



RN
SCHW
$x_s = 0.1$
$\mathcal{T}$
L
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
5
10
15
20

CONCLUSÕES

- As diferenças entre Schw e RN são maiores quando $x_s \equiv 2Mw$ é pequeno

- O tunelamento é maior quanto menor for o parâmetro de impacto

- O valor de $x_s \equiv 2Mw$ é fundamental para definir o comportamento da matriz de espalhamento e, portanto, do tunelamento