

# Towards a Beth-Uhlenbeck EoS for compact stars and supernovae

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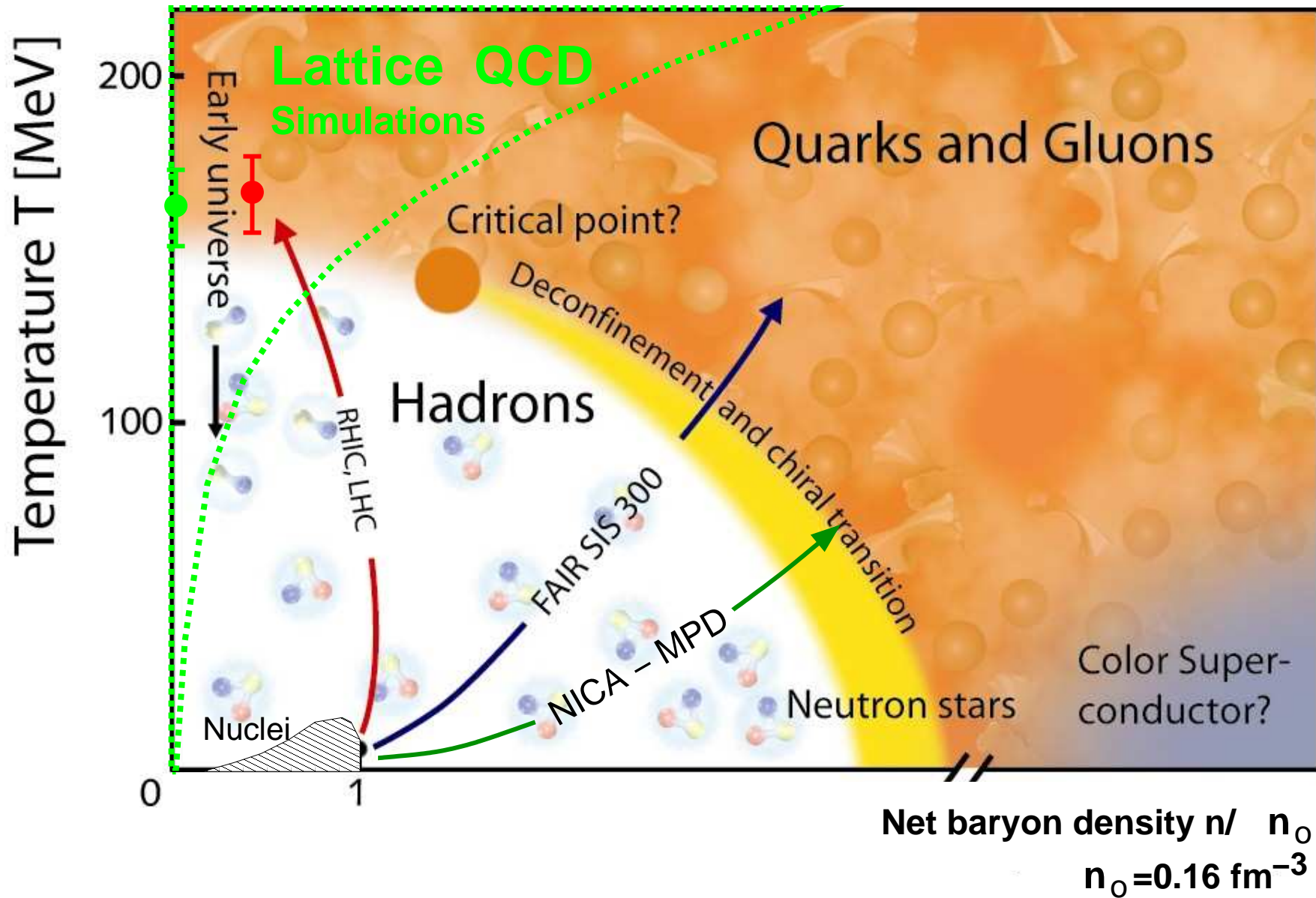
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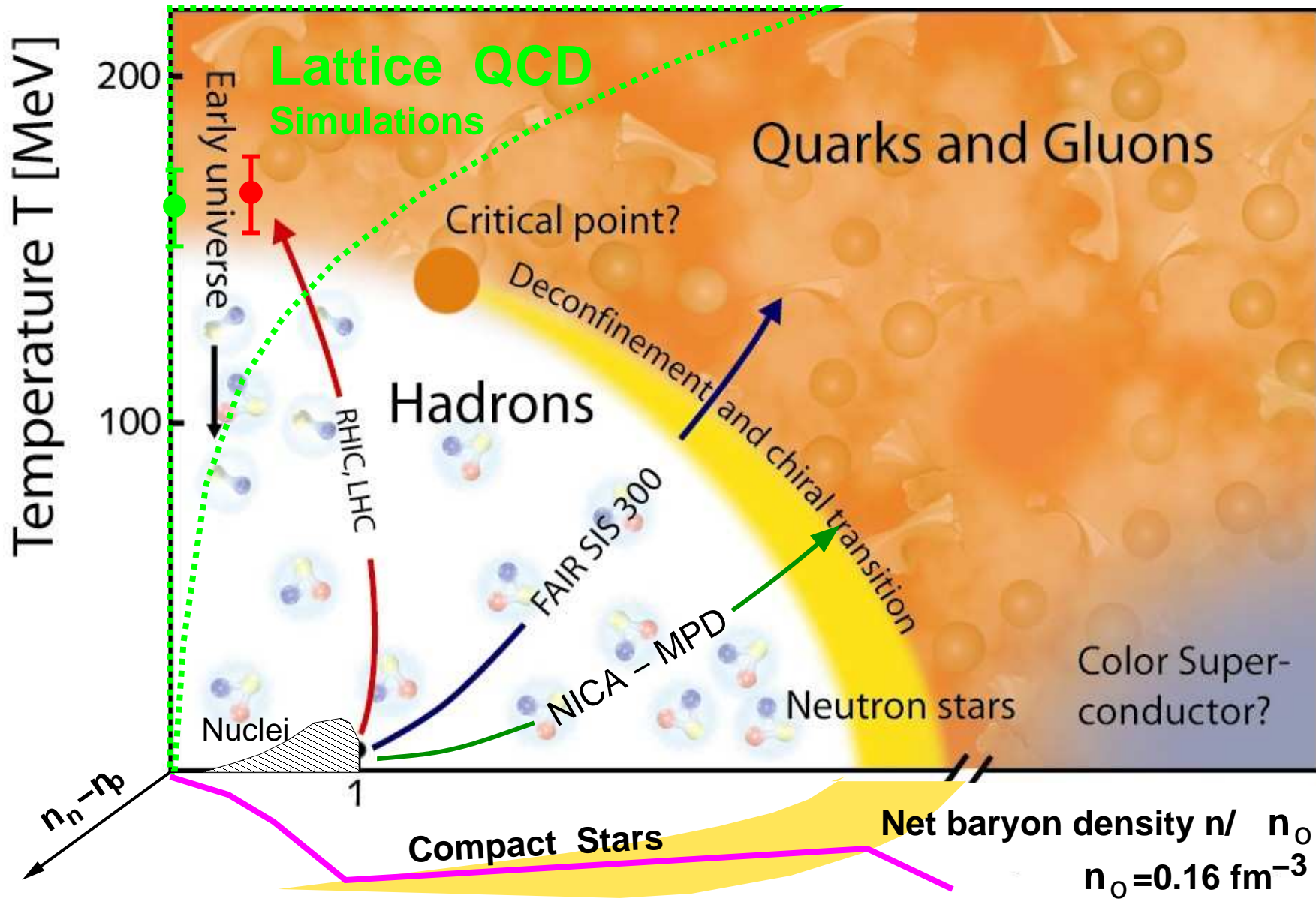
- Pauli blocking for bound states (clusters) in nuclear matter
- Pauli blocking for nucleons: quark substructure effect
- Consequences for Compact Stars and Supernovae:
  - Symmetry energy and cluster fractions
  - Nucleon dissociation - limits of the hadronic world
- Perspectives: Relativistic formulation (Daniel Zablocki)



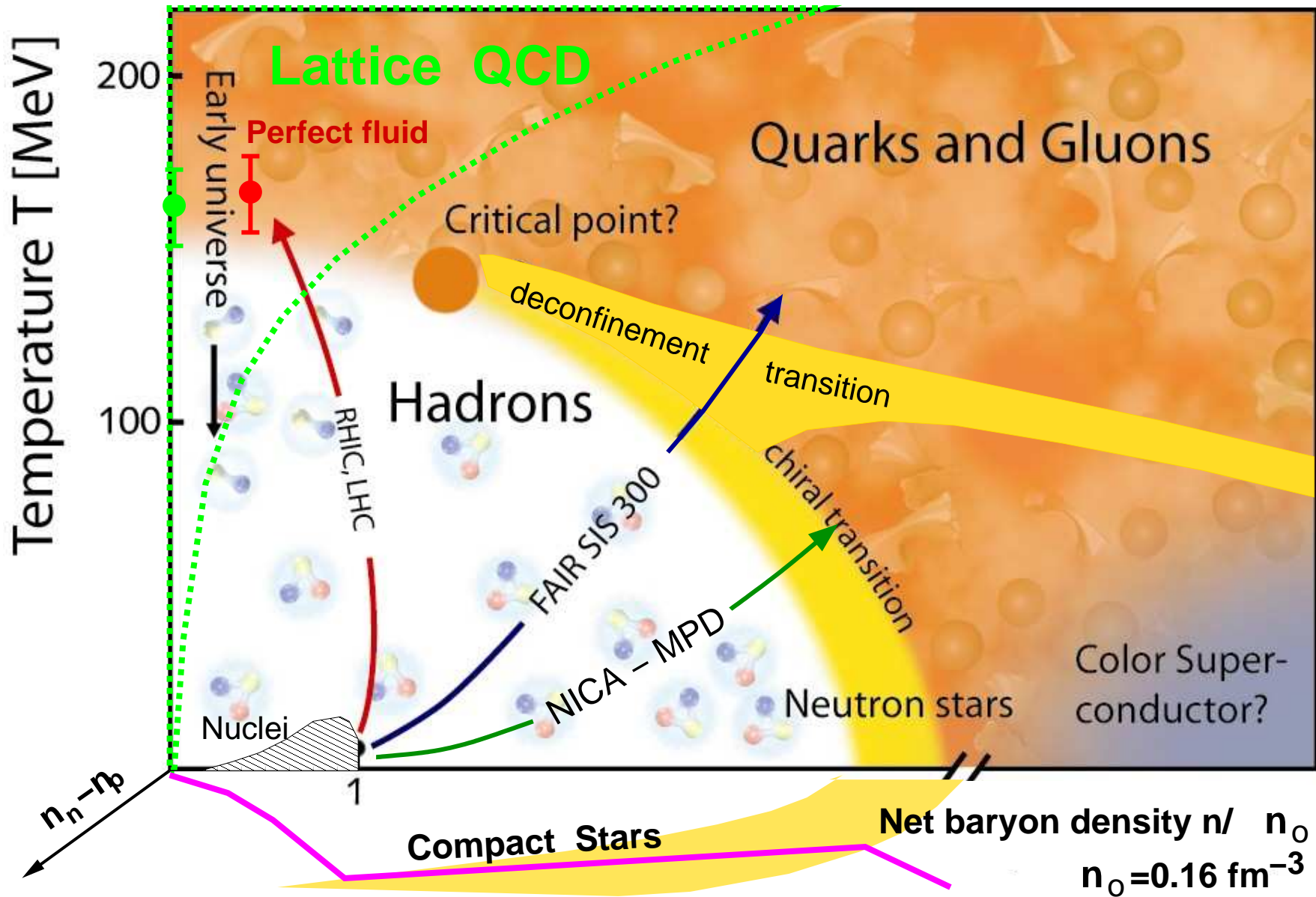
# Extreme States of Matter - The Phase Diagram



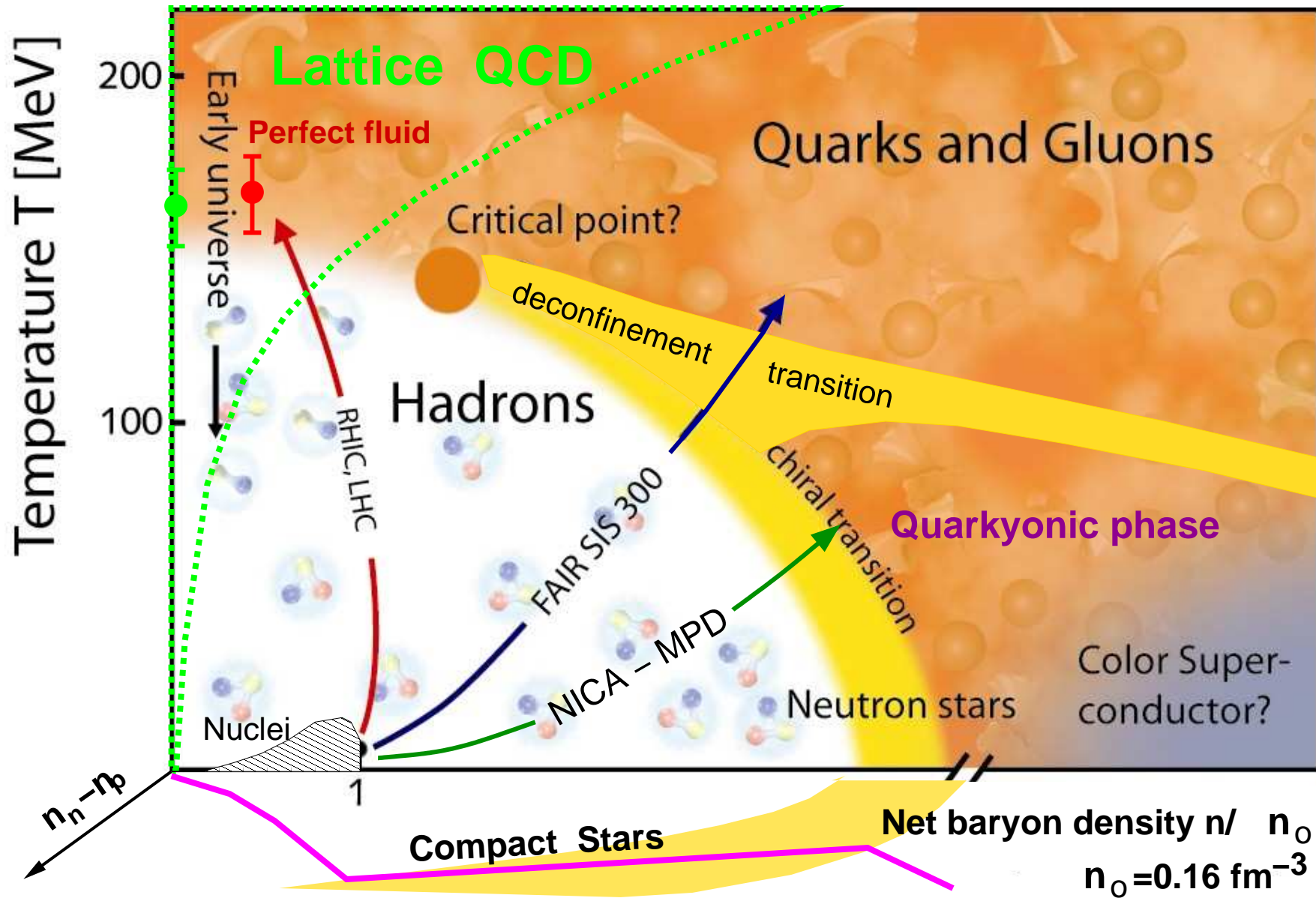
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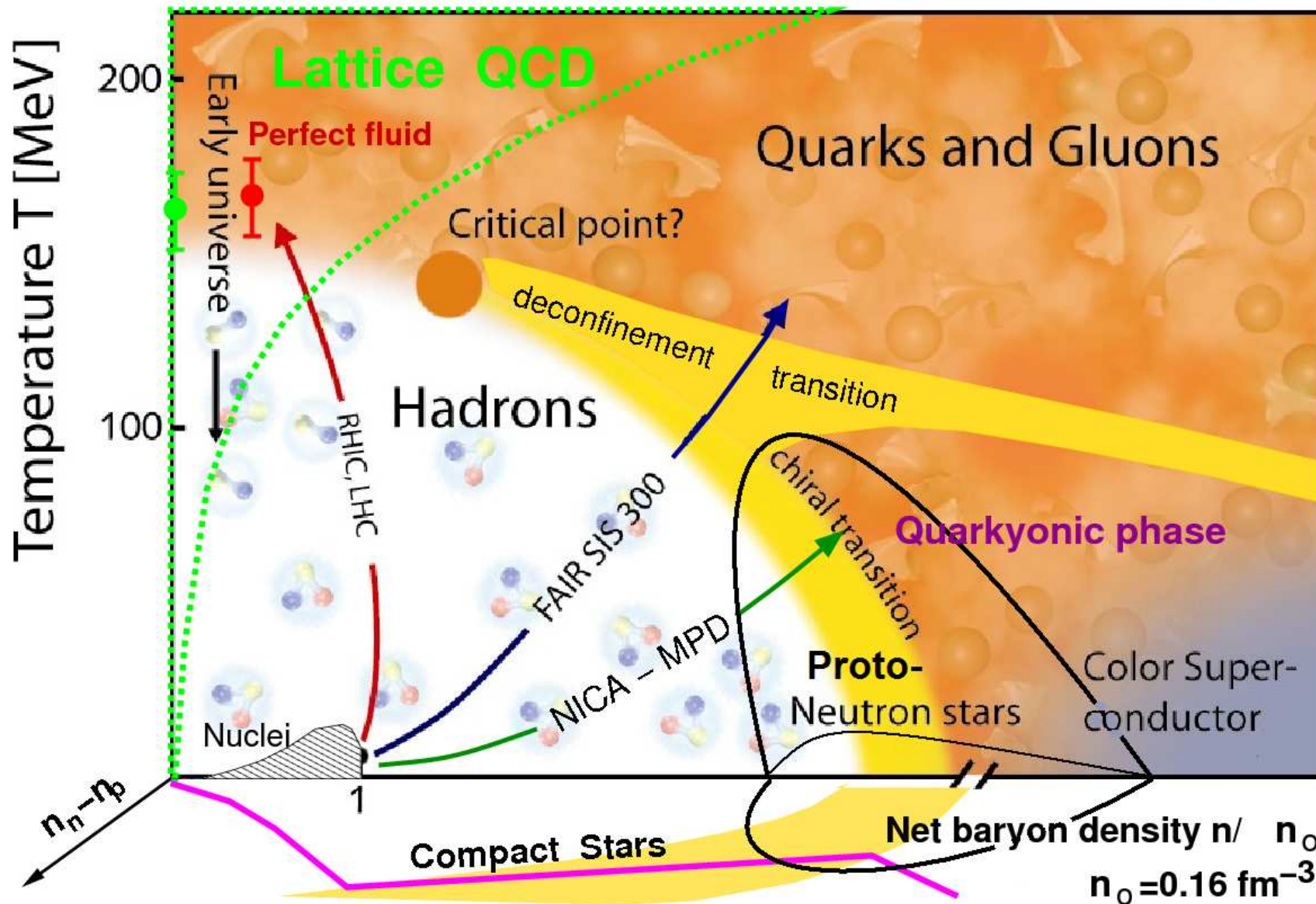
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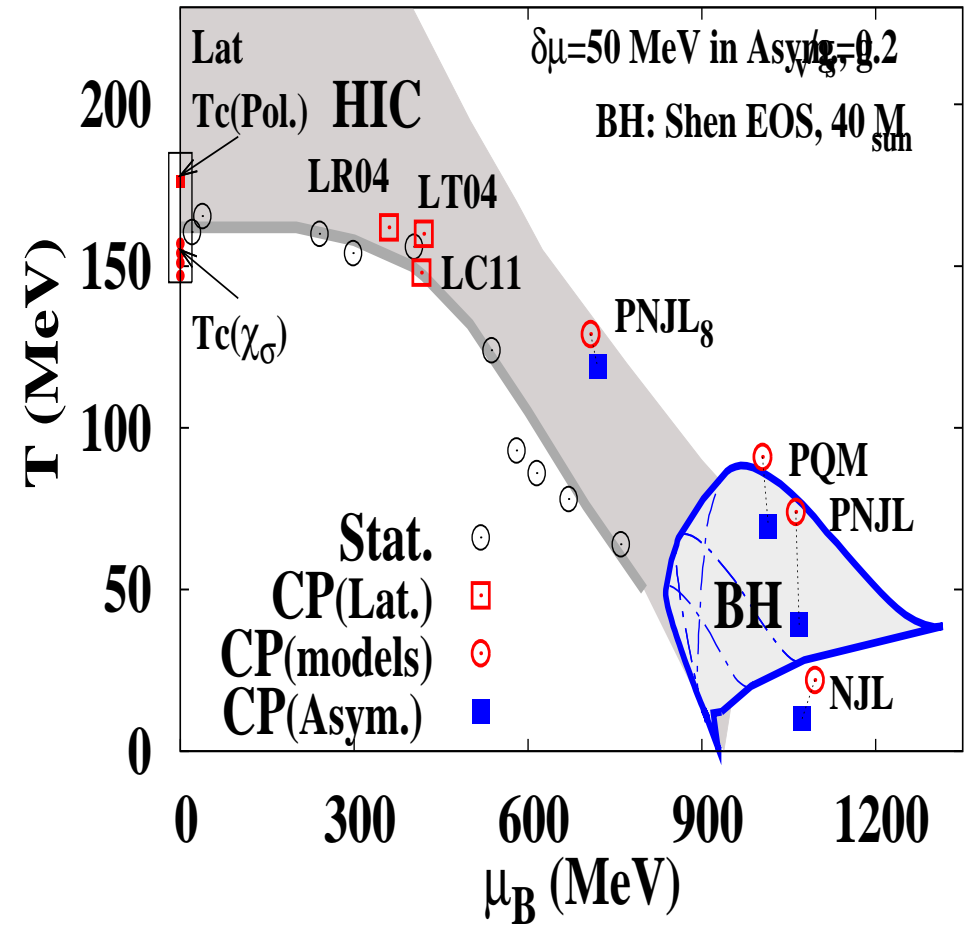
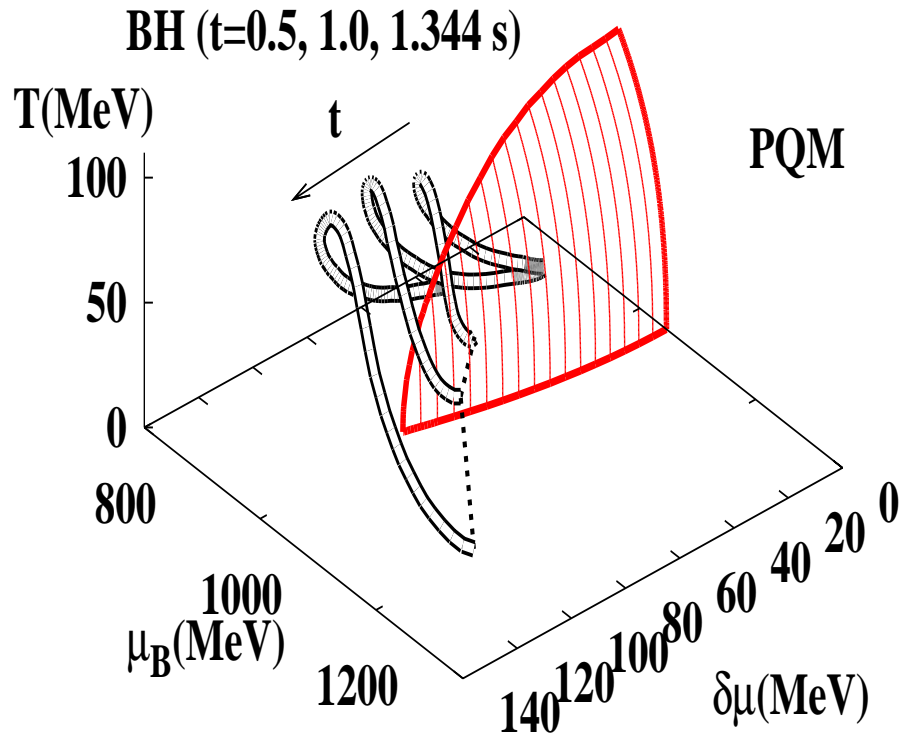
# Extreme States of Matter - The Phase Diagram



# Extreme States of Matter - The Phase Diagram



# Sweeping the critical point in SN collapse & BH formation



A. Ohnishi, H. Ueda, T. Nakano, M. Ruggieri, K. Sumiyoshi, Phys. Lett. B **704**, (2011) 284.

# Motivation and Tasks

**EoS for Supernova and Merger Simulations:** Wide range of parameters!

- $10^{-8} \leq n/n_0 \leq 10$
- $0 \leq T \leq 200 \text{ MeV}$
- $0 \leq Y_p \leq 0.6; \beta = 1 - 2Y_p$

Commonly used EoS:

- Lattimer-Swesty, NPA 535 (1991):  
Skyrme-type model LD modeling of nuclei embedded in nucleon gas
- Shen, Toki et al., Prog. Theor. Phys. 100 (1998):  
RMF model (TM1),  $\alpha$  particles with excluded volume procedure

Recent development:

- Horowitz-Schwenk, NPA 776 (2006): virial expansion, nucleons and  $\alpha$ 's  
uses experimental data for BE and scattering phase shifts, exact limit for low densities  $n/n_0 < 10^{-3}$

**Tasks of the present work:**

- medium effects on light clusters from quantum statistical approach
- realistic description of high-density matter (DD-RMF)
- thermodynamics, liquid-gas phase transition (instability region)



# Theory of nuclear matter with clusters (I)

Total nucleon density:

$$n_{\tau}(T, \tilde{\mu}_p, \tilde{\mu}_n) = \frac{1}{\Omega} \sum_1 \langle a_1^{\dagger} a_1 \rangle \delta_{\tau, \tau_1} = 2 \int \frac{d^3 k_1}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f_{1,Z}(\omega) S_1(1, \omega)$$

Distribution functions:  $f_{A,Z}(\omega) = (\exp \{ \beta [ \omega - Z \tilde{\mu}_p - (A - Z) \tilde{\mu}_n ] \} - (-1)^A)^{-1}$

Cluster decomposition of nucleon densities

$$n_p^{\text{tot}}(T, \tilde{\mu}_p, \tilde{\mu}_n) = \frac{1}{\Omega} \sum_{A,\nu,K} Z f_{A,Z} [E_{A,\nu}^{\text{qu}}(K; T, \tilde{\mu}_p, \tilde{\mu}_n)]$$

$$n_n^{\text{tot}}(T, \tilde{\mu}_p, \tilde{\mu}_n) = \frac{1}{\Omega} \sum_{A,\nu,K} (A - Z) f_{A,Z} [E_{A,\nu}^{\text{qu}}(K; T, \tilde{\mu}_p, \tilde{\mu}_n)]$$

Mass fractions of clusters:  $X_{A,Z} = \frac{A}{\Omega n} \sum_{\nu,K} f_{A,Z} [E_{A,\nu}^{\text{qu}}(K; T, \tilde{\mu}_p, \tilde{\mu}_n)]$

Thermodynamical potential  $F$  by integration of  $\mu(n)$  [inverted  $n(\mu)$ ]

→ all thdyn. functions (EoS)

$$F(T, n, Y_p^{\text{S}}) / \Omega = \int_0^n dn' \tilde{\mu}(T, n', Y_p^{\text{S}})$$

# Theory of nuclear matter with clusters (II)

Single-nucleon quasiparticle dispersion in effective mass approximation:

$$E_{\tau}^{\text{qu}}(k) = \Delta E_{\tau}^{\text{SE}}(0) + \frac{k^2}{2m_{\tau}^*} + \mathcal{O}(k^4)$$

From density-dependent RMF theory follows

$$E_{n,p}^{\text{qu}}(0) = \sqrt{[m - \Sigma_{n,p}(T, n, \pm\delta)]^2 + k^2} + \Sigma_{n,p}^0(T, n, \pm\delta)$$

$\Sigma_{n,p}$  – scalar self energy;  $\Sigma_{n,p}^0$  time component of vector self energy

$$\Delta E_{n,p}^{\text{SE}}(k) = \Sigma_{n,p}^0(T, n, \pm\delta) - \Sigma_{n,p}(T, n, \pm\delta) ; \quad m_{n,p}^* = m - \Sigma_{n,p}(T, n, \pm\delta)$$

Quasiparticle energies for **clusters** from A-particle Schrödinger equation in perturbation theory

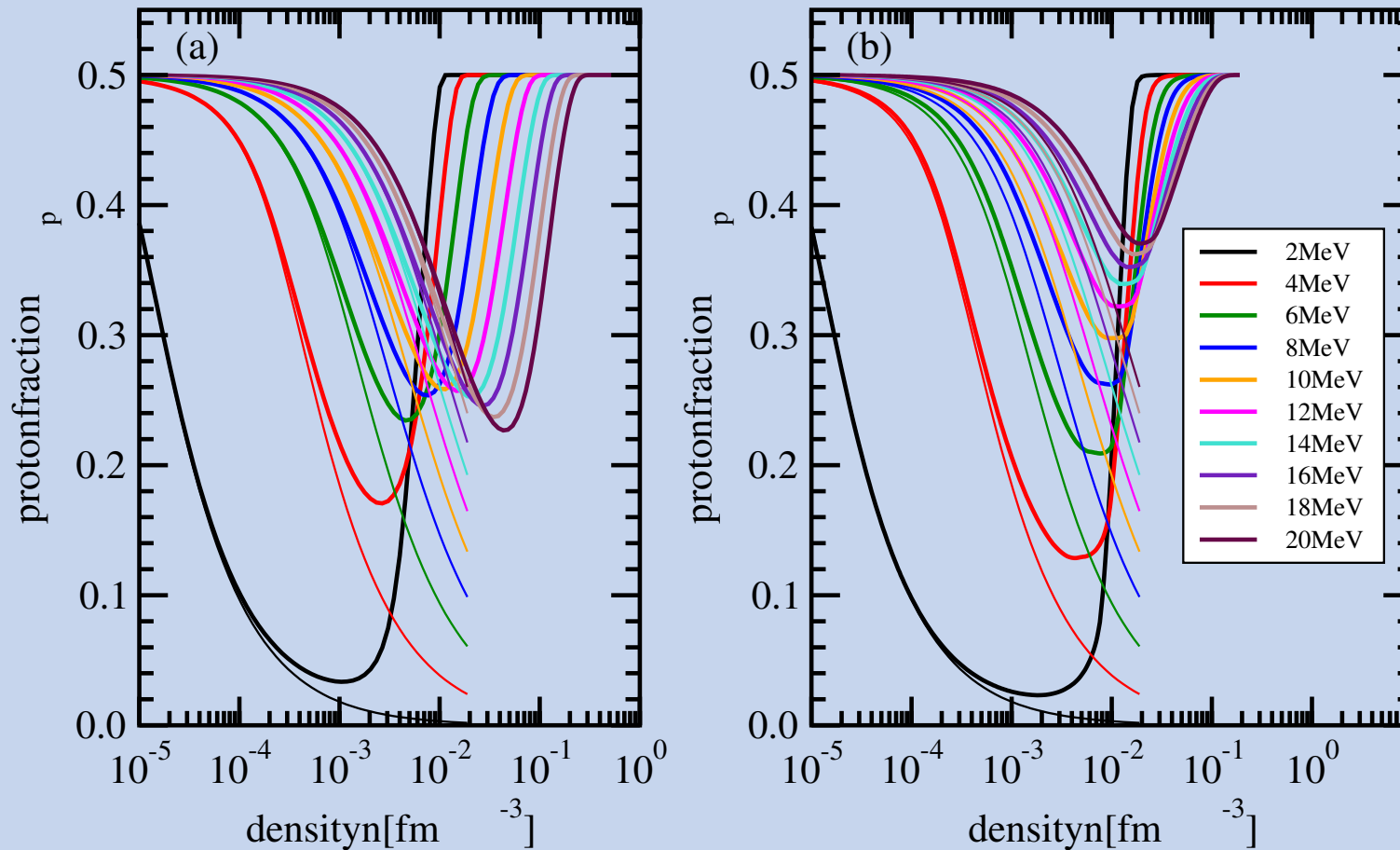
$$E_{A,\nu}^{\text{qu}}(K) = E_{A,Z}^{\text{qu}}(K) = E_{A,Z}^{(0)} + \frac{K^2}{2Am} + \Delta E_{A,Z}^{\text{SE}}(K) + \Delta E_{A,Z}^{\text{Pauli}}(K) + \Delta E_{A,Z}^{\text{Coul}}(K) + \dots$$

Important effect for cluster binding energy in medium: **Pauli shift**:

$$\Delta E_{A,Z}^{\text{Pauli}}(K) \approx \Delta E_{A,Z}^{\text{Pauli}}(0) \exp\left(-\frac{K^2}{g_{A,Z}}\right) ; \quad g_i(T, n, Y_p) = \frac{g_{i,1} + g_{i,2}T + h_{i,1}n}{1 + h_{i,2}n}$$

Typel, Röpke, Klähn, D.B., Wolter, Phys. Rev. C 81, 015803 (2010)

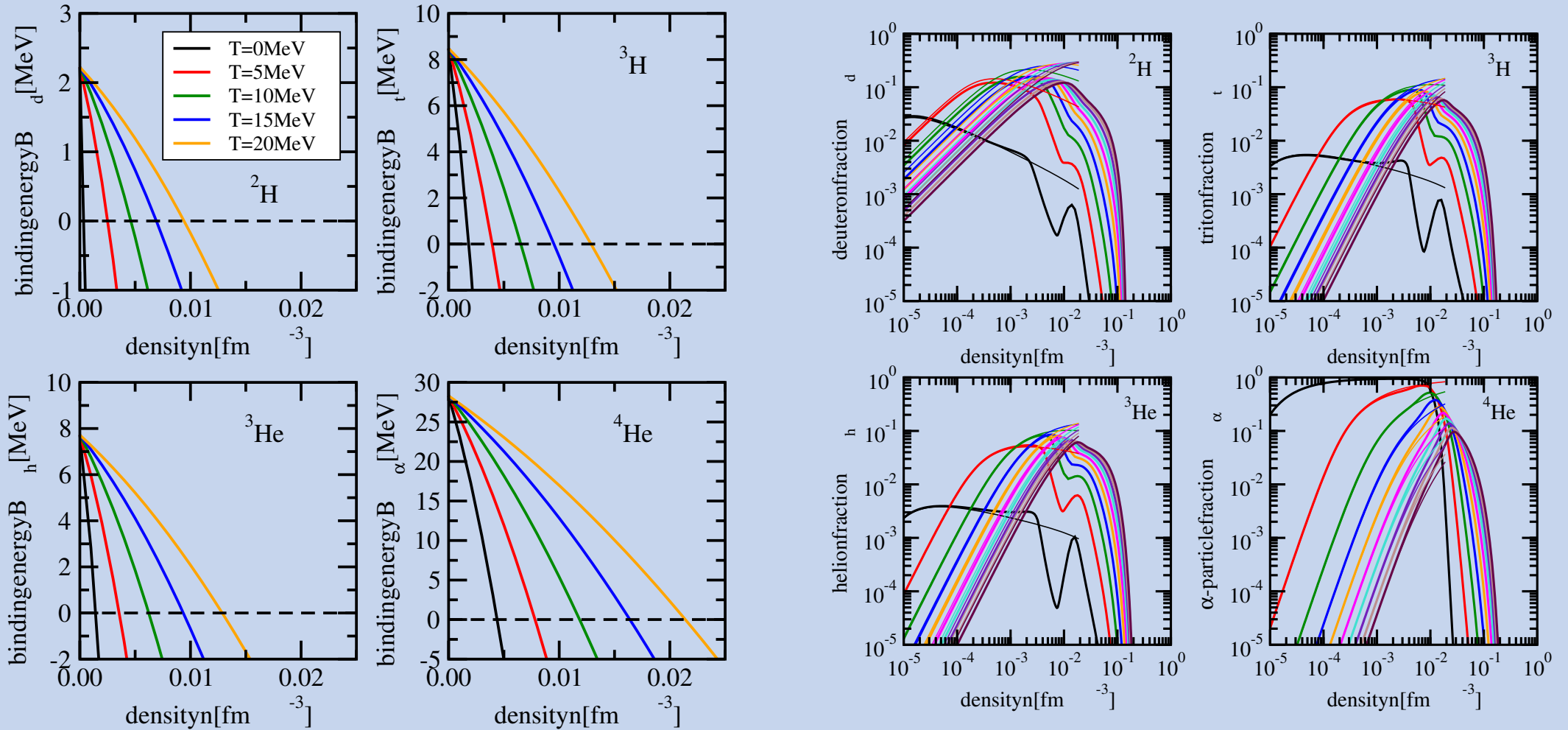
# Proton fraction (dissociation degree) in nuclear matter



Proton fraction  $X_p$  in symmetric nuclear matter in generalized RMF model (a) and quantum statistical approach (b). Thin lines show the nuclear statistical equilibrium (NSE) model for comparison.

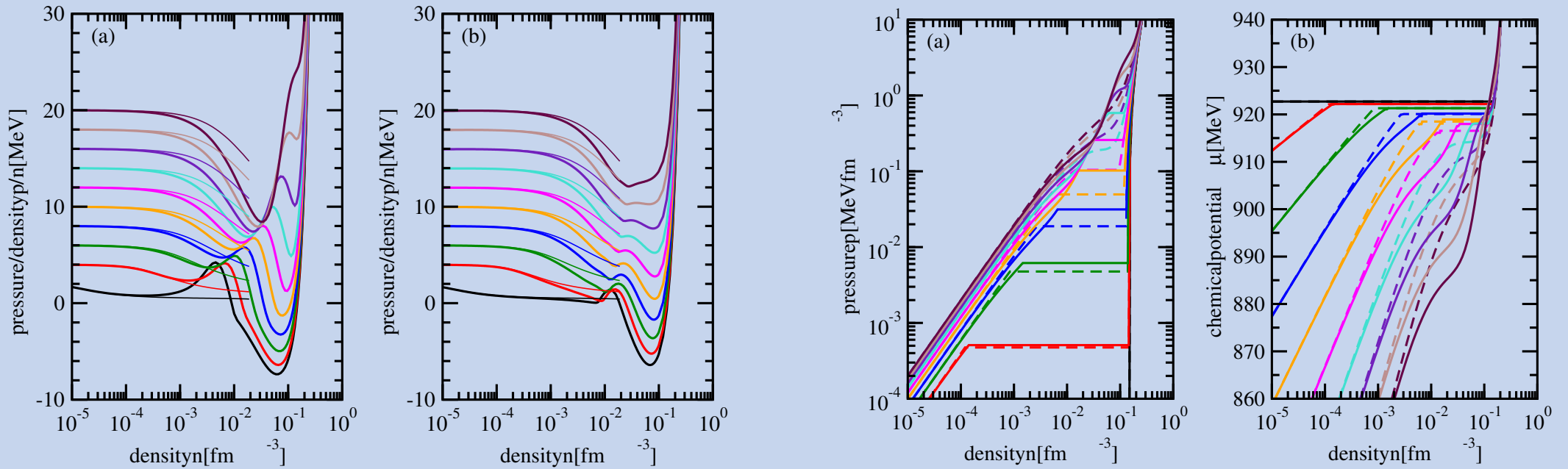
Typel, Röpke, Klähn, D.B., Wolter, arxiv:0908.2344; Phys. Rev. C 81, 015803 (2010)

# Cluster binding energies and fractions in nuclear matter



Typel, Röpke, Klähn, D.B., Wolter, arxiv:0908.2344; Phys. Rev. C 81, 015803 (2010)

# Liquid-gas phase transition in symmetric nuclear matter

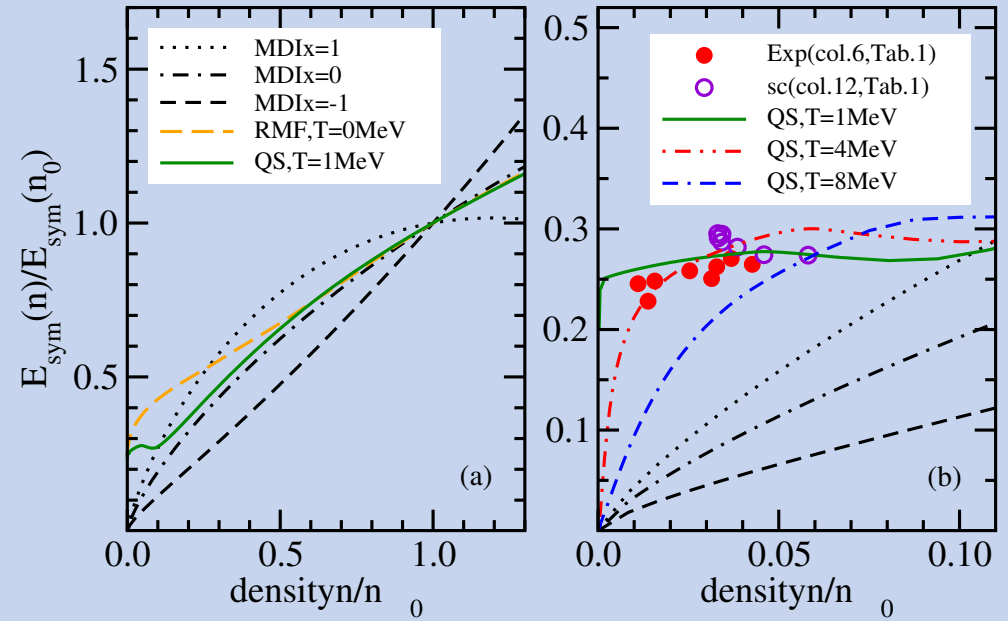
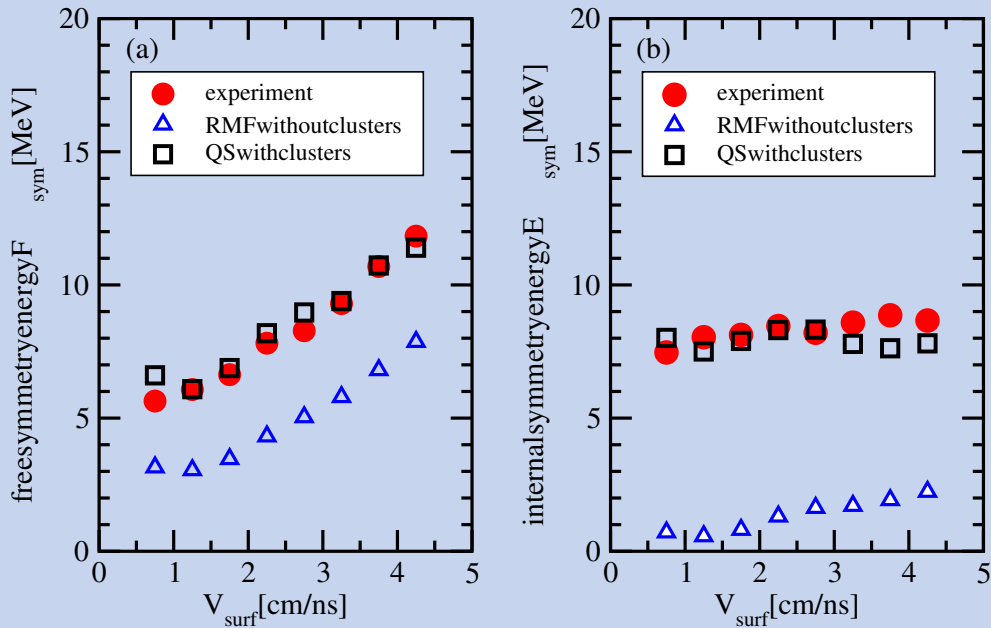


Typel, Röpke, Klähn, D.B., Wolter, arxiv:0908.2344; Phys. Rev. C 81, 015803 (2010)

# Nuclear matter symmetry energy with clusters

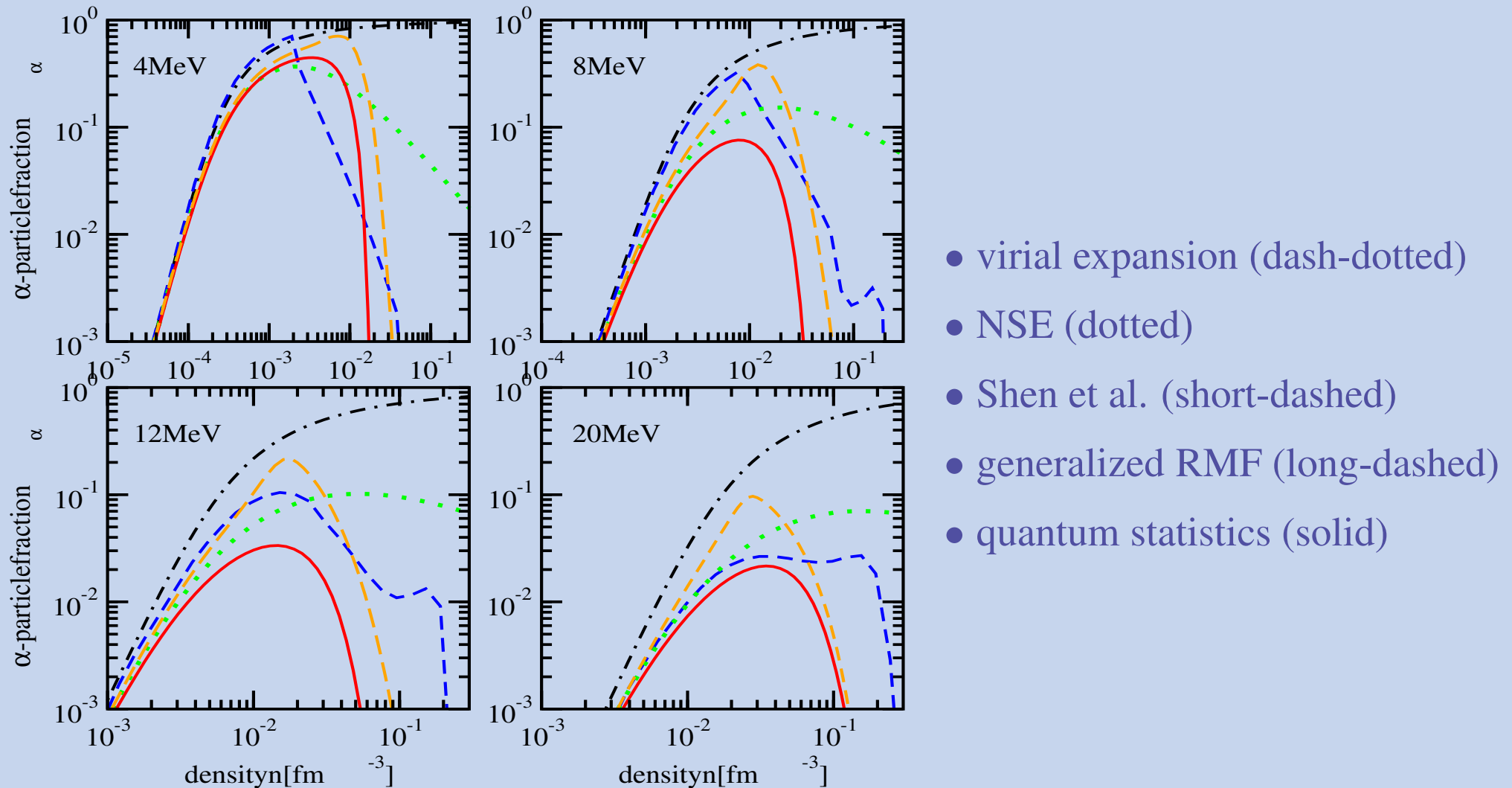
Nuclear (internal) symmetry energy:

$$E_{\text{sym}}(n, T) = \frac{E(n, 1, T) + E(n, -1, T)}{2} - E(n, 0, T)$$



Natowitz, Röpke, Typel, D.B., ..., Wolter, arxiv:1001.1102 [nucl-th]; PRL (2010).

# Comparison with standard SN EoS (Shen et al.)



$\alpha$  particle fractions in symmetric nuclear matter as functions of density at four temperatures.

**Typel, Röpke, Klähn, D.B., Wolter, arxiv:0908.2344; Phys. Rev. C 81, 015803 (2010)**

# Generalized Beth-Uhlenbeck approach to clustered quark matter

One-particle Green function ( $a = \text{quark, diquark}$ ) approximation:

$$G_a(1, z_a) = [z_a - E_a(1) - \Sigma_a(1, z_a)]^{-1}, \quad E_a(1) = E_a + p_1^2/(2m_a)$$

Spectral function

$$A_a(1, E) = i[G_a(1, E + i0) - G_a(1, E - i0)] = \frac{2\Im\Sigma_a(1, E - i0)}{[E - E_a(1) - \Re\Sigma_a(1, E)]^2 + [\Im\Sigma_a(1, E - i0)]^2}$$

mean occupation number  $n_a(1)$  and distribution function  $f_a(E)$

$$n_a(1) = \eta_a \int \frac{dE}{2\pi} f_a(E) A_a(1, E), \quad f_a(E) = \eta_a [\exp[(E - \mu_a)/T] + \eta_a]^{-1}$$

Thermodynamics: via density to pressure as thermodynamical potential

$$n(T, \mu_1, \dots, \mu_c) = \frac{1}{\Omega} \sum_{a=1}^c \sum_1 n_a(1),$$
$$p(T, \mu_1, \dots, \mu_c) = \sum_a \int_{-\infty}^{\mu_a} d\mu' n_a(T, \mu_1 \dots \mu' \dots \mu_c)$$

$\implies$  Generalized Beth-Uhlenbeck EoS for (n.r.) Quark Matter.

**D.B., H. Grigorian, G. Röpke, in preparation (2012)**



# Quark matter beyond meanfield, nucleons as bound states (clusters)

Hartree-Fock:  $\Im\Sigma(1, z)$  vanishes, spectral function  $\delta$  – shaped at quasiparticle energies:

$$e_a(1) = E_a(1) + v_a(1), \quad v_a(1) = \Re\Sigma_a(1, e_a(1))$$

Consider contributions up to first order in  $\Im\Sigma(1, z)$

$$A_a(1, E) = A_a^{free}(1, E) + A_a^{corr}(1, E), \quad A_a^{free}(1, E) = 2\pi\delta(E - e_a(1)),$$
$$A_a^{corr}(1, E) = 2 \int dE' \Im\Sigma_a(1, E' - i0) [\delta(E - e_a(1)) - \delta(E - E')] \frac{d}{dE'} \frac{\mathcal{P}}{E' - e_a(1)}$$

Bound states: selfenergy in terms of the T–matrix

$$\Sigma_a(1, z_\nu^a) = T \sum_{2,b} \sum_{z_\nu^b} T_{ab}(12, 12, z_\nu^a + z_\nu^b) G_b(2, z_\nu^b)$$
$$= \Sigma^{HF}(1) + \sum_{2,b} \int dE \Im T_{ab}(12, 12, E + i0) \int \frac{dE'}{2\pi} A_b(2, E') \frac{[f_b(E') - f_{ab}(E)]}{E - E' - z_a}$$

Two-particle distribution function  $f_{ab}(E)$  describes **fermionic** quark-diquark states,  $\eta_{ab} = -\eta_a\eta_b$ ,

$$f_{ab}(E) = \eta_{ab} [\exp(E - \mu_a - \mu_b)/T + \eta_{ab}]^{-1}$$

**D.B., H. Grigorian, G. Röpke, in preparation (2012)**

# Quark-diquark matter as nonideal two-component plasma

**T-matrix** from solution of the Bethe–Salpeter equation:

$$T_{ab}(12, 1'2', E) = V(12, 1'2') + \sum_{343'4'} V(12, 34)G_{ab}(34, 3'4', E)T_{ab}(3'4', 1'2', E)$$

$V(12, 1', 2')$  - interaction potential model, intermediate propagation given by Green's function

$$\begin{aligned} G_{ab}(12, 1'2', Z_{ab}) &= \sum_{z_a} G_a(1, z_a)G_b(2, Z_{ab} - z_a)\delta_{11'}\delta_{22'} \\ &= \delta_{11'}\delta_{22'} \int \frac{d\omega}{2\pi} \frac{d\omega'}{2\pi} \frac{1 - f_a(\omega) - f_b(\omega')}{\omega + \omega' - Z} A_a(1, \omega) A_b(2, \omega') \\ &= \frac{1 - n_a(1) - n_b(2)}{e_a(1) + e_b(2) - Z_{ab}} \delta_{11'}\delta_{22'} + \Delta G_{ab}(12, 1'2', Z_{ab}) \end{aligned}$$

$$n_a(1) = n_a^{free}(1) + \sum_{2,b} n_{ab}(12), \quad n_{ab}(12) = \eta_{ab} \int \frac{dE}{2\pi} f_{ab}(E) D_{ab}(12, E)$$

Two-particle spectral density (Generalized Beth-Uhlenbeck)

$$D_{ab}(12, E) = 2\pi\delta(E - E_{\alpha,P}) + \sin^2 \delta_{ab}(P, E) \frac{d}{dE} \delta_{ab}(P, E).$$

**D.B., H. Grigorian, G. Röpke, in preparation (2012)**

# Separable potential model of the nucleon

Schrödinger equation equivalent to Bethe–Salpeter equation:

$$(E_{\alpha,P} - e_{ab}^0(p, P))\Psi_{\alpha,P}(p) = \sum_{p'} V_{ab}(p, p')\Psi_{\alpha,P}(p')$$

Bilinear expansion to the  $T$ -matrix

$$T_{ab}(12, 1'2', E) = \sum_{\alpha} T_{ab,\alpha}(p, p', P, E), = \sum_{\alpha} \frac{\Psi_{\alpha,P}(p)\Psi_{\alpha,P}^*(p')}{E_{\alpha,P} - E} (e_{ab}^0(p, P) - E)(e_{ab}^0(p', P) - E_{\alpha,P})$$

Separable confining-type model potential

$$V(p, p') = (2\pi)^3 C \delta^{(3)}(\vec{p} - \vec{p}') - V_0 W(p, \beta) W(p', \beta)$$

Binding energy  $E_B$  and two-particle quasi-energies  $e_{ab}^0(p, P)$  (continuum edge)

$$E_B = -\frac{p_B^2}{2m} = E_{\alpha,P} - M - \frac{P^2}{2M} - C, \quad e_{ab}^0(p, P) = M + \frac{p^2}{2m} + \frac{P^2}{2M} + C$$

**D.B., H. Grigorian, G. Röpke, in preparation (2012)**

# Separable potential model of the nucleon - Parametrization

Wave function and loop integral  $J(E, P) = \sum V_{ab}(p, p)G_{ab}(p, P, E)$

$$\Psi_0(p) = \mathcal{N} \frac{V_0 W(p, \beta)}{p^2 + p_B^2}, \quad J(p_B, P; V_0, \beta) = \frac{mV_0}{\pi^2} \int_0^\infty \frac{W(p, \beta)^2}{p^2 + p_B^2} p^2 dp = 1$$

Separable Yamaguchi potential for  $W(p, \beta) = \beta^2 / (p^2 + \beta^2)$

Mass  $M_N = M + C(V_0, \beta) - |E_B(V_0, \beta)| = 939 \text{ MeV}$  and radius of the nucleon  $\langle r^2 \rangle = 0.7 \text{ fm}^2$

$$4\pi(1+x)^2 = mV_0\beta, \quad \frac{(1+x^3)(1+x)^3 - 16x^3}{8x^2(1-x^2)^2} = \beta^2 R^2 / 4, \quad x = p_B / \beta$$

$V_0[\text{GeV}^{-2}]$	$\beta[\text{fm}^{-1}]$	$C[\text{GeV}]$	$E_B[\text{GeV}]$	$n_{\text{Mott}}[n_0];\text{SNM}$	$n_{\text{Mott}}[n_0];\text{PNM}$
150	3.51997	0.0867326	0.147833	3.8496	1.92465
200	3.03763	0.115714	0.176814	4.1836	2.09181
250	2.73902	0.143152	0.204252	4.46062	2.23005
450	2.16666	0.24039	0.30149	5.19448	2.59723
500	2.08783	0.262325	0.323425	5.96763	2.65924
650	1.91431	0.324144	0.385244	5.61316	2.80667

**D.B., H. Grigorian, G. Röpke, in preparation (2012)**

# Nucleon in the medium - quark substructure effects

In-medium T-matrix describes quark-diquark substructure effects

$$T_{ab}(p, p', P, E) = \frac{V_{ab}(p, p')}{1 - J(E, P)}, \quad J(E, P) = \sum_{a,b,p} V_{ab}(p, p) G_{ab}(p, P, E).$$

Two-particle energies (angle-averaged) in the medium

$$e_{ab}(p, P) = e_{ab}^0(p, P) + u_{ab}(p, P), \quad u_{ab}(p, P) = \langle v_a(1) + v_b(2) \rangle_{pP}$$

Generalized (angle-averaged) Pauli blocking operator

$$Q(p, P) = 1 - \langle n_a^{(B)} \rangle_{pP} - \langle n_b^{(B)} \rangle_{pP}$$

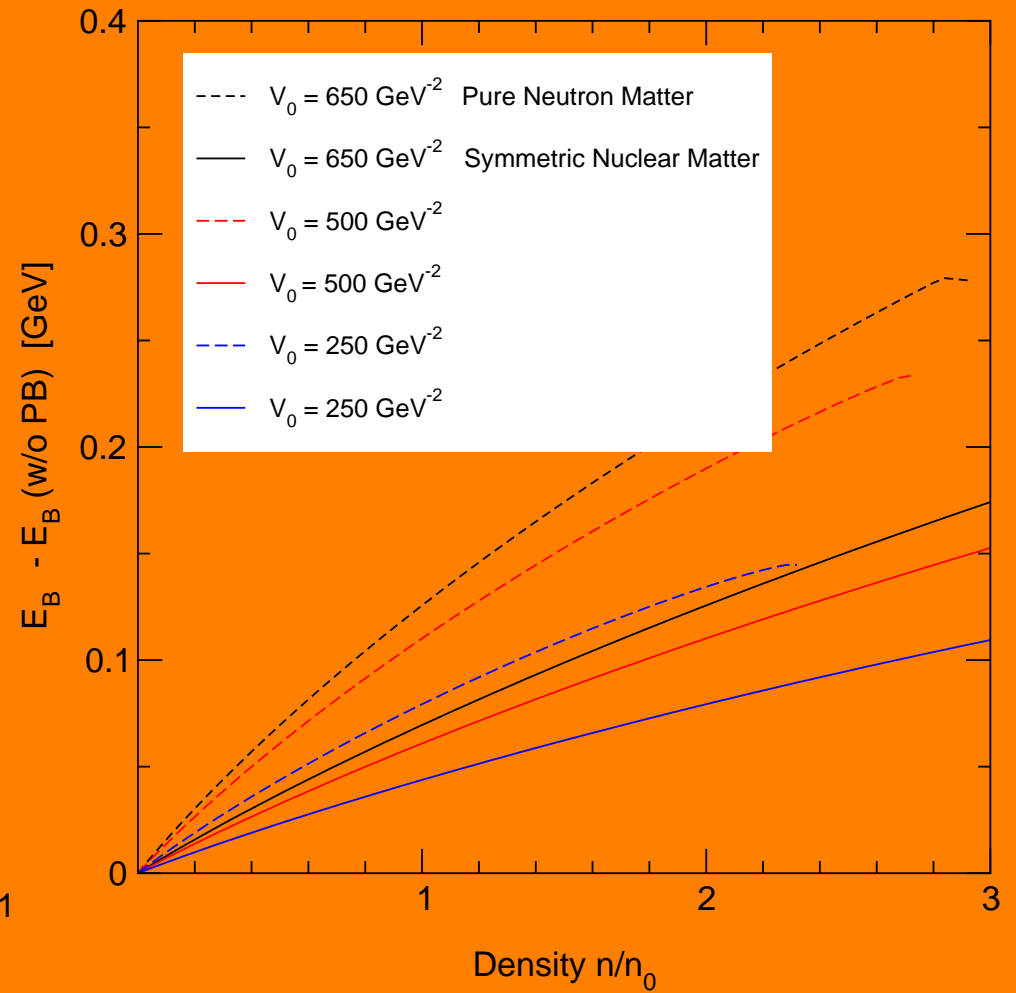
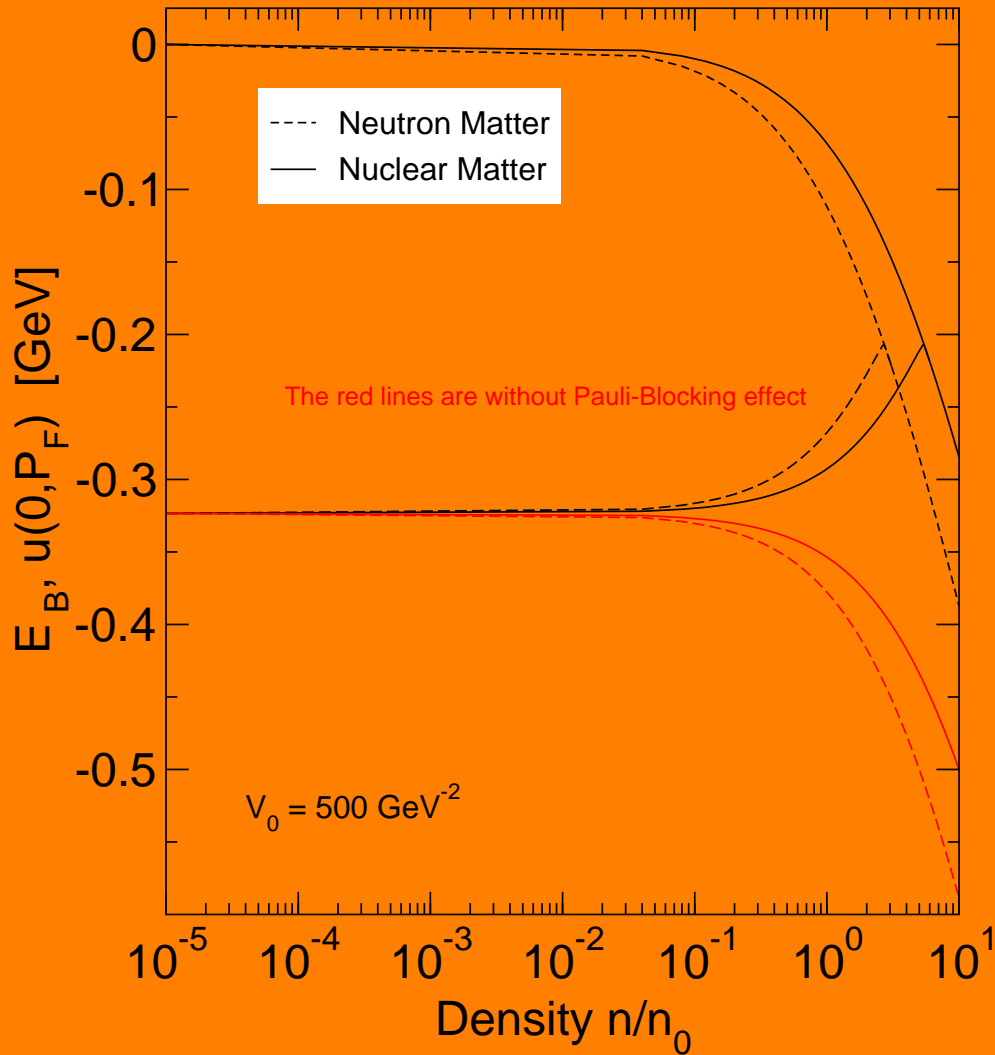
$$J_R(E, P) = \int_0^\infty \frac{p^2 dp}{2\pi^2} V_0 W^2(p) \frac{Q(p, P)}{e_{ab}(p, P) - E}; \quad e_{ab}(p, P) > E$$

Pair distribution function

$$n_{ab}^{(B)}(p, P) = \eta_{ab} f_{ab}(E_B + E_0(P)) |\Psi_P(p)|^2, \quad \Psi_P(p) = \frac{V_0 \mathcal{N} W(p) \sqrt{Q(p, P)}}{e_{ab}(p, P) - E_B - E_0(P)}$$

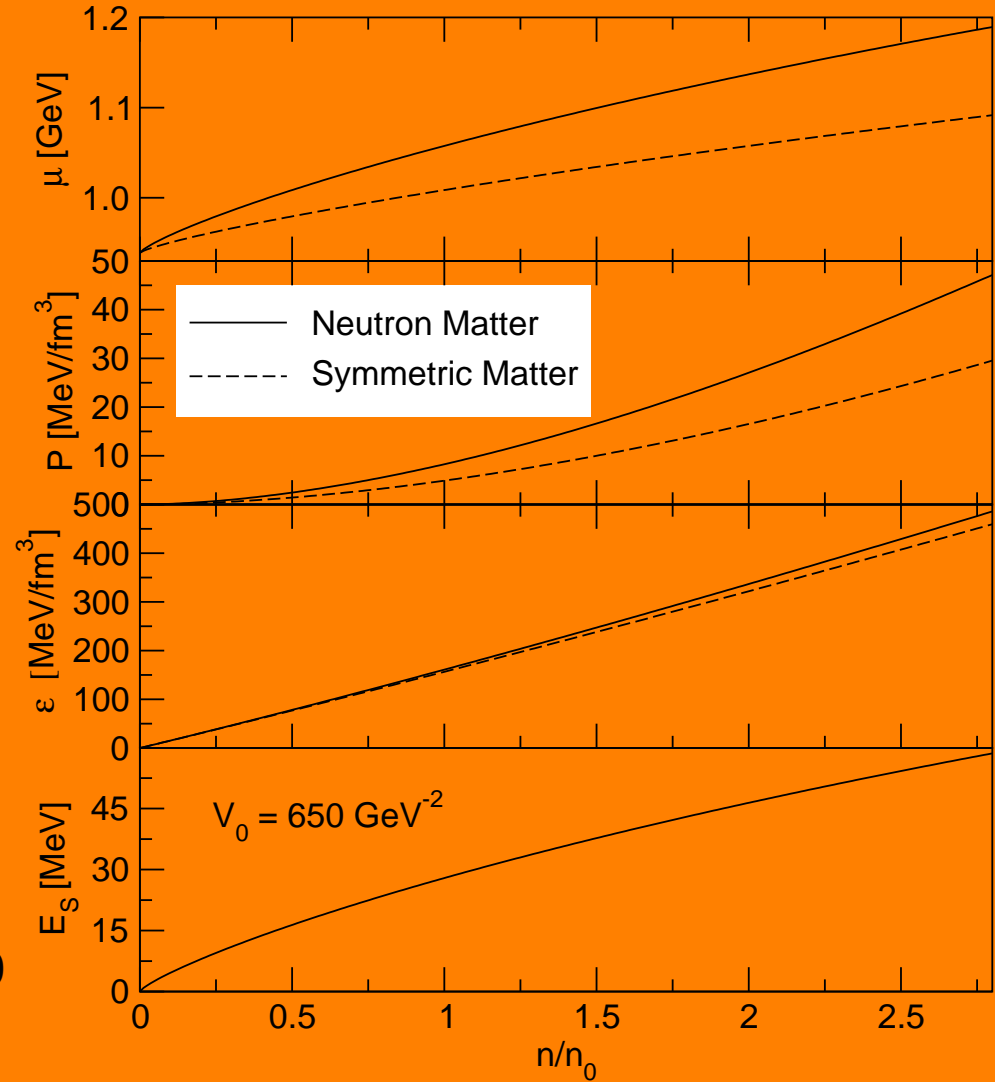
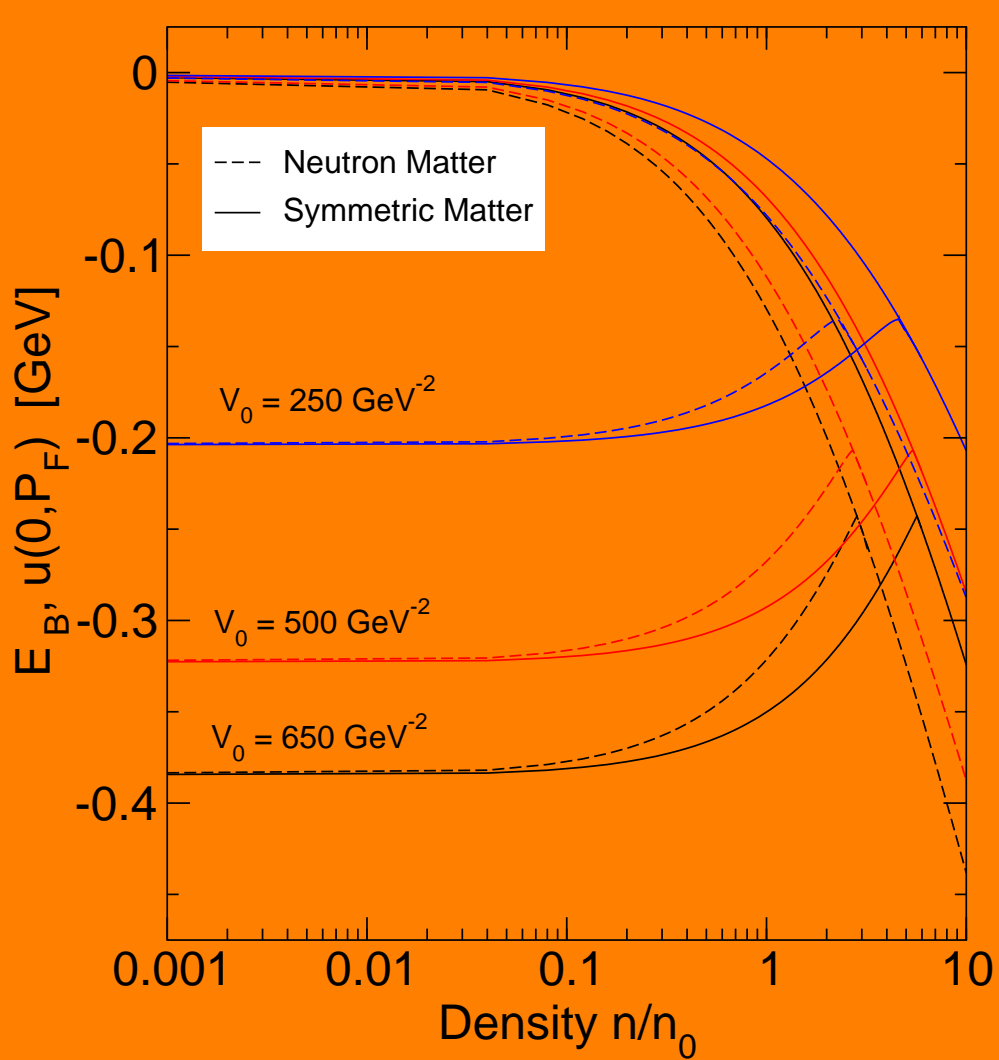
Nucleon Mott dissociation follows when bound state energy equals continuum.

# Nucleon dissociation



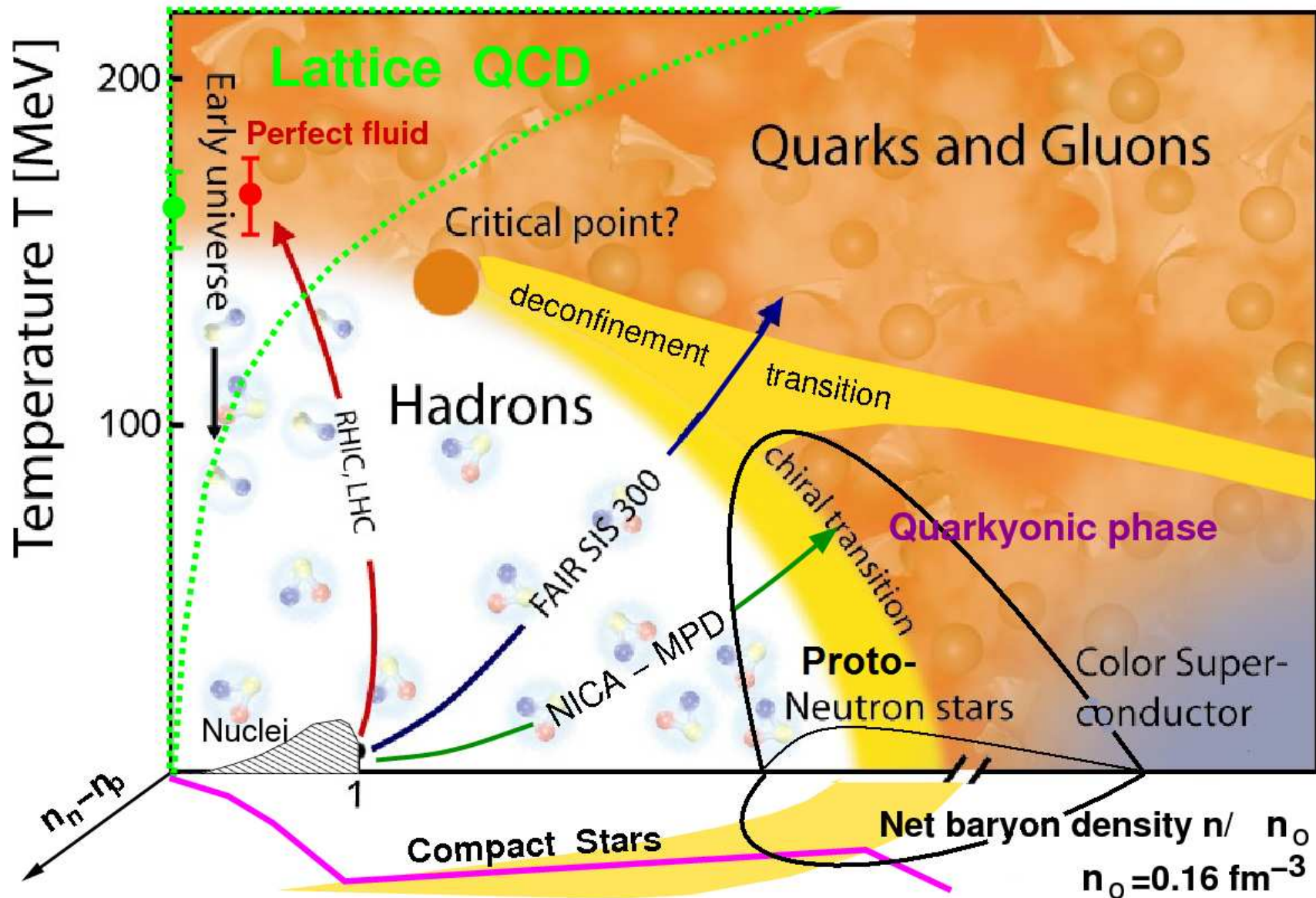
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# Nuclear matter towards deconfinement



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# Extreme States of Matter - The Phase Diagram





# Quantum Field Theory for chiral Quark Matter

- Partition function for chiral Quark Field theory coupled to Polyakov-loop potential

$$Z[T, V, \mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp \left\{ - \int^{\beta} d^4x \{ \bar{\psi} [i\gamma^{\mu}\partial_{\mu} - m - \gamma^0(\mu + i\lambda_3\phi_3)]\psi - \mathcal{L}_{\text{int}} + U[\Phi(\phi_3)] \} \right\}$$

- Current-current coupling (4-fermion interaction) and KMT determinant

$$\mathcal{L}_{\text{int}} = \sum_{M=\pi,\sigma,\dots} G_M (\bar{\psi}\Gamma_M\psi)^2 + \sum_D G_D (\bar{\psi}^C\Gamma_D\psi)^2 + \mathcal{L}_{\text{det}}$$

- Bosonisation (Hubbard-Stratonovich Transformation)

$$Z[T, V, \mu] = \int \mathcal{D}\phi_M \mathcal{D}\Delta_D^{\dagger} \mathcal{D}\Delta_D \exp \left\{ -\frac{\phi_M^2}{4G_M} - \frac{|\Delta_D|^2}{4G_D} - \frac{K\phi_u\phi_d\phi_s}{16G_S^3} + \frac{1}{2}\text{Tr} \ln S^{-1} + U[\Phi(\phi_3)] \right\}$$

- Collective (stochastic) Fields: Mesons ( $\phi_M$ ) and Diquarks ( $\Delta_D$ )

- Systematic Evaluation: **Mean field** + **Fluctuations**

- Mean-field Approximation: **Order parameter** for Phase transitions (Gap equations)
- Fluctuations (2. Order): **Hadronic Correlations** (Bound- & Scattering states)
- Fluctuations of higher Order: Hadron-Hadron **Interaction**

# Phase diagram for 3-Flavor Quark Matter

1. Introduction
2. Hadronic Cooling
3. Quark Substructure and Phases
4. Hybrid Star Cooling
5. Summary

Thermodynamic Potential  $\Omega(T, \mu) = -T \ln Z[T, \mu]$

$$\Omega(T, \mu) = \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2 + |\Delta_{us}|^2 + |\Delta_{ds}|^2}{4G_D} - T \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \text{Tr} \ln \left( \frac{1}{T} S^{-1}(i\omega_n, \vec{p}) \right) + \Omega_e - \Omega_0.$$

Inverse Nambu – Gorkov Propagator  $S^{-1}(i\omega_n, \vec{p}) = \begin{bmatrix} \gamma_\mu p^\mu - M(\vec{p}) + \mu\gamma^0 & \hat{\Delta}(\vec{p}) \\ \hat{\Delta}^\dagger(\vec{p}) & \gamma_\mu p^\mu - M(\vec{p}) - \mu\gamma^0 \end{bmatrix},$

$$\Delta_{k\gamma} = 2G_D \langle \bar{q}_{i\alpha} i\gamma_5 \epsilon_{\alpha\beta\gamma} \epsilon_{ijk} g(\vec{q}) q_{j\beta}^C \rangle. \quad \hat{\Delta}(\vec{p}) = i\gamma_5 \epsilon_{\alpha\beta\gamma} \epsilon_{ijk} \Delta_{k\gamma} g(\vec{p}).$$

Fermion Determinant (Tr ln D = ln det D)

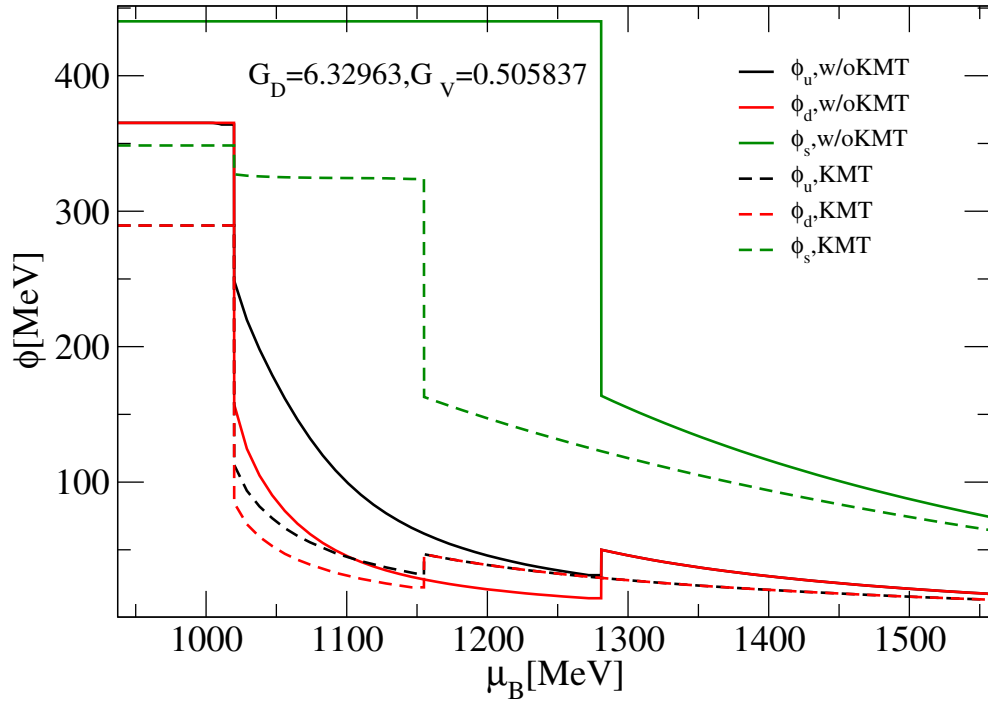
$$\ln \det \left( \frac{1}{T} S^{-1}(i\omega_n, \vec{p}) \right) = 2 \sum_{a=1}^{18} \ln \left( \frac{\omega_n^2 + \lambda_a(\vec{p})^2}{T^2} \right).$$

Result for the thermodynamic Potential (Meanfield approximation)

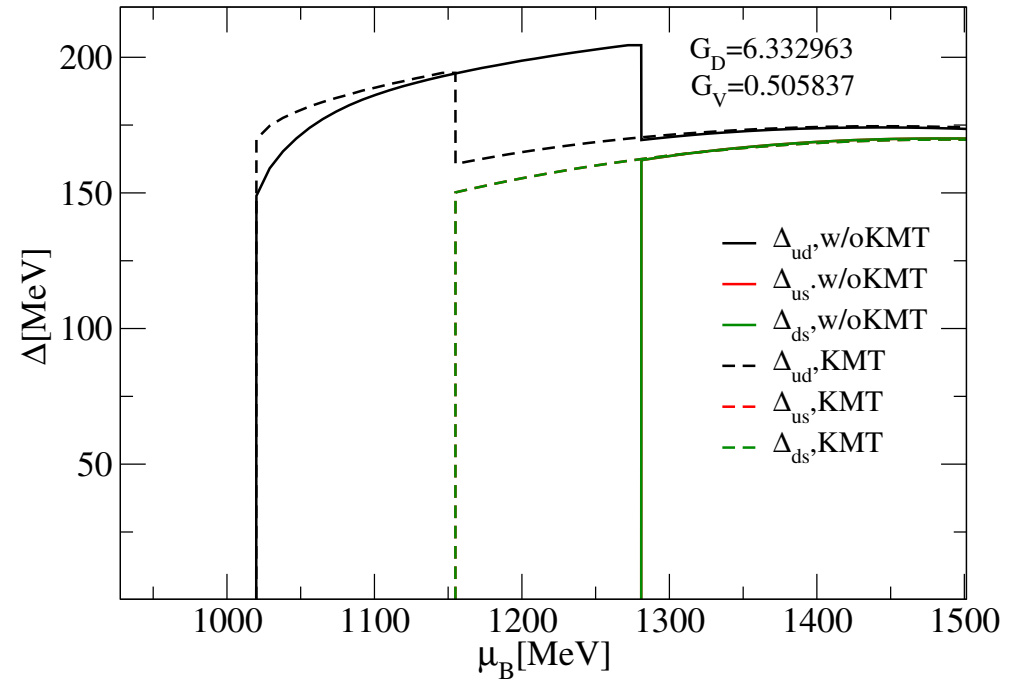
$$\Omega(T, \mu) = \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2 + |\Delta_{us}|^2 + |\Delta_{ds}|^2}{4G_D} - \int \frac{d^3p}{(2\pi)^3} \sum_{a=1}^{18} \left[ \lambda_a + 2T \ln \left( 1 + e^{-\lambda_a/T} \right) \right] + \Omega_e - \Omega_0.$$

Neutrality constraints:  $n_Q = n_8 = n_3 = 0$ ,  $n_i = -\partial\Omega/\partial\mu_i = 0$ ,  
Equations of state:  $P = -\Omega$ , etc.

# Chiral and Diquark Gaps w/wo KMT determinant



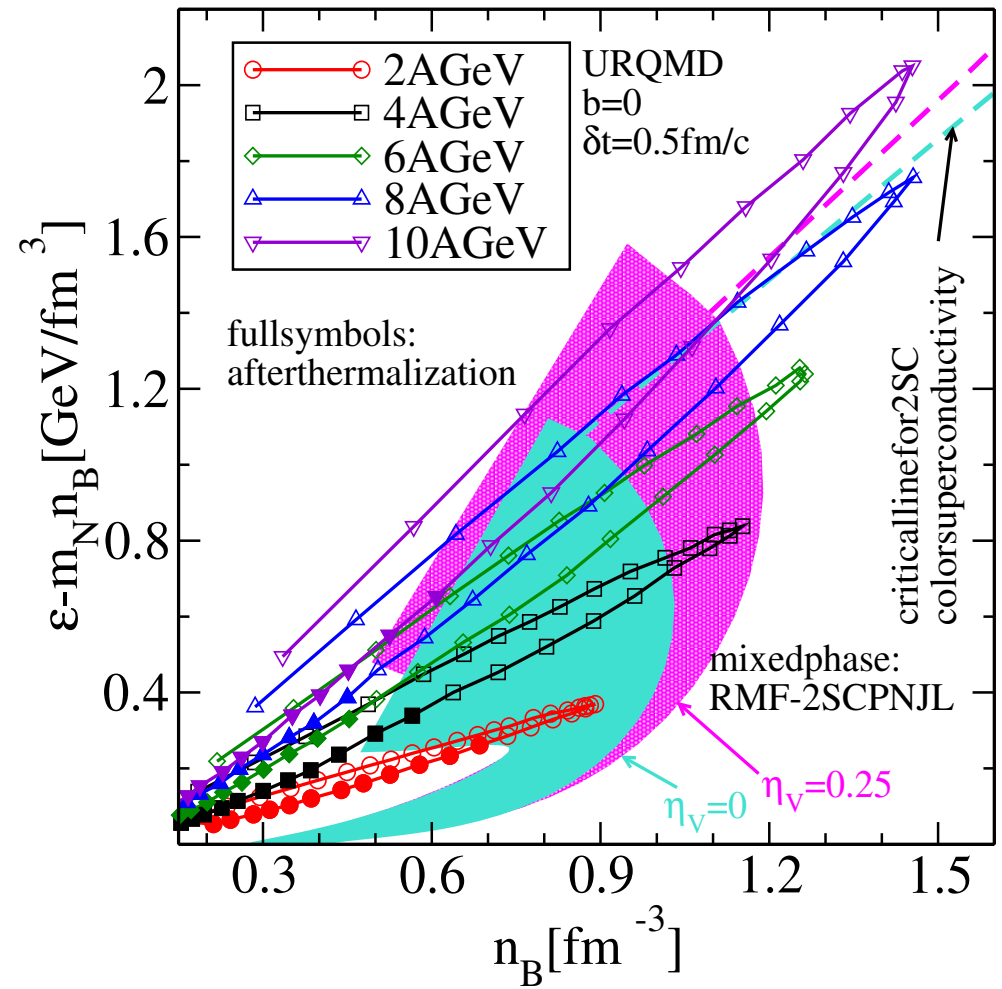
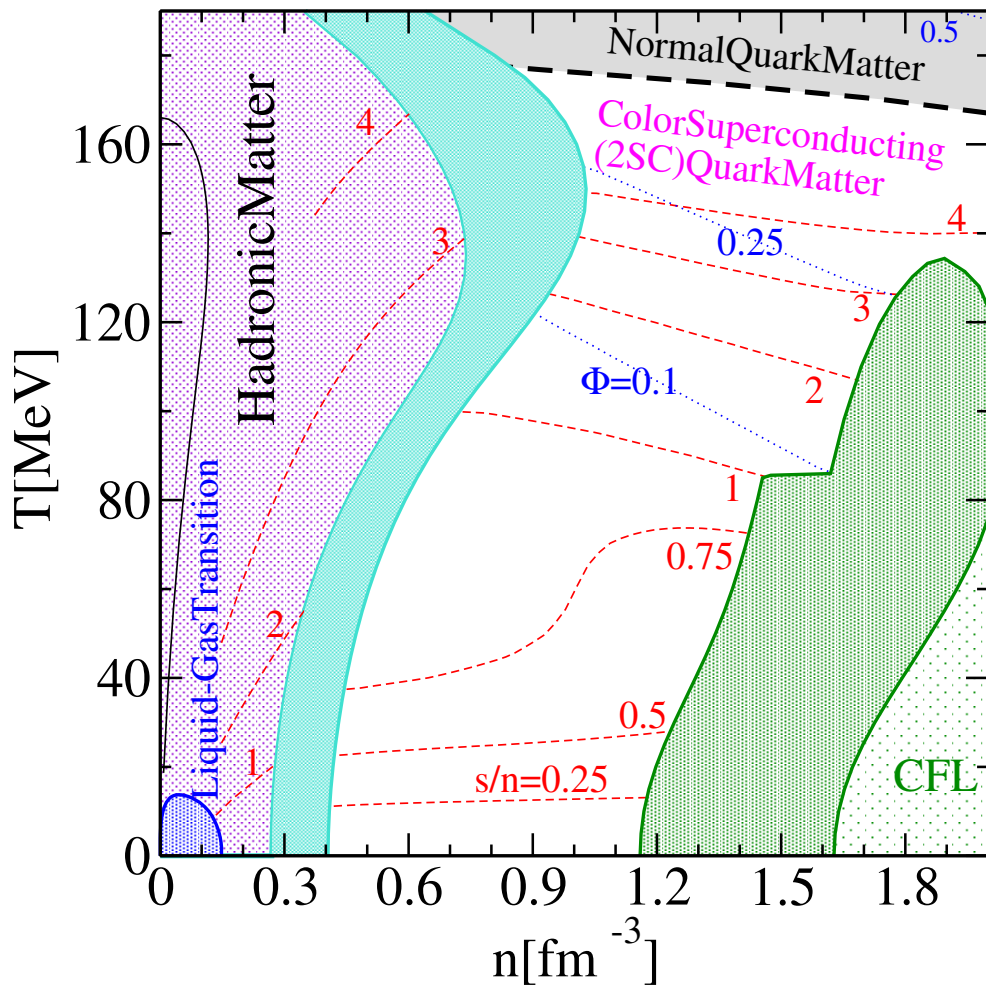
Dynamical quark masses with and without KMT determinant interaction at  $T = 0$ ; pion properties and quark mass fixed at  $\mu = 0$



Diquark condensates with and without KMT determinant at  $T = 0$ , strong diquark coupling case

**Rüster et al, PRD 72 (2005) 034004; Blaschke et al, PRD 72 (2005) 065020;**  
**D.B., Łastowiecki et al. JPG 37 (2010) 094063; PTPS 186 (2010) 81**

# Phase diagrams for the CBM and NICA experiments

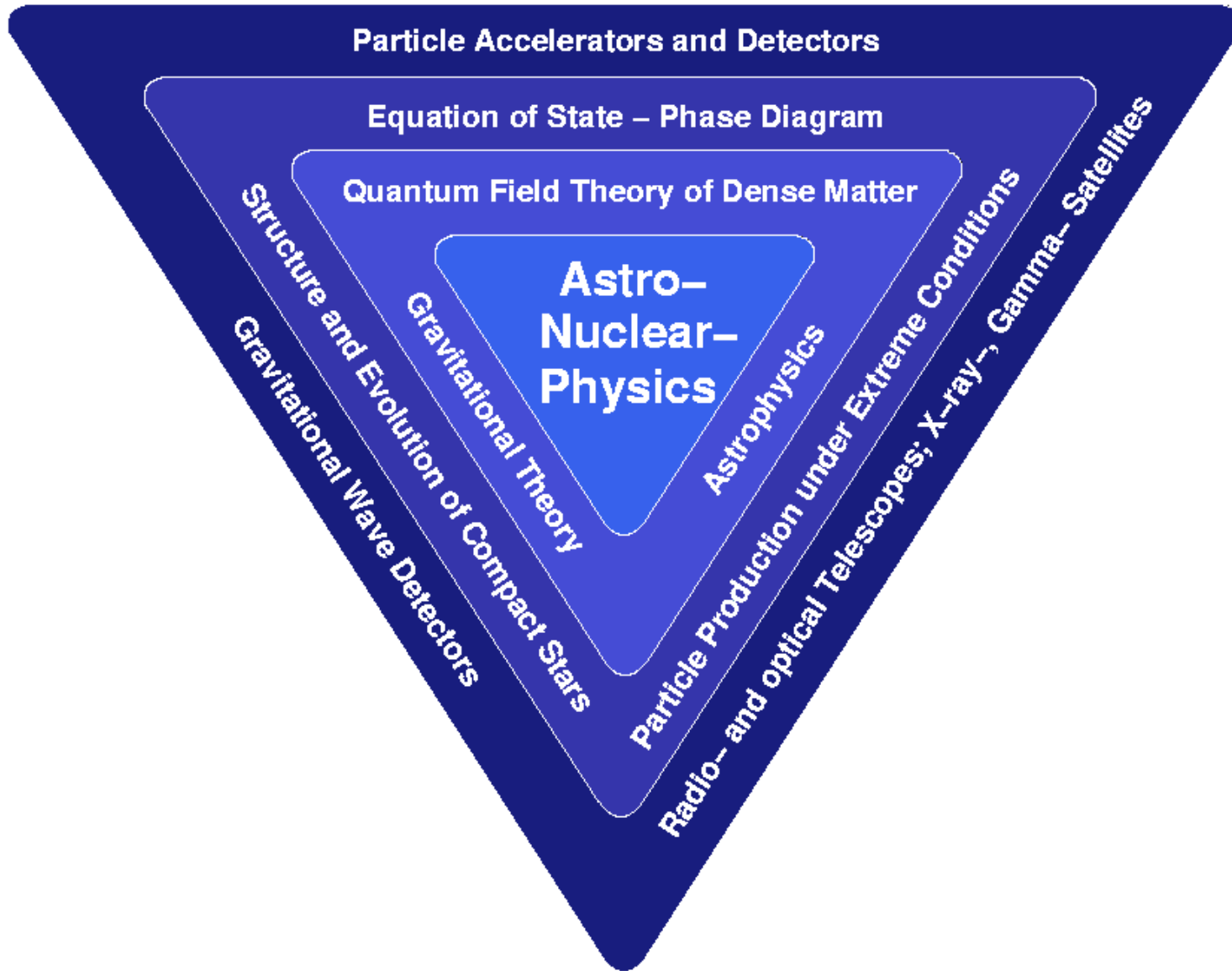


Phase diagram for isospin-symmetric matter (left); trajectories for heavy-ion collisions (right)

D.B., F. Sandin, V. Skokov, S. Typel, Acta Phys. Pol. Supp. 3, 741 (2010).



# New ways to understand Dense QCD Matter



<http://www.esf.org/compstar>

CompStar Online Supernova EoS → coming soon!

# CompOSE - CompStar Online Supernova EoS

Reference manual  
version 1.00

## CompOSE

CompStar Online Supernovæ Equations of State

*harmonising the concert of nuclear physics and astrophysics*

compose.obspm.fr

CompOSE Core Team

November 27, 2012

### General Requirements:

- Densities:  $10^{-8} \leq n/n_0 \leq 10$
- Temperatures:  $0 \leq T \leq 200$  MeV
- Proton fractions:  $0 \leq Y_p \leq 0.6$ ;  $\beta = 1 - 2Y_p$

### New Developments:

- Dissolution of clusters due to Pauli blocking
- Realistic high-density modeling: DD-RMF/3FSC PNJL
- Thermodynamics of 1<sup>st</sup> order PT; pasta phases

### I. For Contributors:

- How to prepare EoS tables
- How to submit EoS tables
- Extending CompOSE

### II. For Users:

- Hadronic EoS: Statistical, Skyrme, DBHF, ...
- Quark Matter EoS: Bag, PNJL, ...
- Phase transition: Maxwell, Gibbs, Pasta, ...