On the cosmological effects of the Weyssenhoff spinning fluid in the Einstein-Cartan framework

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Introduction

The Einstein-Cartan theory is a straightforward and natural way to introduce torsion in metric gravity.

$$S = \int \sqrt{-g} d^4x \left\{ -\frac{1}{\kappa^2}(\tilde{R} - 2\Lambda) + \mathcal{L}_{\text{source}} \right\}, \quad (1)$$

where $\kappa^2 = 16\pi G = 3.38 \times 10^{-37}$ GeV^{-2}.

But Einstein gravity satisfies the known gravitational tests, so why to include torsion?

1) Torsion is a degree of freedom present in low energy string theory.

2) It can be related to theories with Lorentz symmetry violation.

3) Torsion effects are restricted to very high energy densities, having negligible effects on gravitational tests.

Gravity with torsion:
Hehl, von der Heyde, Kerlick and Nester, Rev. Mod. Phys. (1976).
See also Shapiro, Phys. Rept. (2002).

It is interesting to study the effects of torsion in early cosmology. For a review, see Puetzfield, New Astron. Rev. (2005).

There are some well known effects, such as singularity avoidance and accelerated expansion.

Some references:
Kopczynski (1973), Trautman (1973), Hehl, von der Heyde and Kerlick (1974).
Also: Gasperini (1986), Obukhov and Korotky (1987), Shapiro (2002).

Theory

We shall consider for the source terms the axial current coupling, $J^\mu S_\mu$, and also a special spinning fluid (Weyssenhoff, 1947).

It is natural to assume $J^\mu = < \bar{\psi}\gamma^5\gamma^\mu\psi >$.

There are papers with the spinning fluid (Ray and Smalley, Gasperini, Szydlowski and Krawiec, etc.)

But no one includes both, spinning fluid and axial current.

We take the metric $g^{\mu\nu}$ and torsion

$$T^\mu{}_{\alpha\beta} := \tilde{\Gamma}^\mu{}_{\alpha\beta} - \tilde{\Gamma}^\mu{}_{\beta\alpha} \qquad (2)$$

as independent quantities in the variational proceedure.

The dynamical equations in the metric $ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$ are

$$G_{\mu\nu} = \kappa^4 \left\{ -3g_{\mu\nu}J^2 + \frac{1}{16}g_{\mu\nu}\sigma^2 - \frac{1}{8}u_\mu u_\nu \sigma^2 \right\} + \frac{\kappa^2}{2}\{(\rho + p)u_\mu u_\nu - p g_{\mu\nu}\} + \Lambda g_{\mu\nu}\,,$$

where $\sigma^2 =< S_{\mu\nu}S^{\mu\nu} >$, $< S_{\mu\nu} >= 0 = S_{\mu\nu}u^\nu$ and $J_\mu = (J(t), 0, 0, 0)$. Let $p = \rho/3$:

$$\frac{3\dot{a}^2}{a^2} = \kappa^4 \left\{ -3J^2 - \frac{\sigma^2}{16} \right\} + \frac{\kappa^2}{2}\rho + \Lambda$$

$$-\frac{\dot{a}^2}{a^2} - \frac{2\ddot{a}}{a} = \kappa^4 \left\{ 3J^2 - \frac{\sigma^2}{16} \right\} + \frac{\kappa^2}{6}\rho - \Lambda\,.$$

One has also

$$\frac{\ddot{a}}{a} = \kappa^4 \left\{ -J^2 + \frac{\sigma^2}{24} \right\} - \frac{\kappa^2}{6}\rho + \Lambda/3\,.$$

The energy-momentum conservation reads

$$\dot{\rho}a + 4\dot{a}\rho = \kappa^2 \left\{ \frac{1}{8a^5}\frac{d}{dt}\left(a^6\sigma^2\right) + 6a\frac{d}{dt}(J^2) \right\} .$$

Assuming the natural ansatz for the sources scale dependence, $\sigma^2 = \sigma_0^2 a^{-6}$ and $J^2 = J_0^2 a^{-6}$, we get the following first order non-linear system:

$$\begin{aligned} \frac{\dot{a}^2}{a^2} &= -\kappa^4 \left(\frac{J_0^2}{a^6} + \frac{\sigma_0^2}{48a^6} \right) + \frac{\kappa^2}{6}\rho + \frac{\Lambda}{3} , \\ \frac{d\rho}{da} &= -\frac{4\rho}{a} - \frac{36\kappa^2 J_0^2}{a^7} . \end{aligned}$$

The last equation admits the radiation-like solution $\rho \propto a^{-4}$ only if $J_0^2 = 0$.

Solutions

The system has two remarkable particular solutions, given by $\rho = \rho_0\, a^{-6}$:

(A) Case $\theta := \kappa^4(2J_0^2 - \sigma_0^2/48) > 0$:

$$a(t) = \left\{ \sqrt{\frac{3\theta}{\Lambda}} \sinh\left(\sqrt{3\Lambda}\, t\right) \right\}^{1/3} ; \qquad (3)$$

(B) Case $\theta < 0$:

$$a(t)^3 + \sqrt{\frac{3\theta}{\Lambda} + a(t)^6} = \sqrt{-\frac{3\theta}{\Lambda}} \exp(\sqrt{3\Lambda}\, t) \,. \quad (4)$$

These solutions restrict the present value of $\rho(t_0) = \rho_0$ to $\rho_0 = 18\kappa^2 J_0^2$.

Measurement of ρ_0 determines J_0^2.

Some features of solutions (A) and (B)

Solution (A):

No singularity avoidance, accelerated expansion only for $a > a_i$, with a_i determined by the sign of $\ddot{a}$.

Greater θ means greater a_i.

Solution (B):

Accelerated expansion all the time, singularity avoidance.

Condition $\dot{a}^2 > 0$ implies a fine tunning between parameters σ_0^2 and J_0^2: $|\theta| < 1.67 \times 10^{-84}$ GeV^2.

Let $|\theta| = 1.67 \times 10^{-84}$ GeV^2. Then, the minimum $a_{\rm min}$ will be given by $(-3\theta/\Lambda)^{1/6} \sim 1$!

Of course, $a_{\rm min}$ should be extremely small, so that $|\theta|$ must be less than, say, 10^{-300} GeV^2.

General solutions

In this case, $\rho_0 \neq 18\kappa^2 J_0^2$, such that condition $\dot{a}^2 > 0$ is not a fine tunning condition, but an upper bound one for both σ_0^2 and J_0^2.

One can integrate numerically the system, starting from an arbitrary point ρ_0, say, $\rho_0 = 10^{-54}$ GeV^4.