Phase diagram of strongly interacting matter under strong magnetic fields.

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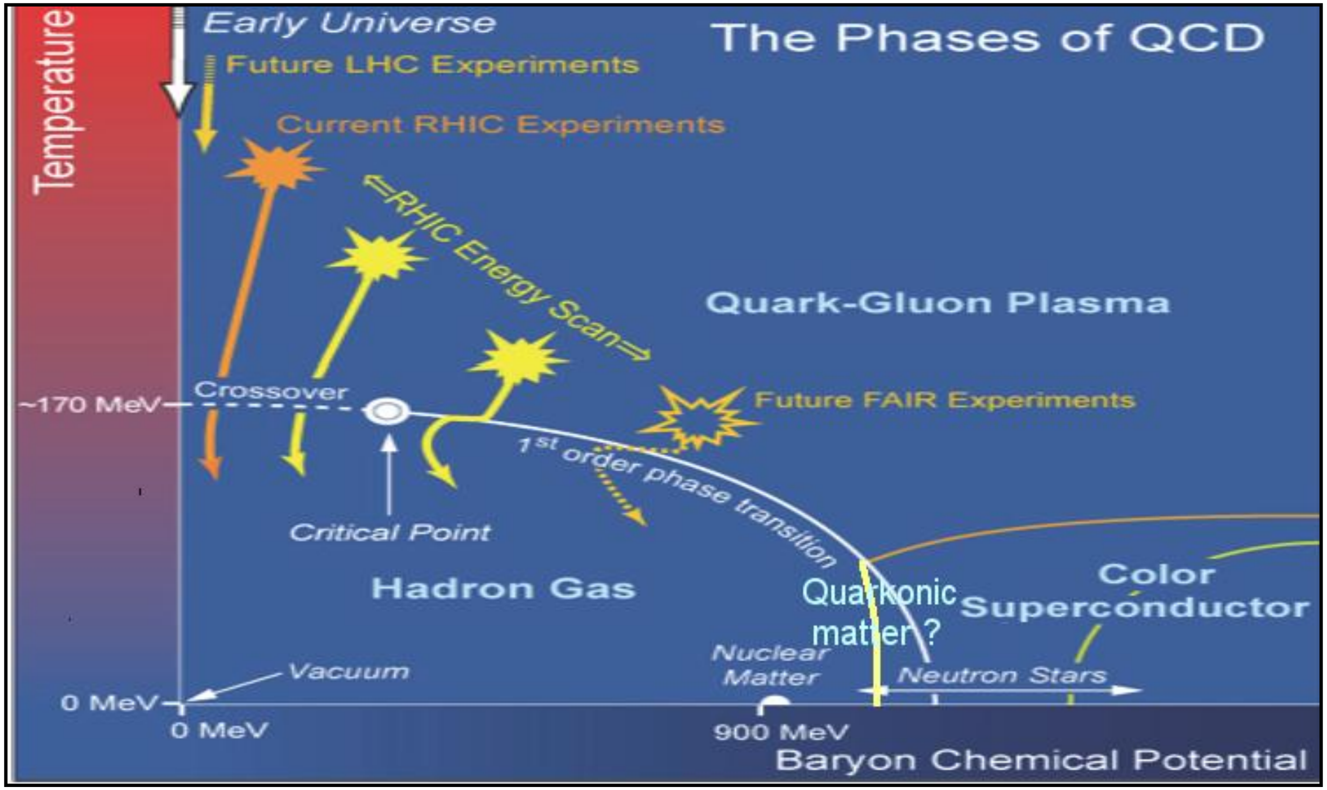
PLAN OF THE TALK

- Introduction
- The PNJL and the EPNJL models under strong magnetic fields
- Results
- Outlook & Conclusions

Introduction

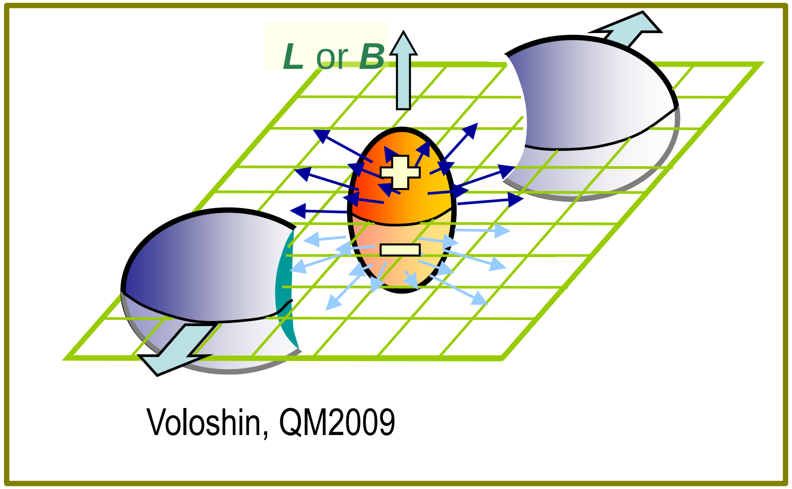
The understanding of the behavior of strongly interacting matter at finite T and/or density is of fundamental interest and has important applications in cosmology, in the astrophysics of neutron stars and in the physics of RHIC.

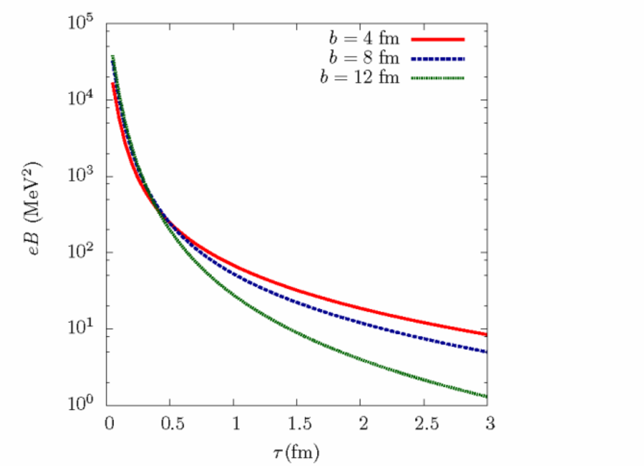


Recently, there has been quite a lot of interest in investigating how this phase diagram is affected by the presence of strong magnetic fields . The main motivation for this is their possible existence in physically relevant situations:

High magnetic fields in non-central relativistic heavy ion collisions

(Kharzev, McLerran, Warringa (08))





Magnetic field at the center of a gold-gold collision, for different impact parameters Here the center of mass energy is 200 GeV per nucleon pair

$eB \sim 10^4 - 10^5$ MeV^2 $\rightarrow$ $B \sim 10^{19}$ G

Compact Stellar Objects: magnetars are estimated to have $B \sim 10^{14}$-10^{15} G at the surface. It could be much higher in the interior (Duncan and Thompson (92/93))

Several theoretical/phenomenological questions arise:

- How does the QCD phase diagram look like when one includes a non-zero uniform B ?
- Are there modifications in the nature of the phase transitions ?
- Do chiral and deconfinement transitions behave differently ?
- Which is the fate of the critical point(s) ?
- …..

This has been investigated in a variety of approaches. For example [not an exhaustive list !]

- NJL and relatives (Klevansky, Lemmer (89); Klimenko et al. (92,..); Gusynin, Miransky, Shokovy (94/95); Ferrer, Incera et al (03..), Hiller, Osipov (07/08); Menezes et al (09);Fukushima, Ruggieri, Gatto (10) [PNJL]; …)
- χPT (Shushpanov, Smilga (97); Agasian, Shushpanov (00); Cohen, McGady, Werbos (07);….)
- Linear Sigma Model and MIT bag model: (Fraga, Mizher (08), Fraga, Palhares (12))
- Lattice QCD [at $\mu = 0$] (D'Elia (10/11), Bali et al (11/12))

(E)PNJL model

PNJL model is a synthesis of {
- NJL model (CHIRAL DYNAMICS)
- Polyakov loop dynamics (CONFINEMENT)

• NJL model: simplest model with chiral quark interactions. Local scalar and pseudoscalar four-fermion couplings + UV regularization prescription

NJL (Euclidean) lagrangian
Nambu, Jona-Lasinio, PR (61)

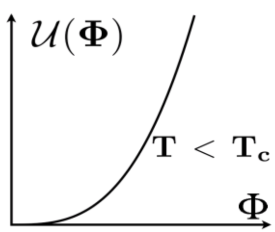
$$\mathcal{L}^{E}_{NJL} = \bar{\psi}\,(-i\partial\!\!\!/ + m_c)\,\psi(x) - \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}\ i\gamma_5\vec{\tau}\psi\ \right)^2\right]$$

• Polyakov loop
Polyakov, PLB (78)

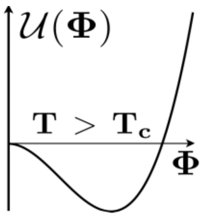
$$\Phi(\vec{x}) = \frac{1}{N_c} Tr\left[i\int_0^{1/T} d\tau\ A_4(\vec{x},\tau)\right]$$

pure gauge → Z(3) symmetry

$\mathcal{U}(\Phi)$ Effective potential



confinement:
Z(3) symmetry
not broken
$\langle\Phi\rangle = 0$



deconfinement:
Z(3) symmetry
spontaneously
broken
$\langle\Phi\rangle \neq 0$

Fukushima (03), Megias,Ruiz Arriola, Salcedo (06), Ratti, Thaler, Weise (06),...

In the quark sector the effects of finite T and μ are considered by using the Matsubara formalism

$$p_4 \to (2n+1)\pi T - i\mu \qquad ; \qquad \int dp_4 \to 2\pi T \sum_{n=-\infty}^{\infty}$$

and the coupling to the color background fields associated with the PL is introduced by using

$$\partial_4 \to \partial_4 - i\phi$$ where $$\phi = \phi_3\lambda_3 + \phi_8\lambda_8 = \mathrm{diag}(\phi_r, \phi_g, \phi_b)$$ and $$\Phi = \frac{1}{3} Tr_c\left[\exp\left(i\,\phi / T\right)\right]$$

In the standard PNJL model the quark-quark coupling constant G is independent of the PL. To account for further correlations between the quark and gluon degrees of freedom a PL dependent G might be introduced. This leads to the so-called Entangled PJL (EPNJL) model (Sakai, Sasaki, Kouno, Yahiro (10))

$$G(\Phi) = \left[1 - \alpha_1 \Phi\Phi^* - \alpha_2\left(\Phi^3 + \Phi^{*3}\right)\right] G$$

α_1, α_2 are chosen to be $\alpha_1 = \alpha_2 = 0.2$ so as to reproduce lattice results for the phase diagram at imaginary chemical potential.

For the Polyakov Loop effective potential we take (Roessner, Ratti, Weise (07))

$$\frac{U(\Phi,T)}{T^4} = -\frac{1}{2}a(T)\,\Phi\Phi^* + b(T)\,\ln\left(1-6\,\Phi\Phi^* + 4\left(\Phi^3+\Phi^{*3}\right) - 3\left(\Phi\Phi^*\right)^2\right)$$

where

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 \quad ; \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

and $a_0 = 3.51$, $a_1 = -2.47$, $a_2=15.2$, $b_3 = -1.75$. This form of the potential and parameters have been shown to describe well the behavior of the PL found in pure gauge lattice calculations.

In the original work by Roessner et al. m was taken to T_0 =270 MeV in order to reproduce the lattice value for the critical temperature in the pure gauge theory. It was latter suggested (Schaefer, Pawlowski, Wambach (07)) that the effect of the finite current quark mass m on the PL potential can be taken into account by a running of T_0 with m and μ. For two light quarks it was estimated $T_0 = 208\ (30)$ MeV. In our calculations we will consider both values of T_0 but, for simplicity, ignore any dependence of T_0 on μ.

The coupling of the quark fields to an external constant and homogenous magnetic field in the z-direction is done using minimal coupling i.e.

$$\vec{\partial} = \vec{\partial} - ie\vec{A} \qquad \vec{A} = \frac{B}{2}(-y, x, 0)$$

As well-known, within the Mean Field Approximation that we use in what follows, this leads to the following modifications

$$E_p = \sqrt{p^2 + M^2} \quad \rightarrow \quad E^f_{p_z,k} = \sqrt{p_z^2 + k|q_f|B + M^2}$$

$$\int \frac{d^3p}{(2\pi)^3} \quad \rightarrow \quad \frac{|q_f|B}{2\pi} \sum_{k=0}^{\infty} \alpha_k \int \frac{dp_z}{2\pi}$$

$k = 0, 1, 2, ..$ Landau levels

q_f Charge for each quark flavor

$\alpha_k = 2 - \delta_{k0}$ Degeneracy

The resulting thermodynamical potential in the mean feld approximation (MFA) reads

$$\Omega_{MFA}(M,\Phi) = U\left(\Phi,T\right) + \frac{\left(M-m_0\right)^2}{4G\left(\Phi\right)} - \frac{N_c N_f}{\pi^2}\int_0^{\Lambda} dp\; p^2\,\sqrt{p^2+M^2}$$

$$- N_c \sum_{f=u,d} \frac{(q_f B)^2}{2\pi^2}\left\{\xi'(-1,x_f) - \frac{1}{2}(x_f^2 - x_f)\log[x_f] + \frac{x_f^2}{4}\right\}$$

$$- \frac{T}{2\pi}\sum_{s=\pm,k,c,f} \alpha_k\; q_f B \int_{-\infty}^{+\infty}\frac{dp_z}{2\pi}\log\left(1+\exp\left[-\frac{E_f(p_z,n_1)+s\mu+i\,\phi_c}{T}\right]\right)$$

where $x_f = \dfrac{M^2}{2q_f B}$ and $\xi(y,x) = zeta\;\; de\;\; Riemann$

For μ = 0 one has ϕ_8=0. In order to have a real Ω_{MFA} for finite values of μ we set ϕ_8=0 also in that case. Then

$$\Phi = \frac{1+2\cos(\phi_3/T)}{3}$$

Then, we solve numerically the gap equations given by

$$\frac{\partial \Omega_{MFA}}{\partial M} = \frac{\partial \Omega_{MFA}}{\partial \Phi} = 0$$

to obtain M and Φ for each value of T, μ and B

Cross over transitions are defined by the peak of the corresponding susceptibilities

$$\chi_{ch} = dM / dT$$

chiral

$$\chi_{PL} = d\Phi / dT$$

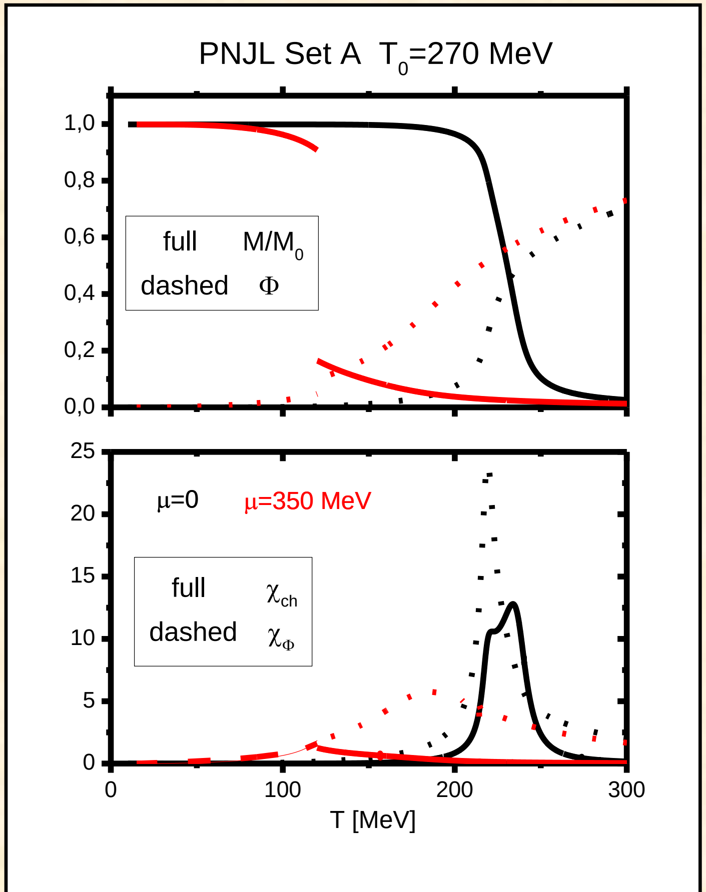
deconfinement

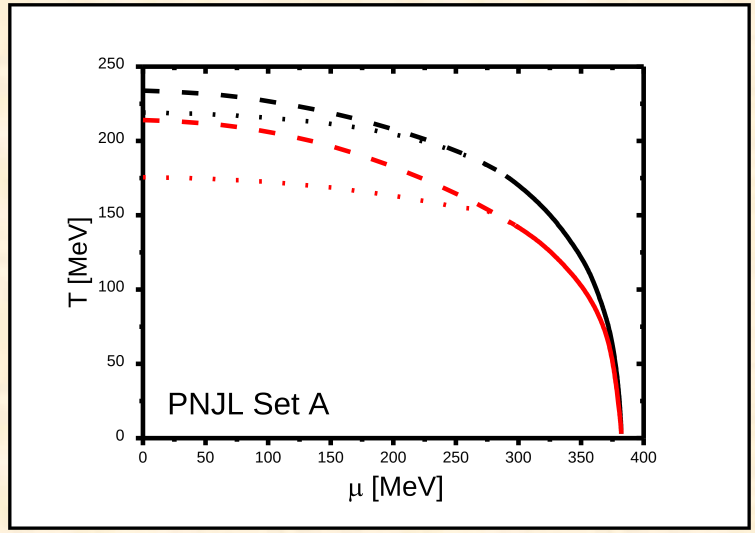
The model parameters in the quark sector are chosen to reproduce the empirically known values of m_π and f_π at $T=\mu=B=0$ as well as phenomenological reasonable values of M_0 (i.e. M at $T=\mu=B=0$)

	$G\,\Lambda^2$	Λ [MeV]	m_c [MeV]	M_0 [MeV]
Set A	2.44	587.9	5.6	400
Set B	2.19	631.5	5.5	340

Results

Typical results for B=0

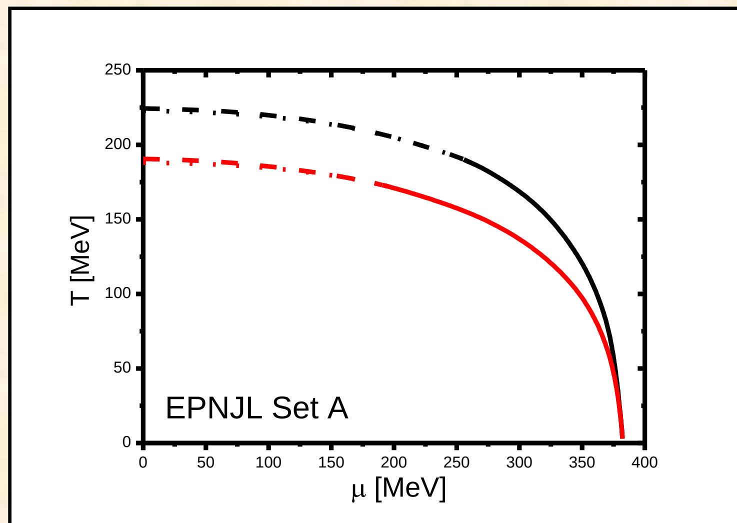




Full Line
1st Order

Dotted line
Deconf COv

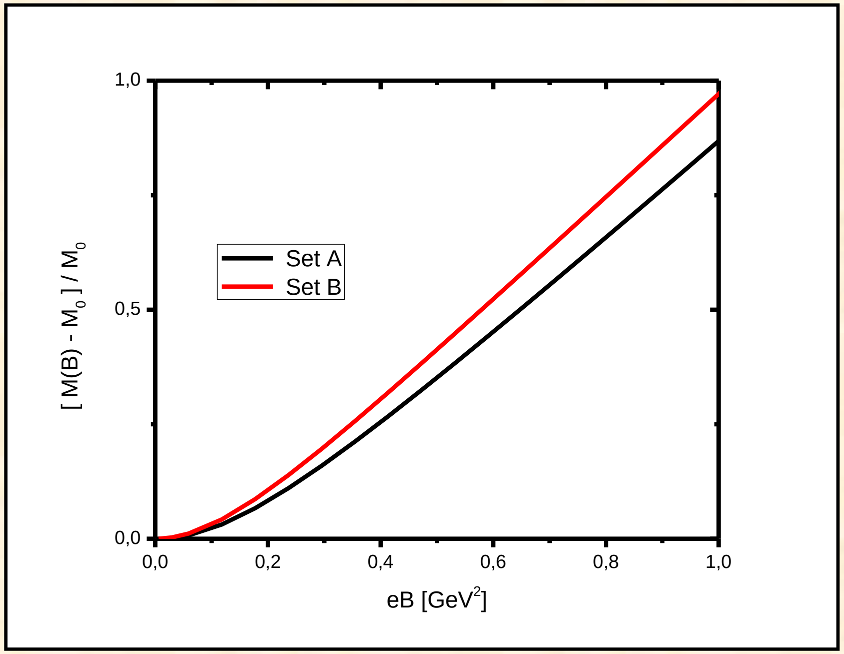
Dashed Line
Chiral rest
COv

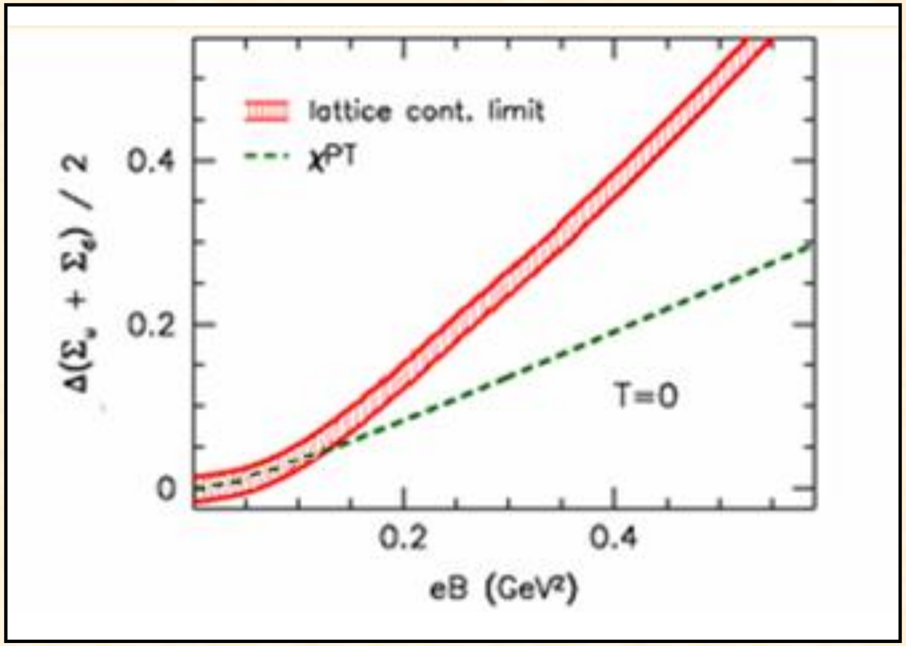


Black Line
T0=270 MeV

Red line
T0=208 MeV

Magnetic catalysis (μ=T=0)





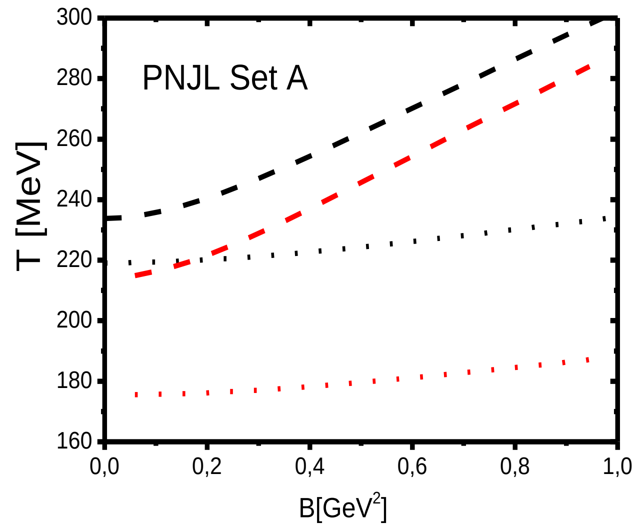
$eB = 1\text{GeV}^2 \rightarrow \text{B} = 1.69 \text{ x } 10^{20} G$

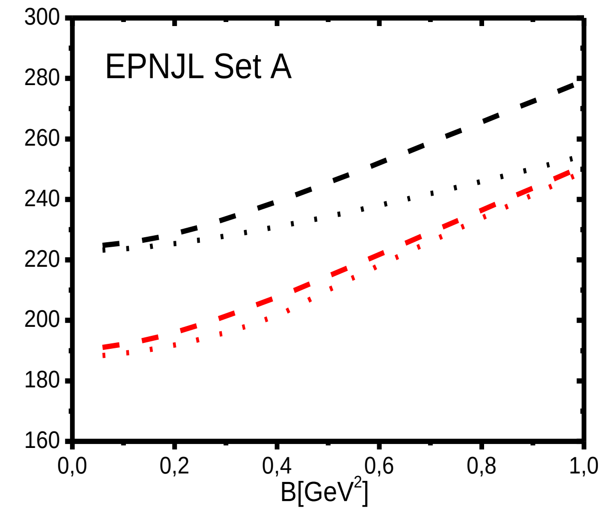
Lattice Bali et al (12)
χPT Cohen et al(07)

At T=0 there is an enhancement of the condensate with B: Magnetic catalysis (Gusynin, Miransky, Shokovy (94/95))

Critical temperatures for $\mu=0$

Dotted	Deconf.	Black	T_0=270 MeV
Dashed	Chiral	Rest Red	T_0=208 MeV



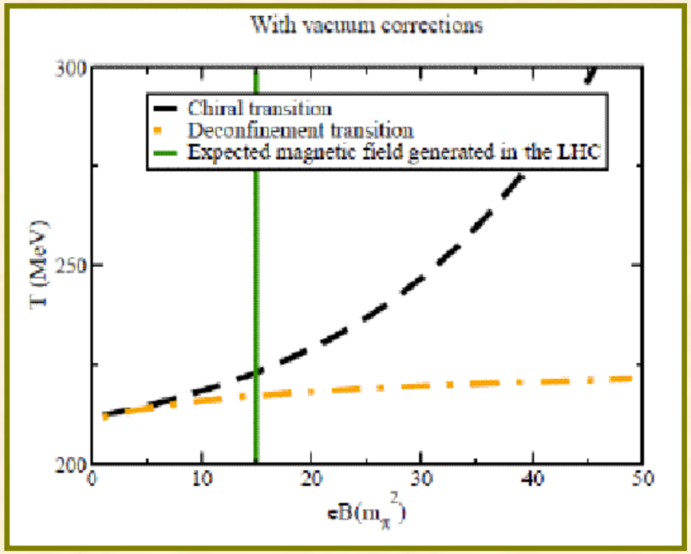


Similar results obtained for Set B

$eB = 1\text{GeV}^2 \rightarrow \text{B} = 1.69 \text{ x } 10^{20} G$

- PNJL results in agreement with calculation by Ruggieri, Gatto (10)
- Both critical temperatures increase with B.
- In PNJL the splitting between the temperatures for chiral restoration and deconfinement increases with B.
- In EPNJL both critical temperatures are quite similar, specially for lower value of T_0

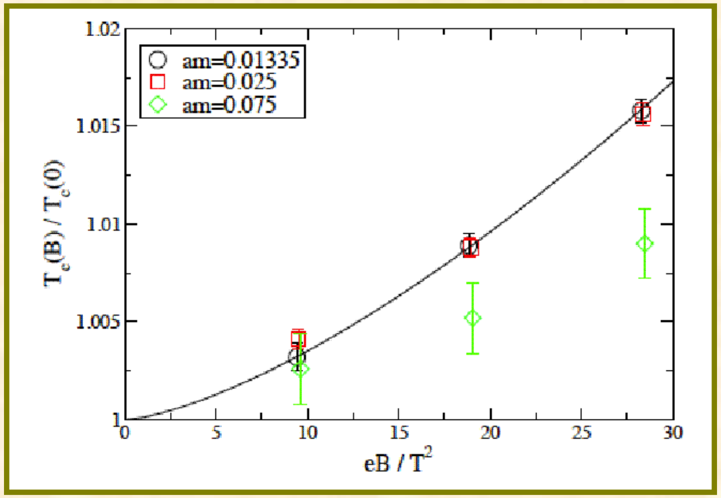
Comparison with result of other approaches



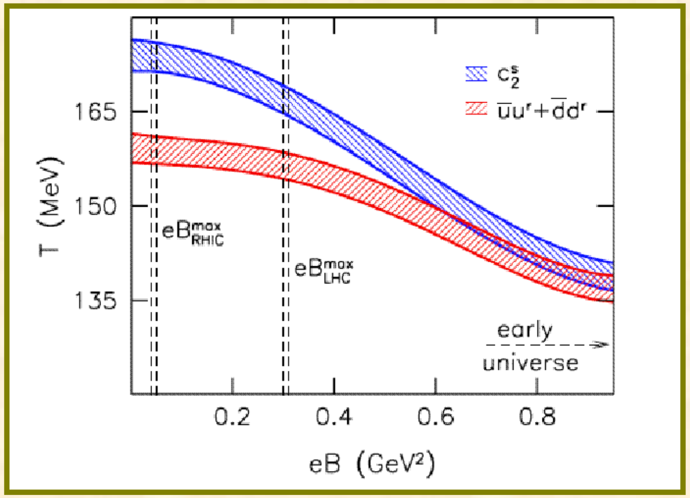
LSM Mizhner, Fraga (10)

Most models lead to an enhancement of critical temperatures with B

$$eB = 1\text{GeV}^2 = 51\, m_\pi^{\,2} \rightarrow \text{B} = 1.69 \text{ x } 10^{20} G$$

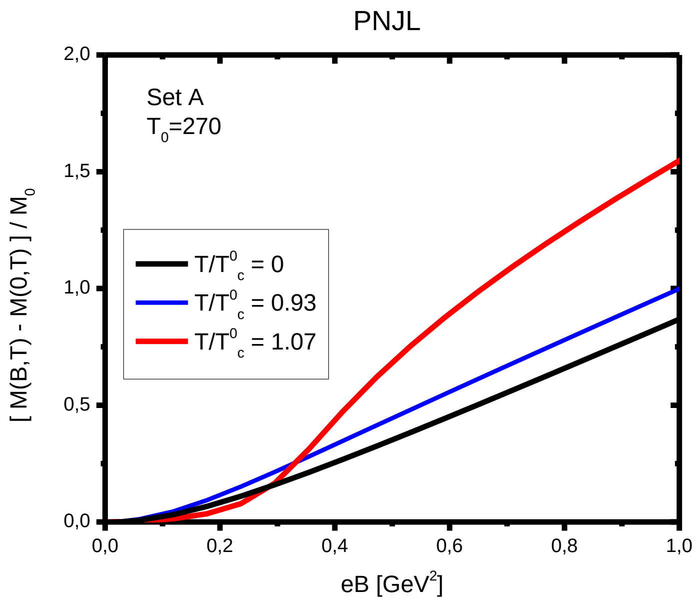


Lattice D'Elia et al (10)



Lattice Bali et al (12)

Condensates as functions of B for various T



Similar results for EPNJL and other parameterizations

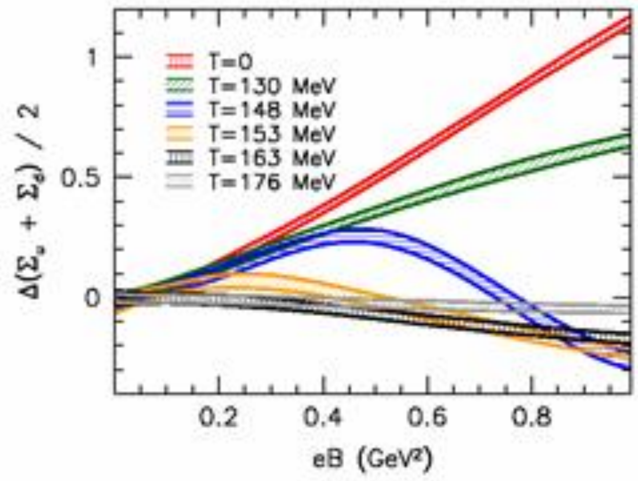


FIG. 2. Continuum extrapolated lattice results for the change of the condensate as a function of B, at six different temperatures.

Lattice results from Bali et al (12)

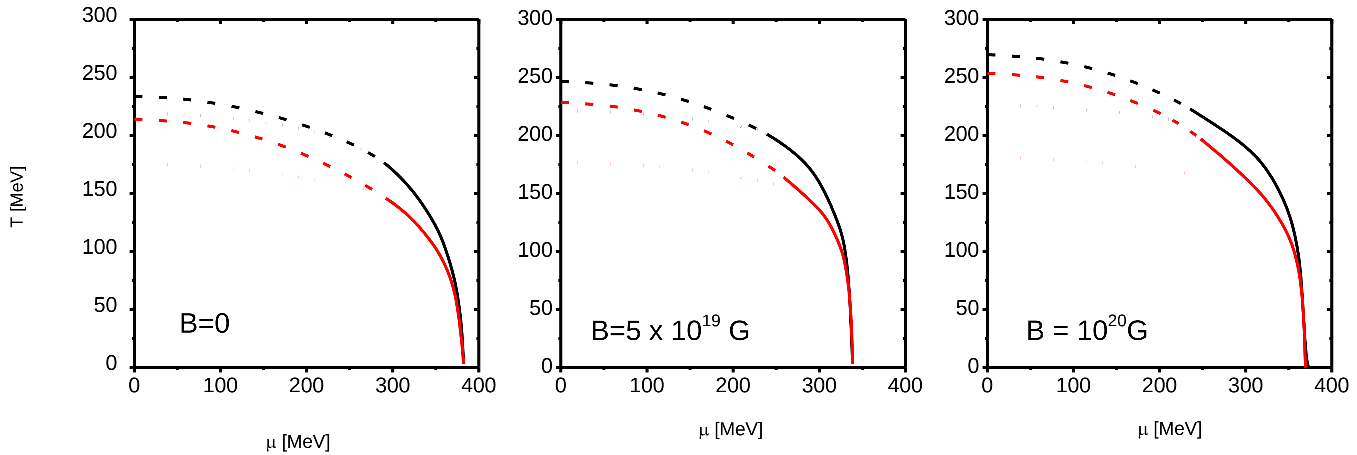
$$eB = 1\text{GeV}^2 \rightarrow \text{B} = 1.69 \text{ x } 10^{20} G$$

As they stand these models fail to reproduce lattice behavior of condensate as a function of B for T close and above T_c

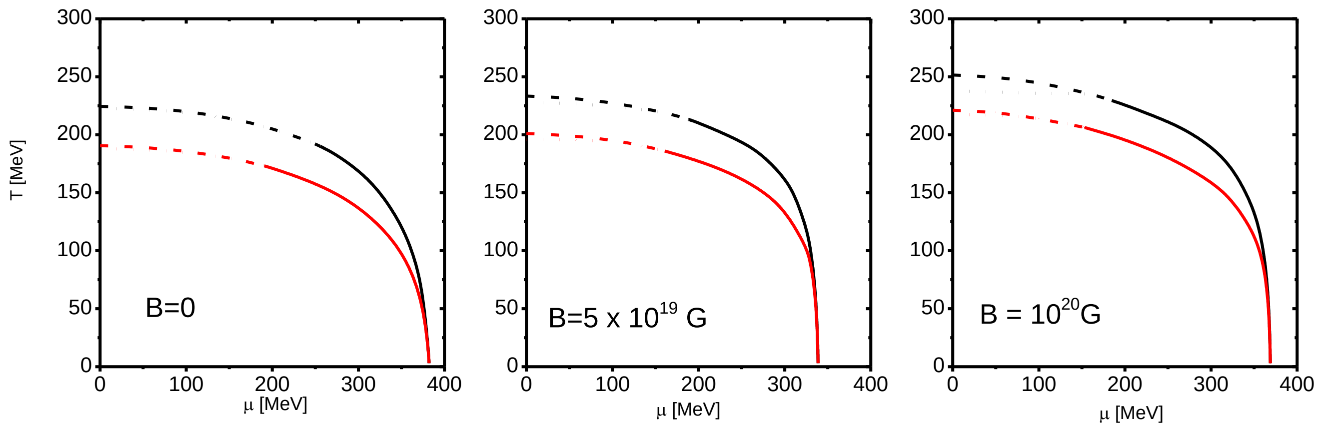
Results for Set A
Phase diagrams for different values of B

Dotted	Deconf.	Black	T_0=270 MeV
Dashed	Chiral	Rest Red	T_0=208 MeV

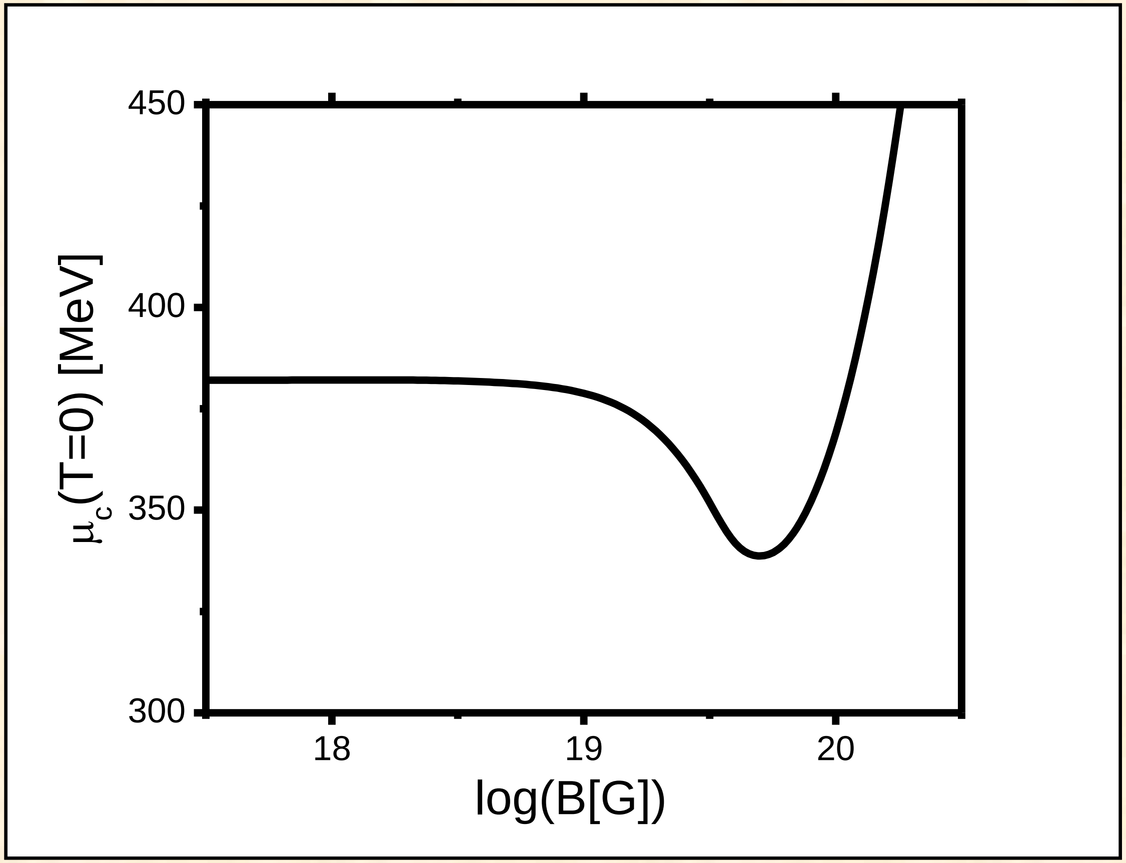
PNJL model



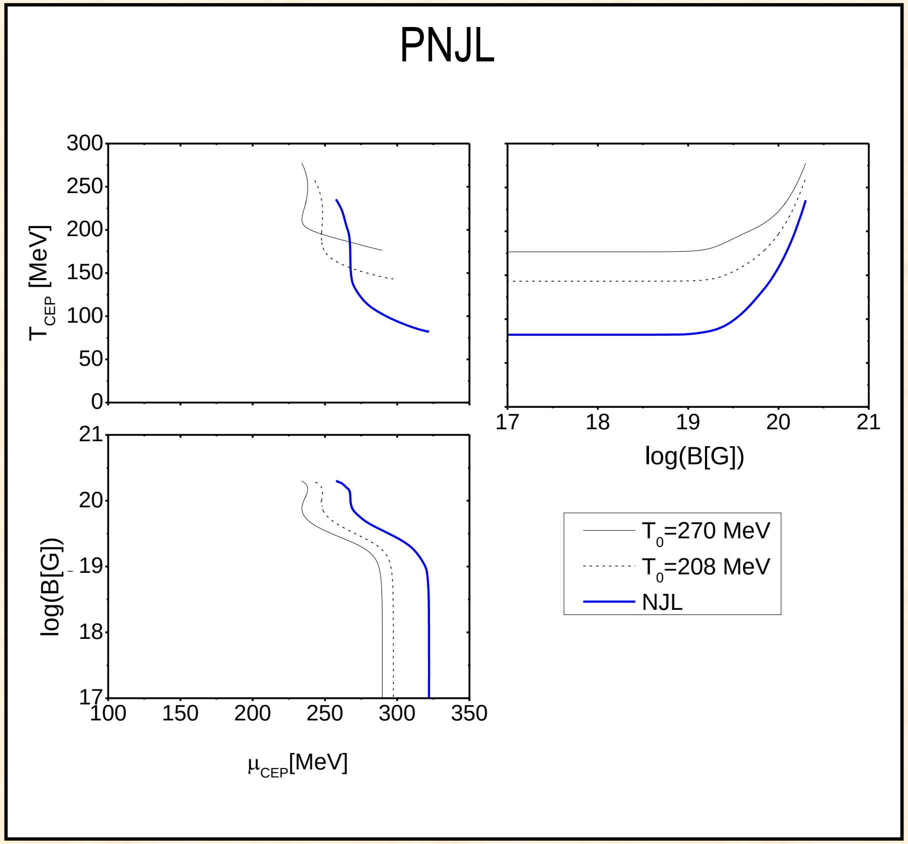
EPNJL model

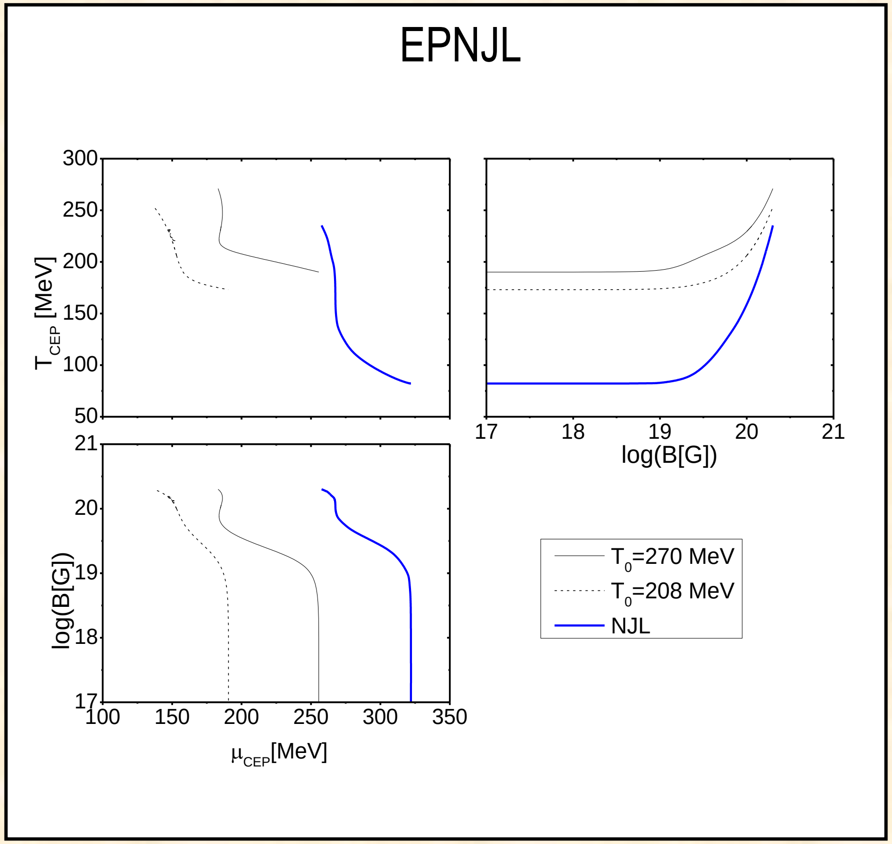


μ_c at T=0 as function of B (Set A)



Position of the CEP as of B (Set A)



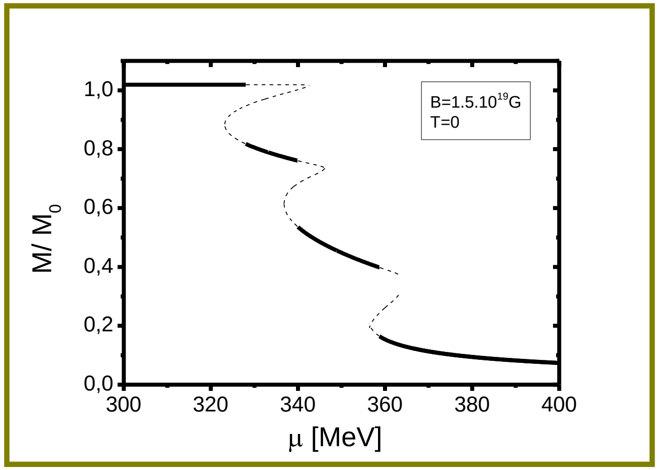


In PNJL CEP moves to higher values of T as compared with NJL.

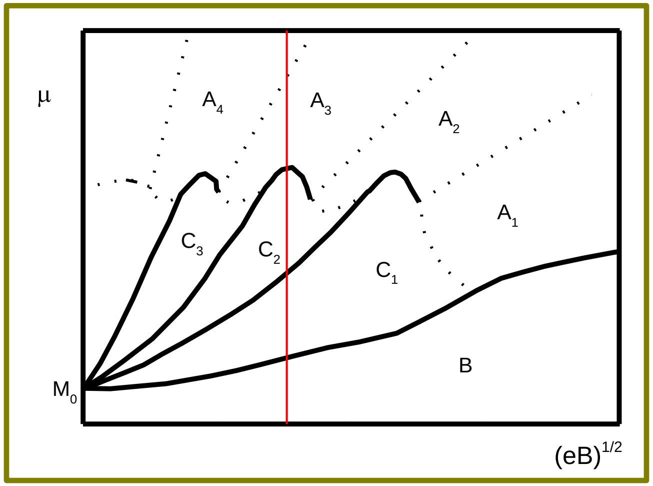
Compared to PNJL, in EPNJL CEP moves to lower values of μ. Effect is larger for lower T_0

Results for Set B

Appearence of intermediate phases even at T=0 for finite values of B (Klimenko et al (00))



Schematic μ-B phase diagram T=0 in chiral limit

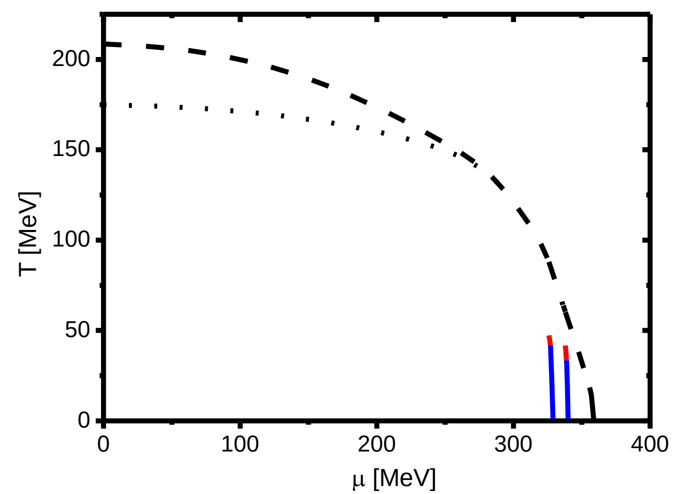


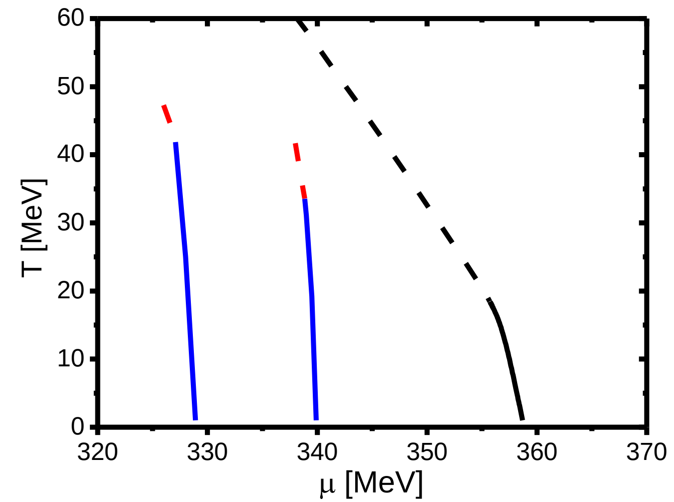
$A_i \rightarrow M=0\ (\rho \neq 0)$

$C_i \rightarrow M=M0\ (\rho \neq 0)$

$B \rightarrow M=M(m)\ (\rho=0)$

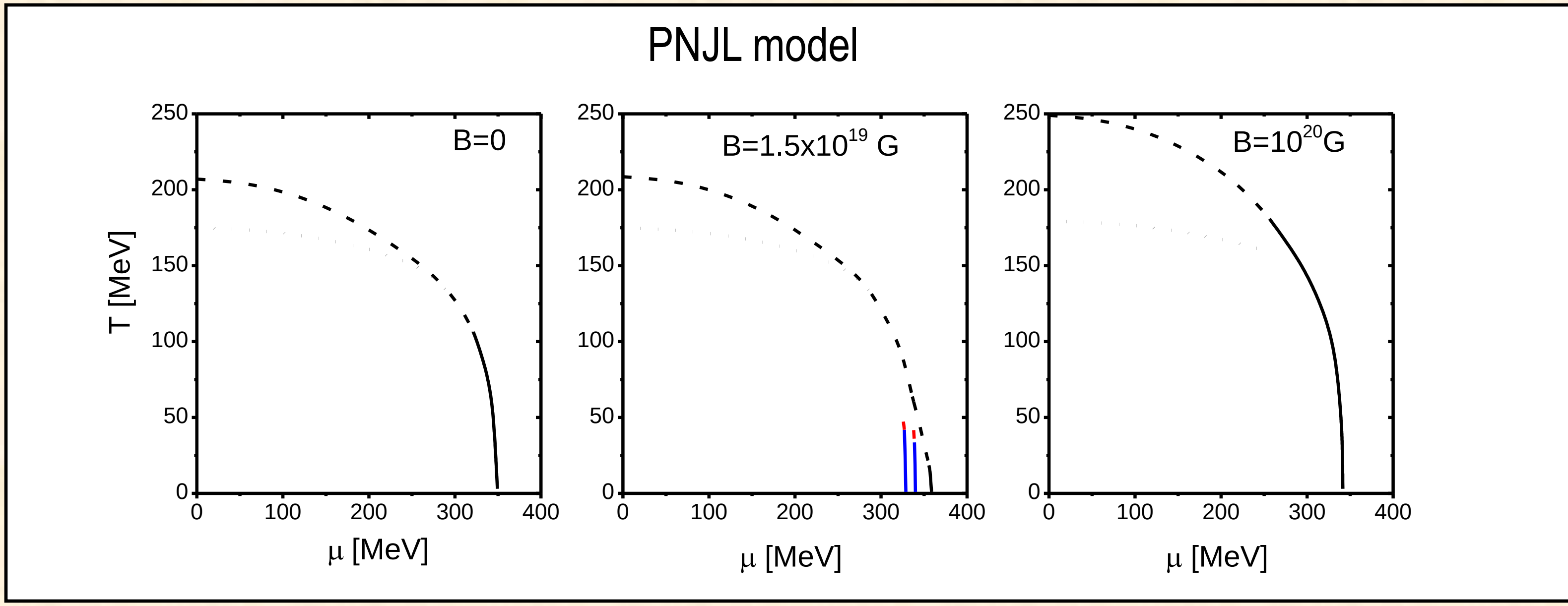
Connected with van Alphen- de Hass effect

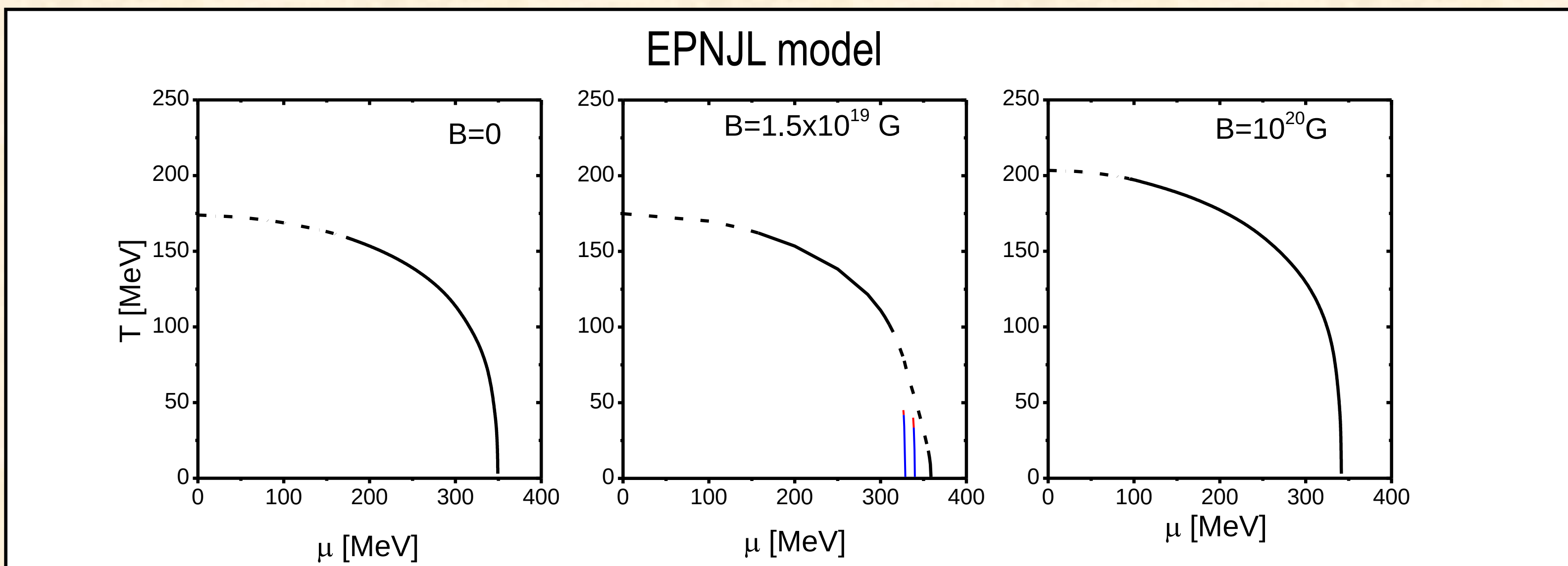




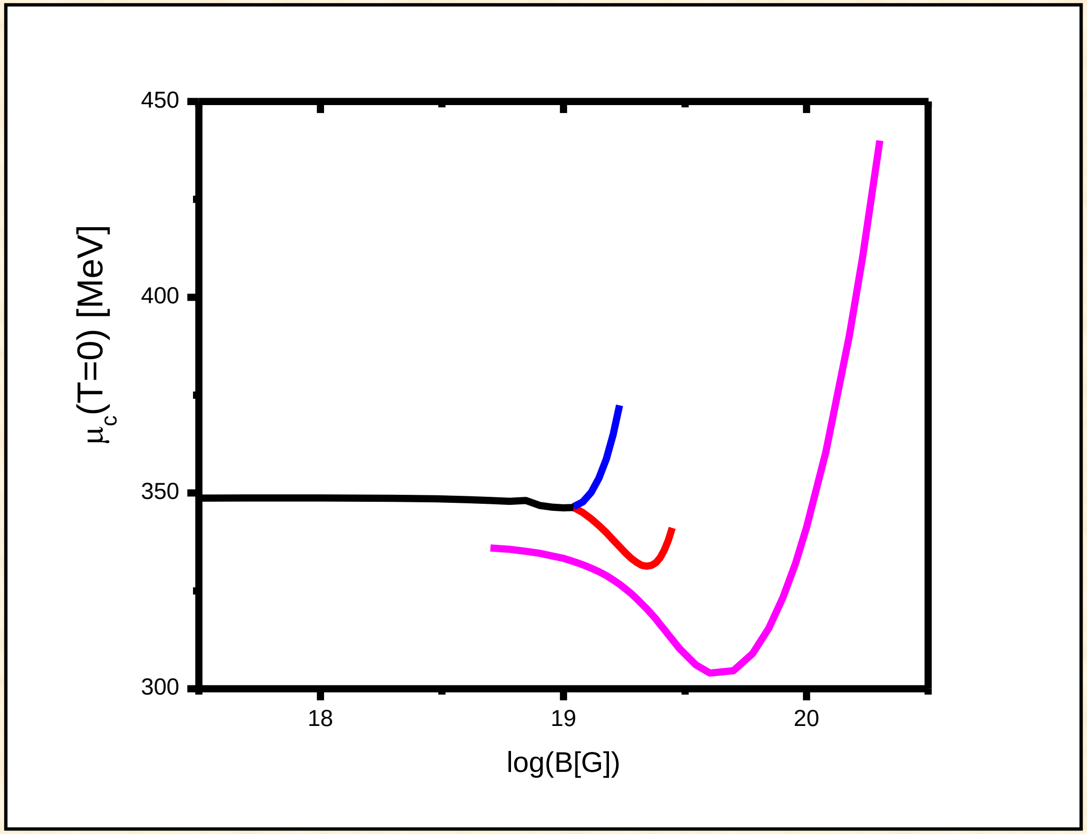
For our Set B this occurs for $0 < B < 3 \times 10^{19}$G

Phase diagrams for different values of B (set B)



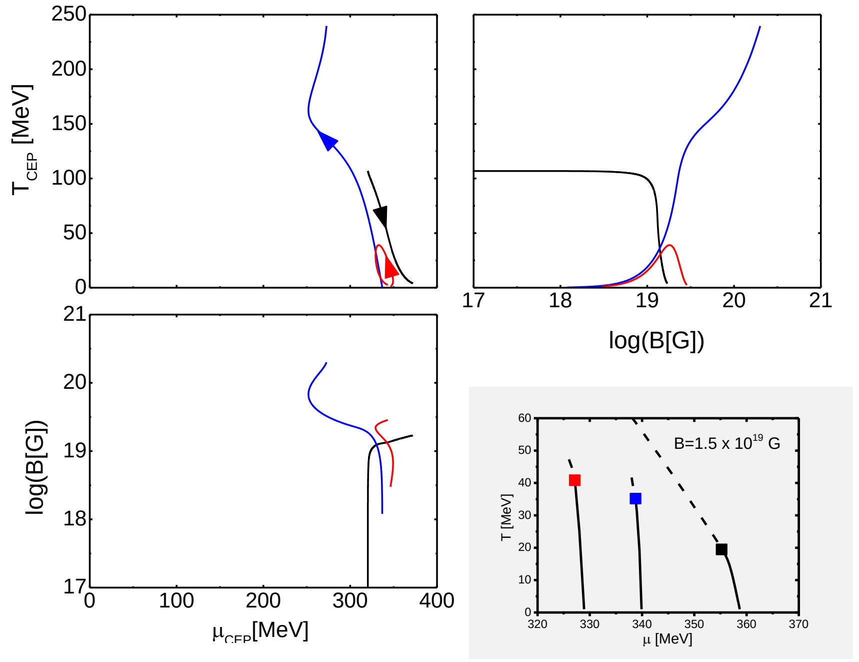


μ_c at T=0 as function of B (Set B)

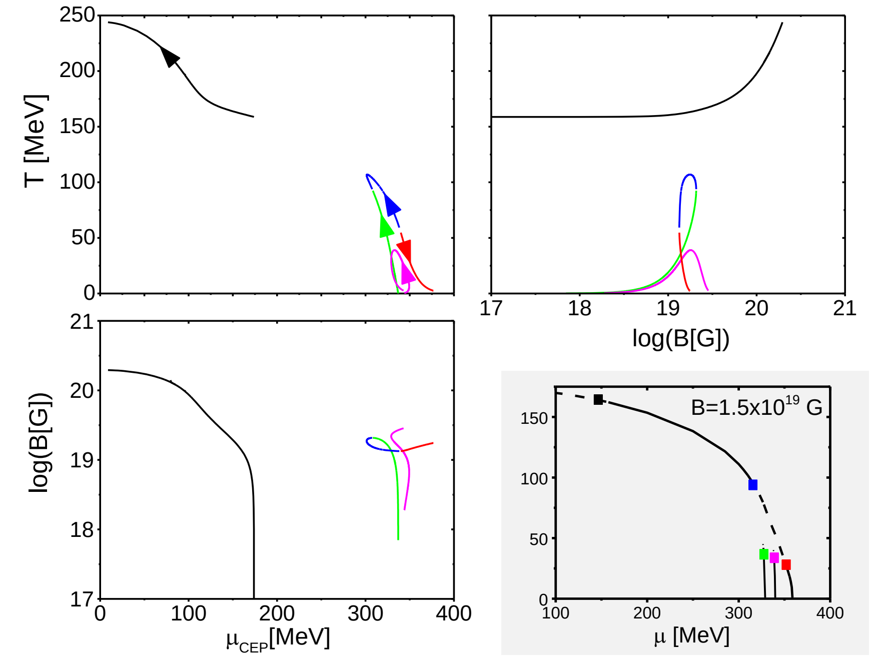


Position of CEP as a function of B – Set B:

PNJL



EPNJL



Summary & Conclusions

• We have analyzed the effect of a strong magnetic field on quark matter as described by (E)PNJL-type. These model provide a simultaneous dynamical description of the DECONFINEMENT and CHIRAL cross-over transitions.

•They are able to describe the enhancement of the quiral condensate with B at T=0. However, as most of the present available models they fail to reproduce recent lattice QCD results for T_c vs B at μ=0. What is missing ?

•In EPNJL there is no splitting at μ=0 between chiral restoration and deconfinement transitions as functions of B. Similarly for a given B both transitions lines coincide up to the critical point.

•The detailed form of the phase diagram, particularly at low T, is rather different dependending on the parameterization used for the quark sector. For parametrizations leading to $M_0 <\sim 350$ MeV there is a quite rich structure due to the subsequent population of the Landau levels as μ increases. In particular several CEP are found.

•Possible extensions and applications: more realistic non-local models, EOS, etc