

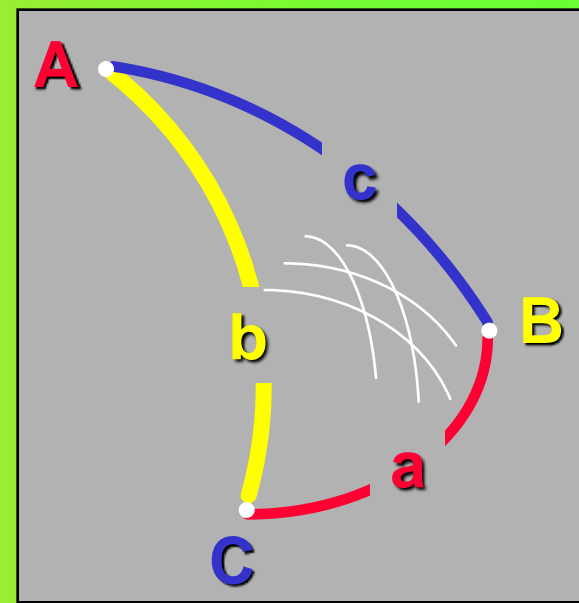
Ângulo entre dois astros

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Trigonometria esférica

Resumo das Fórmulas de Trigonometria Esférica



Co-seno

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

Seno

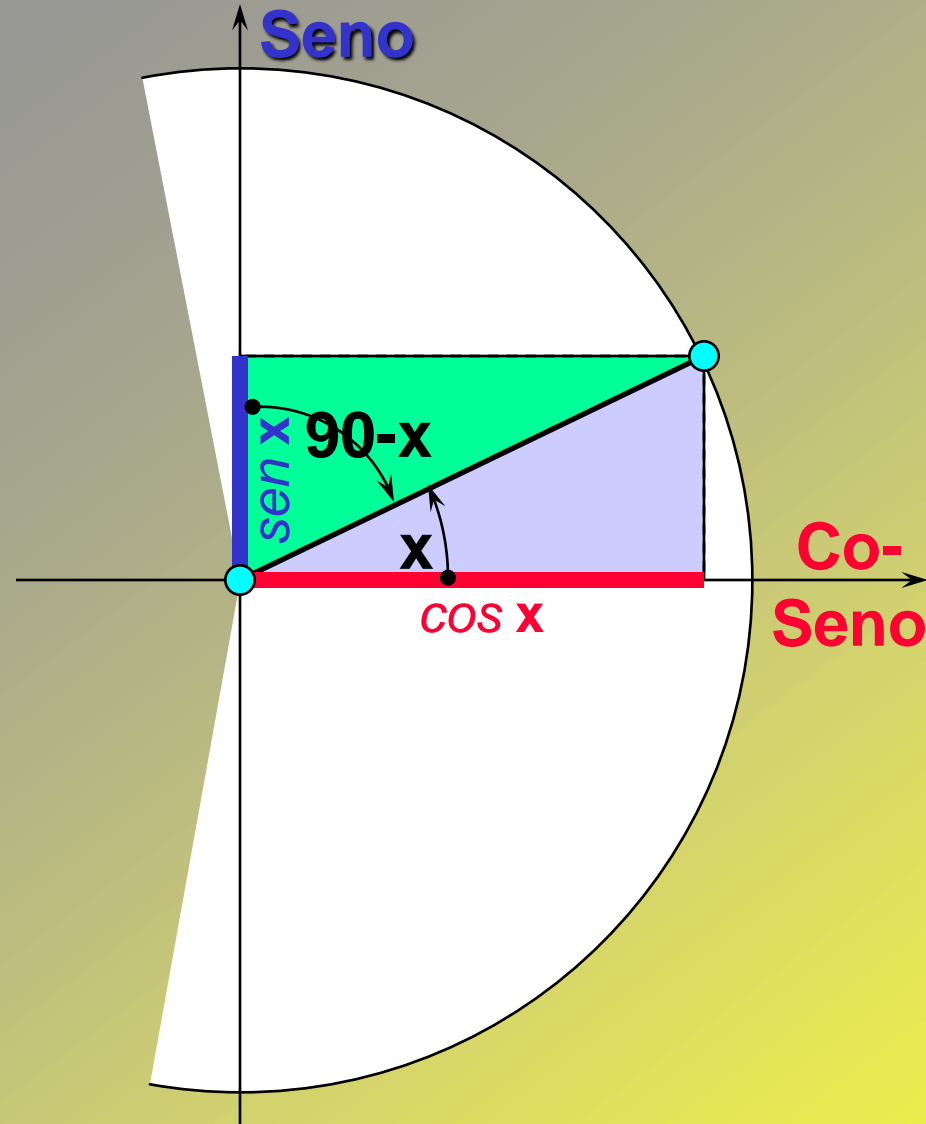
$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Seno & Co-seno

$$\sin a \cdot \cos B = \cos b \cdot \sin c - \sin b \cdot \cos c \cdot \cos A$$

Ângulo entre 2 cidades na Esfera Terrestre

Seno & Co-seno de $(90^\circ - x)$



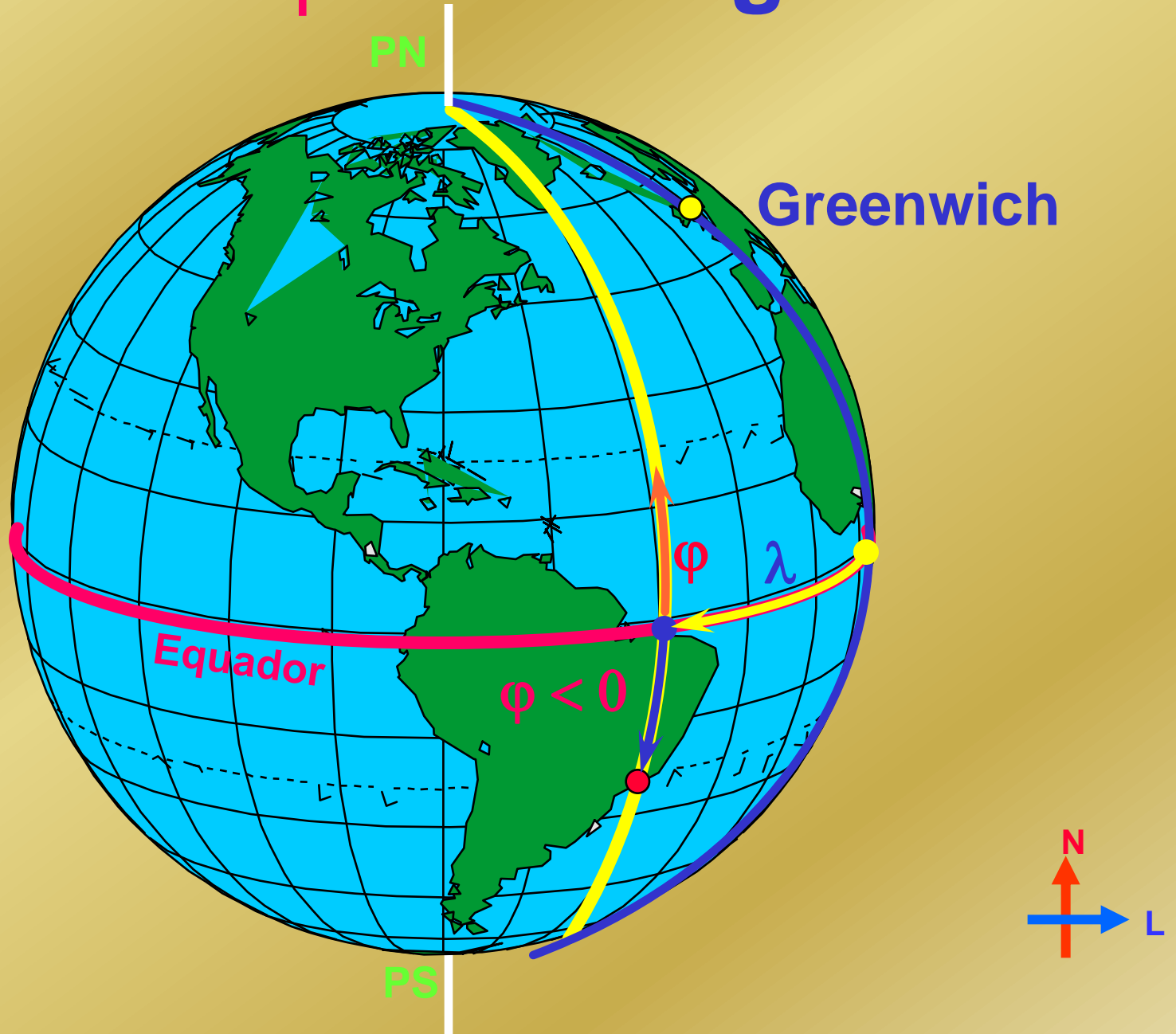
$$\text{sen } (90^\circ - x) = \text{cos } x$$

$$\text{cos } (90^\circ - x) = \text{sen } x$$

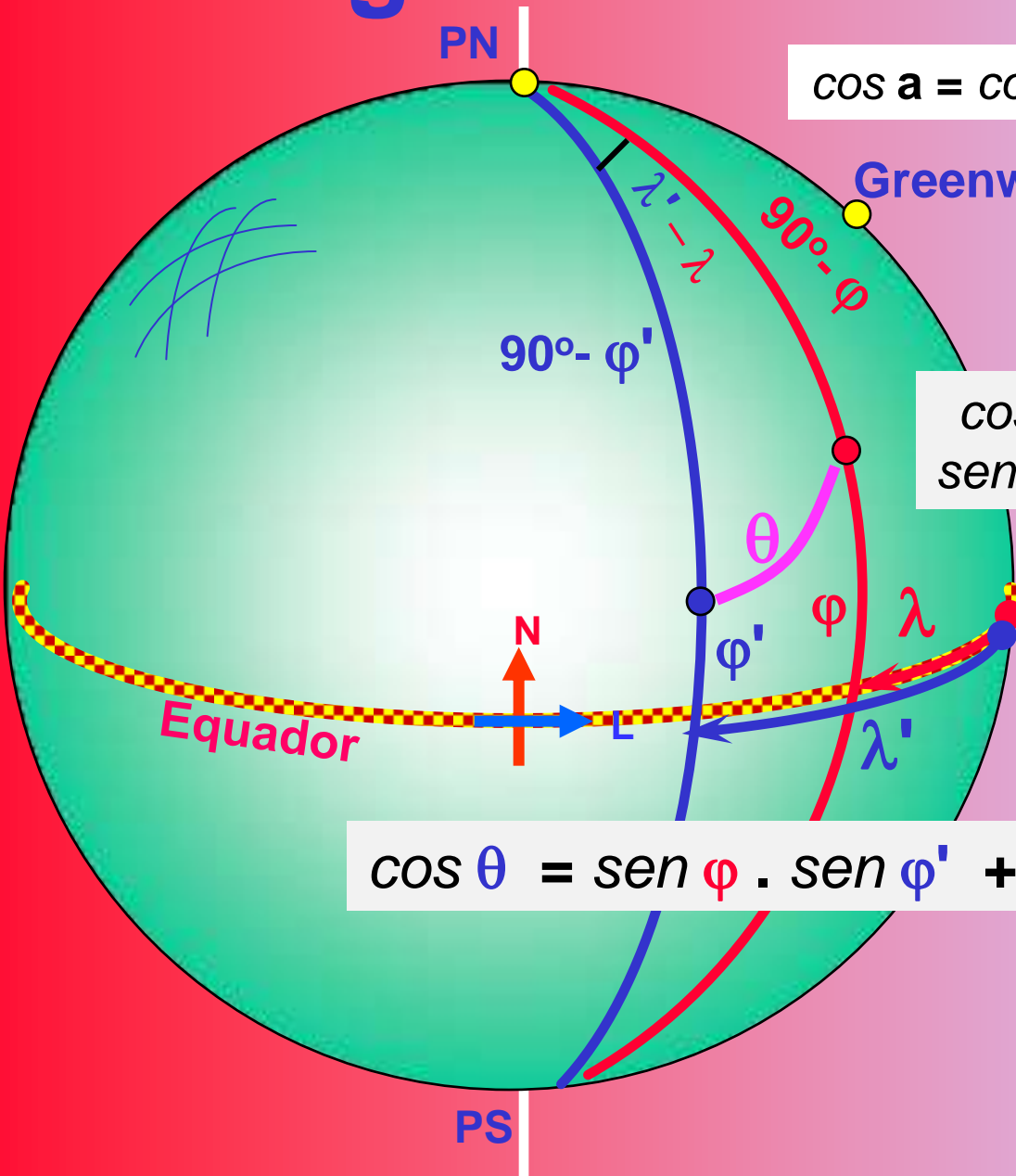
Latitude φ

e

Longitude λ



Ângulo entre duas cidades



$$\cos a = \cos b \cdot \cos c + \text{sen } b \cdot \text{sen } c \cdot \cos A$$

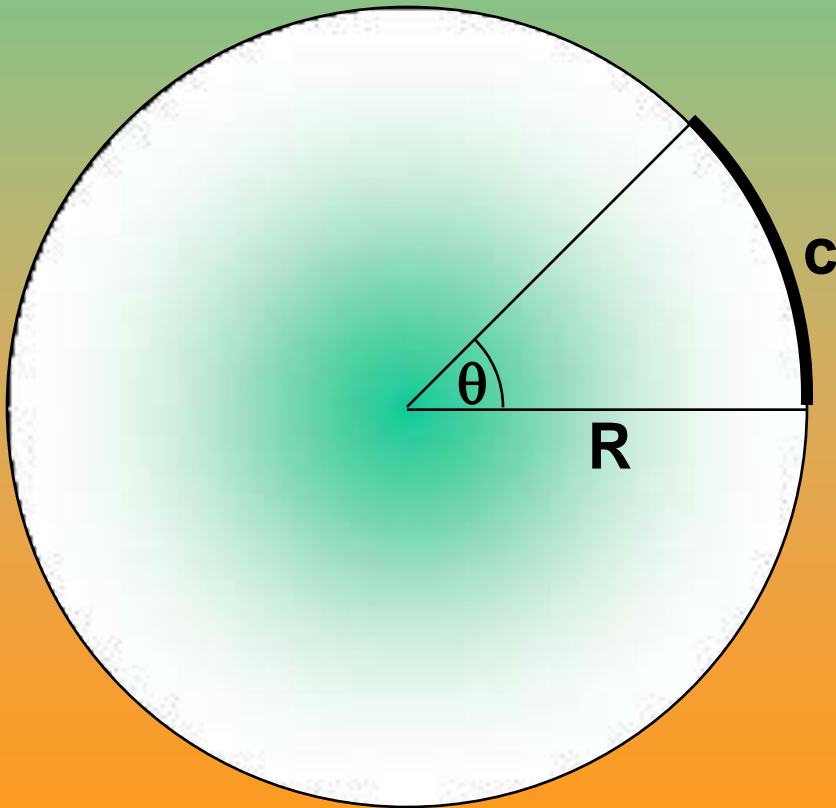
Co-seno

$$\cos \theta = \cos (90 - \varphi) \cdot \cos (90 - \varphi') + \text{sen } (90 - \varphi) \cdot \text{sen } (90 - \varphi') \cdot \cos (\lambda' - \lambda)$$

$$\cos \theta = \text{sen } \varphi \cdot \text{sen } \varphi' + \cos \varphi \cdot \cos \varphi' \cdot \cos (\lambda' - \lambda)$$

Distância entre duas cidades

Comprimento de um arco de circunferência



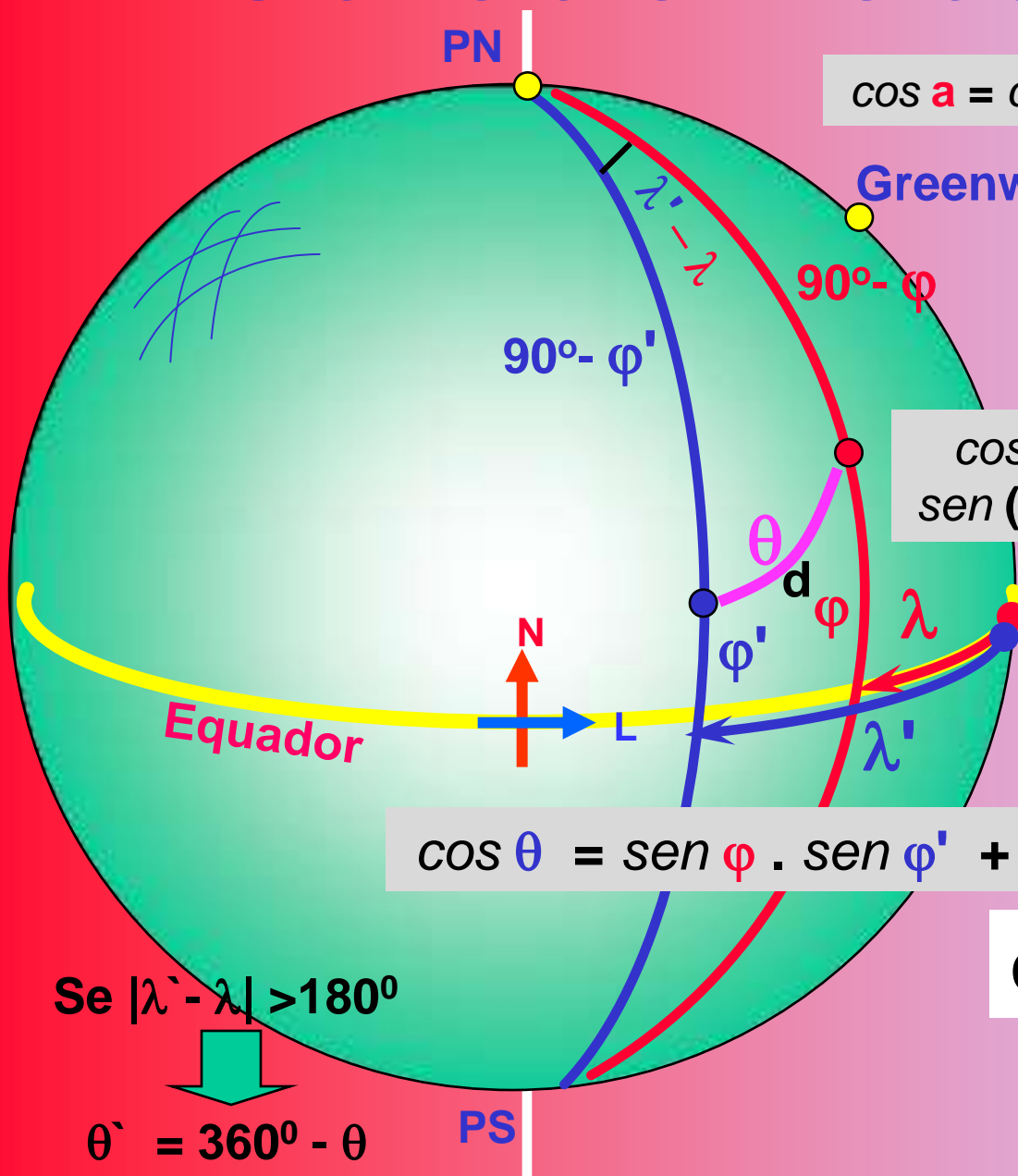
$$360^{\circ} \Rightarrow 2\pi R$$

$$\theta \Rightarrow c$$

$$c = 2\pi R \cdot \theta^{\circ} / 360^{\circ}$$

$$c = R \cdot \theta^{\circ} \cdot \pi / 180^{\circ}$$

Distância entre duas cidades



$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

Co-seno

$$\cos \theta = \cos (90 - \varphi) \cdot \cos (90 - \varphi') + \sin (90 - \varphi) \cdot \sin (90 - \varphi') \cdot \cos (\lambda' - \lambda)$$

$$\cos \theta = \sin \varphi \cdot \sin \varphi' + \cos \varphi \cdot \cos \varphi' \cdot \cos (\lambda' - \lambda)$$

$$d = 2\pi R \cdot \theta^\circ / 360^\circ$$

Se $|\lambda' - \lambda| > 180^\circ$



$$\theta' = 360^\circ - \theta$$

Ângulo entre 2 astros no Sistema Altazimutal

Coordenadas Altazimutais

$$0^\circ \leq A < 360^\circ$$

$$[\tilde{N}] -90^\circ \leq h \leq +90^\circ [Z]$$

$$[Z] 0^\circ \leq z \leq 180^\circ [\tilde{N}]$$

★ A,h

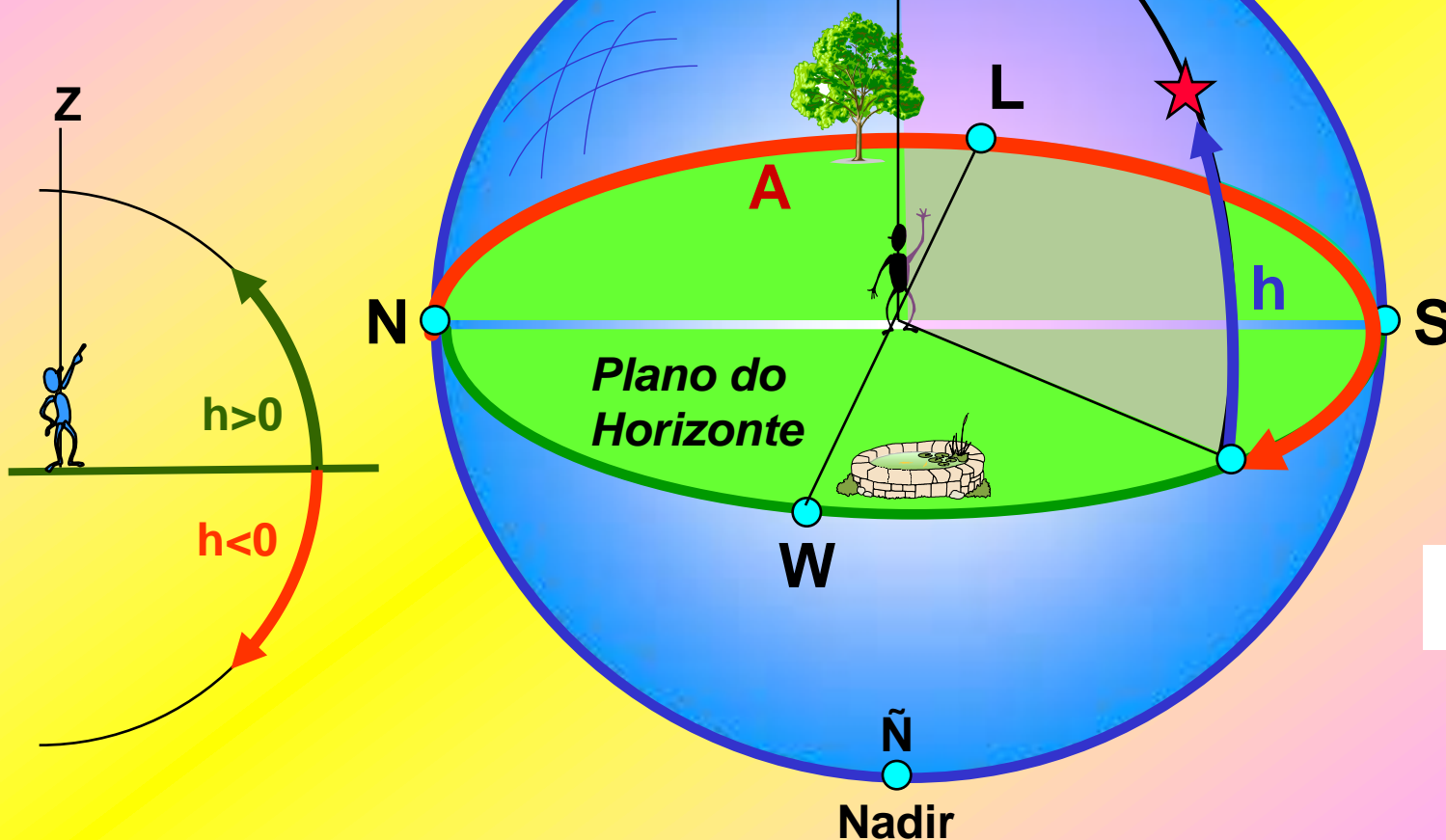
★ A,z

$$h + z = 90^\circ$$

A = azimuth

h = altura

z = distância zenital



Ângulo entre dois astros

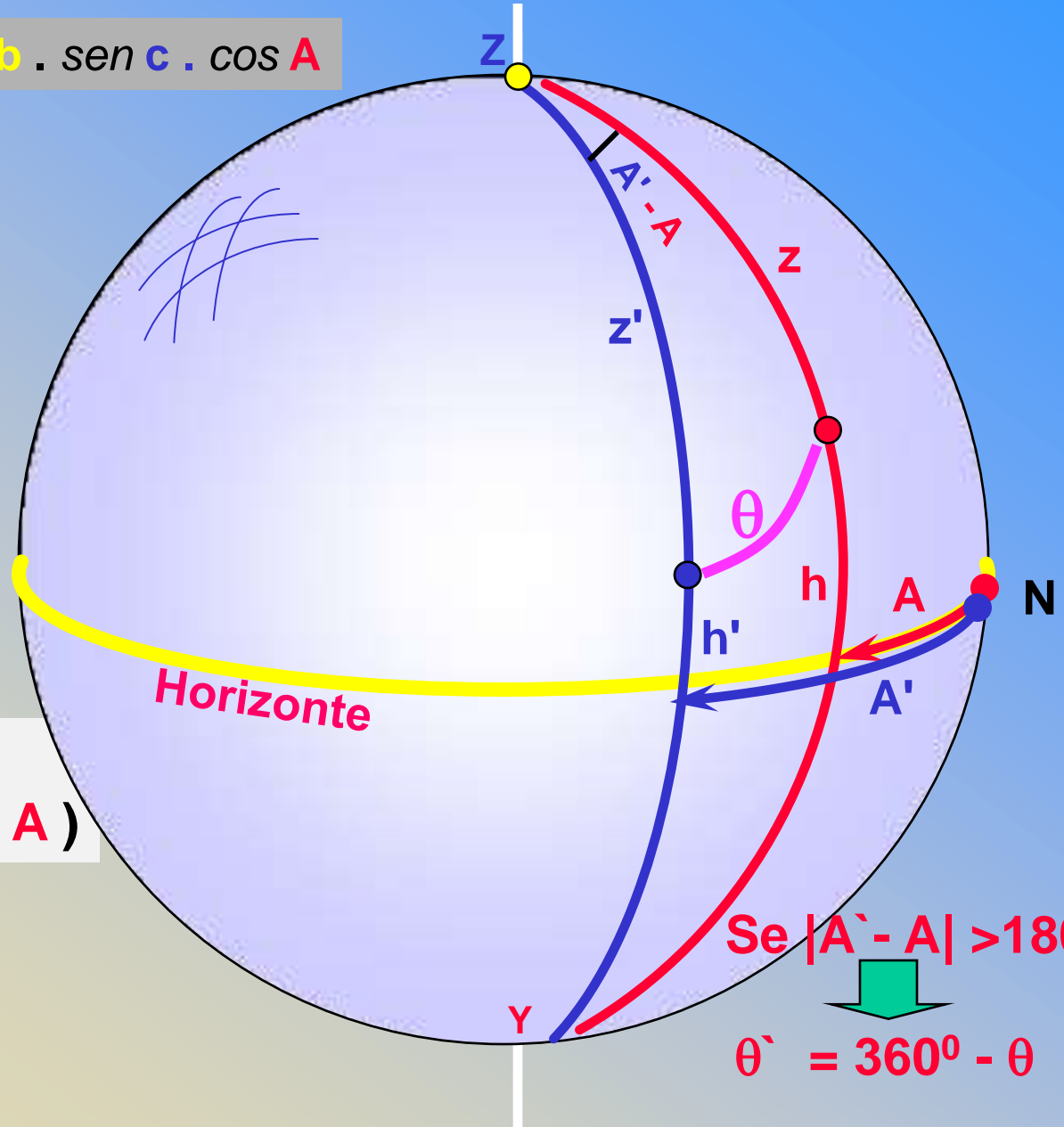
$$\cos a = \cos b \cdot \cos c + \text{sen } b \cdot \text{sen } c \cdot \cos A$$

Co-seno

Dados:

A, z

A', z'

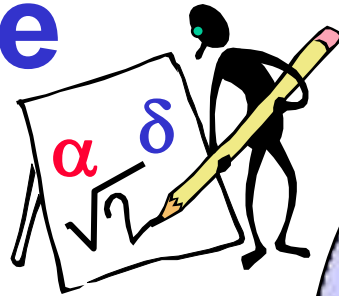


$$\cos \theta = \cos z \cdot \cos z' + \text{sen } z \cdot \text{sen } z' \cdot \cos (A' - A)$$

Se $|A' - A| > 180^\circ$

$$\theta' = 360^\circ - \theta$$

Ângulo entre 2 astros



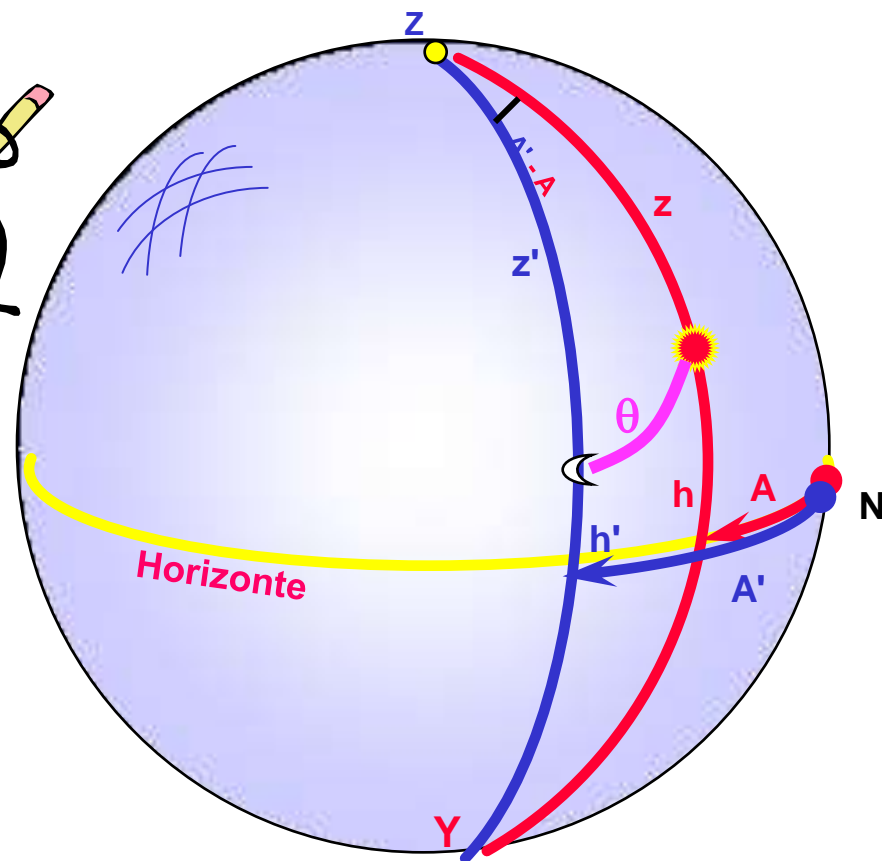
Dados:

$$A1 = 4^{\circ} 05' 06.000'' = 4.08500000^{\circ}$$

$$Z1 = 10^{\circ} 20' 30.000'' = 10.34166667^{\circ}$$

$$A2 = 280^{\circ} 50' 30.000'' = 280.84166667^{\circ}$$

$$Z2 = 40^{\circ} 10' 10.000'' = 40.16944444^{\circ}$$



$$\cos \theta = \cos z \cdot \cos z' + \sin z \cdot \sin z' \cdot \cos (A' - A)$$

$$\cos \theta = \cos Z1 * \cos Z2 + \sin Z1 * \sin Z2 * \cos(A2-A1)$$

$$\cos \theta = 0.98375475 * 0.76414014 + 0.17951767 * 0.64505027 * 0.11765295$$

$$\cos \theta = 0.76535046$$

$$\theta = 40^{\circ} 03' 42.551'' = 40.06181965^{\circ}$$

Como $|A_2 - A_1| > 180^{\circ}$

$$\theta' = 360^{\circ} - \theta$$

$$\theta' = 319^{\circ} 56' 17,449''$$

Ângulo entre 2 astros no Sistema Equatorial

Sistema Equatorial de Coordenadas

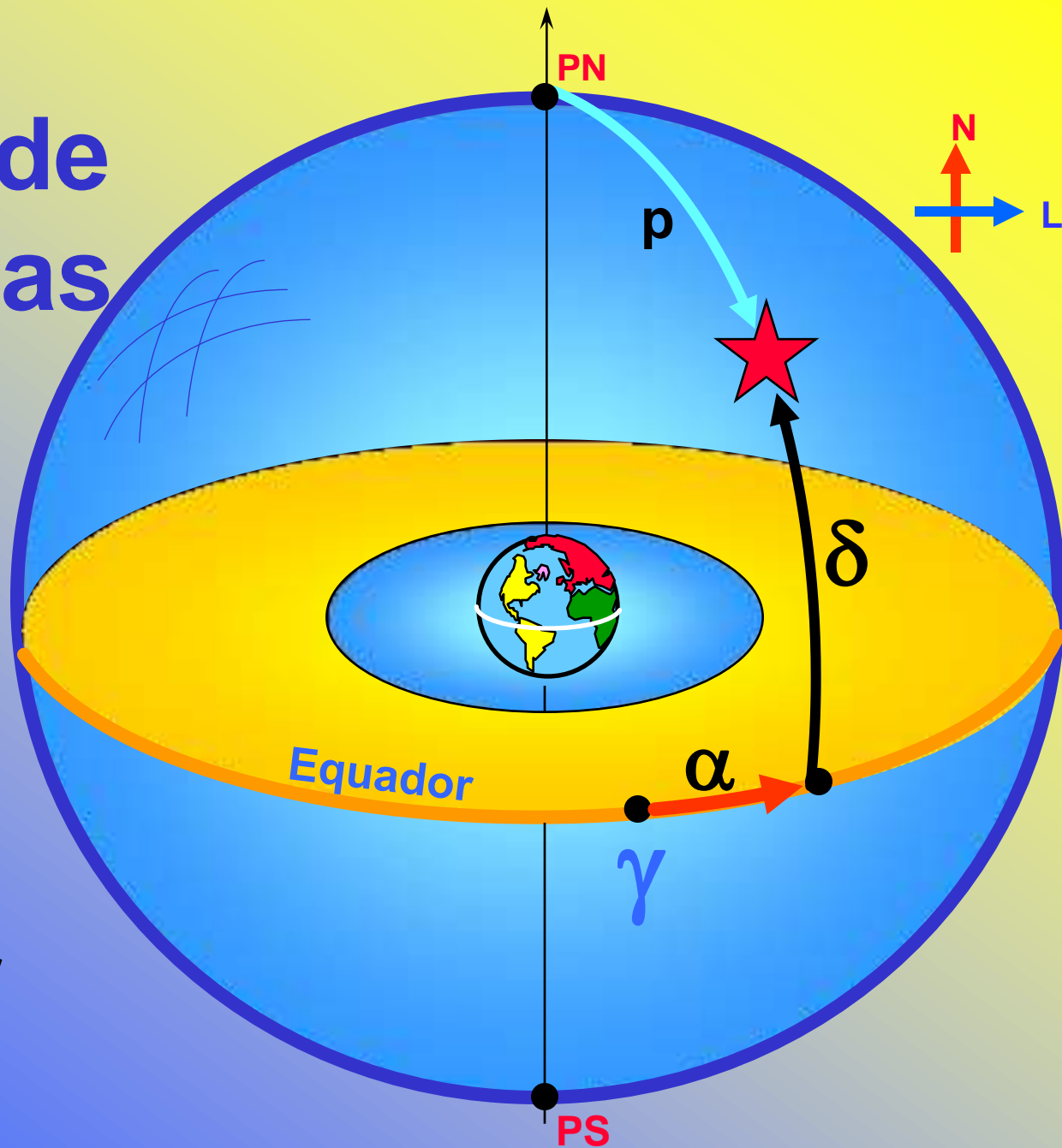
$$p + \delta = 90^\circ$$

★ (α, δ)

α = ascensão reta

δ = declinação

p = distância polar



Ângulo entre dois astros

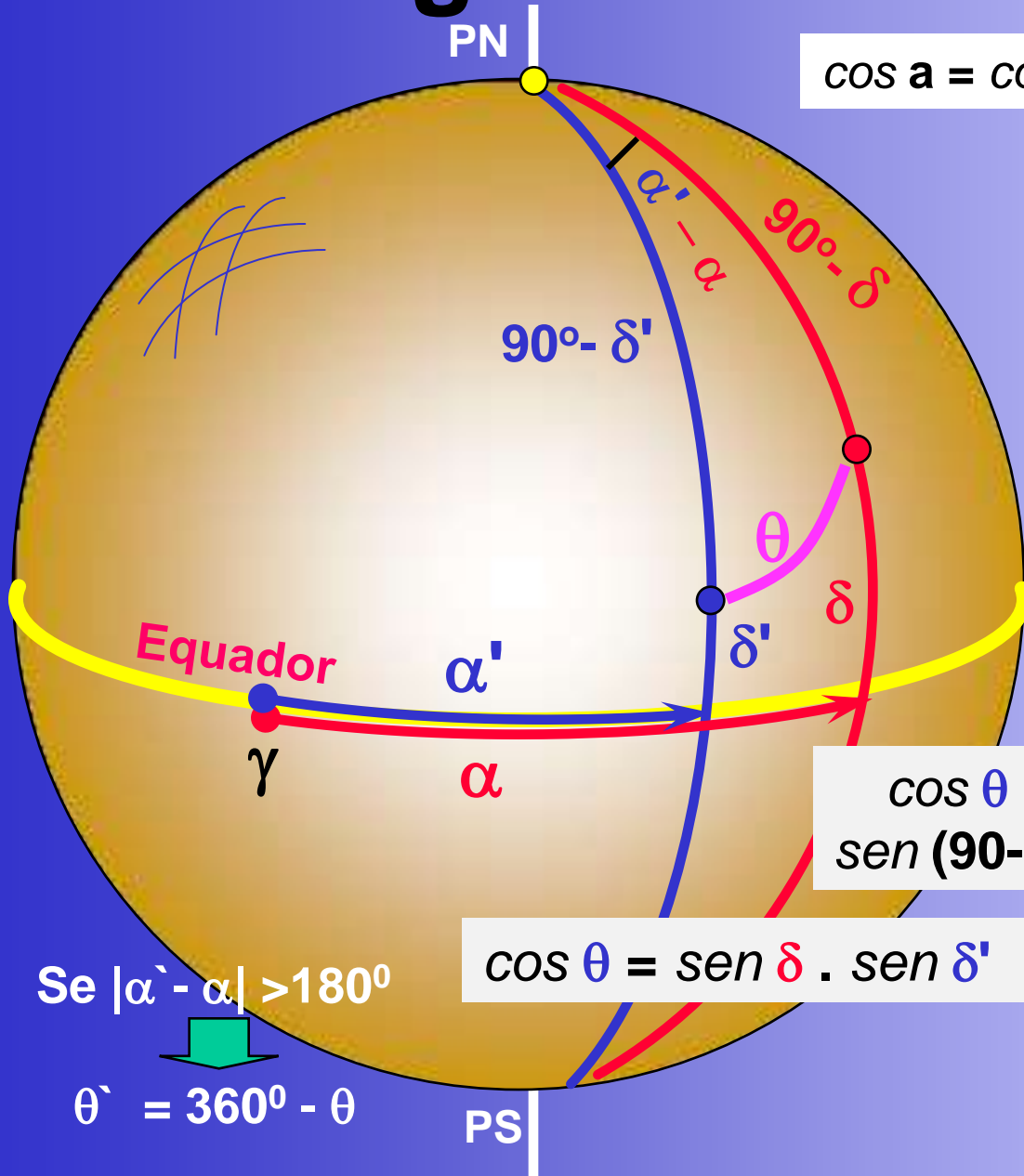
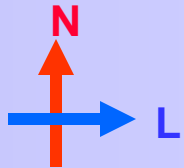
$$\cos a = \cos b \cdot \cos c + \text{sen } b \cdot \text{sen } c \cdot \cos A$$

Co-seno

Dados:

α , δ

α' , δ'



$$\cos \theta = \cos (90 - \delta) \cdot \cos (90 - \delta') + \text{sen } (90 - \delta) \cdot \text{sen } (90 - \delta') \cdot \cos (\alpha' - \alpha)$$

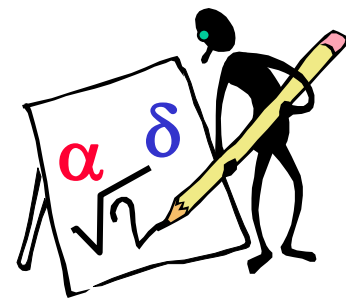
$$\cos \theta = \text{sen } \delta \cdot \text{sen } \delta' + \cos \delta \cdot \cos \delta' \cdot \cos (\alpha' - \alpha)$$

Se $|\alpha' - \alpha| > 180^\circ$

$\theta' = 360^\circ - \theta$

Ângulo entre 2 astros

Dados



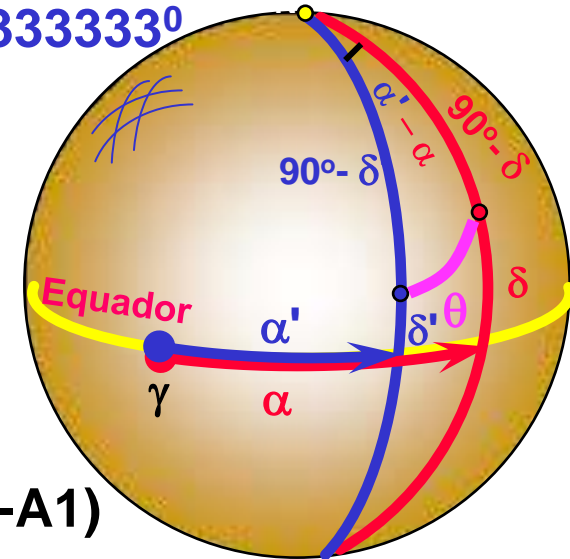
★ Alfa1 = 3h 10m 50.000s = 3.18055556 h = 47.70833333^o

★ Delta1 = 20^o 30' 40.000" = 20.51111111^o

★ Alfa2 = 5h 30m 20.000s = 5.50555556 h = 82.58333333^o

★ Delta2 = -10^o 15' 20.000" = -10.25555556^o

$$\cos \theta = \sin \delta \cdot \sin \delta' + \cos \delta \cdot \cos \delta' \cdot \cos (\alpha' - \alpha)$$



$$\cos \theta = \sin D1 \cdot \sin D2 + \cos D1 \cdot \cos D2 \cdot \cos(A2 - A1)$$

$$\cos \theta = 0.35038902 \cdot -0.17803896 + 0.93660426 \cdot 0.98402344 \cdot 0.82041144$$

$$\cos \theta = 0.69373233$$

$$\theta = 46^{\circ} 04' 25.368'' = 46.07371342^{\circ}$$

Como $|\alpha_2 - \alpha_1| < 180^{\circ}$



$$\theta = 46^{\circ} 04' 25.368''$$

Fim