A General Relativistic Model for the Gravitational Field of Active Galactic Nuclei Surrounded by a Disk

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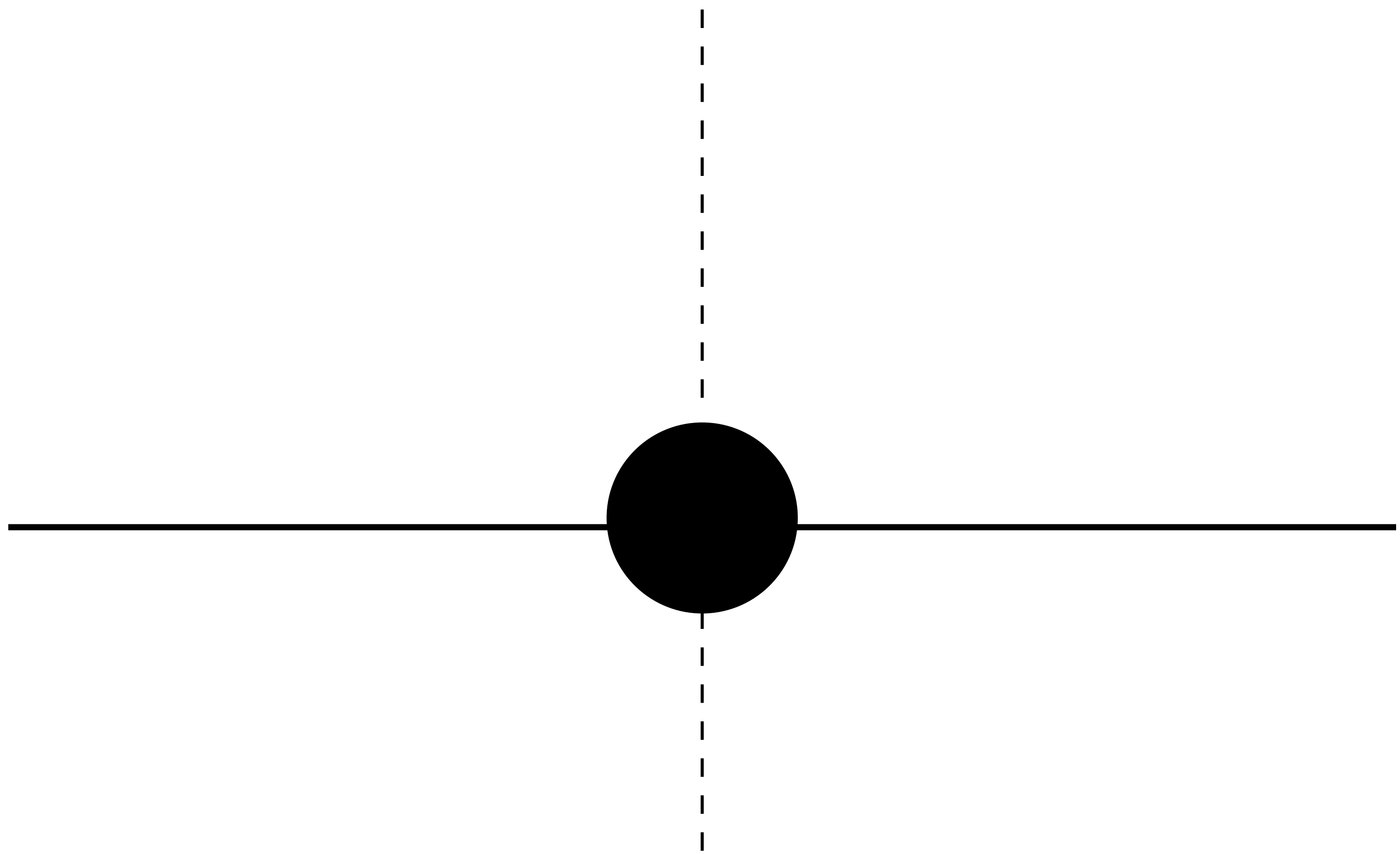
Abstract

- A Newtonian model of the gravitational field of an AGN + disk
- Relativistic model (Exact solution of EFE)
- Stability of the disk
- Orbits of test particles
- Conclusions
- Work in progress

Sketch of an AGN+disk



Weyl Solutions of EFE

Metric for axially symmetric static space-time

$$\mathrm{d}s^2 = -e^{\phi}\mathrm{d}t^2 + e^{\nu-\phi}(\mathrm{d}r^2 + \mathrm{d}z^2) + r^2 e^{-\phi}\mathrm{d}\varphi^2,$$

$$\phi_{,rr} + \frac{\phi_{,r}}{r} + \phi_{,zz} = 0,$$

$$\nu[\phi] = \frac{1}{2}\int r\left[(\phi_{,r}^2 - \phi_{,z}^2)\mathrm{d}r + 2\phi_{,r}\phi_{,z}\mathrm{d}z\right].$$

ϕ, ν functions of r, z.
Good: one equations is Laplace's eq. (linear), gives Newtonian images
Not so good: Scharzschild (BH) is represented by a bar of density 1/2.

N.B. The other equation is non linear.

Metric potential ϕ, bar and BH

The metric potential ϕ_R of a finite rod of linear mass density λ lying on the z axis and located along $[c_1, c_2]$ is

$$\phi_R = -2\lambda \ln \left[\frac{c_2 - z + \sqrt{r^2 + (c_2 - z)^2}}{c_1 - z + \sqrt{r^2 + (c_1 - z)^2}} \right]. \quad (1)$$

For the Scharzschild solution, BH at $z = 0$

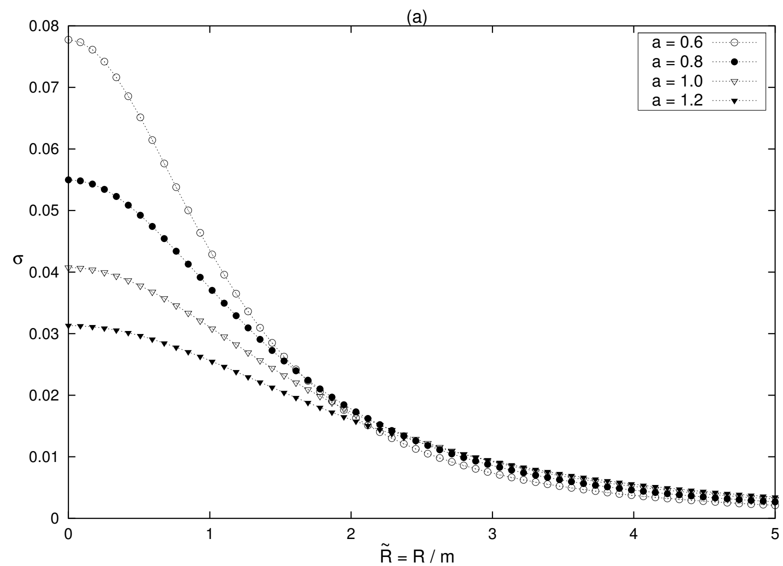
$$\phi_{BH} = \ln \left(\frac{r_1 + r_2 - 2M}{r_1 + r_2 + 2M} \right), \quad (2)$$

where $r_1^2 = (M - z)^2 + r^2$ and $r_2^2 = (M + z)^2 + r^2$. BH of mass M is represented by a rod with length $2M$.

Metric potential ϕ disk

$$\phi_D = -\frac{2m}{\sqrt{r^2 + (|z| + a)^2}}. \tag{3}$$

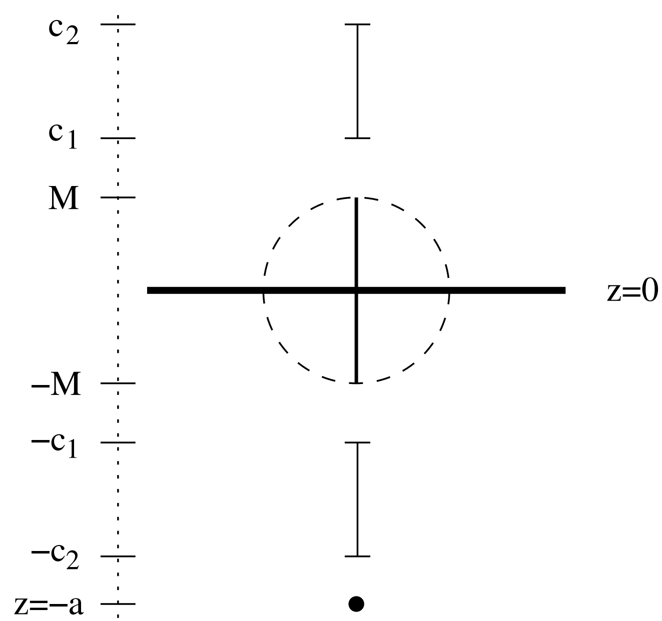
Disk density



The Superposition

$$\phi = -2\lambda \ln\left(\frac{\mu_3}{\mu_4}\right) + \ln\left(\frac{\mu_1}{\mu_2}\right) - 2\lambda \ln\left(\frac{\mu_5}{\mu_6}\right) + \phi_D,$$

$$\mu_k = \alpha_k - z + \sqrt{r^2 + (\alpha_k - z)^2},$$



The function ν

Using solitonic techniques ν can be computed in the general case. In the special case $c_1 = M$ $(\mu_4 = \mu_2$ and $\mu_5 = \mu_1)$, and on $z = 0$, we have

$$\nu[\phi_{R1} + \phi_{BH} + \phi_{R2} + \phi_D^+] =$$

$$\ln\left[\frac{r^{16\lambda^2-8\lambda+2}(r^2+\mu_1\mu_6)^{8\lambda^2-4\lambda}(r^2+\mu_2\mu_3)^{8\lambda^2-4\lambda}}{(r^2+c_2^2)^{4\lambda^2}(r^2+M^2)^{4\lambda^2-4\lambda+1}(r^2+\mu_1\mu_3)^{8\lambda^2-4\lambda}(r^2+\mu_2\mu_6)^{8\lambda^2-4\lambda}}\right]$$

$$-\frac{m^2r^2}{(r^2+a^2)^2} + \frac{8\lambda m}{(a^2-c_2^2)(a^2-M^2)\sqrt{r^2+a^2}}\left[c_2(a^2-M^2)\sqrt{r^2+c_2^2}\right.$$

$$\left.-M(a^2-c_2^2)\sqrt{r^2+M^2} + (M-c_2)(a^2+Mc_2)\sqrt{r^2+a^2}\right]$$

$$+4mM\frac{(\sqrt{r^2+M^2}-\sqrt{r^2+a^2})}{(a^2-M^2)\sqrt{r^2+a^2}}.$$

Disk Stability General

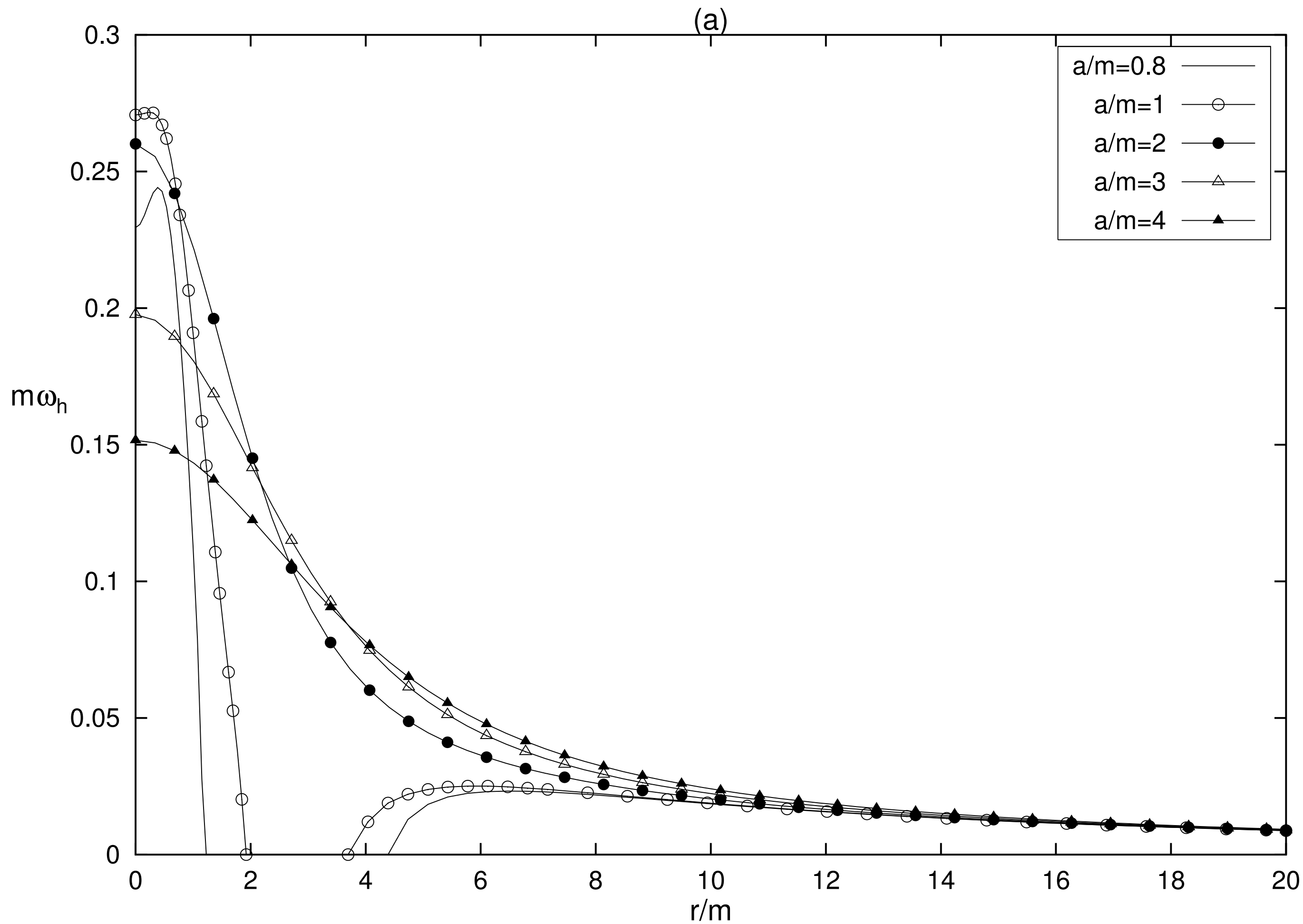
Using the perturbed equations for equatorial circular geodesics, the epicyclic frequency with respect to infinity ω_h and the vertical oscillation frequency with respect to infinity ω_v for the metric Weyl are:

$$\omega_h^2 = \frac{e^{2\phi-\nu}}{2 - r\phi_{,r}} \left(\phi_{,rr} + r\phi_{,r}^3 - 3\phi_{,r}^2 + \frac{3}{r}\phi_{,r} \right),$$

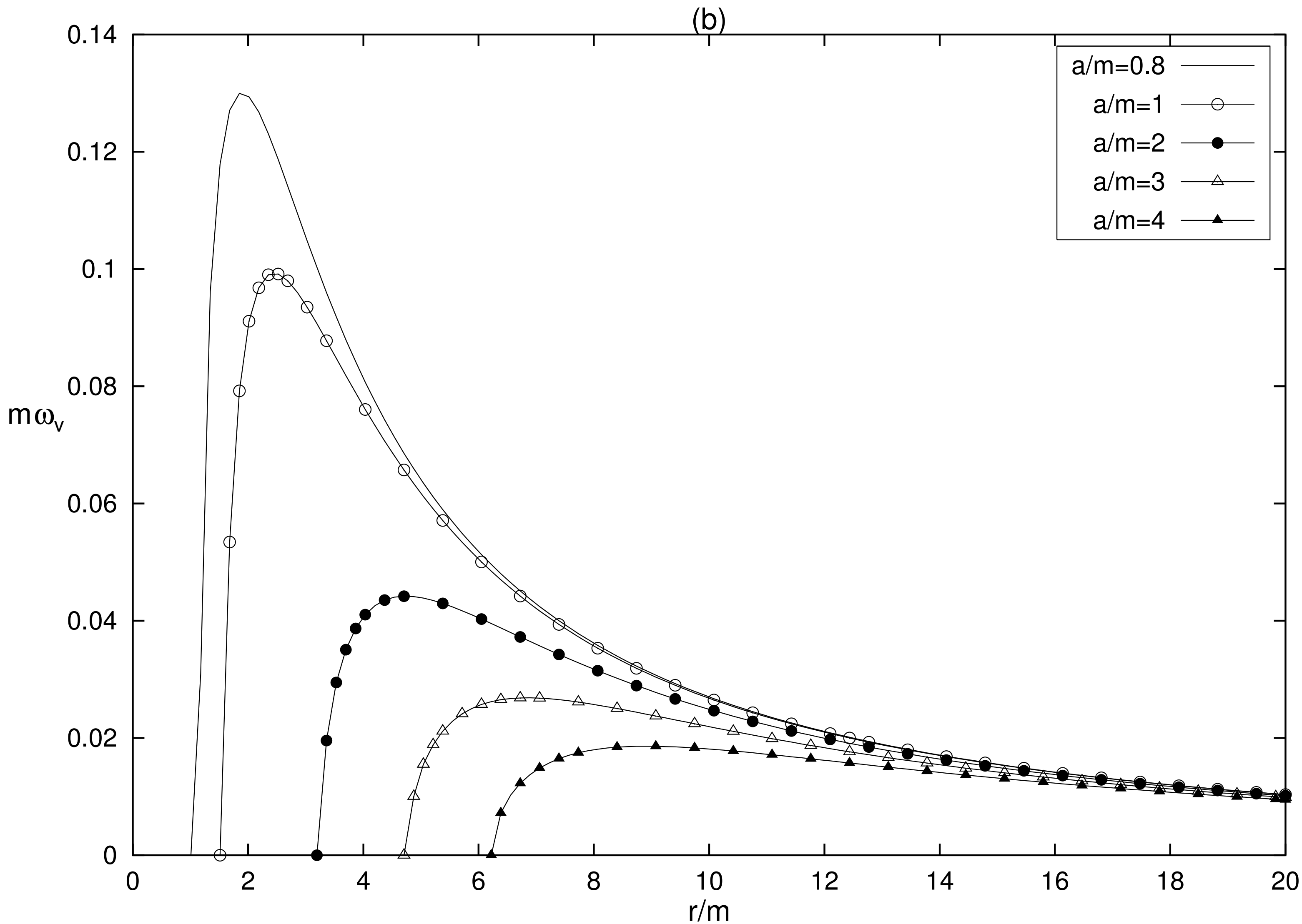
$$\omega_v^2 = \frac{e^{2\phi-\nu}}{2 - r\phi_{,r}} \left[\phi_{,zz} - 2\phi_z^2(1 - r\phi_{,r}) \right].$$

Stable horizontal and vertical orbits are only possible where $\omega_h^2 > 0$ and $\omega_v^2 > 0$, respectively.
N.B. The condition $\omega_h^2 > 0$ is equivalent to the Rayleigh stability criteria.

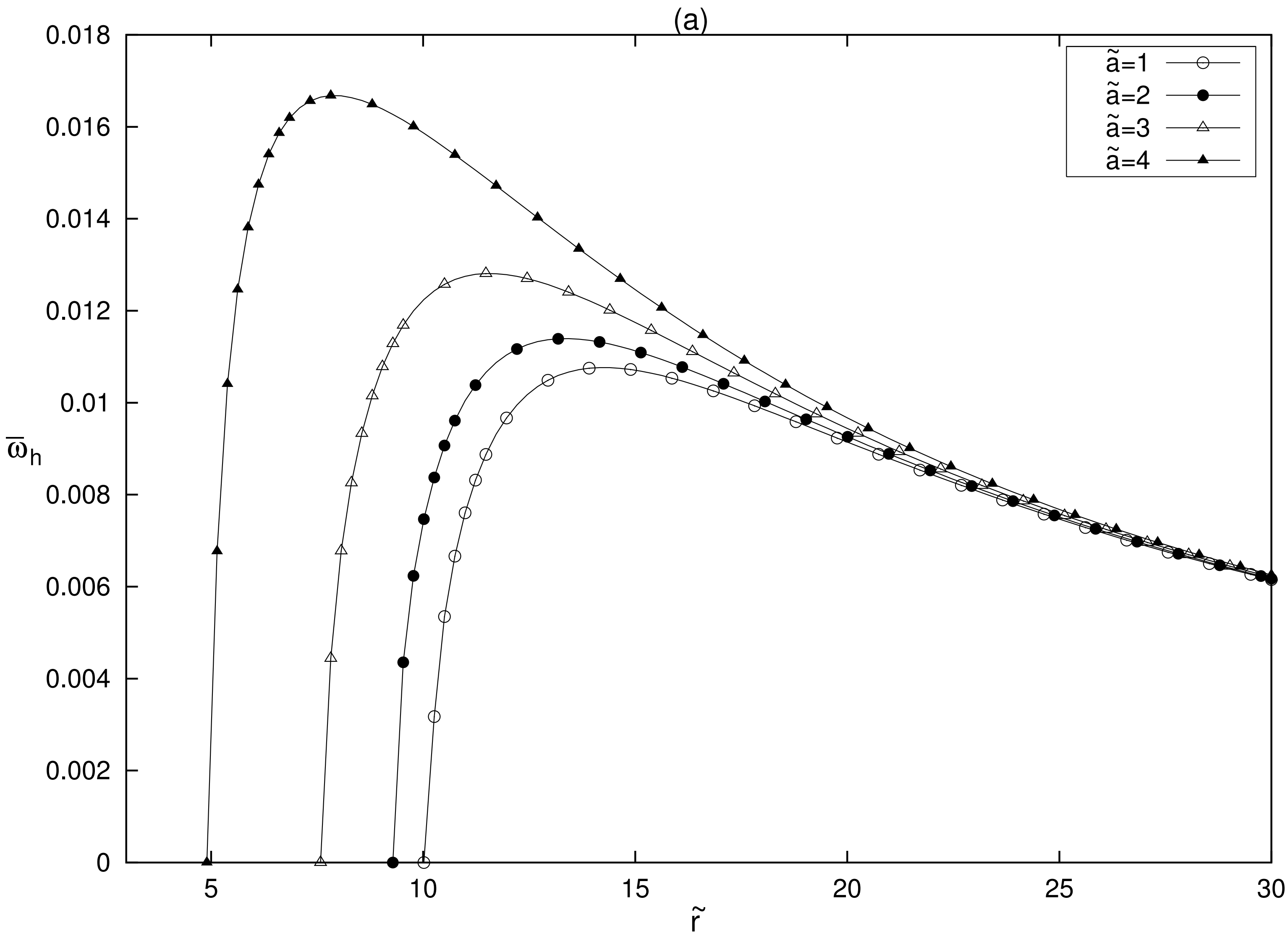
Stability of Isolated Disk (Horizontal)



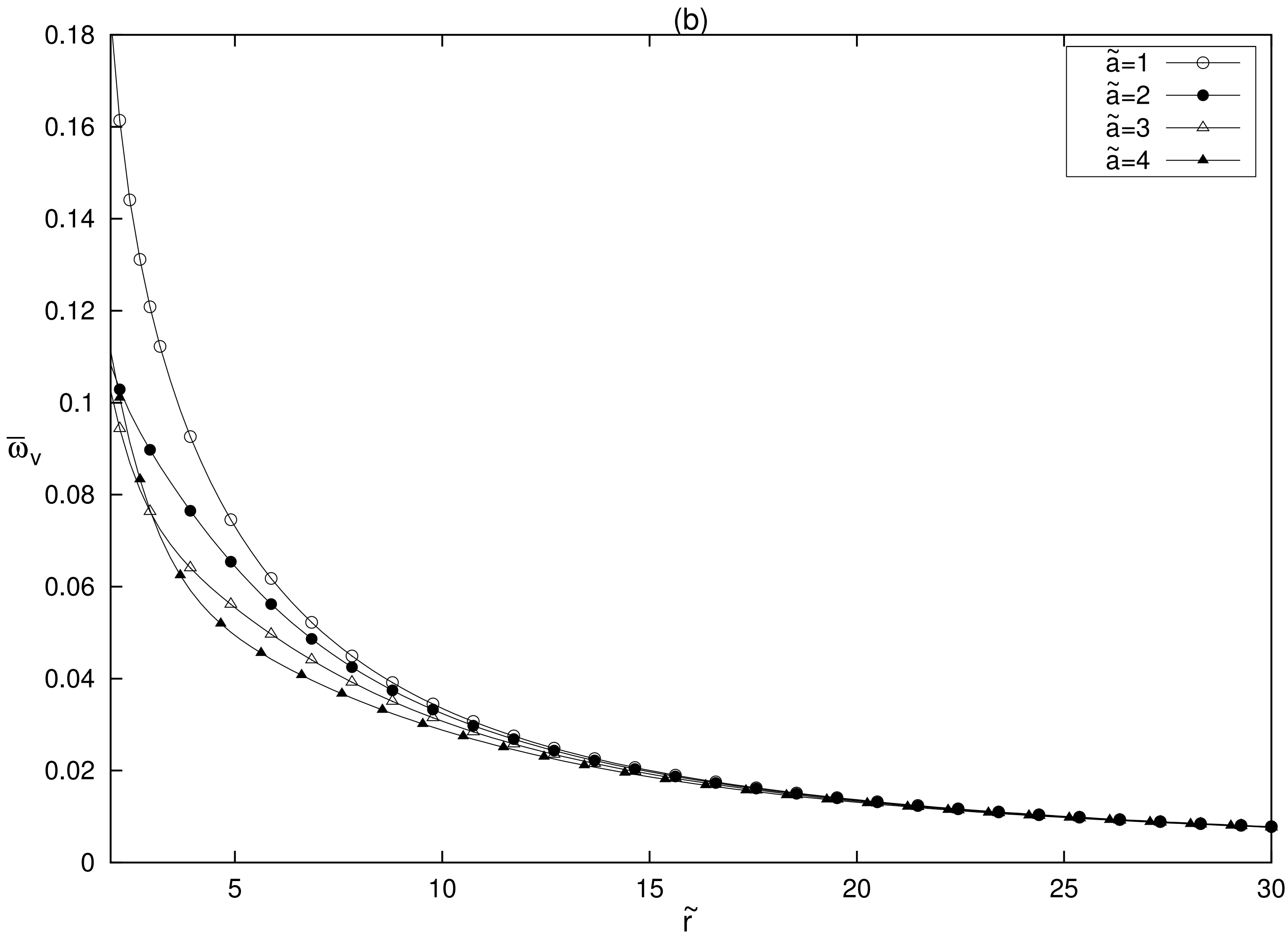
Stability of an Isolated Disk (Vertical)



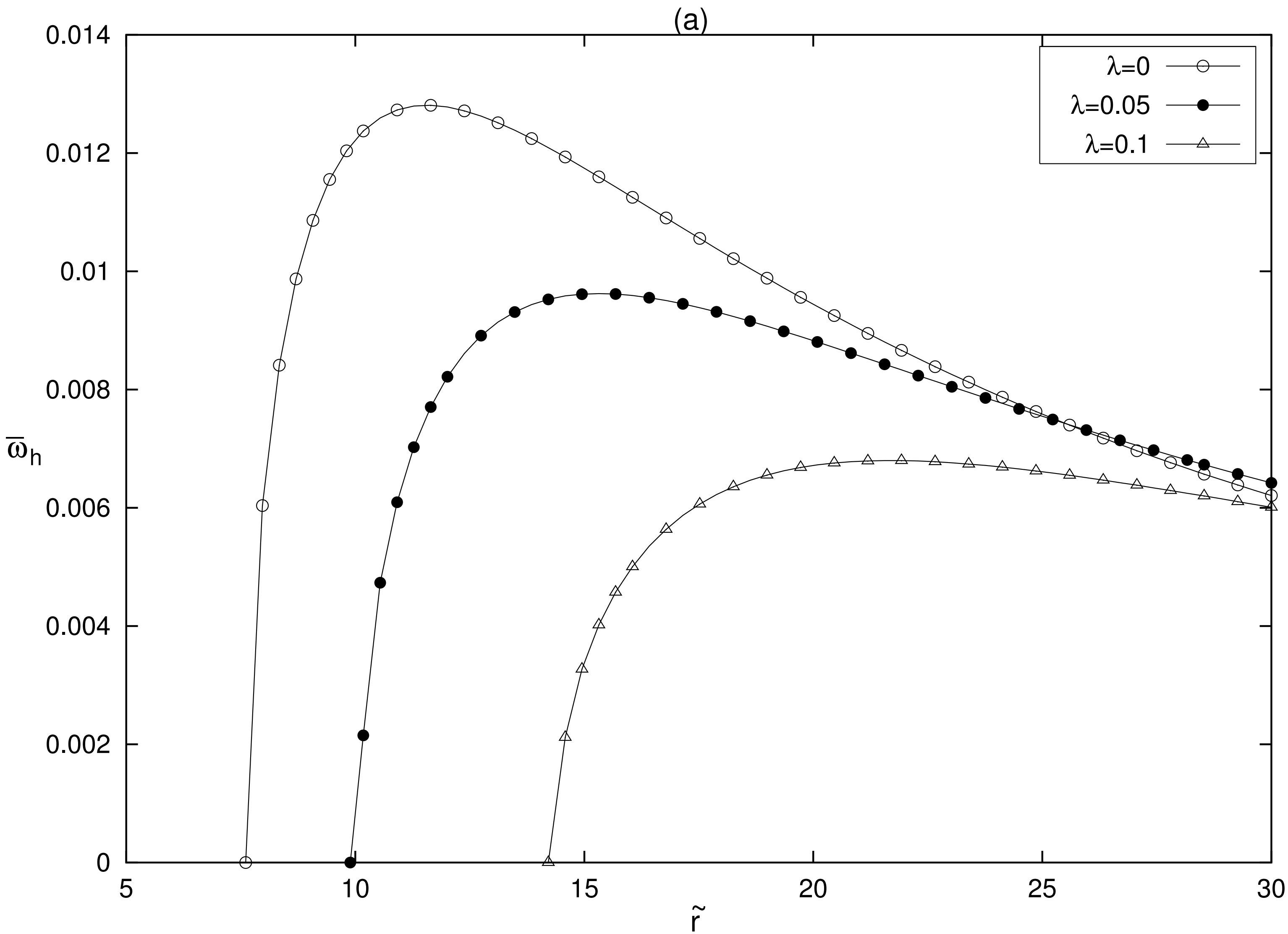
Stability of a Disk+ BH (Horizontal)



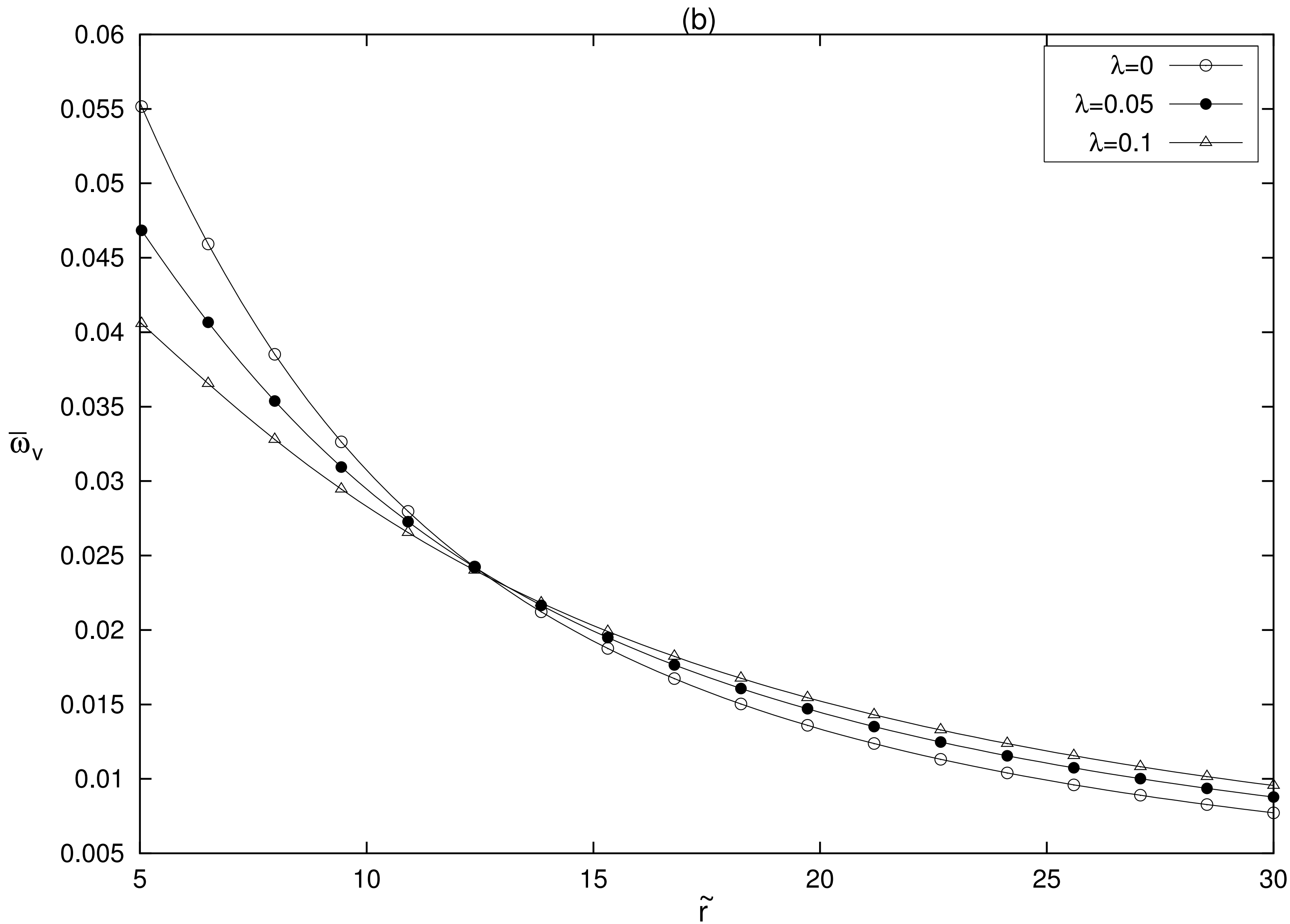
Stability of Disk + BH (Vertical)



Stability of an Disk + BH+ Bar (Horizontal



Stability of Disk + BH+ Bars (Vertical)



Vertical Geodesics

Defining the orthonormal tetrad $\{V^a, W^a, Y^a, Z^a\}$

$$V^a = e^{-\phi/2}(1,0,0,0),\; W^a = e^{(\phi-\nu)/2}(0,1,0,0),$$

$$Y^a = e^{(\phi-\nu)/2}(0,0,1,0),\; Z^a = \frac{e^{\phi/2}}{r}(0,0,0,1),$$

the tetrad components of the four-velocity v^a read

$$v^a = \gamma(1, v\sin\psi\cos\chi, v\sin\psi\sin\chi, v\cos\psi), \tag{5}$$

with $\gamma = 1/\sqrt{1-v^2}$. The specific energy and angular momentum of the test particle are

$$\mathcal{E} = e^{\phi}\dot{t} = e^{\phi/2}\gamma,\; h = r^2 e^{-\phi}\dot{\varphi} = r e^{-\phi/2}\gamma v\cos\psi.$$

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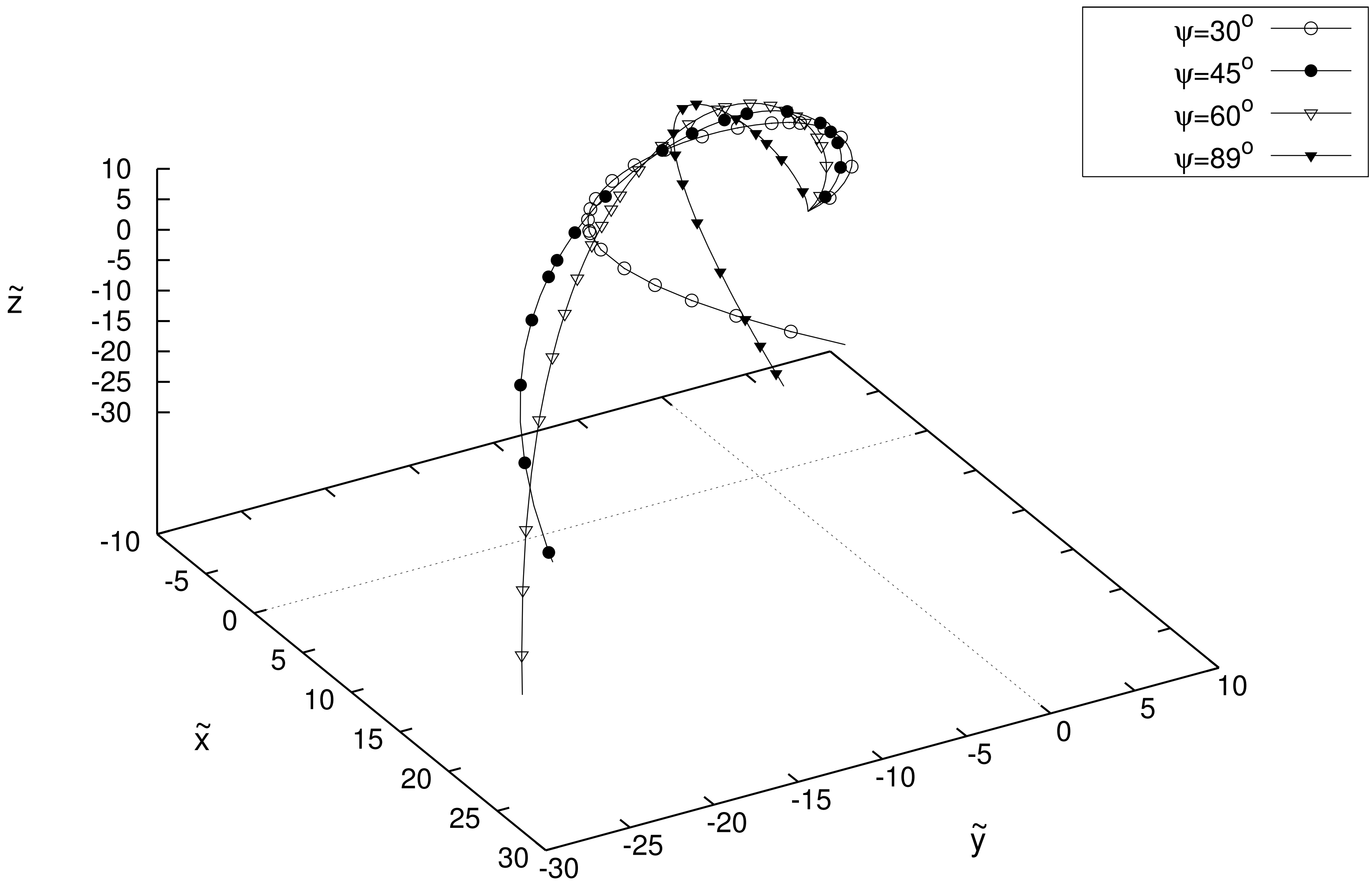
As initial conditions we take a position at radius r_0 on the disk's plane and components of the four-velocity $v_0^a = \gamma(1, 0, v_0 \sin\psi, v_0 \cos\psi)$, where v_0 is equal to the tangential velocity of circular orbits at radius r_0. We choose initial radii such that the energy is slightly higher than the escape energy.

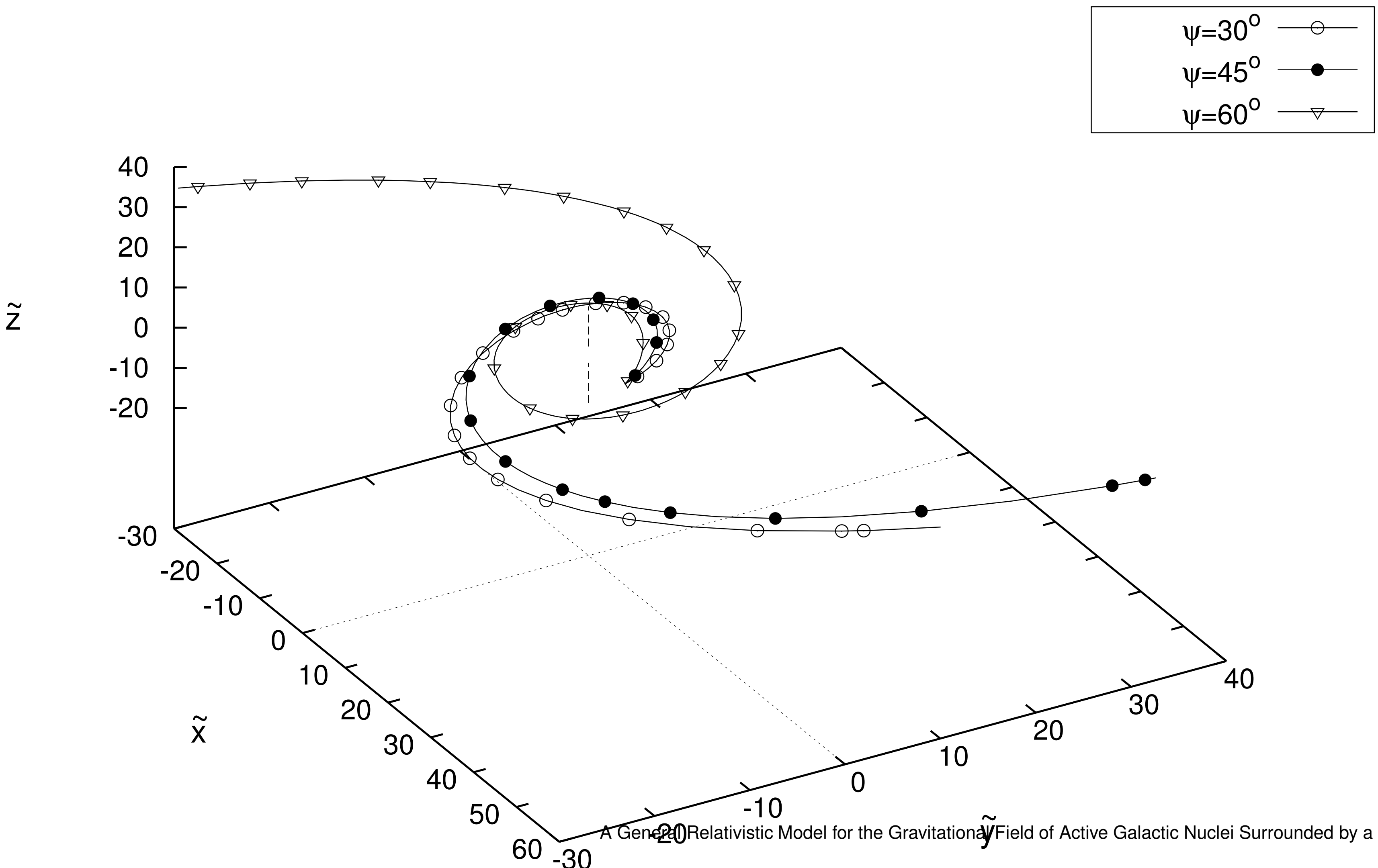
Geodesics Disk + BH

The parameters are $\alpha = 1$, $\tilde{a} = 3$, $\tilde{r}_0 = 3.9$, $\mathcal{E} \approx 1.01$.



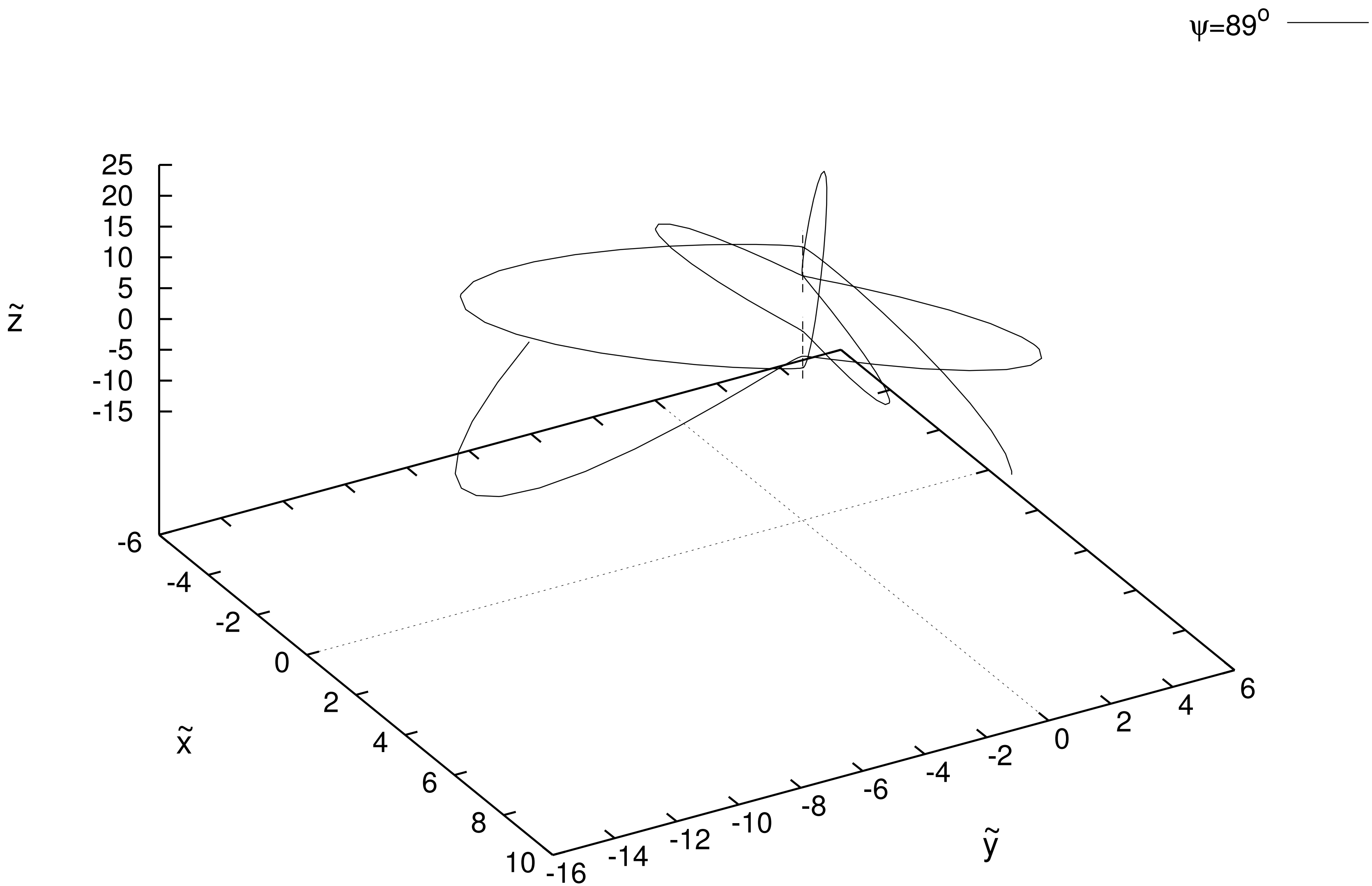
Geodesics Disk + BH+Bars I

The parameters are $\alpha = 1$, $\tilde{a} = 3$, $\tilde{r}_0 = 7.43$, $\lambda = 0.1$, $\tilde{c}_2 = 11$, $\mathcal{E} \approx 1.01$



Geodesics Disk + BH+Bars II

Same parameters and $\psi = 89^o$



Conclusions

- The black hole desestabilizes the disk in the horizontal direction whereas the opposite is true for the vertical direction.
- The more massive the rods, the larger are the disk's unstable regions in the horizontal direction. The rods also tend to lower the vertical oscillation frequencies near the disk's center, but unstable regions do not appear.
- For vertical geodesics the rods have little effect on the trajectories, but this is not true as the particles approach the z axis.
- The orbit in the last figure even suggests that we can expect chaotic behavior for orbits that pass very near the rods.

Work in Progress

- Thick Miyamoto Nagai type of Disks.
- Fokker-Plank equations for Newtonian disks