Quantum Corrections and Modified Gravity

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Abstract.

Thinking about possible applications,
we review recent achievements concerning
semiclassical corrections to gravity.

We analyze three distinct possibilities:
massless fields, heavy massive fields
and intermediate masses case.

The difference with string theory
is indicated.

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The backgrounds of QFT of matter in curved space-time have been mainly developed in the 70-ies and 80-ies.

The minimal consistency requirements include renormalizability. Renormalizable theory has

classical action of vacuum

$$S_{vac} = S_{HE} + S_{HD}\,,$$

$$L_{HE} = -\frac{1}{16\pi G}\,(R + 2\Lambda)\,,$$

$$L_{HD} = a_1 C^2 + a_2 E + a_3 \Box R + a_4 R^2\,,$$

$C_{\mu\nu\alpha\beta}$ is *Weyl tensor*, $E = R^2_{\mu\nu\alpha\beta} - 4R^2_{\mu\nu} + R^2$ is *Gauss-Bonnet* integrand.

$G,\ \Lambda,\ a_{1,2,3,4}$ are **vacuum parameters**.

None of them can be zero! However the effects of $a_{1,2,3,4}$ are Planck-supressed in IR.

On quantum level all vacuum parameters are renormalized and become running constants, that is

depend on the energy scale.

Renormalization is quite simple, but what about relevant finite corrections?

1. How to evaluate quantum corrections?

2. Which form they may have?

One of the problems due to the presence of external gravity is the choice of the vacuum state. E.g. for the black hole case there are 3 choices of vacuum (Boulware, Unruh and Hartle-Hawking) and no good understanding of the relation between them.

Effective Action (EA) method.

Advantages: relative simplicity,
direct interpretation via path integral,
explicit description of renormalization
and
separation of local and non-local corrections.

The effective action is given by

$$\Gamma[\phi] = S[\phi] + \sum_{n=1}^{\infty} \hbar^n \bar{\Gamma}^{(n)}[\phi]\,,$$

n being the number of loops. 1-loop term:

$$\bar{\Gamma}^{(1)}[\phi] = \frac{i}{2}\ln \mathrm{Det}\, S_2[\phi], \qquad S_2 = \left.\frac{\delta^2 S[\phi,\varphi]}{\delta\varphi^2}\right|_{\varphi=0}$$

In the case of interest $\phi = g_{\mu\nu}$, the quantum fields φ are fermions, vectors and scalars.

The practical calculation can be performed by different methods:

1) ζ-regularization, which is especially useful on deSitter-like spaces;

2) The local momentum representation or Schwinger-DeWitt method & generalizations;

3) Feynman diagrams using linearized gravity approach.

Schwinger-DeWitt expansion

$$\bar{\Gamma}^{(1)} = \frac{i}{2}\,\mathrm{Tr}\int_0^\infty \frac{ds}{s}\, e^{-im^2 s}\cdot \hat{U}_0 \sum_{k=0}^{\infty}(is)^k \hat{a}_k .$$

$$\hat{a}_k = \mathcal{O}\left(R^k_{....}\right) .$$

Logarithmic divergences in $d=4$ correspond to the $\lim_{x'\to x}$ tr $\hat{a}_2(x',x)$.

E.g., a real scalar with the mass m

$$\bar{\Gamma}^{(1)}_{div} = -\frac{1}{\varepsilon}\int d^4x\sqrt{-g}\,\Big\{\frac{1}{120}\,C^2_{\mu\nu\alpha\beta} - \frac{1}{360}\,E$$

$$+\frac{1}{2}\left(\xi-\frac{1}{6}\right)^2 R^2 + m^2\left(\xi-\frac{1}{6}\right)R + \frac{1}{2}\,m^4\Big\} .$$

An example of finite (nonlocal) corrections:

$$\frac{1}{4(4\pi)^2}\int C_{\mu\nu\alpha\beta}\left[\frac{8A}{15a^4}+\frac{2}{45a^2}+\frac{1}{150}\right]C^{\mu\nu\alpha\beta},$$

where

$$A = 1-\frac{1}{a}\ln\left(\frac{2+a}{2-a}\right), \qquad a^2 = \frac{4\nabla^2}{\nabla^2-4m^2} .$$

For the massless conformal fields the best option is to integrate the conformal anomaly.

$$< T^\mu_\mu >= -\,(wC^2 + bE + c\nabla^2 R)\,.$$

w, b, c depend on $N_0,\, N_{1/2},\, N_1$ - numbers of fields of different spin (helicity).

The equation for EA can be solved exactly:

$$-\frac{2}{\sqrt{-g}}\, g_{\mu\nu}\frac{\delta\bar{\Gamma}_{ind}}{\delta g_{\mu\nu}} =< T^\mu_\mu >$$

(Deser,Duff & Isham, 76; Davies, 78; Reigert; Fradkin & Tseytlin, 84).

$$\bar{\Gamma}_{ind} = S_c[\bar{g}_{\mu\nu}] + \frac{3c+2b}{36}\int d^4x\sqrt{-g}R^2$$

$$+\int d^4x\sqrt{-\bar{g}}\{w\sigma\bar{C}^2 + b\sigma(\bar{E} - \frac{2}{3}\bar{\nabla}^2\bar{R}) + 2b\sigma\bar{\Delta}_4\sigma\}\,,$$

where $g_{\mu\nu} = \bar{g}_{\mu\nu}\,\times\, e^{2\sigma}$ and $\Delta_4 = \nabla^4 + ...$, $S_c[g_{\mu\nu}]$ is an unknown conformal functional.

$\bar{\Gamma}_{ind}$ can be written in the covariant non-local form and in the covariant local form using two auxiliary scalars (Jacksenaev & Sh., 94).

We now get a hint what the general EA is: **many nonlocalities should be there!**

For isotropic and homogeneous metrics $S_c[g_{\mu\nu}]$ has no importance.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} \cdot a^2(\eta)$$

where $d\bar{s}^2 = \bar{g}_{\mu\nu}dx^\mu dx^\nu$ the $S_c[\bar{g}_{\mu\nu}]$ is just a constant. In this particular case we obtain an exact one-loop EA.

For a more general background, e.g.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} \cdot a^2(\eta) + h_{\mu\nu}(\eta, \vec{r}) \, ,$$

the anomaly-induced EA is an approximation.

Using EA or $< T_\mu^\mu >$, one can derive the dynamical equation for the grav. waves (Starobinsky, 83; Fabris, Pelinson & Shapiro, 2001; Hawking, Hertog & Real, 2001).

In the black hole case one can reproduce the result for Hawking radiation using anomaly (Christensen & Fulling, 1977)

and even classify vacuum states (Boulware, Unruh and Hartle-Hawking), choosing initial data for the Green functions insertions (Balbinot, Fabbri & Shapiro, 1999).

Quantum effects of fields with **small masses** (compared to curvature) didn't attract much attention. There is no regular approximation for this case.

A simple but useful Ansatz has been developed recently (Shapiro & Sola, 2002).

The idea is to construct the **conformal formulation** for massive fields and use it to derive generalized anomaly-induced EA for massive fields.

Conformally non-invariant terms:

$$m_s^2\,\varphi^2\,, \quad m_f\,\bar{\psi}\psi\,, \quad \mathcal{L}_{EH} = R + 2\Lambda\,.$$

Replacing dim. parameters by the scalar χ:

$$m_{s,f} \to \frac{m_{s,f}}{M}\,\chi\,, \quad M_P^2 \to \frac{M_P^2}{M^2}\,\chi^2\,, \quad \Lambda \to \frac{\Lambda}{M^2}\,\chi^2\,.$$

M is a new parameter. In the IR $\chi \sim M$.

Massive terms get replaced by $\bar{\psi}\chi\psi$ and $\chi^2\varphi^2$ interactions - all conformal.

In the gravity sector

$$\mathcal{L}^{*}_{EH} = -\frac{M_P^2}{16\pi M^2}\left\{R\chi^2 + 6(\partial\chi)^2 + \frac{2\Lambda\,\chi^4}{M^2}\right\}.$$

The new theory is conformal invariant.

The new scalar χ absorbs the conformal degree of freedom. Fixing $\chi \to M$ we come back to the original formulation.

The **conformal anomaly** becomes

$$< \mathcal{T} > = \,-\{wC^2 + bE + c\Box R +$$

$$+\frac{f}{M^2}\,[R\chi^2 + 6(\partial\chi)^2] + \frac{g}{M^4}\,\chi^4\},$$

$f,\, g$ are β-functions for $1/16\pi G$ and $\Lambda/8\pi G$.

This **new anomaly** can be integrated, the result is compatible with the renormalization group correction.

This method of deriving EA enables one to construct an interesting inflationary model. Unfortunately this approximation is not reliable in the last period of inflation, most relevant from the physical point of view.

The case of heavy fields $(m^2 \gg |R|)$ is specially interesting for the cosmological applications . This situation is typical for the late universe, if we are interested in the quantum effects on the vacuum energy.

The lowest order local terms in the EA can be derived using Schwinger-DeWitt method, but they depend on renormalization condition and play restricted role in the applications. The higher order terms are negligible.

The non-local terms are most important !

In order to derive the non-local part of EA we need to apply a mass-dependent renormalization scheme. For a while it has been done only within the linearized gravity framework (Gorbar & Shapiro, 2003).

In this way we can observe the decoupling for the higher derivative terms, but not for the cosmological constant or Einstein terms.

The very $\exists$ of EA teaches us a lesson.
Covariance: odd in metric derivatives terms are completely **ruled out.**

If, e.g., cosmologists will discover $\rho_\Lambda \sim H$, this will be an indication to new unknown physics (strings, quintessence etc).

Even worst situations hold for the intermediate regime $(m^2 \gg |R|)$. In this case the higher order terms in the Schwinger-DeWitt expansion have the same order of magnitude as the first terms, so even for local terms there is no reliable approximation.

Good news:
It seems there isn't a big difference between one-loop and multiloop corrections.
In both cases relevant terms are infinite series of (probably) infinite products of non-local terms like Green functions and local ones like curvatures.

Furthermore, at the tree level the SSB is producing ∞ many nonlocalities.

Consider the vacuum sector of a theory with **Spontaneous Symmetry Breaking (SSB).**

The theory of a charged scalar φ

$$\mathcal{L} = |\partial_\mu \varphi|^2 + (\mu_0^2 + \xi R)\,|\varphi|^2 - \lambda\,|\varphi|^4 \,.$$

At the classical level the VEV for φ is v:

$$-\Box v + \mu_0^2 v + \xi R v - \, 2\lambda v^3 = 0.$$

For $\xi = 0$ the vacuum solution is constant

$$v_0^2 \,=\, \mu_0^2/2\,\lambda\,.$$

For $\xi \neq 0$ the derivatives can not be ignored.

For the general case of a non-constant scalar curvature the solution can be presented as the power series in curvature (or ξ).

$$v(x) \,=\, v_0 \,+\, v_1(x) \,+\, v_2(x) \,+\, \dots\,.$$

For the first order term $v_1(x)$

$$-\,\Box v_1 \,+\, \mu^2 v_1 \,+\, \xi R\, v_0 \,-\, 6\lambda v_0^2\, v_1 \,=\, 0\,,$$

and the solution is

$$v_1 \,=\, \frac{\xi v_0}{\Box + 4\lambda v_0^2} R\,.$$

In a similar way, we find

$$v_2 = \frac{\xi^2\, v_0}{\Box + 4\lambda v_0^2}\, R\, \frac{1}{\Box + 4\lambda v_0^2}\, R$$

$$- \frac{6\,\lambda\,\xi^2\, v_0^3}{\Box + 4\lambda v_0^2} \left(\frac{1}{\Box + 4\,\lambda v_0^2}\, R \right)^2 , \quad \text{etc.}$$

One can continue the expansion of v to any order. The induced vacuum action is $S_{ind} =$

$$= \int d^4x \sqrt{g} \left\{ \lambda v_0^4 + \xi R v_0^2 + R \frac{\xi^2 v_0^2}{\Box + 4\lambda v_0^2} R + ... \right\} .$$

First terms: induced Λ and Einstein-Hilbert action. One has to sum them with the corresponding vacuum terms.

Other (∞ many) terms are quasi-local in IR.

E.g.

$$\frac{1}{\Box + 4\lambda v_0^2} \approx \frac{1}{4\lambda v_0^2} \left\{ 1 - \frac{\Box}{4\lambda v_0^2} + ... \right\} .$$

The non-localities coming from the SM are perhaps irrelevant. But this is not true for the hypothetic light scalars ...

Concluding observations.

- Since the regular methods fail, we can do phenomenology and construct useful models for the quantum corrections - in the way this is actually done for string theory.

- At the same time, any new information about the form of the quantum corrections is significant and important.

- The importance of quantum corrections to the gravitational action is absolute. Different from quantum gravity, strings, etc, these corrections exist **for sure**. Indeed they can be negligible in relevant cases, but this is not certain until now.

- **Until we investigate better the quantum corrections from matter fields, perhaps nothing definite can be stated about manifestations of a qualitatively new physics**.